CS109: Probability for Computer Scientists
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Lecture Topics: 2.1 Discrete Probability

[**Tags**: PSet1 Q8a, Equally Likely Outcomes]

1. Suppose you went trick-or-treating (as an adult) and were able to nab 50 total candies, 13 of which are kit-kats. Your responsible parent says you can only eat 6 of them tonight. Let X be the number of kit-kats you grabbed out of 6. What is $P(X = k)$ for valid values of k ($k \in \{0,1,2,\dots,6\}$)?

Solution: Watch lecture ☺

[**Tags**: Counting, Equally Likely Outcomes]

2. Suppose we have 13 chairs (in a row) with 8 TA's, and 5 professors to be seated. Suppose all seatings are equally likely. What is the probability that every professor has a TA to their immediate left and right?

Solution:

The problem mentions all seatings are equally likely, so let Ω be the set of all seatings of the 13 people, and E be the event (set) that every professor has a TA to their immediate left and right.

Because of the equally likely assumption, we know $P(E) = |E|/|\Omega|$, so we just need to count the size of these two sets.

Then, $|\Omega| = 13!$ since it's just the number of ways to arrange 13 people with no restrictions.

Counting $|E|$ is a bit tricker. Imagine we just arrange all 8 TA's in order(forget about the chairs): there are $8!$ ways to do so. Now, there are 7 spaces between them, and a professor will be sitting between 2 TA's if and only if they sit in one of these 7 spaces (without any other professor). So pick 5 out of 7 locations for the professors, for a total of $P(7,5)$ ways, or choose 5 spots and assign each of the 5 professors there $\binom{7}{5} \cdot 5!$. So our answer by the product rule is $|E| = 8! \cdot \binom{7}{5} \cdot 5!$.