



# Modeling

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# Announcements

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- Midterm next Tuesday.
  - Practice and info is on [cs109.stanford.edu](https://cs109.stanford.edu)
  - Midterm review Thurs 7p.

# Where are we in CS109?

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## Overview of Topics



Counting  
Theory



Core  
Probability



Random  
Variables



Probabilistic  
Models



Uncertainty  
Theory



Machine  
Learning



YOU  
ARE  
HERE

# Where are we locally?

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**Discrete  
Models:**

General Case,  
Multinomial

**Continuous  
Models:**

General Case,  
Multi-Gauss

**Inference**

Conclusions  
from  
Observations

**Modelling:**

Make your own!

**General**

**Inference:**

Use computers  
to infer

# Last Week

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## **Joint Distribution** *noun*

The probability of a simultaneous assignment to ***all*** the random variables in a probabilistic model.

***Eg:***

$$P(X = x, Y = y)$$

$$f(X = x, Y = y)$$

$$P(X = x, Y = y, \dots, Z = z)$$

# Monday: Inference

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## **Inference** *noun*

An updated belief about a random variable (or multiple) based on conditional knowledge regarding another random variable (or multiple) in a probabilistic model.

TLDR: conditional probability with random variables.

# All the Bayes Belong to Us

M,N are discrete. X, Y are continuous

OG Bayes

$$p_{M|N}(m|n) = \frac{P_{N|M}(n|m)p_M(m)}{p_N(n)}$$

Mix Bayes #1

$$f_{X|N}(x|n) = \frac{P_{N|X}(n|x)f_X(x)}{P_N(n)}$$

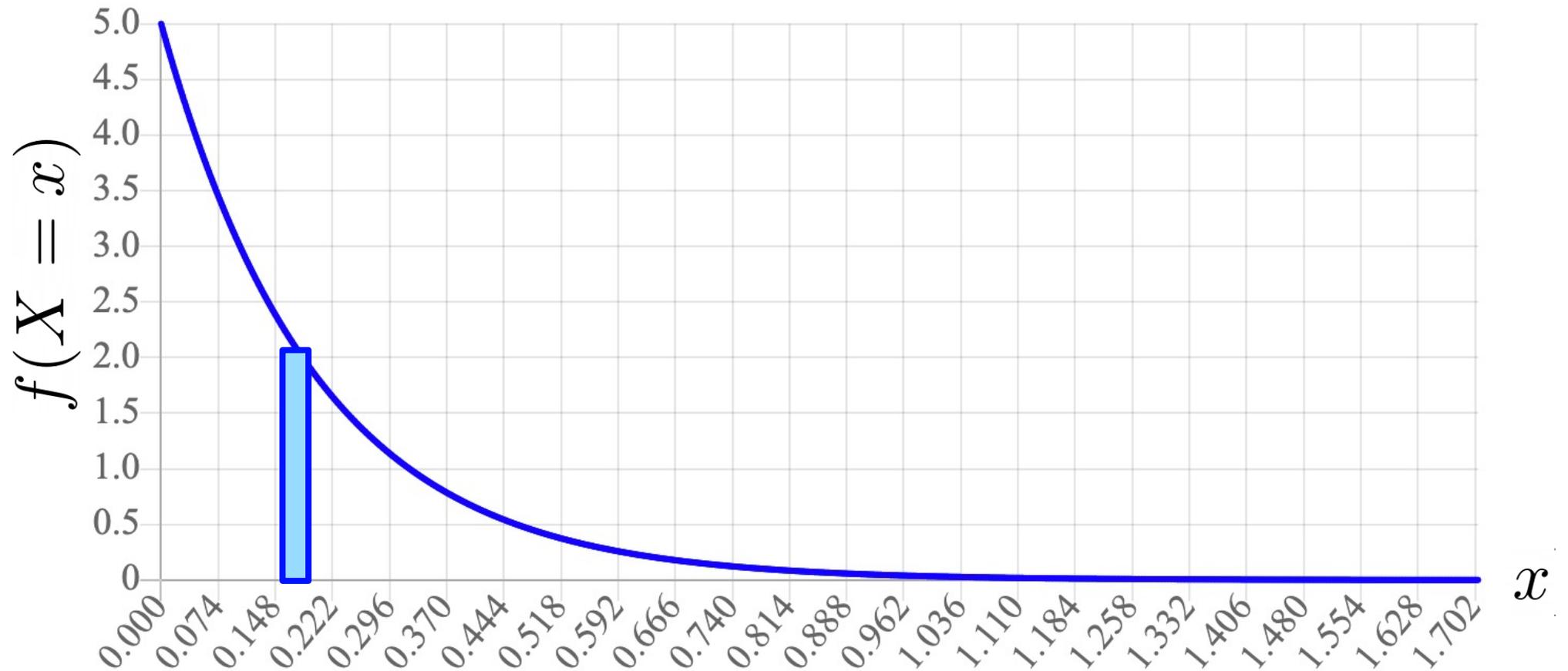
Mix Bayes #2

$$p_{N|X}(n|x) = \frac{f_{X|N}(x|n)p_N(n)}{f_X(x)}$$

$$f_{X|Y}(x|y) = \frac{f_{Y|X}(y|x)f_X(x)}{f_Y(y)}$$

# Proof is based on this claim

$$P(X = x) = f(X = x) \cdot \epsilon_x$$



# Medium Inference

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Q: At birth, girl elephant weights are distributed as a Gaussian with mean = 160kg, std = 7kg. At birth, boy elephant weights are distributed as a Gaussian with mean = 165kg, std = 3kg. All you know about a newborn elephant is that it is 163kg. What is the probability that it is a girl?



Notation: These are all the same

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$$P(X = x, Y = y)$$

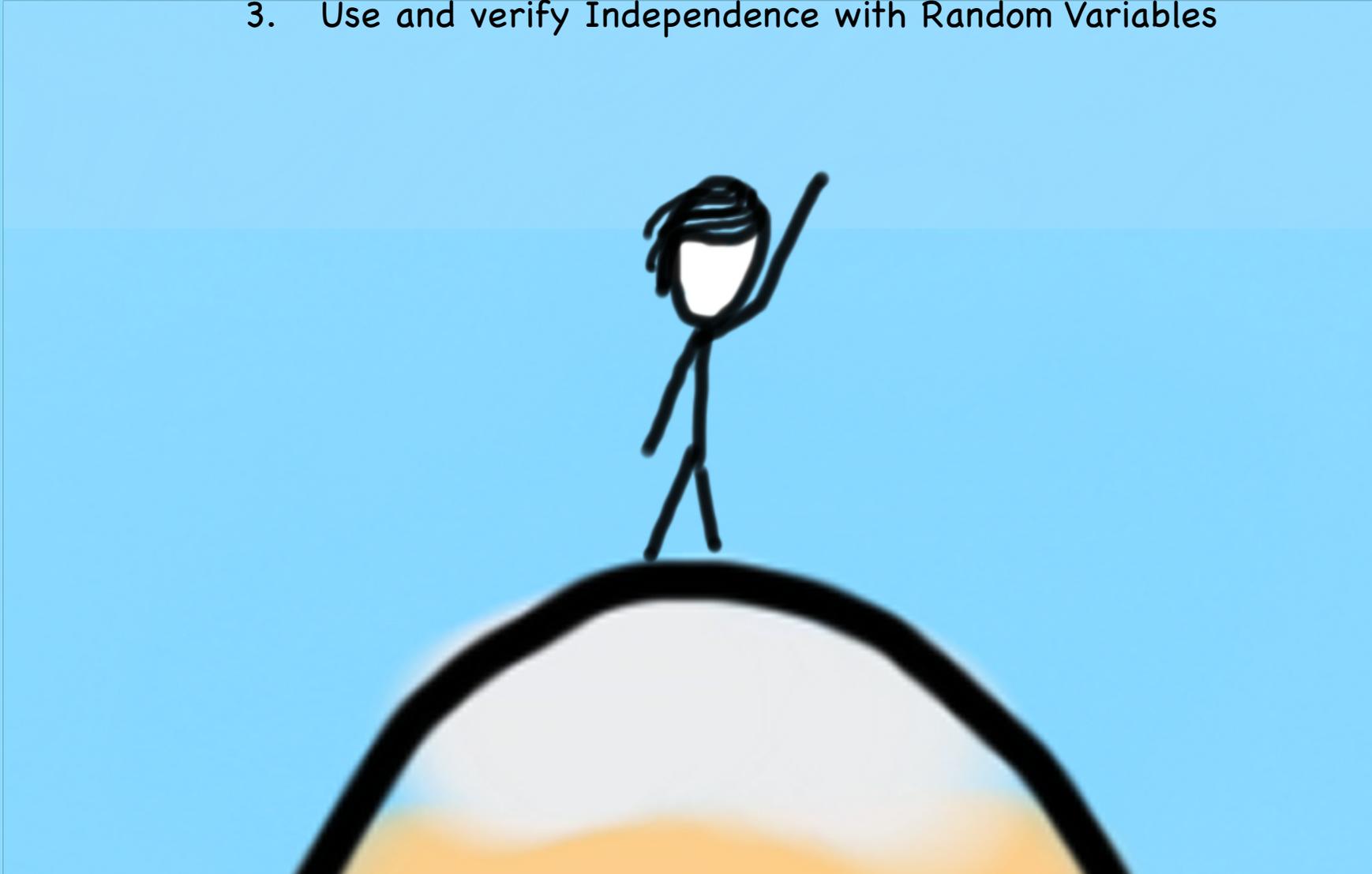
$$P_{X,Y}(x, y)$$

$$P(x, y)$$

So it begins...

# Learning Goals

1. Perspective on the artform of how to design probabilistic models
  2. How to calculate Correlations
  3. Use and verify Independence with Random Variables

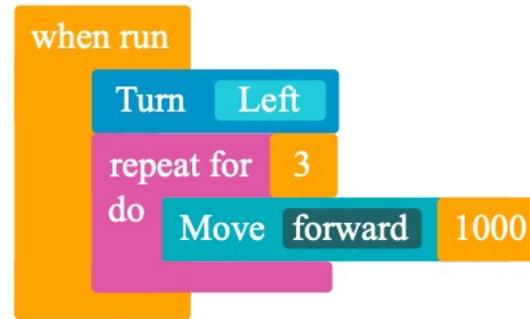


# Computers Couldn't Understand Code

60,000 students attempted this problem  
37,000 unique solutions



Challenge

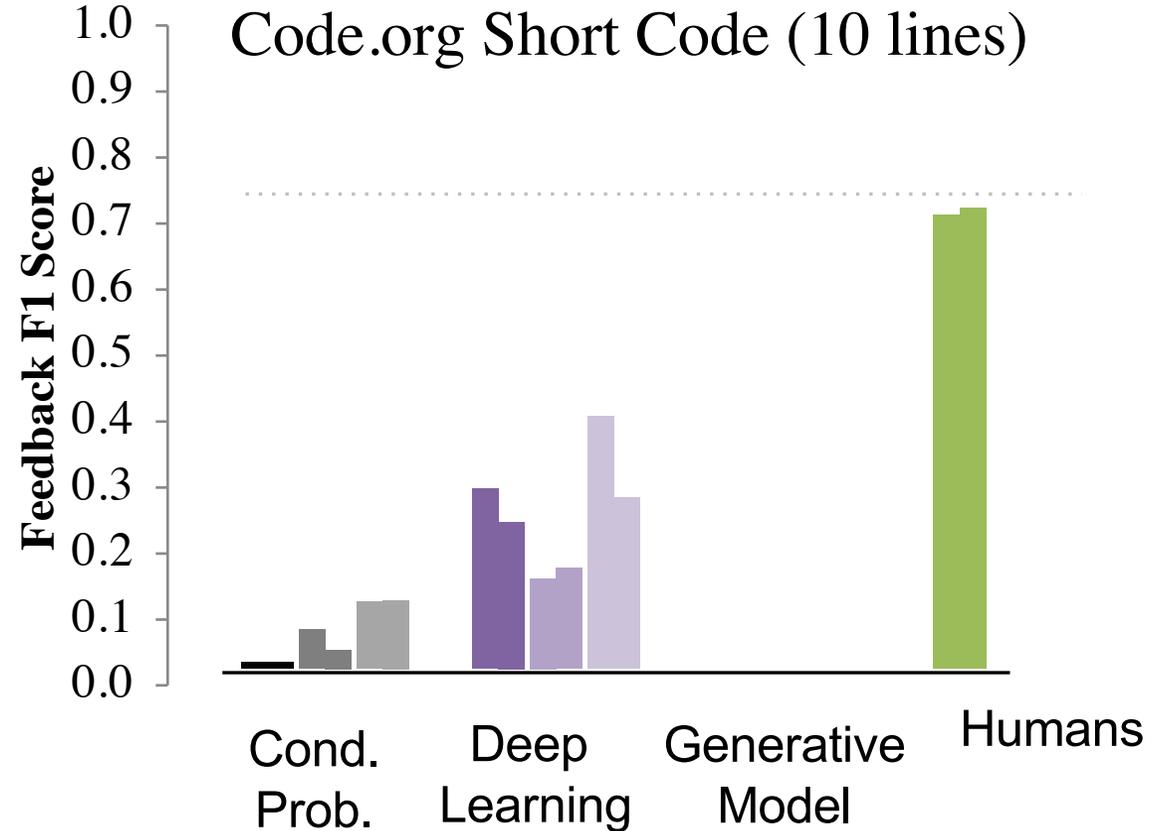


Student Code

You need to  
move and  
turn in your  
loop

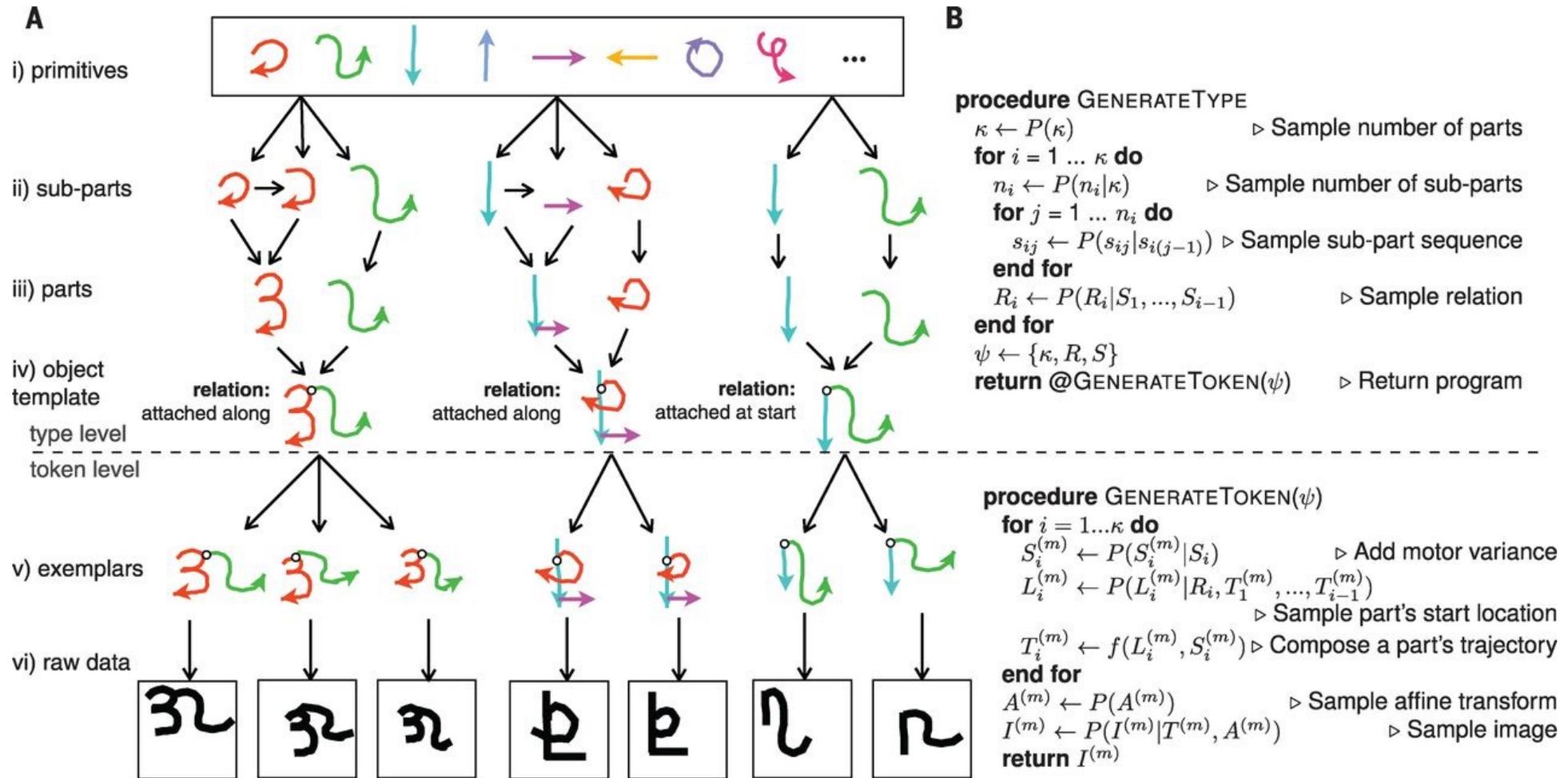
Insight

# Computers Couldn't Understand Code

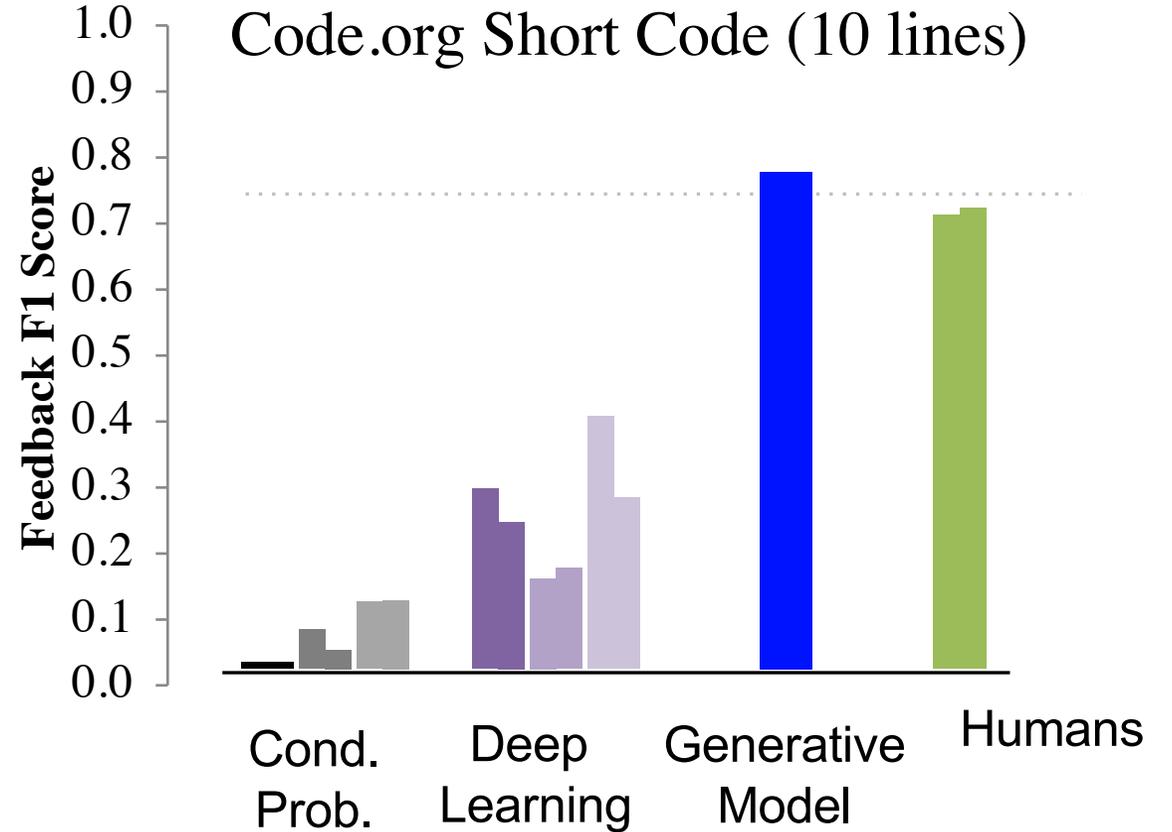


# Generative Model of Characters

Lake et al, 2015

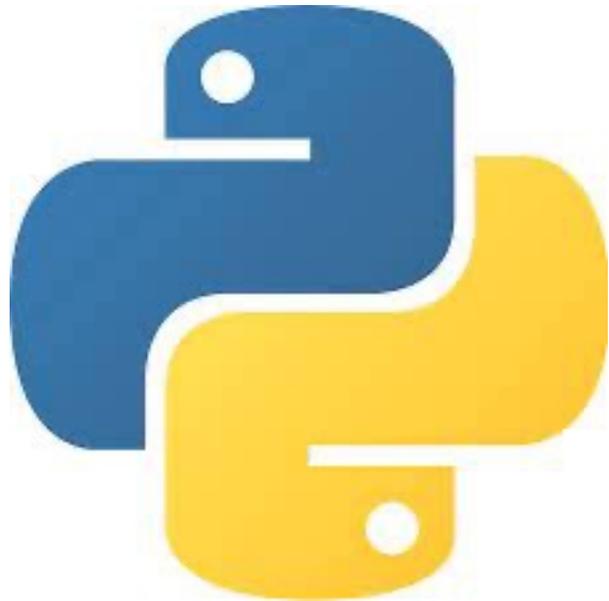


# Computers Couldn't Understand Code



Lets talk about how to make a model

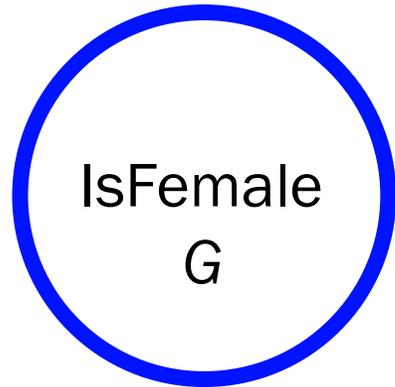
# Model Version #1: Python That Outputs a **Joint** Sample



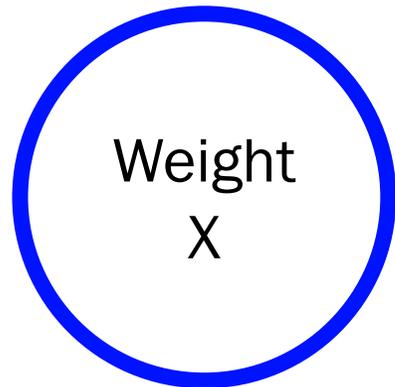
Sample Baby  
Elephant

Sex: Female  
Weight: 161kg

# Model Version #2: Bayesian Network



$G = 1$  is  $\text{Bern}(p = 0.5)$



$X | G = 1$  is  $\text{N}(\mu = 160, \sigma^2 = 7^2)$

$X | G = 0$  is  $\text{N}(\mu = 165, \sigma^2 = 3^2)$

Does this define the joint?

$$f(G = g, X = x)$$

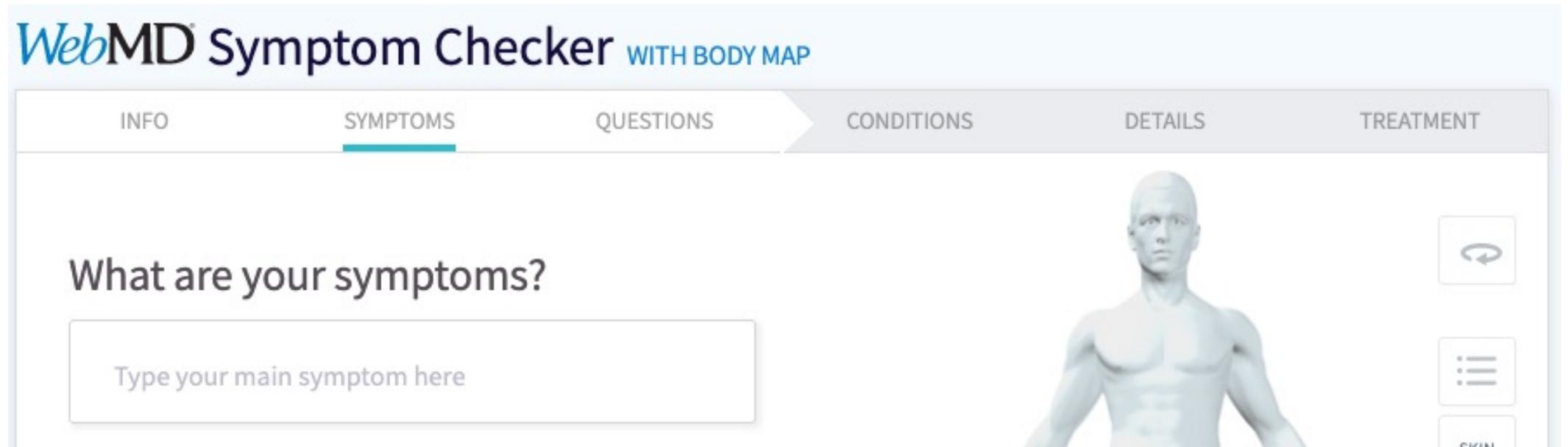
$$= f(X = x | G = g) P(G = g)$$

# Why You Need a Model

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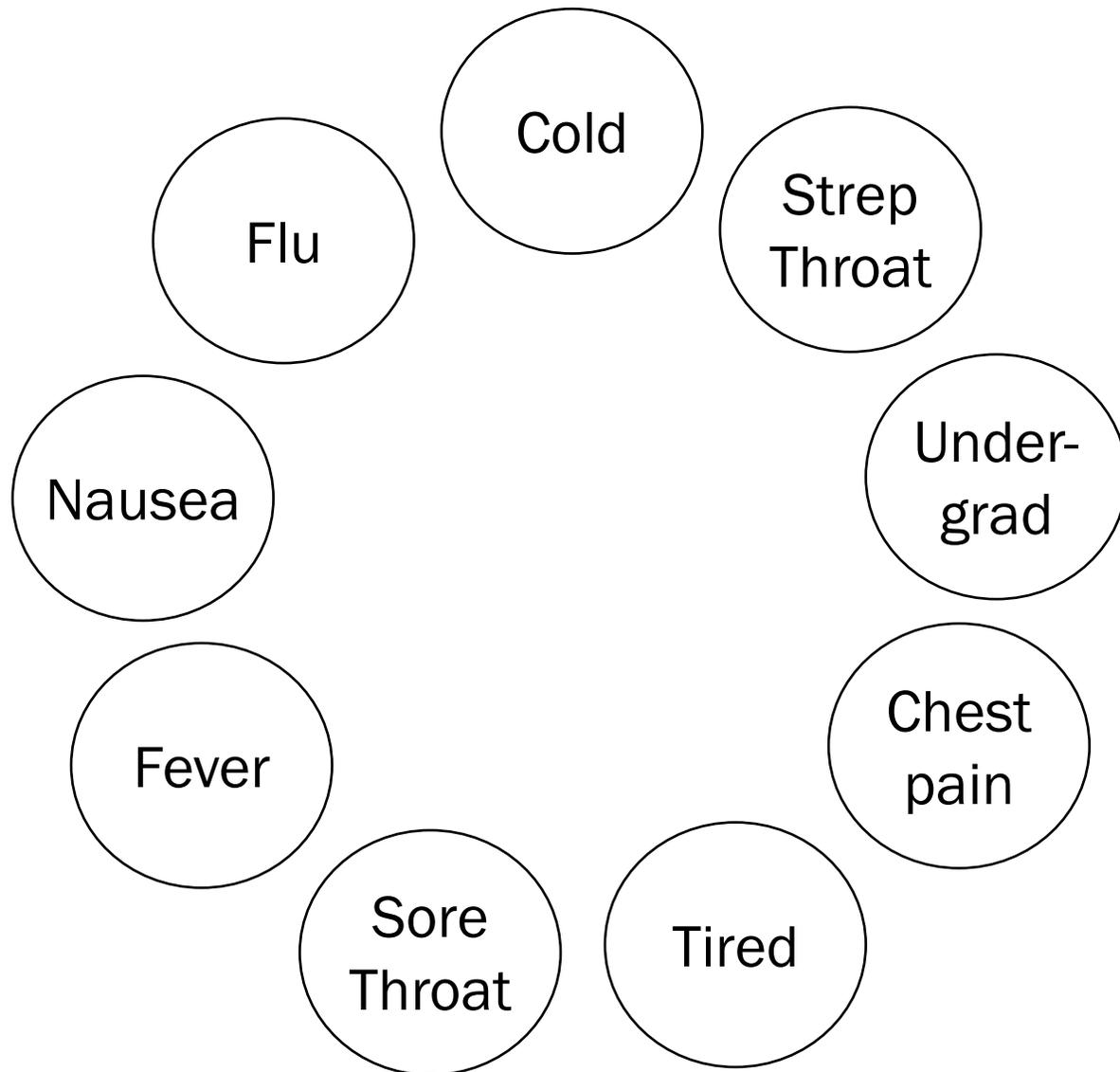
*Web*MD<sup>®</sup>

# Inference



# Inference

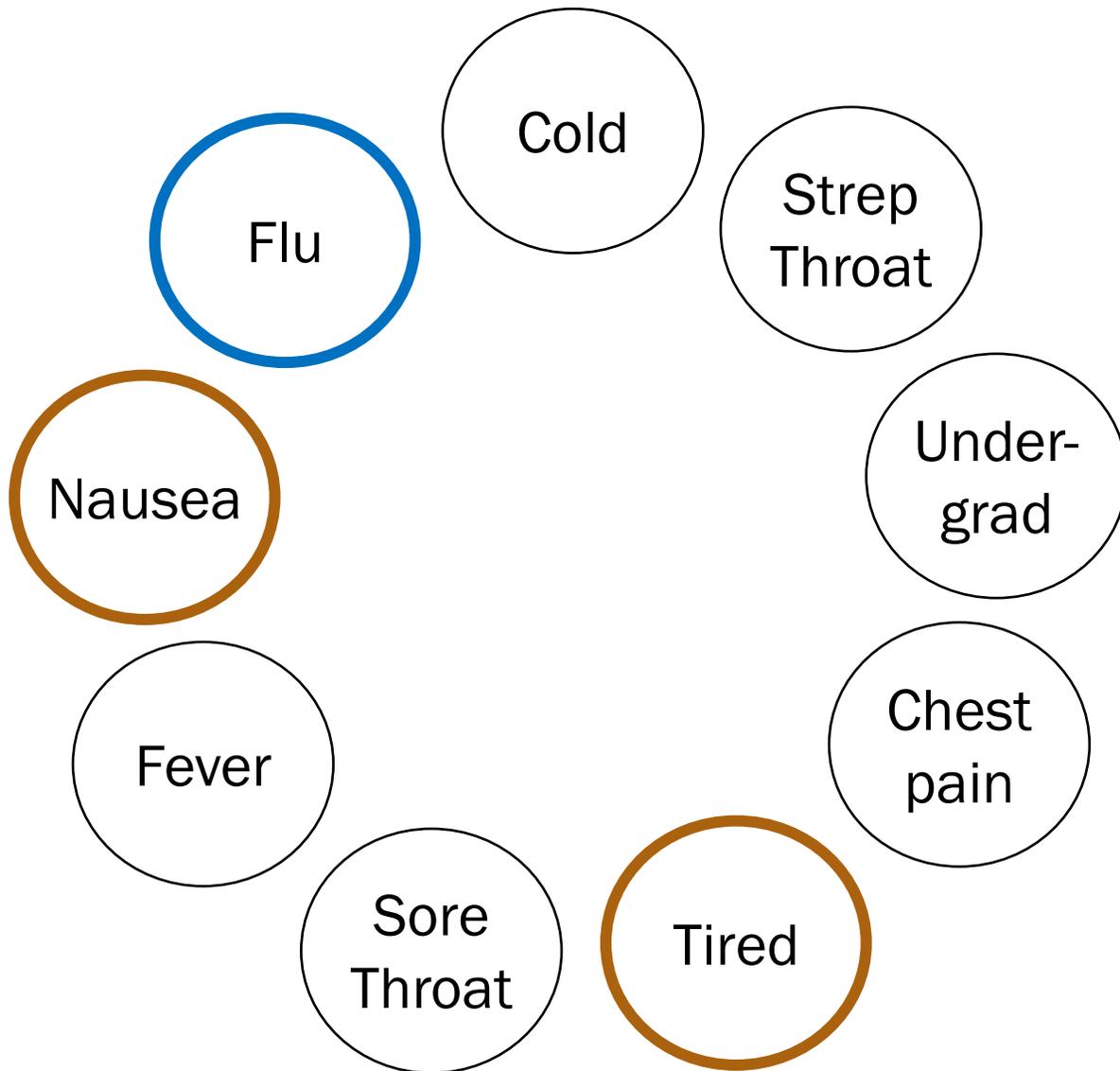
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## Inference question:

Given the values of some random variables, what are the conditional distributions of some other random variables?

# Inference

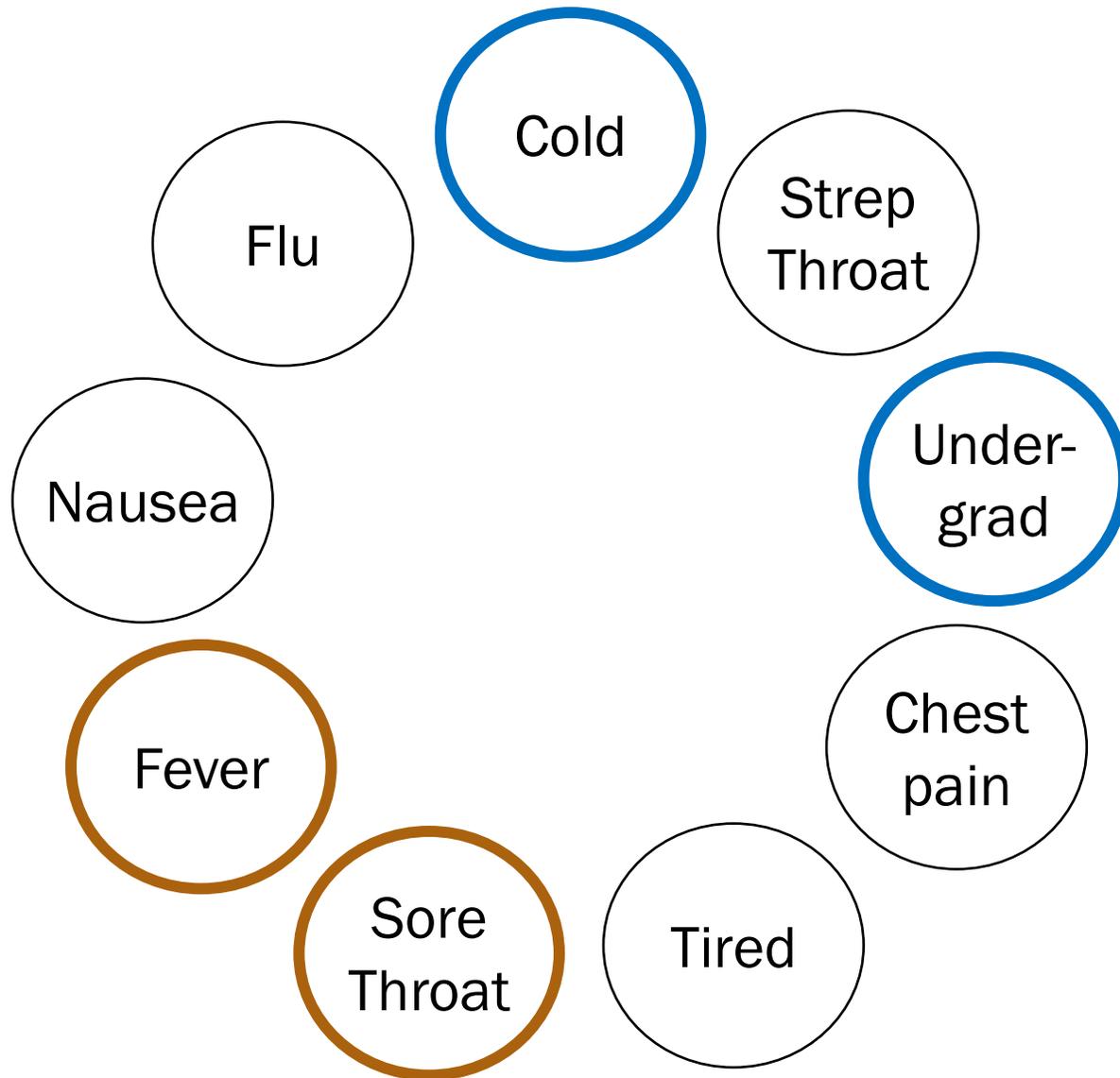


One inference question:

$$P(F = 1 | N = 1, T = 1)$$

$$= \frac{P(F = 1, N = 1, T = 1)}{P(N = 1, T = 1)}$$

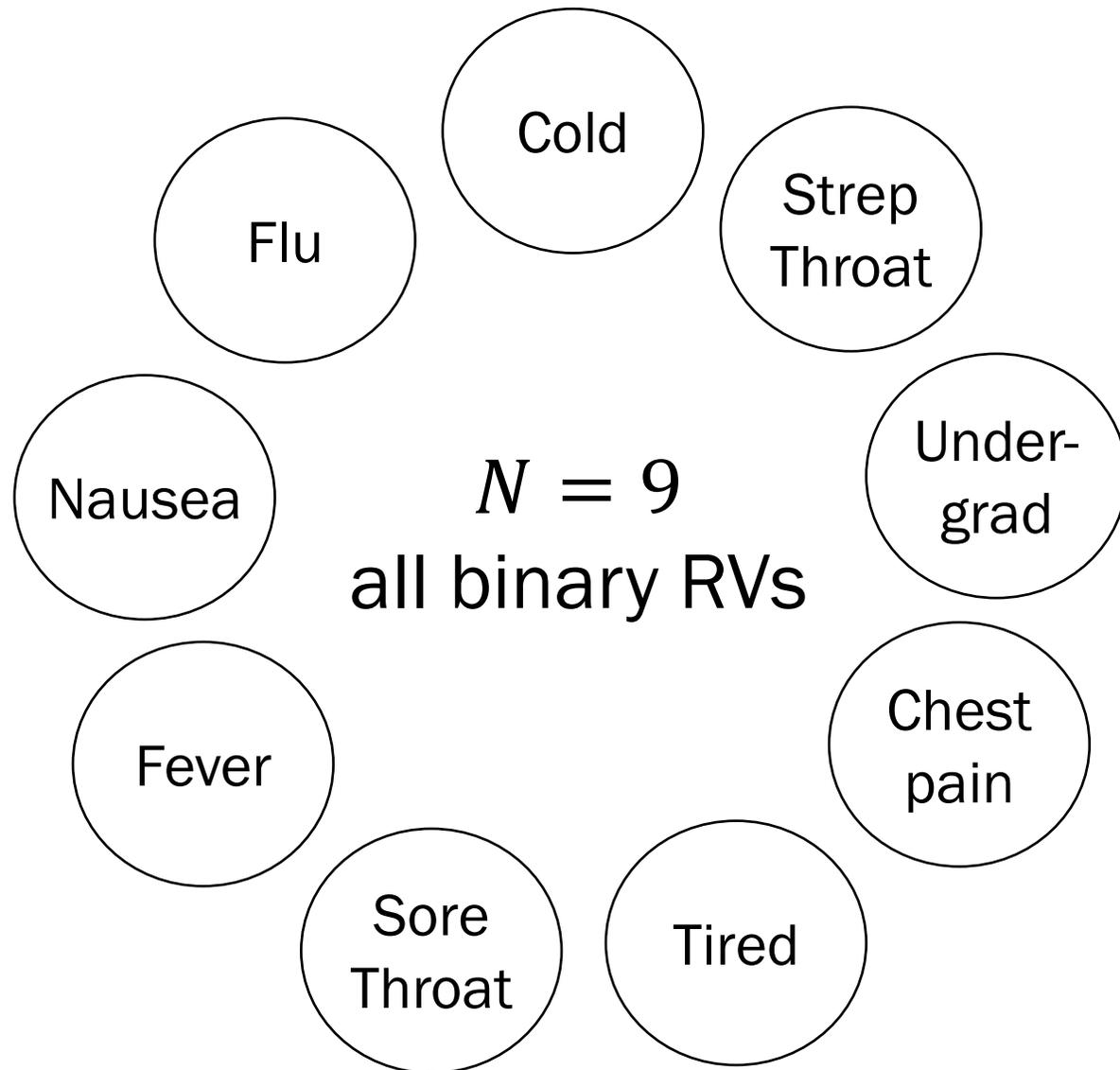
# Inference



Another inference question:

$$P(C_o = 1, U = 1 | S = 0, F_e = 0) \\ = \frac{P(C_o = 1, U = 1, S = 0, F_e = 0)}{P(S = 0, F_e = 0)}$$

# Inference



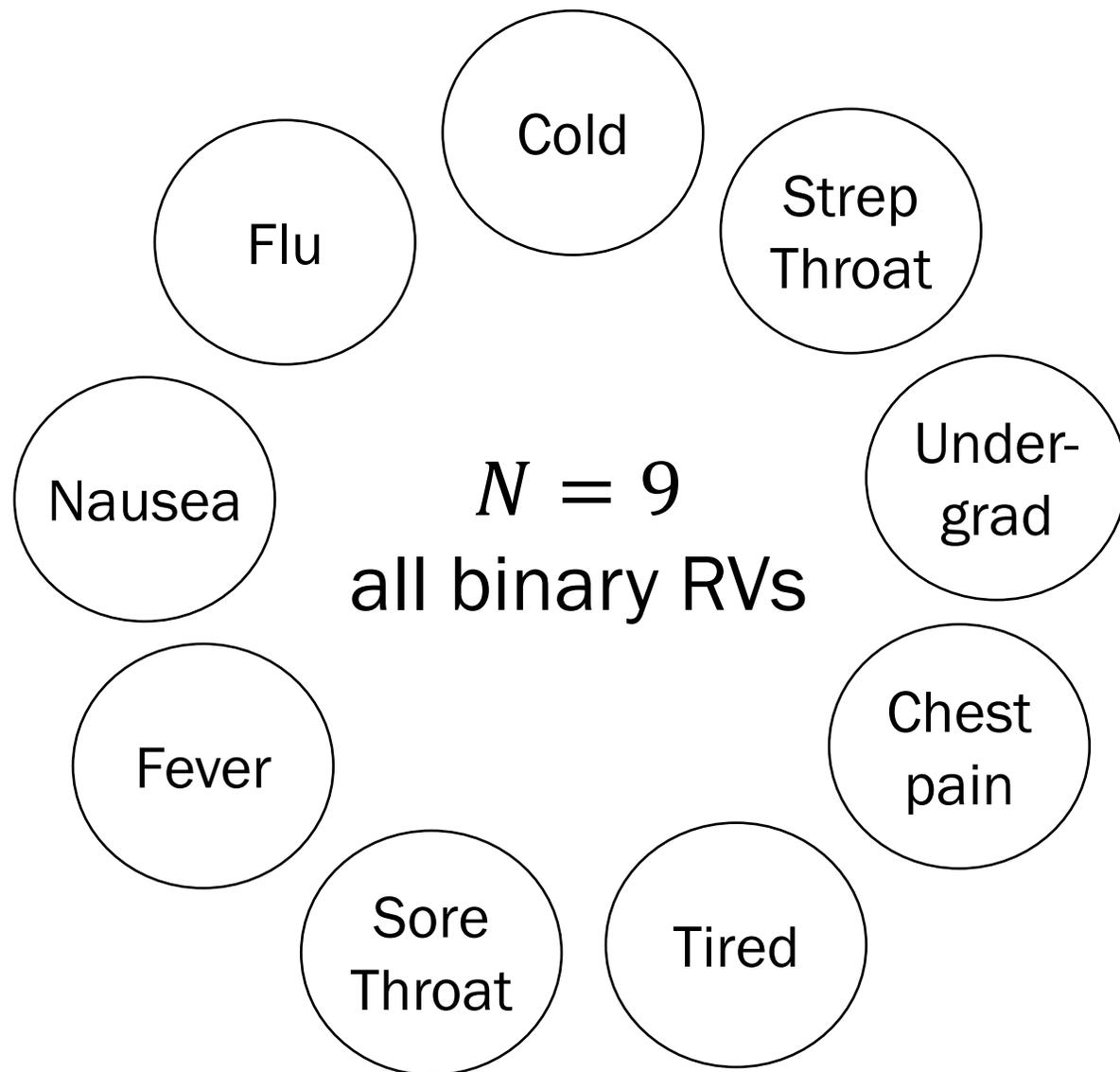
If we knew the **joint distribution**, we can answer all probabilistic inference questions.

What is the size of the joint probability table?

- A.  $2^{N-1}$  entries
- B.  $N^2$  entries
- C.  $2^N$  entries
- D. None/other/don't know



# Inference



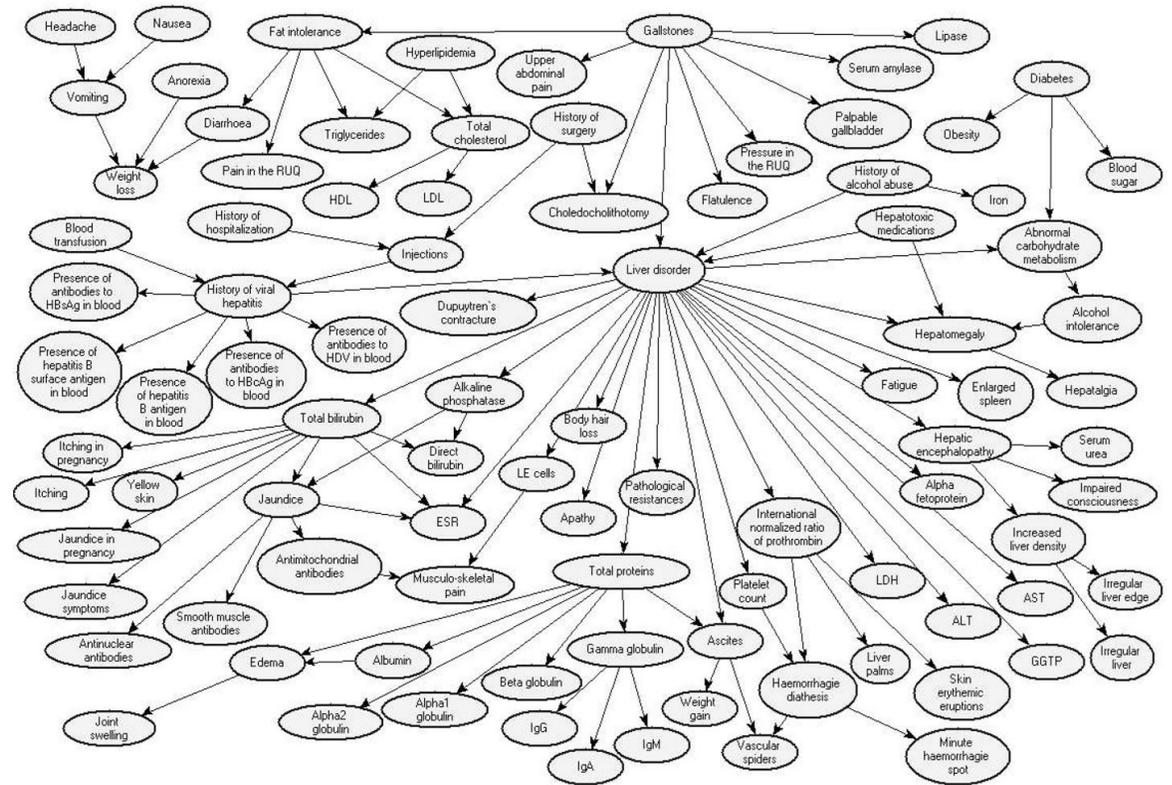
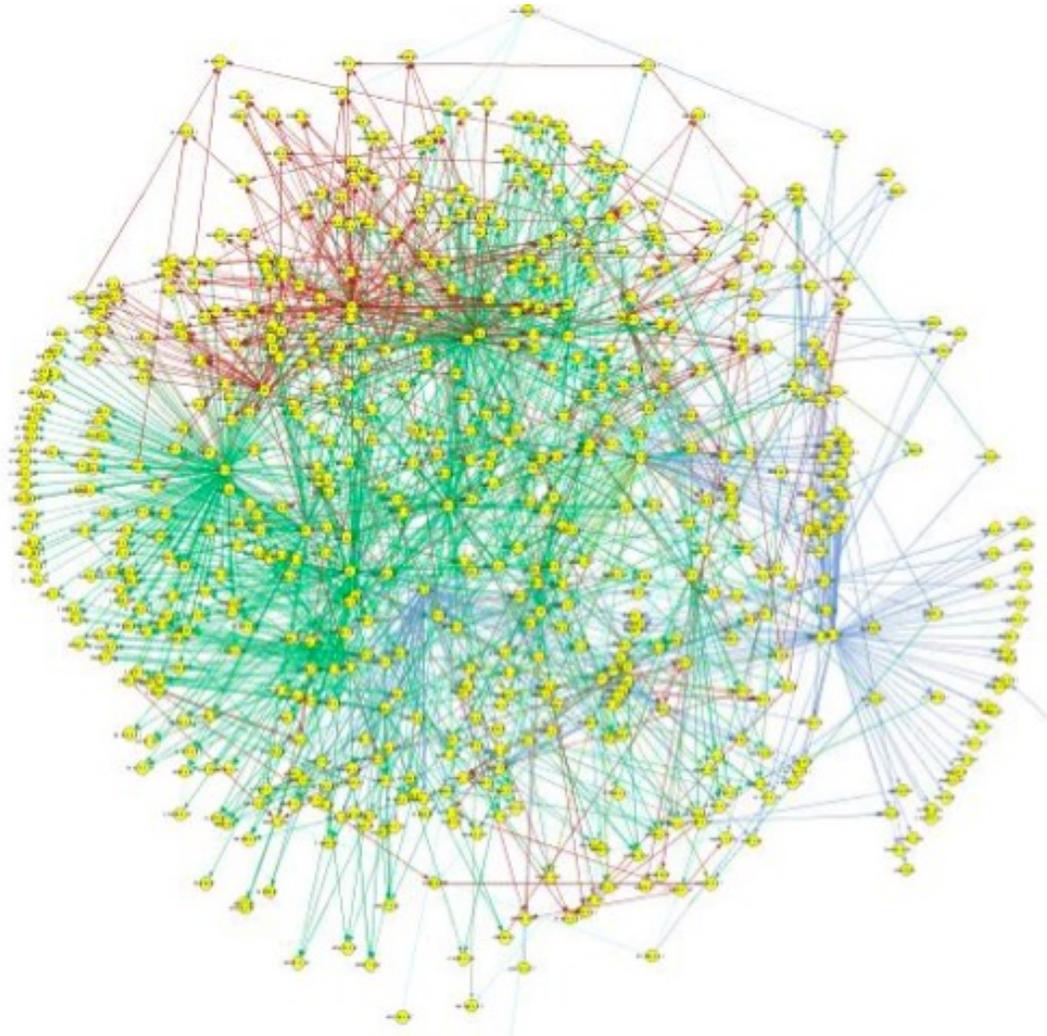
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- B.  $N^2$  entries
- C.  $2^N$  entries**
- D. None/other/don't know

Naively specifying a joint distribution is, in general, intractable.

# N can be large...



# Bayesian Networks

# A simpler WebMD

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Flu

Under-  
grad

Fever

Tired

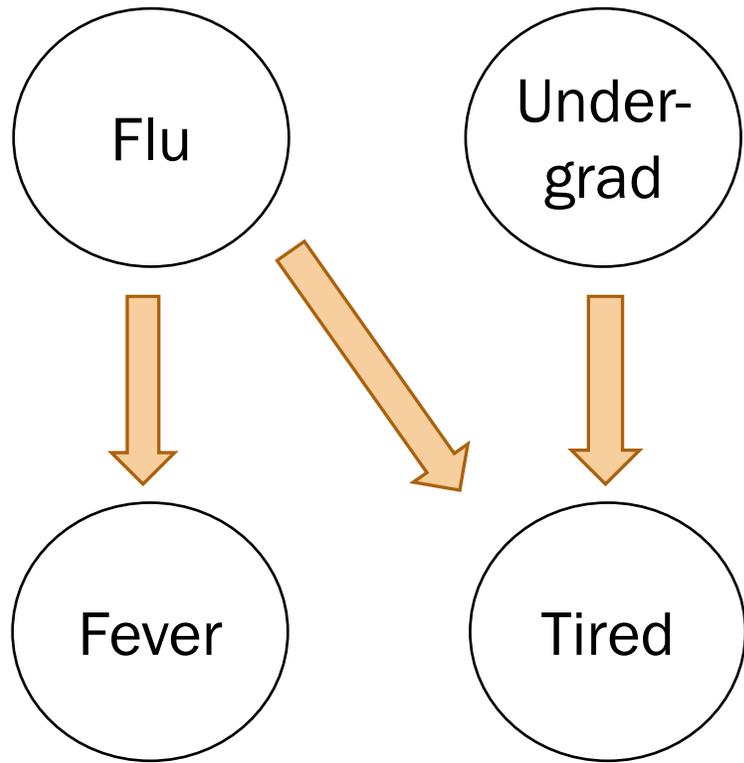
Great! Just specify  $2^4 = 16$  joint probabilities...?

$$P(F_{lu} = a, F_{ev} = b, U = c, T = d)$$

What would a Stanford flu expert do?

Describe the joint distribution using causality!

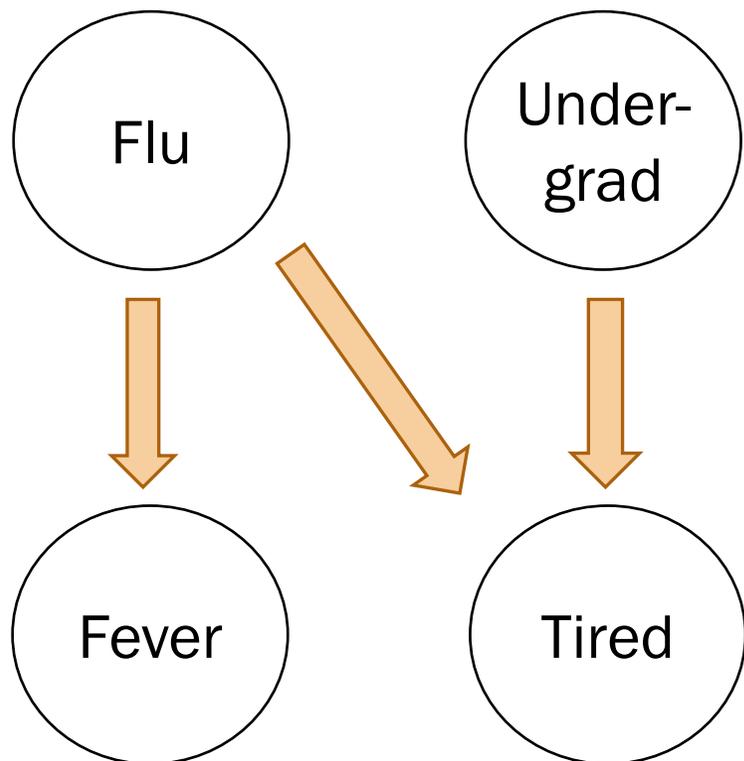
# Constructing a Bayesian Network



What would a Stanford flu expert do?

- ✓ 1. Describe the joint distribution using causality.
2. Provide  $P(\text{values}|\text{parents})$  for each random variable
3. Implicitly assumes independences.

# Constructing a Bayesian Network



$$\begin{aligned} P(T = 1 | F_{lu} = 0, U = 0) \\ P(T = 1 | F_{lu} = 0, U = 1) \\ P(T = 1 | F_{lu} = 1, U = 0) \\ P(T = 1 | F_{lu} = 1, U = 1) \end{aligned}$$

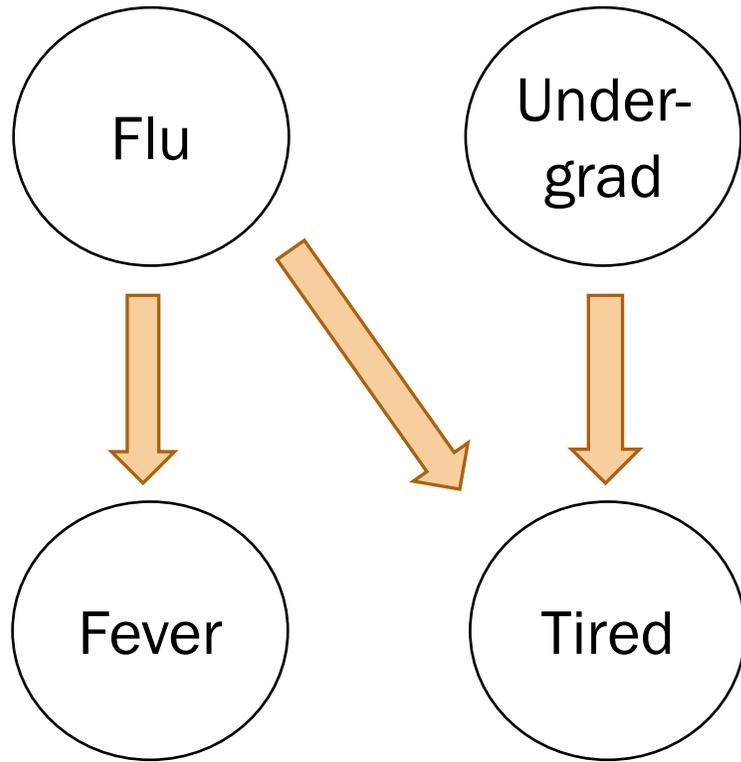
In a Bayesian Network,  
Each random variable is caused by  
its **parents**. Def  $P(\text{node} \mid \text{parents})$

- Node: random variable
- Directed edge: causality

Examples:

- $P(F_{lu} = 1)$
- $P(U = 0)$
- $P(F_{ev} = 1 | F_{lu} = 1), P(F_{ev} = 1 | F_{lu} = 0)$
- $P(T = 1 | F_{lu} = 0, U = 0) \dots$

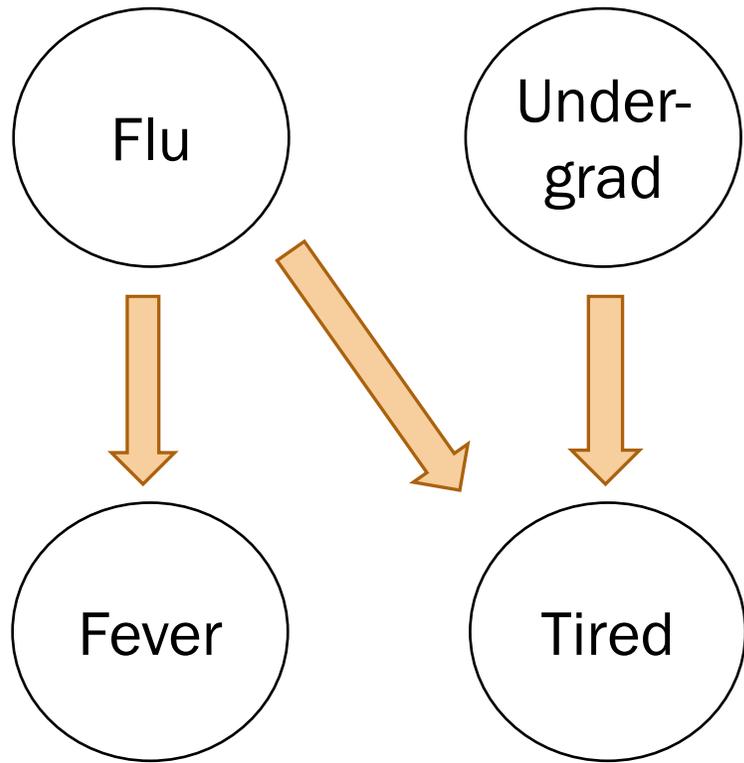
# Constructing a Bayesian Network



What would a Stanford flu expert do?

- ✓ 1. Describe the joint distribution using causality.
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# Constructing a Bayesian Network



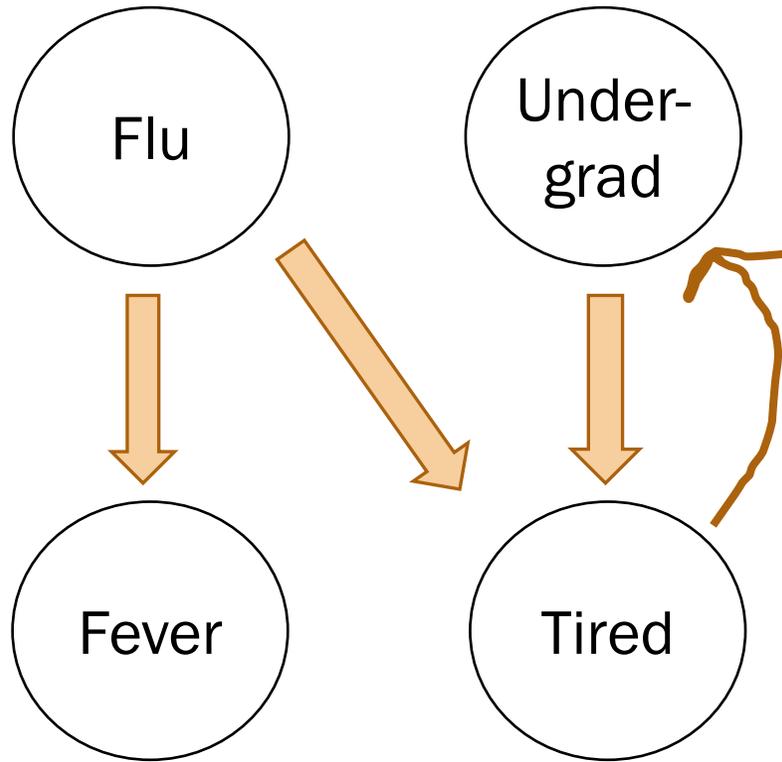
This model assumes that Flu and being an Undergraduate are independent.

Advanced: it also assumes that fever and tired are conditionally independent given Flu.

You need to tell a generative story. The independence assumptions come for free.

# Bug: Constructing a Bayesian Network

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Must be acyclic!

# Independence of RVs

# Independent discrete RVs

Recall the definition of independent events  $E$  and  $F$ :

$$P(EF) = P(E)P(F)$$

Two discrete random variables  $X$  and  $Y$  are **independent** if:

for all  $x, y$ :

$$P(X = x, Y = y) = P(X = x)P(Y = y)$$

Different notation,  
same idea:

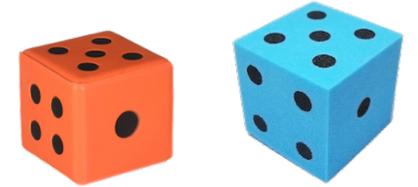
$$p(x, y) = p(x)p(y)$$

- Intuitively: knowing value of  $X$  tells us nothing about the distribution of  $Y$  (and vice versa)
- If two variables are not independent, they are called **dependent**.

# Dice (after all this time, still our friends)

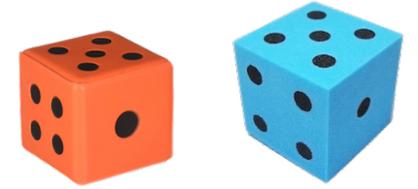
Let:  $D_1$  and  $D_2$  be the outcomes of two rolls  
 $S = D_1 + D_2$ , the sum of two rolls

- Each roll of a fair, 6-sided die is an independent trial.
  - Random variables  $D_1$  and  $D_2$  are independent.
1. Are events  $(D_1 = 1)$  and  $(S = 7)$  independent?
  2. Are events  $(D_1 = 1)$  and  $(S = 5)$  independent?
  3. Are random variables  $D_1$  and  $S$  independent?



# Dice (after all this time, still our friends)

Let:  $D_1$  and  $D_2$  be the outcomes of two rolls  
 $S = D_1 + D_2$ , the sum of two rolls



- Each roll of a 6-sided die is an independent trial.
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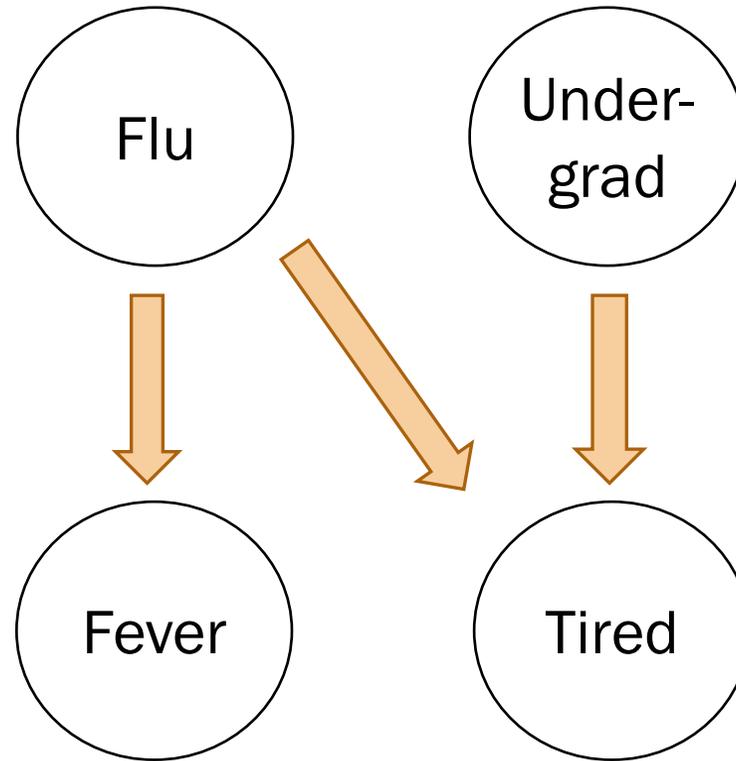
1. Are events  $(D_1 = 1)$  and  $(S = 7)$  independent? 

2. Are events  $(D_1 = 1)$  and  $(S = 5)$  independent? 

3. Are random variables  $D_1$  and  $S$  independent? 

All events  $(X = x, Y = y)$  must be independent for  $X, Y$  to be independent RVs.

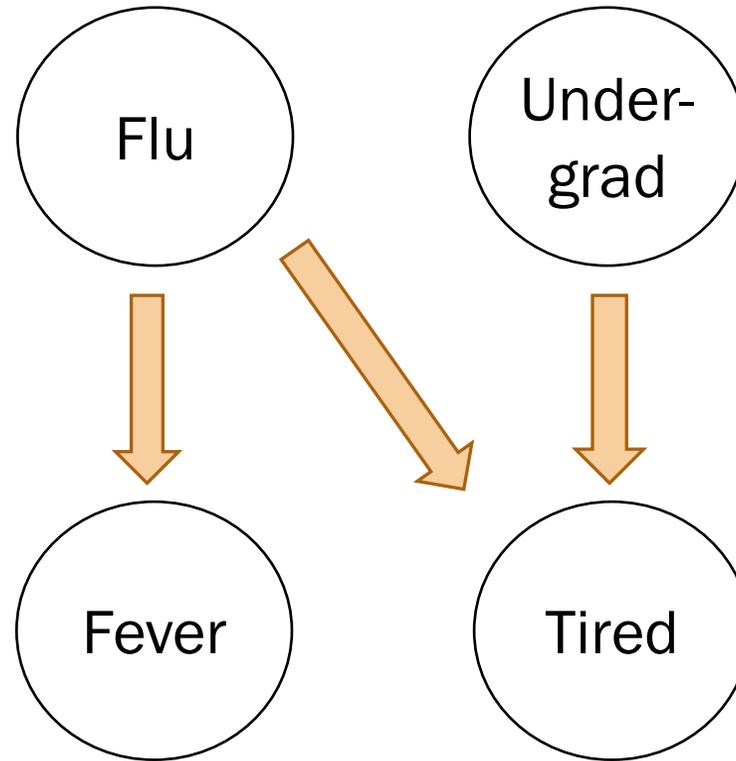
# Independence Assumption in Bayes Net



**Assumes:**

$$P(F_{lu} = 1, U = 0) = P(F_{lu} = 1) P(U = 0)$$

# Independence Assumption in Bayes Net



**Assumes:**  $P(F_{ev} = 1, T = 0 | F_{lu} = 1) = P(F_{ev} = 1 | F_{lu} = 1) P(T = 0 | F_{lu} = 1)$



Bayes Nets tell a generative story.

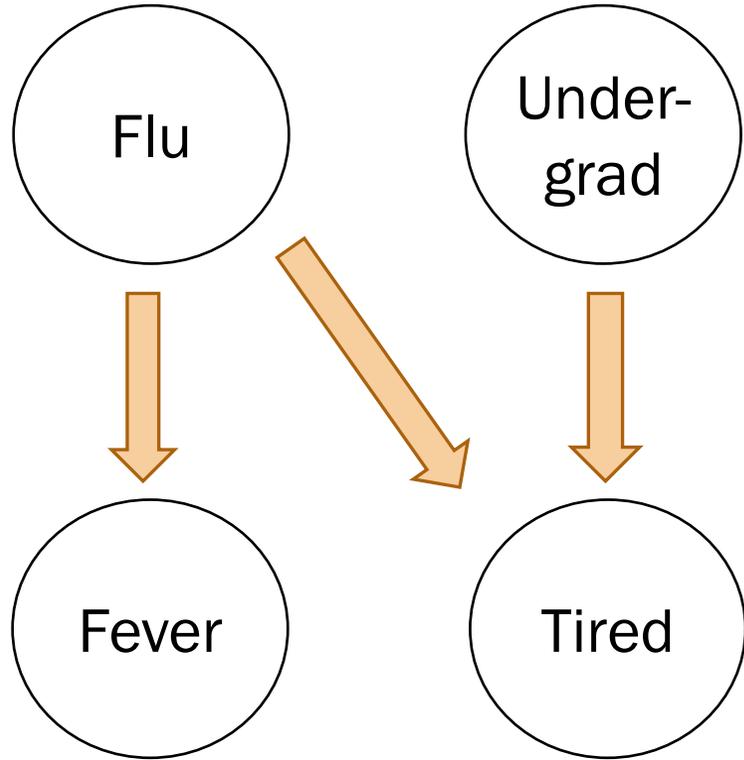
This leads to many independence assumptions

Makes it tractable to represent the join

# Challenge: Exact Inference in a Bayes Net

$$P(Fl = x)$$

$$P(U = u)$$



$$P(Fl = 0 | Fe = 1)$$

$$= \frac{P(Fl = 0, Fe = 1)}{P(Fe = 1)}$$

$$= \frac{P(Fe = 1 | Fl = 0)P(Fl = 0)}{\sum_i P(Fe = 1 | Fl = i)P(Fl = i)}$$

$$P(F = f | Fl = x) P(T = t | U = u, Fl = x)$$

Can I discover independence from  
data?

# ROCK

**The Sound:** Vigorous, defiant, energetic, inventive

**The Roots:** Rhythm & blues, country

**The Pioneers:** Bill Haley, Chuck Berry, Fats Domino, Little Richard, Buddy Holly, Elvis Presley

**The Places:** Cleveland, New Orleans, Detroit, New York City

**The Ensemble:** Electric guitar, bass, drums, keyboard, vocals

"We're a rock group. We're noisy, raucous, emotional and wild."

— Angus Young (c. 1960)  
Lead guitarist of the band AC/DC

# HIP-HOP R&B

**The Sound:** Rhythmic, unvarnished, adaptable, streetwise

**The Roots:** Rhythm & blues, soul, funk, reggae

**The Pioneers:** Afrika Bambaataa, Kool Herc, DJ Hollywood, Grandmaster Flash, Kurtis Blow, Grandmaster Caz

**The Places:** New York City (South Bronx)

**The Ensemble:** Vinyl, turntable, vocals

"The beautiful thing about hip-hop is it's like an audio collage. You can take any form of music and do it in a hip-hop way and it'll be a hip-hop song."

— Tom Mchale (1971)  
Hip-hop artist

# LATIN American

**The Sound:** Syncopated, enthusiastic, diverse, vibrant

**The Roots:** Spain, Africa, Caribbean, South America

**The Pioneers:** Arsenio Rodriguez, Machito, Pérez Prado, Tito Puente, Celia Cruz, Johnny Pacheco

**The Places:** Cuba, Puerto Rico, Mexico, Miami, New York

**The Ensemble:** Congas, bongos, maracas, güiro, guitar, vocals

"The emphasis was dancing and rhythm. I came in with an emphasis on lyrics... telling stories that were familiar to people in Latin America—and everybody identified with the songs."

— Rubén Blades (c. 1960)  
Salsa singer and composer

# Folk

**The Sound:** Grassroots, narrative, sincere, lyrical

**The Roots:** Ballads, immigrant folklore, spirituals, cowboy songs

**The Pioneers:** Lead Belly, Odetta, Woody Guthrie, Pete Seeger, Bob Dylan, Joan Baez

**The Places:** Appalachia, Deep South, Western frontier

**The Ensemble:** Guitar, banjo, fiddle, accordion, vocals

"I find the rhythms [of folk music]. I find the melodies, time-tested by generations of singers. Above all, I find the words... they seemed punchy, straightforward, honest."

— Peter Dinklage (c. 1960)  
Folk musician

# COUNTRY Western

**The Sound:** Genuine, uncomplicated, nostalgic, informal

**The Roots:** European ballads, folk and gospel songs

**The Pioneers:** Uncle Dave Macon, the Carter Family, Jimmie Rodgers, Roy Acuff, Gene Autry, Bill Monroe

**The Places:** Appalachia, Nashville, Chicago, Western U.S.

**The Ensemble:** Fiddle, banjo, guitar, harmonica, accordion, vocals

"Country music is three chords and the truth."

— Hank Williams (1917–1953)  
Country music singer

# CLASSICAL

**The Sound:** Intricate, polished, structured, harmonious

**The Roots:** Sacred music, choral chants, madrigals, dance rhythms

**The Pioneers:** J.S. Bach, Handel, Haydn, Mozart, Beethoven, Brahms

**The Places:** Austria, Germany, France, Italy

**The Ensemble:** Strings, woodwinds, brass, percussion, vocals

"I carry my thoughts about with me a long time... before writing them down. I change many things, discard others, and try again and again until I am satisfied."

— Ludwig van Beethoven (1770–1827)  
Classical music composer

AutoSave OFF Search Sheet

Home Insert Page Layout Formulas Data >> Share ^

Clipboard Font Alignment Number Conditional Formatting > Format as Table > Cell Styles > Cells Editing

C15 fx 3

	A	B	C	D	E	F	G	H	I
1	Music	Dance	Folk	Country	Classical music	Musical	Pop	Rock	Me
2	5	2	1	2	2	1	5	5	
3	4	2	1	1	1	2	3	5	
4	5	2	2	3	4	5	3	5	
5	5	2	1	1	1	1	2	2	
6	5	4	3	2	4	3	5	3	
7	5	2	3	2	3	3	2	5	
8	5	5	3	1	2	2	5	3	
9	5	3	2	1	2	2	4	5	
10	5	3	1	1	2	4	3	5	
11	5	2	5	2	2	5	3	5	
12	5	3	2	1	2	3	4	3	
13	5	1	1	1	4	1	2	5	
14	5	1	2	1	4	3	3	5	
15	5	5	3	2	1	5	5	2	
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26	5	3	1	1	4	3	3	5	
27	5	4	2	1	2	3	5	1	
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32	4	4	3	3	3	3	4	4	
33	4	4	1	3	2	3	5	3	
34	5	3	1	3	2	3	3	4	
35	5	2	2	3	4	5	4	3	

music +

Ready 100%

Why is it harder to find independences here than for bat DNA expression?

	A	B	C	D	E	F	G	H	Me
1	Music	Dance	Folk	Country	Classical music	Musical	Pop	Rock	Me
2	5	2	1	2	2	1	5	5	
3	4	2	1	1	1	2	3	5	
4	5	2	2	3	4	5	3	5	
5	5	2	1	1	1	1	2	2	
6	5	4	3	2	4	3	5	3	
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31	5	3	4	2	3	3	3	4	
32	4	4	3	3	3	3	4	4	
33	4	4	1	3	2	3	5	3	
34	5	3	1	3	2	3	3	4	
35	5	2	2	3	4	5	4	3	

# Dance of the Covariance

# Recall our Ebola Bats



# Bat Data

Gene1	Gene2	Gene3	Gene4	Gene5	Trait
TRUE	FALSE	TRUE	TRUE	FALSE	FALSE
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	TRUE	FALSE	FALSE	FALSE
TRUE	FALSE	TRUE	TRUE	TRUE	FALSE
FALSE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	FALSE	FALSE	TRUE	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	FALSE	FALSE
TRUE	FALSE	TRUE	FALSE	FALSE	FALSE
FALSE	TRUE	FALSE	TRUE	FALSE	FALSE
TRUE	TRUE	FALSE	TRUE	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	FALSE	FALSE
TRUE	FALSE	TRUE	TRUE	TRUE	FALSE
FALSE	FALSE	TRUE	TRUE	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	FALSE	FALSE
TRUE	FALSE	FALSE	...	FALSE	FALSE
TRUE	FALSE	FALSE	TRUE	FALSE	FALSE

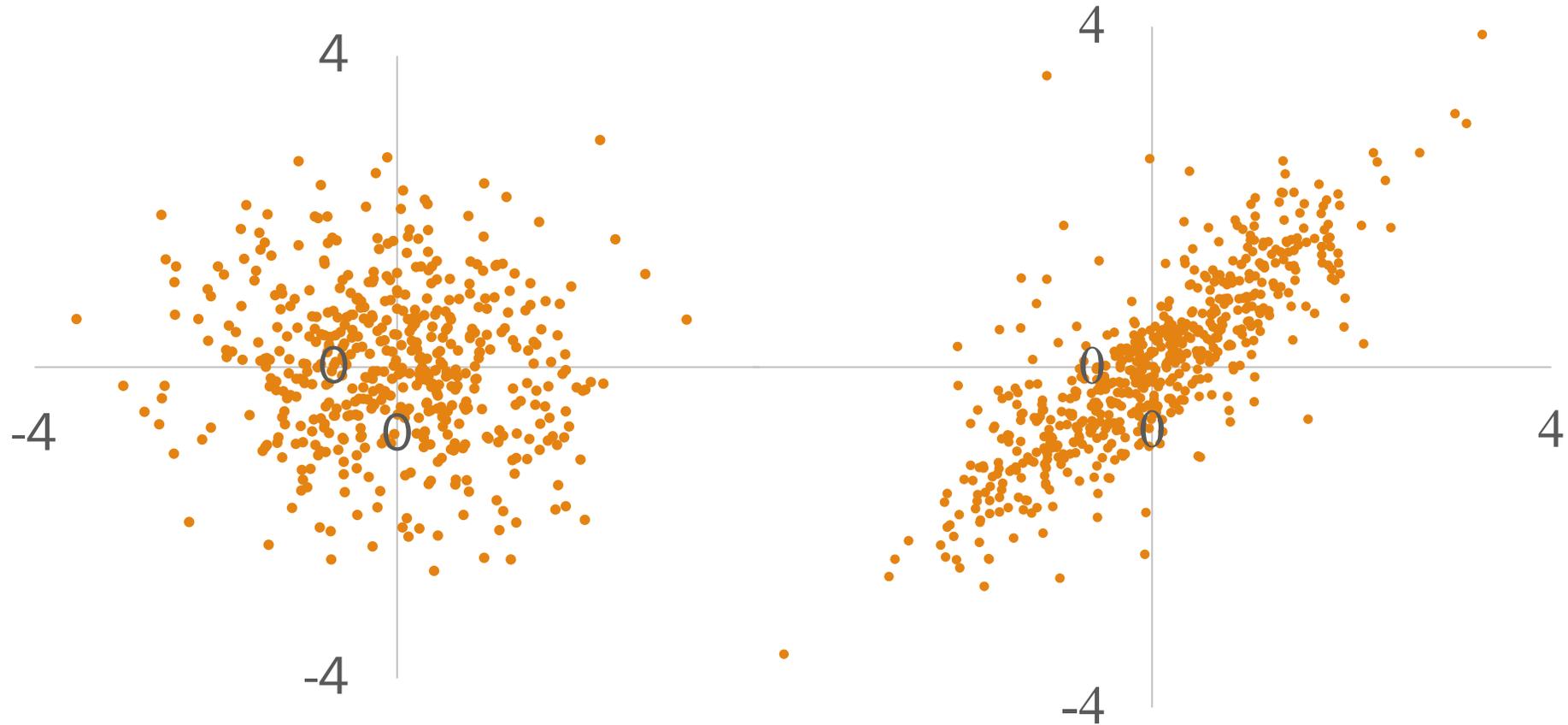
# Expression Amount

---

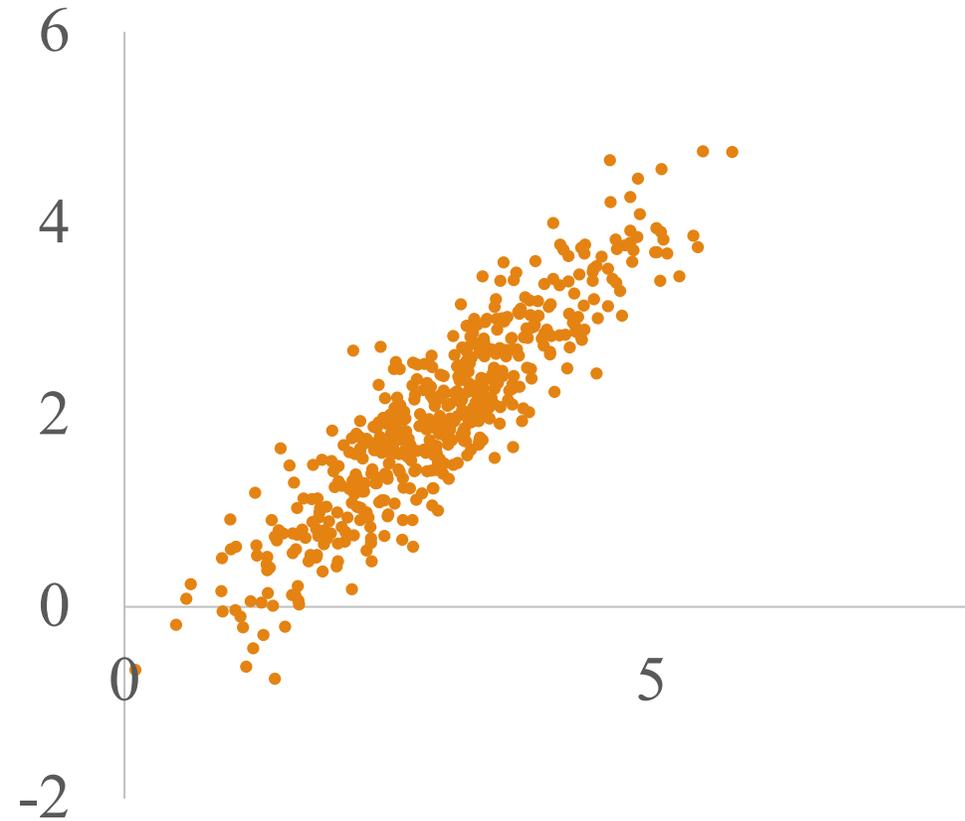
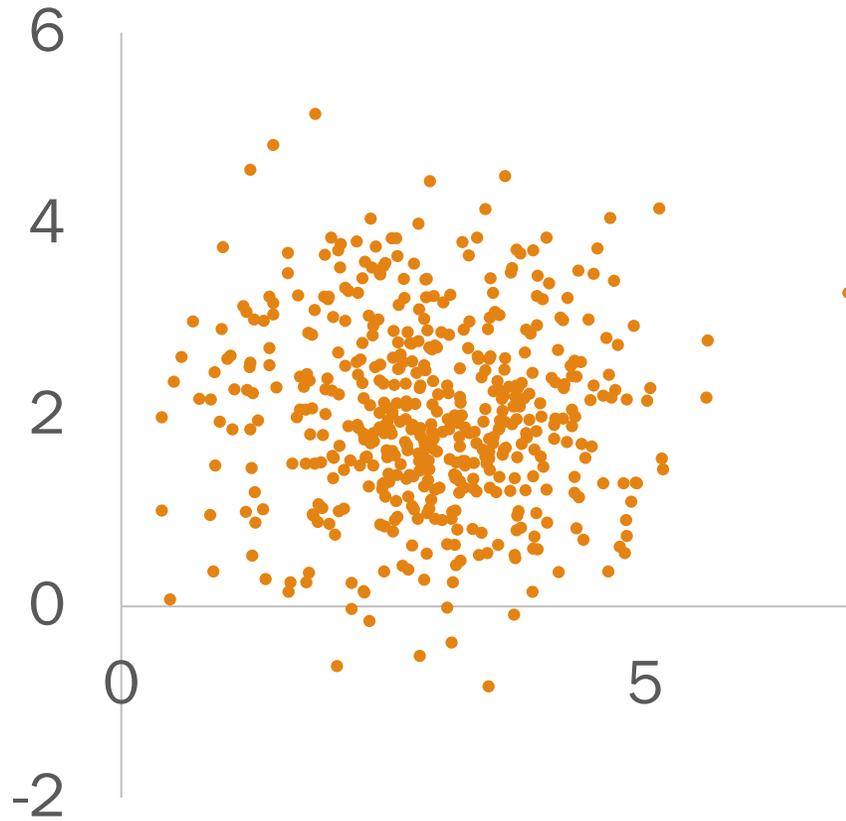
Gene5	Trait
0.76	0.83
0.94	0.85
0.82	0.03
0.94	0.32
0.50	0.10
0.40	0.53
0.90	0.67
0.29	0.71
0.72	0.25
0.15	0.24
0.79	0.98
0.68	0.77
0.71	0.37
0.36	0.18
0.62	0.08
0.59	0.38
0.82	0.76

---

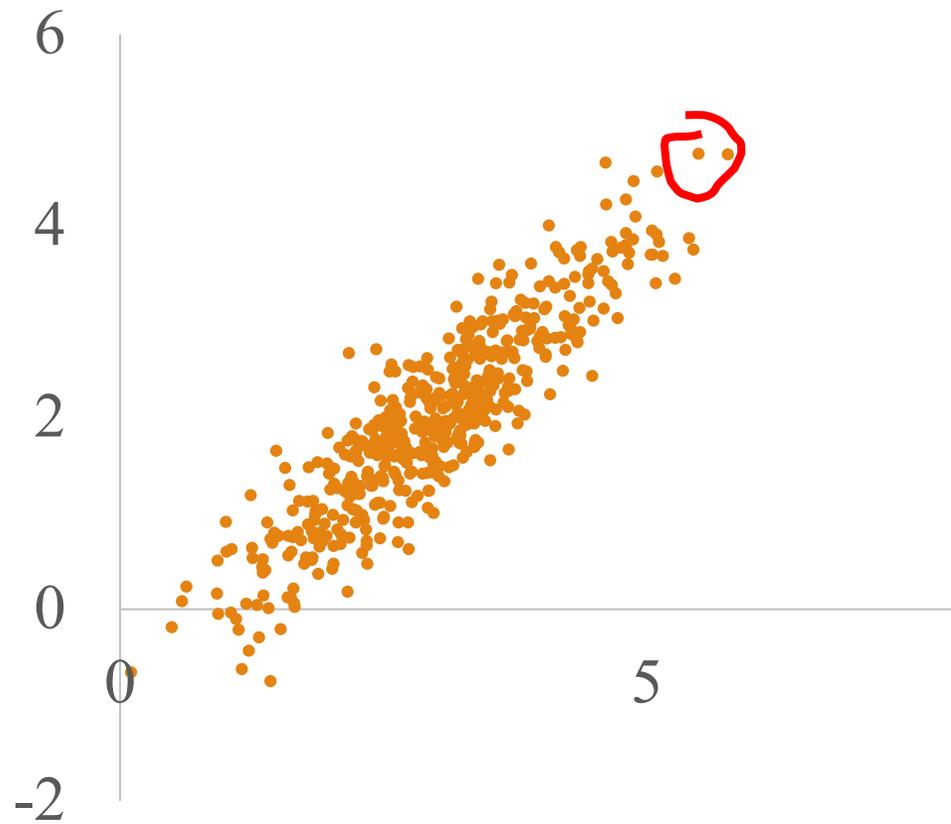
# Spot The Difference



# Spot The Difference



# Vary Together

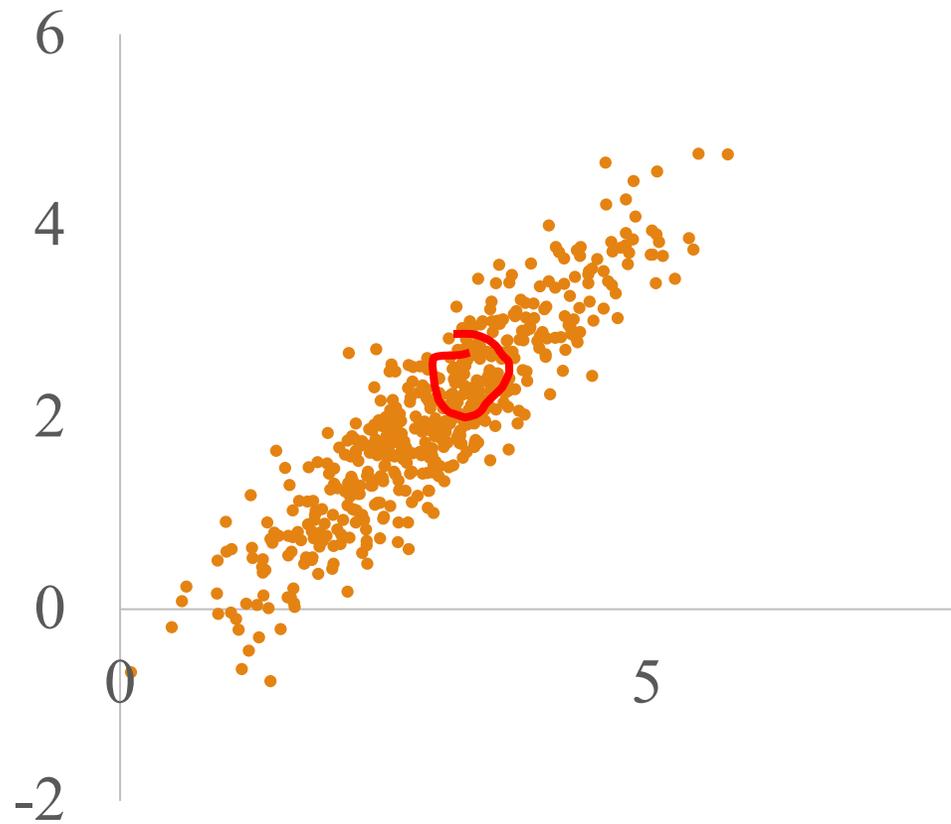


$$x - E[x] = 3$$

$$y - E[y] = 2.6$$

$$(x - E[x])(y - E[y]) = 7.8$$

# Vary Together

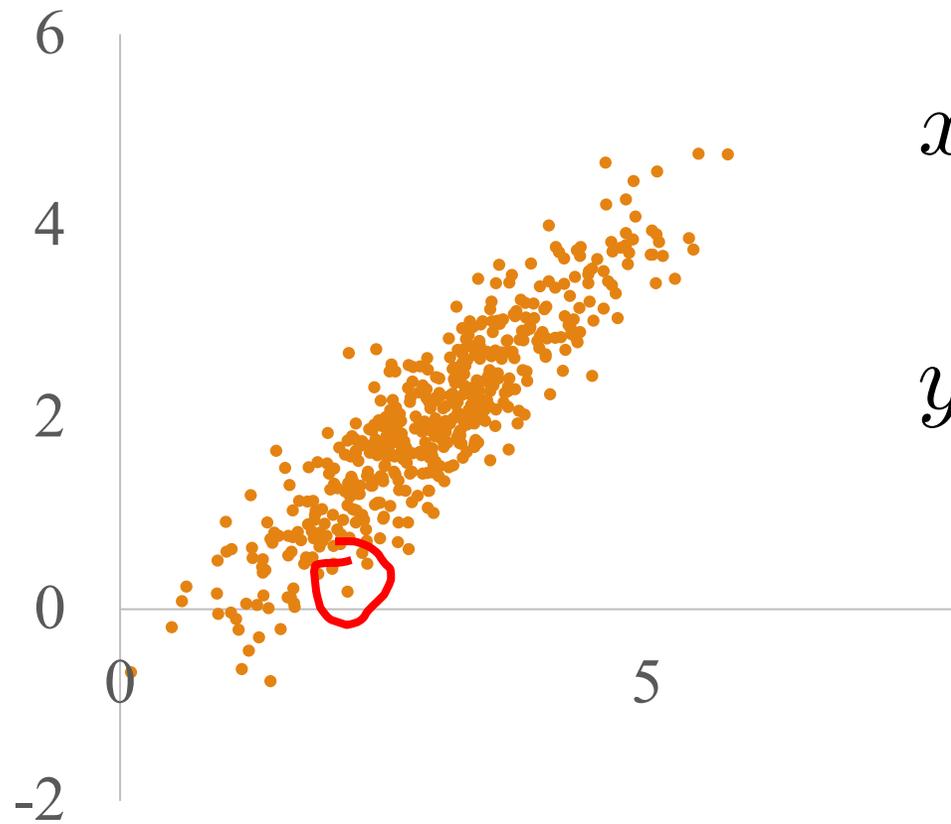


$$x - E[x] \approx 0$$

$$y - E[y] \approx 0$$

$$(x - E[x])(y - E[y]) = 0$$

# Vary Together

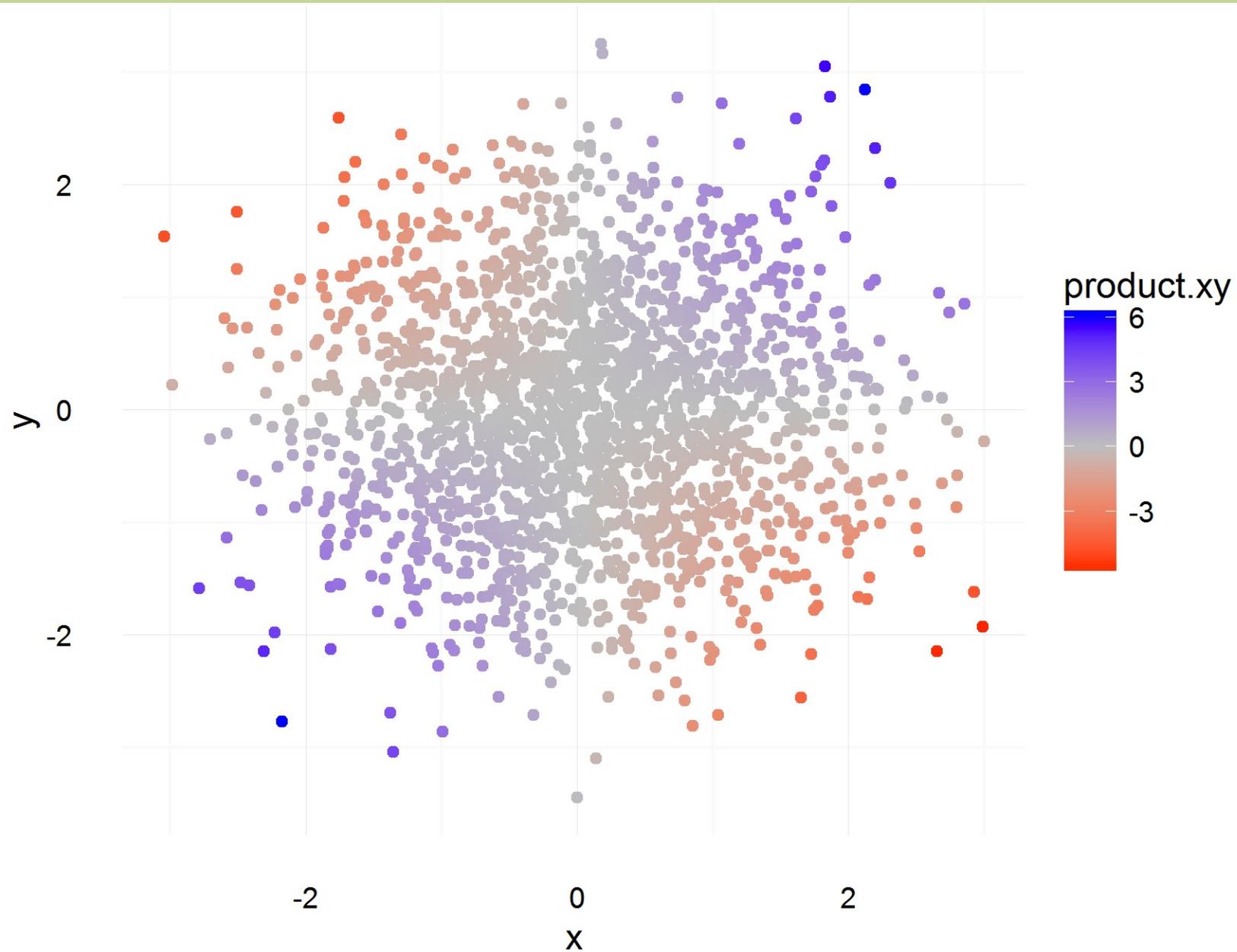


$$x - E[x] = -1.1$$

$$y - E[y] = -2.8$$

$$(x - E[x])(y - E[y]) \approx 3.1$$

# Understanding Covariance



# The Dance of the Covariance

Say  $X$  and  $Y$  are arbitrary random variables

Covariance of  $X$  and  $Y$ :

$$\text{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])]$$

$x$	$y$	$(x - E[X])(y - E[Y])p(x,y)$
Above mean	Above mean	Positive
Bellow mean	Bellow mean	Positive
Bellow mean	Above mean	Negative
Above mean	Bellow mean	Negative

# The Dance of the Covariance

Say  $X$  and  $Y$  are arbitrary random variables

Covariance of  $X$  and  $Y$ :

$$\text{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])]$$

Equivalently:

$$\begin{aligned}\text{Cov}(X, Y) &= E[XY - E[X]Y - XE[Y] + E[Y]E[X]] \\ &= E[XY] - E[X]E[Y] - E[X]E[Y] + E[X]E[Y] \\ &= E[XY] - E[X]E[Y]\end{aligned}$$

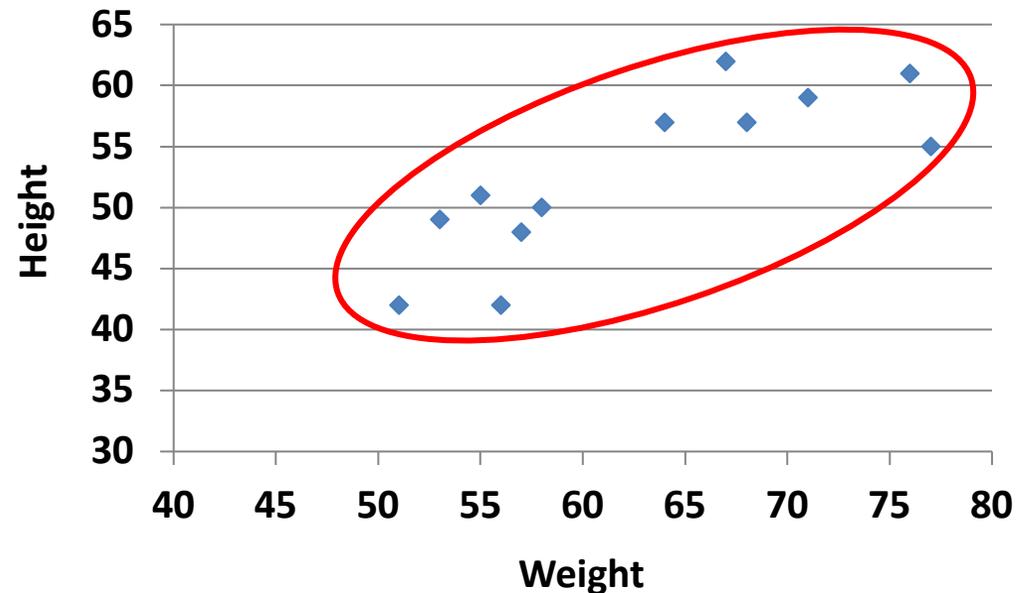
- $X$  and  $Y$  independent,  $E[XY] = E[X]E[Y] \rightarrow \text{Cov}(X, Y) = 0$
- But  $\text{Cov}(X, Y) = 0$  does **not** imply  $X$  and  $Y$  independent!

# Covariance and Data

Consider the following data:

Weight	Height	Weight * Height
64	57	3648
71	59	4189
53	49	2597
67	62	4154
55	51	2805
58	50	2900
77	55	4235
57	48	2736
56	42	2352
51	42	2142
76	61	4636
68	57	3876

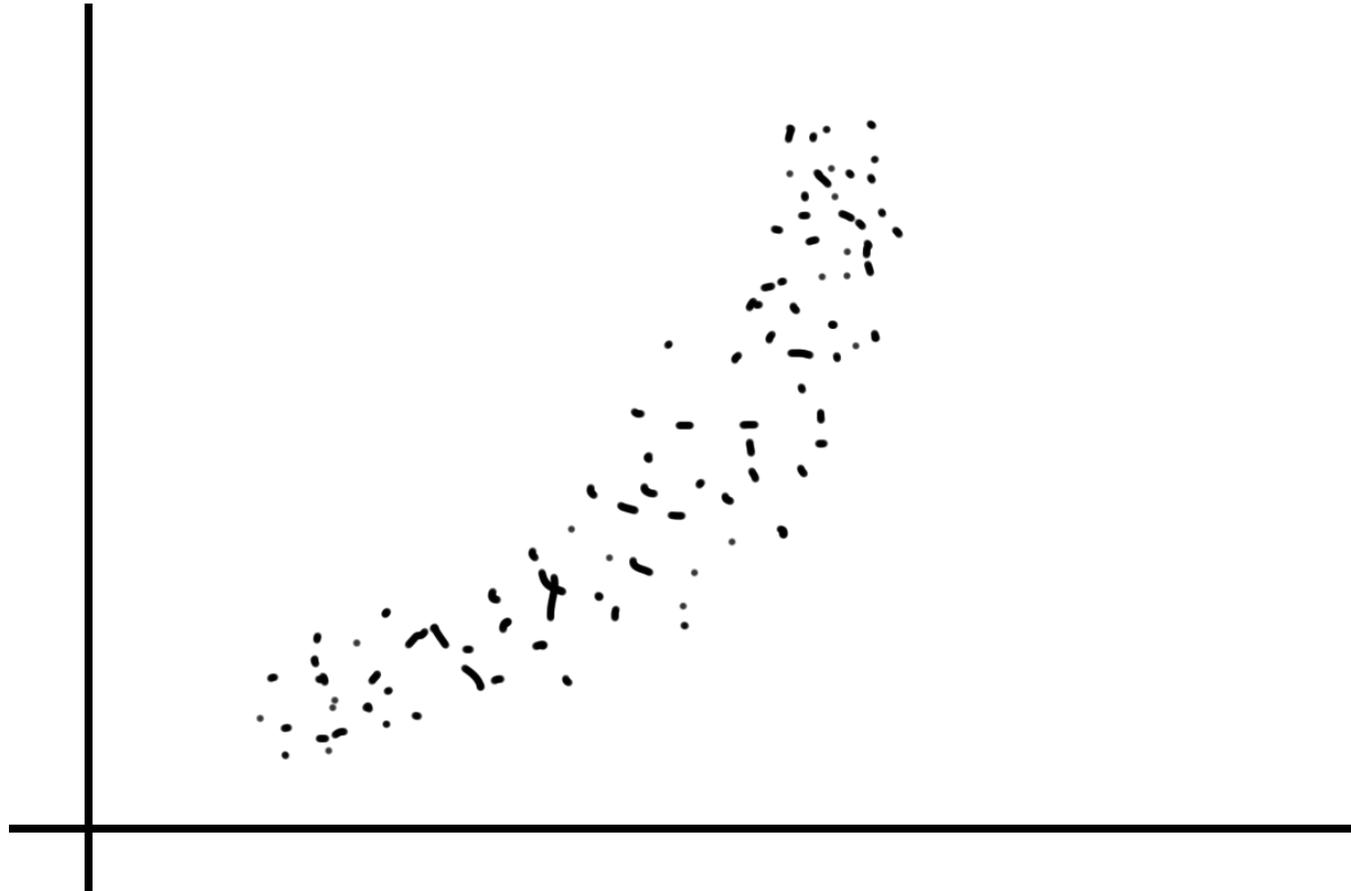
$$\begin{array}{lll} E[W] & E[H] & E[W*H] \\ = 62.75 & = 52.75 & = 3355.83 \end{array}$$



$$\begin{aligned} \text{Cov}(W, H) &= E[W*H] - E[W]E[H] \\ &= 3355.83 - (62.75)(52.75) \\ &= 45.77 \end{aligned}$$

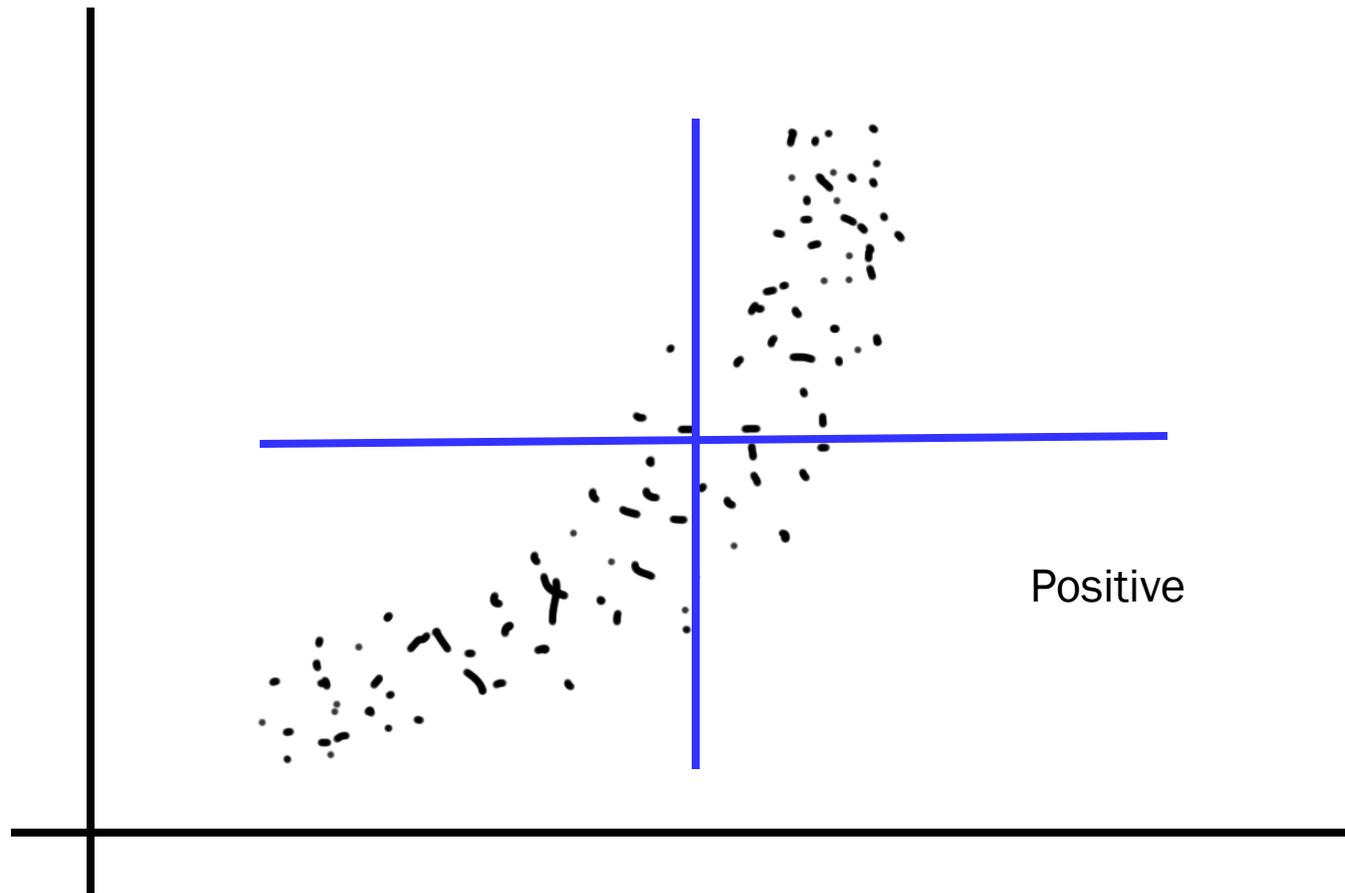
# Covariance

Poll: (a) positive, (b) negative, (c) zero



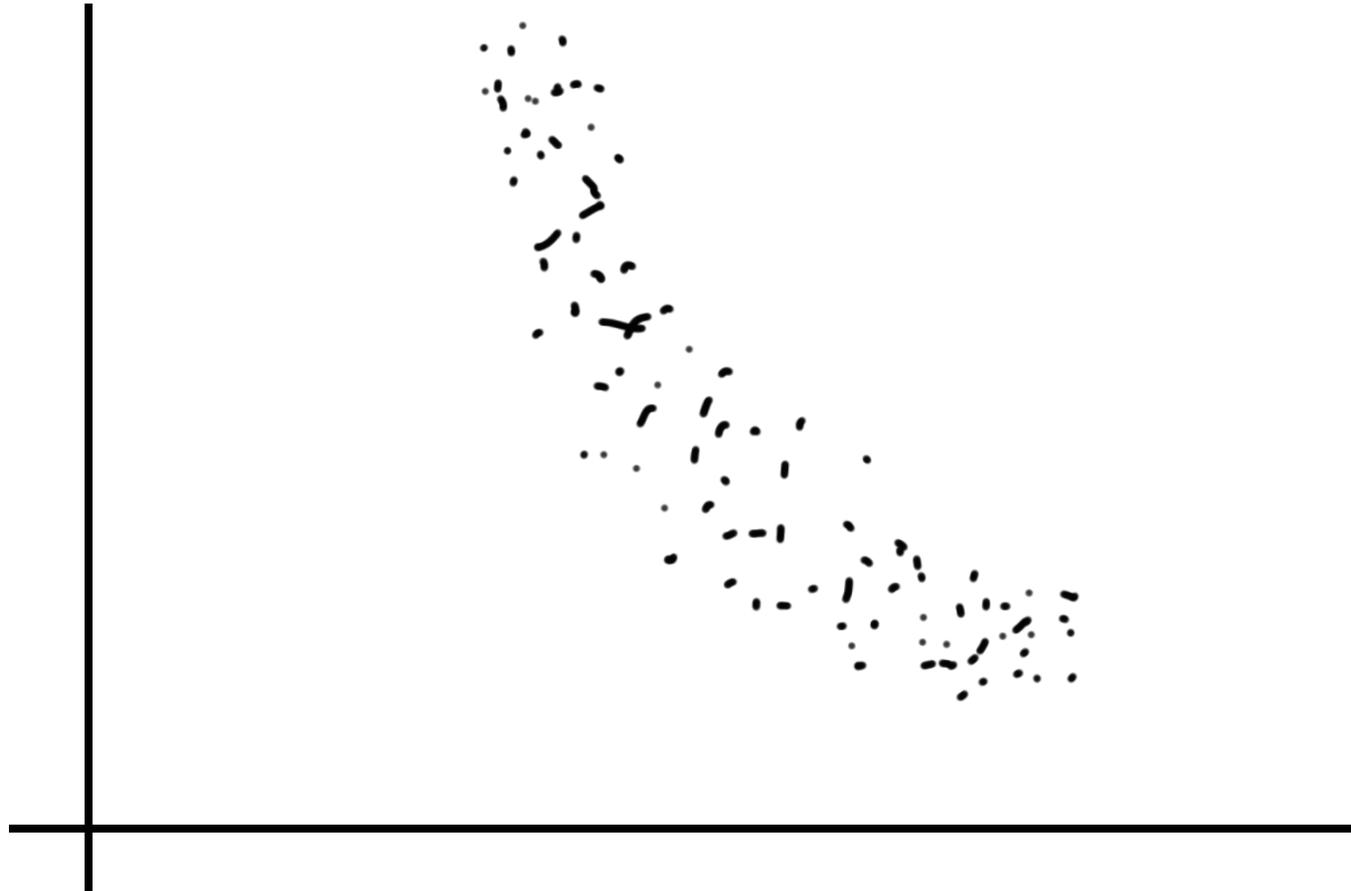
# Covariance

Is the Covariance: (a) positive, (b) negative, (c) zero



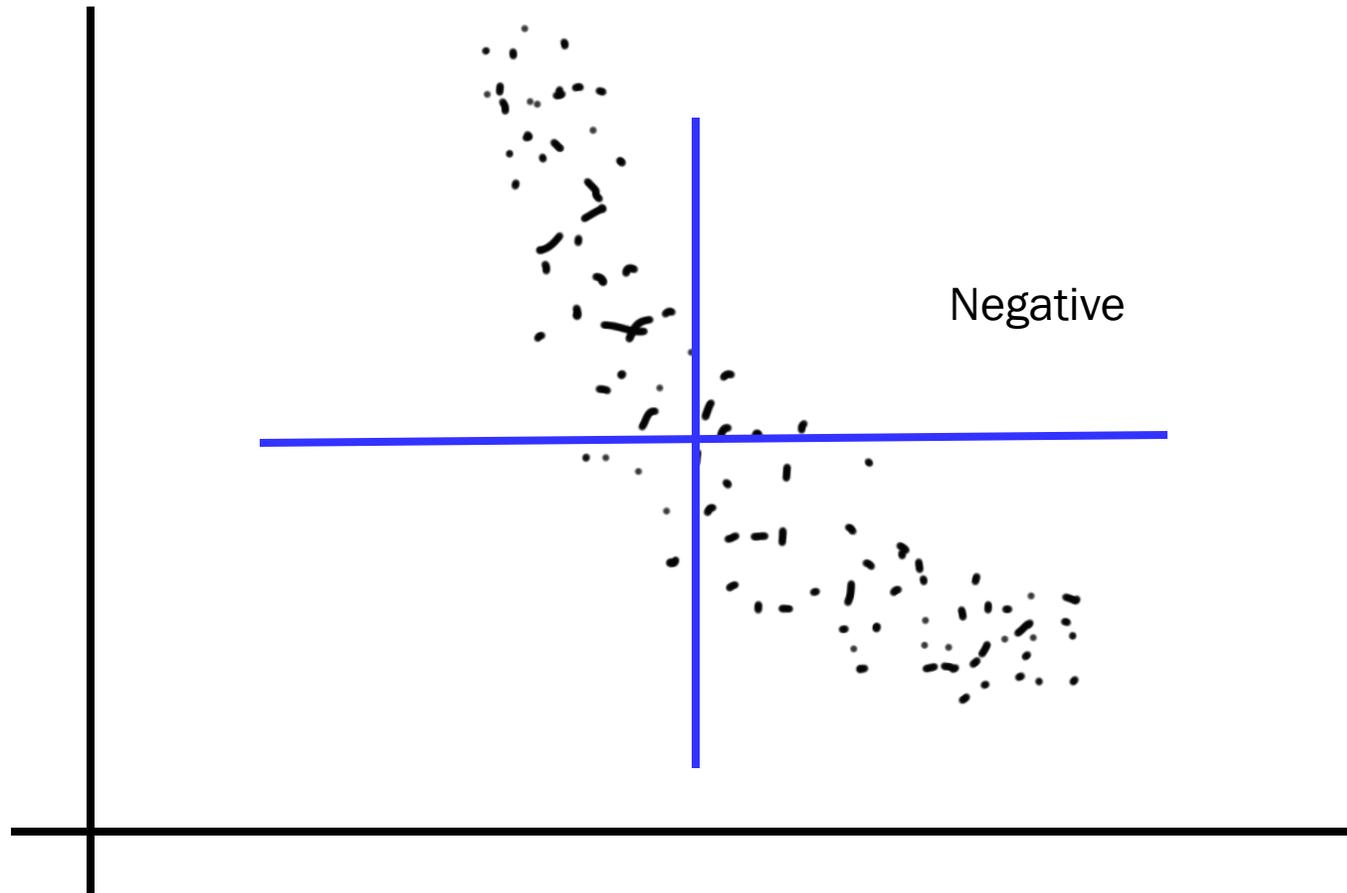
# Covariance

Is the Covariance: (a) positive, (b) negative, (c) zero



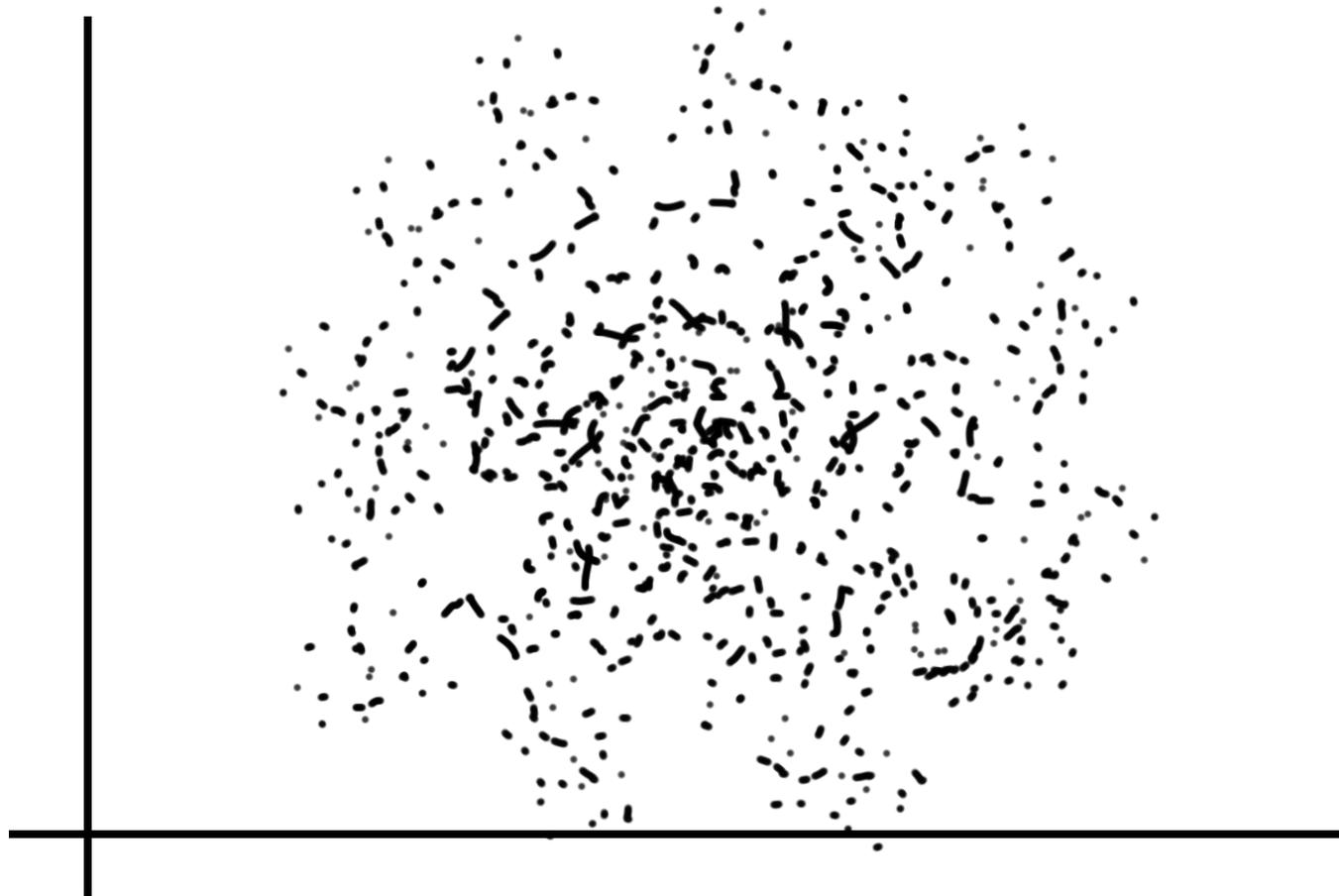
# Covariance

Is the Covariance: (a) positive, (b) negative, (c) zero



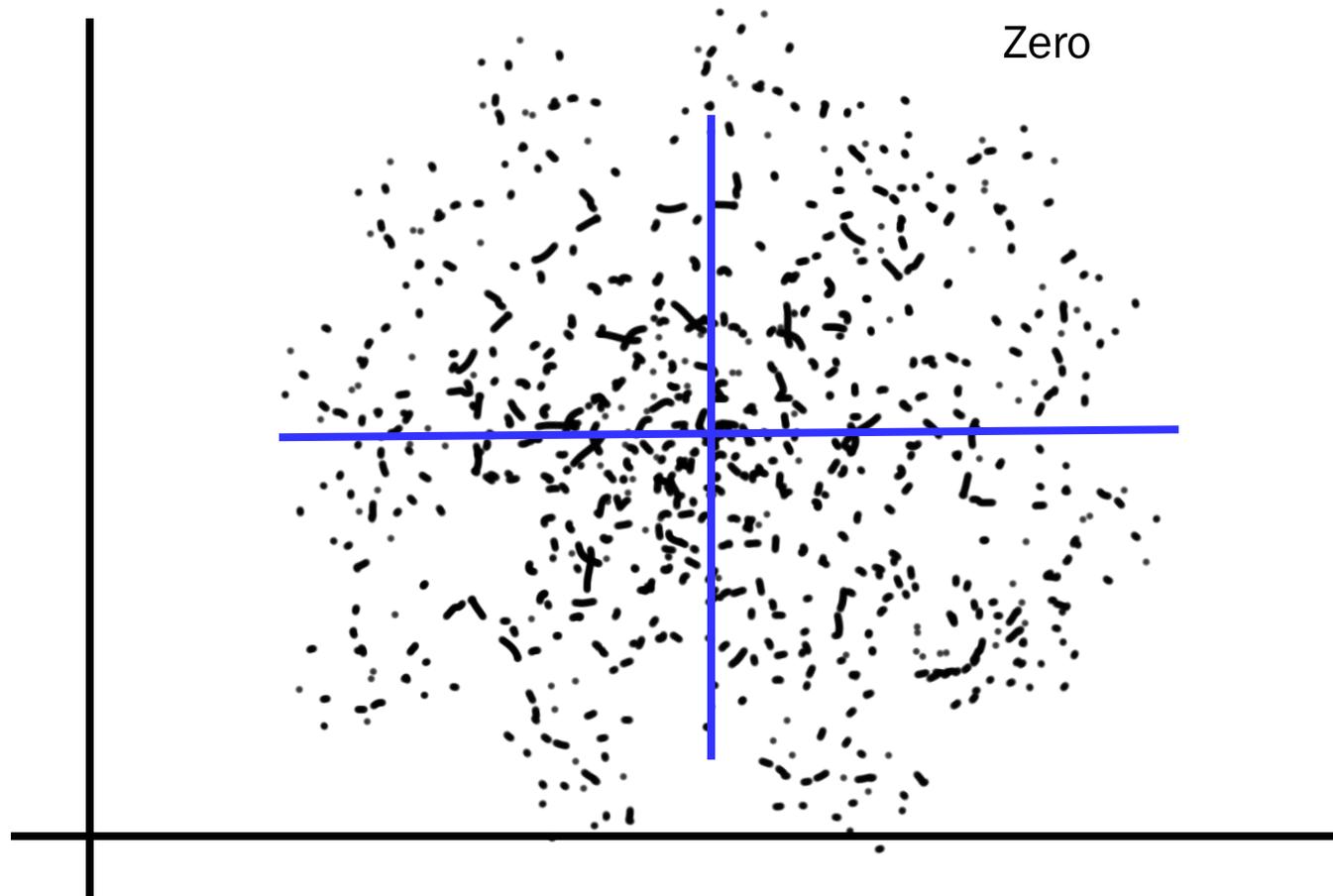
# Covariance

Is the Covariance: (a) positive, (b) negative, (c) zero



# Covariance

Is the Covariance: (a) positive, (b) negative, (c) zero



# Independence and Covariance

X and Y are random variables with PMF:

Y \ X	-1	0	1	$p_Y(y)$
0	1/3	0	1/3	2/3
1	0	1/3	0	1/3
$p_X(x)$	1/3	1/3	1/3	1

$$Y = \begin{cases} 0 & \text{if } X \neq 0 \\ 1 & \text{otherwise} \end{cases}$$

- $E[X] = -1(1/3) + 0(1/3) + 1(1/3) = 0$
- $E[Y] = 0(2/3) + 1(1/3) = 1/3$
- Since  $XY = 0$ ,  $E[XY] = 0$
- $\text{Cov}(X, Y) = E[XY] - E[X]E[Y] = 0 - 0 = 0$

But, X and Y are clearly dependent!

# Properties of Covariance

Say  $X$  and  $Y$  are arbitrary random variables

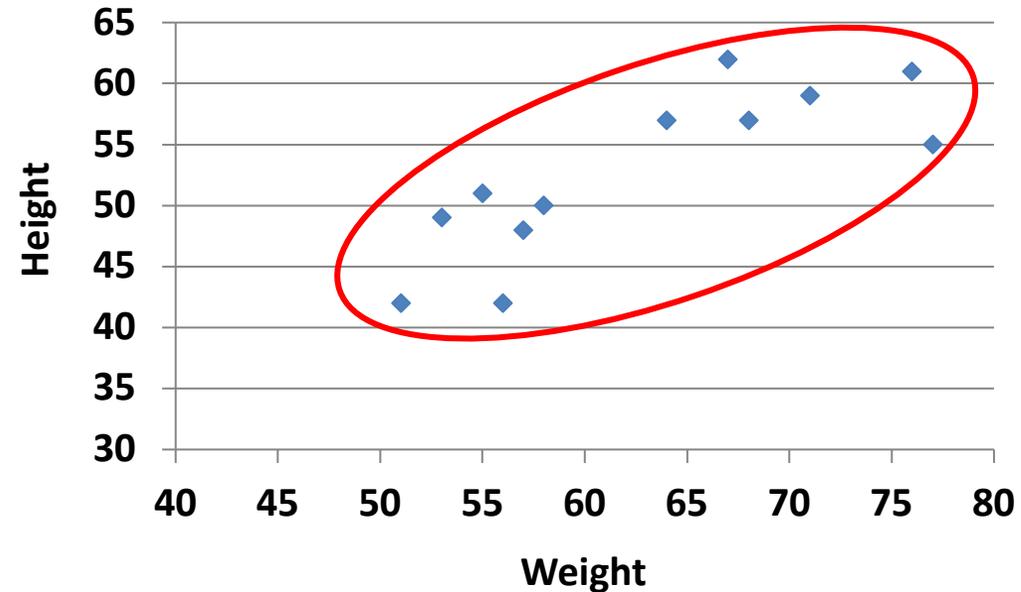
- $\text{Cov}(X, Y) = \text{Cov}(Y, X)$
- $\text{Cov}(X, X) = E[X^2] - E[X]E[X] = \text{Var}(X)$
- $\text{Cov}(aX + b, Y) = a\text{Cov}(X, Y)$

# Correlation

# What is Wrong With This?

Consider the following data:

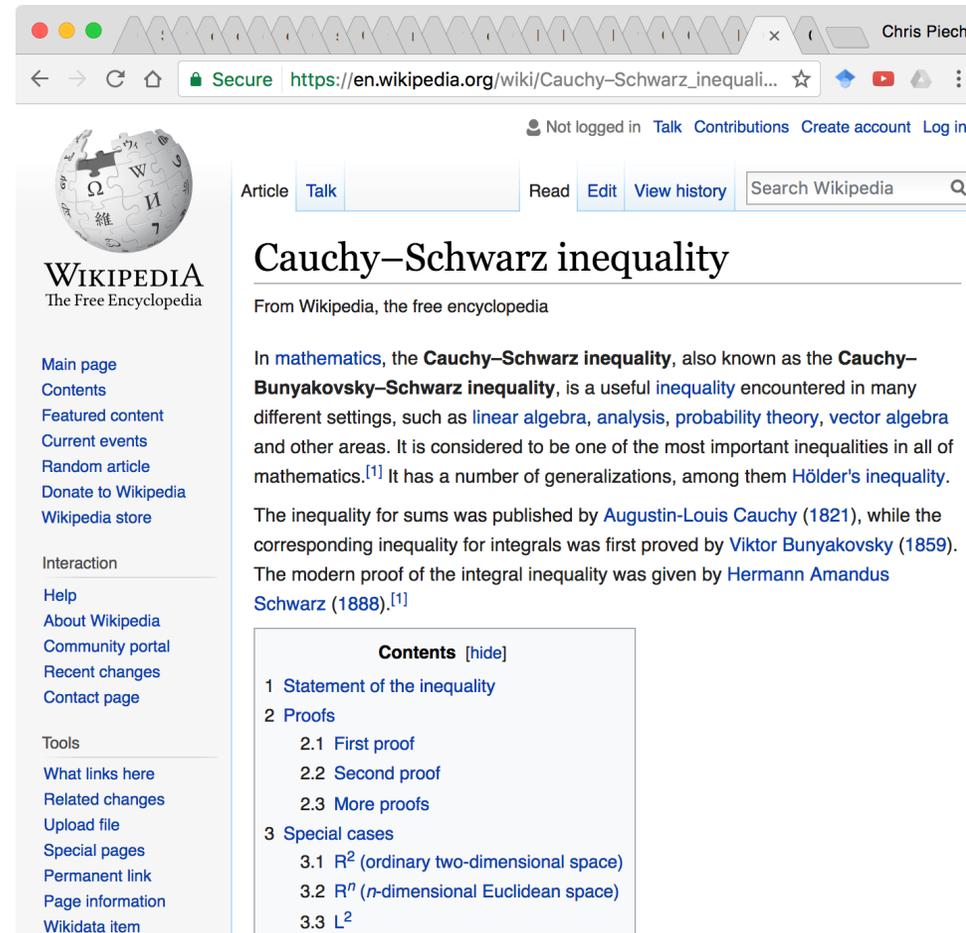
Weight	Height	Weight * Height
64	57	3648
71	59	4189
53	49	2597
67	62	4154
55	51	2805
58	50	2900
77	55	4235
57	48	2736
56	42	2352
51	42	2142
76	61	4636
68	57	3876



$$\begin{array}{lll} E[W] & E[H] & E[W*H] \\ = 62.75 & = 52.75 & = 3355.83 \end{array}$$

$$\begin{aligned} \text{Cov}(W, H) &= E[W*H] - E[W]E[H] \\ &= 3355.83 - (62.75)(52.75) \\ &= 45.77 \end{aligned}$$

# Cauchy Schwarz, a great way to normalize!



The screenshot shows a web browser window displaying the Wikipedia article for the Cauchy–Schwarz inequality. The browser's address bar shows the URL: [https://en.wikipedia.org/wiki/Cauchy–Schwarz\\_inequality](https://en.wikipedia.org/wiki/Cauchy–Schwarz_inequality). The page title is "Cauchy–Schwarz inequality". The article text states: "In **mathematics**, the **Cauchy–Schwarz inequality**, also known as the **Cauchy–Bunyakovsky–Schwarz inequality**, is a useful **inequality** encountered in many different settings, such as **linear algebra**, **analysis**, **probability theory**, **vector algebra** and other areas. It is considered to be one of the most important inequalities in all of mathematics.<sup>[1]</sup> It has a number of generalizations, among them **Hölder's inequality**. The inequality for sums was published by **Augustin-Louis Cauchy** (1821), while the corresponding inequality for integrals was first proved by **Viktor Bunyakovsky** (1859). The modern proof of the integral inequality was given by **Hermann Amandus Schwarz** (1888).<sup>[1]</sup>" Below the text is a "Contents" section with the following items: 1 Statement of the inequality, 2 Proofs (with sub-items 2.1 First proof, 2.2 Second proof, 2.3 More proofs), and 3 Special cases (with sub-items 3.1 R<sup>2</sup> (ordinary two-dimensional space), 3.2 R<sup>n</sup> (n-dimensional Euclidean space), 3.3 L<sup>2</sup>).

$$-\text{Std}(X)\text{Std}(Y) \leq \text{Cov}(X, Y) \leq \text{Std}(X)\text{Std}(Y)$$

# Viva La Correlación

Say  $X$  and  $Y$  are arbitrary random variables

- Correlation of  $X$  and  $Y$ , denoted  $\rho(X, Y)$ :

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$$

- Note:  $-1 \leq \rho(X, Y) \leq 1$
- Correlation measures linearity between  $X$  and  $Y$
- $\rho(X, Y) = 1 \quad \Rightarrow \quad Y = aX + b \quad \text{where } a = \sigma_y/\sigma_x$
- $\rho(X, Y) = -1 \quad \Rightarrow \quad Y = aX + b \quad \text{where } a = -\sigma_y/\sigma_x$
- $\rho(X, Y) = 0 \quad \Rightarrow \quad \text{absence of linear relationship}$ 
  - But,  $X$  and  $Y$  can still be related in some other way!
- If  $\rho(X, Y) = 0$ , we say  $X$  and  $Y$  are “uncorrelated”
  - Note: Independence implies uncorrelated, but **not** vice versa!

# Viva La Correlación

Say  $X$  and  $Y$  are arbitrary random variables

- Correlation of  $X$  and  $Y$ , denoted  $\rho(X, Y)$ :

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$$

Say  $Y = cX$ . Correlation should be 1.

# ROCK

**The Sound:** Vigorous, defiant, energetic, inventive

**The Roots:** Rhythm & blues, country

**The Pioneers:** Bill Haley, Chuck Berry, Fats Domino, Little Richard, Buddy Holly, Elvis Presley

**The Places:** Cleveland, New Orleans, Detroit, New York City

**The Ensemble:** Electric guitar, bass, drums, keyboard, vocals

"We're a rock group. We're noisy, raucous, emotional and wild."

— Angus Young (c. 1960)  
Lead guitarist of the band AC/DC



# HIP-HOP R&B

**The Sound:** Rhythmic, unvarnished, adaptable, streetwise

**The Roots:** Rhythm & blues, soul, funk, reggae

**The Pioneers:** Afrika Bambaataa, Kool Herc, DJ Hollywood, Grandmaster Flash, Kurtis Blow, Grandmaster Caz

**The Places:** New York City (South Bronx)

**The Ensemble:** Vinyl, turntable, vocals

"The beautiful thing about hip-hop is it's like an audio collage. You can take any form of music and do it in a hip-hop way and it'll be a hip-hop song."

— Tom Mchale (1971)  
Hip-hop artist



# LATIN American

**The Sound:** Syncopated, enthusiastic, diverse, vibrant

**The Roots:** Spain, Africa, Caribbean, South America

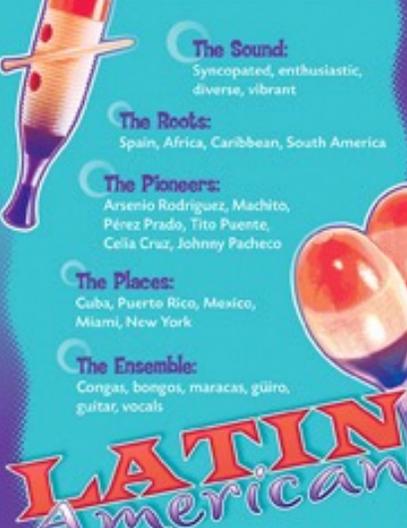
**The Pioneers:** Arsenio Rodriguez, Machito, Pérez Prado, Tito Puente, Celia Cruz, Johnny Pacheco

**The Places:** Cuba, Puerto Rico, Mexico, Miami, New York

**The Ensemble:** Congas, bongos, maracas, güiro, guitar, vocals

"The emphasis was dancing and rhythm. I came in with an emphasis on lyrics... telling stories that were familiar to people in Latin America—and everybody identified with the songs."

— Rubén Blades (c. 1960)  
Salsa singer and composer



# Folk

**The Sound:** Grassroots, narrative, sincere, lyrical

**The Roots:** Ballads, immigrant folklore, spirituals, cowboy songs

**The Pioneers:** Lead Belly, Odetta, Woody Guthrie, Pete Seeger, Bob Dylan, Joan Baez

**The Places:** Appalachia, Deep South, Western frontier

**The Ensemble:** Guitar, banjo, fiddle, accordion, vocals

"I find the rhythms [of folk music]. I find the melodies, time-tested by generations of singers. Above all, I find the words... they seemed punchy, straightforward, honest."

— Peter Dinklage (c. 1960)  
Folk musician



# COUNTRY Western

**The Sound:** Genuine, uncomplicated, nostalgic, informal

**The Roots:** European ballads, folk and gospel songs

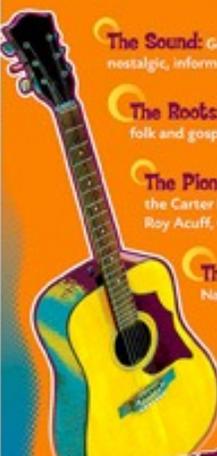
**The Pioneers:** Uncle Dave Macon, the Carter Family, Jimmie Rodgers, Roy Acuff, Gene Autry, Bill Monroe

**The Places:** Appalachia, Nashville, Chicago, Western U.S.

**The Ensemble:** Fiddle, banjo, guitar, harmonica, accordion, vocals

"Country music is three chords and the truth."

— Hank Williams (1917–1953)  
Country music singer



# CLASSICAL

**The Sound:** Intricate, polished, structured, harmonious

**The Roots:** Sacred music, choral chants, madrigals, dance rhythms

**The Pioneers:** J.S. Bach, Handel, Haydn, Mozart, Beethoven, Brahms

**The Places:** Austria, Germany, France, Italy

**The Ensemble:** Strings, woodwinds, brass, percussion, vocals

"I carry my thoughts about with me a long time... before writing them down. I change many things, discard others, and try again and again until I am satisfied."

— Ludwig van Beethoven (1770–1827)  
Classical music composer



AutoSave OFF Search Sheet

Home Insert Page Layout Formulas Data >> Share ^

Clipboard Font Alignment Number Conditional Formatting Format as Table Cell Styles

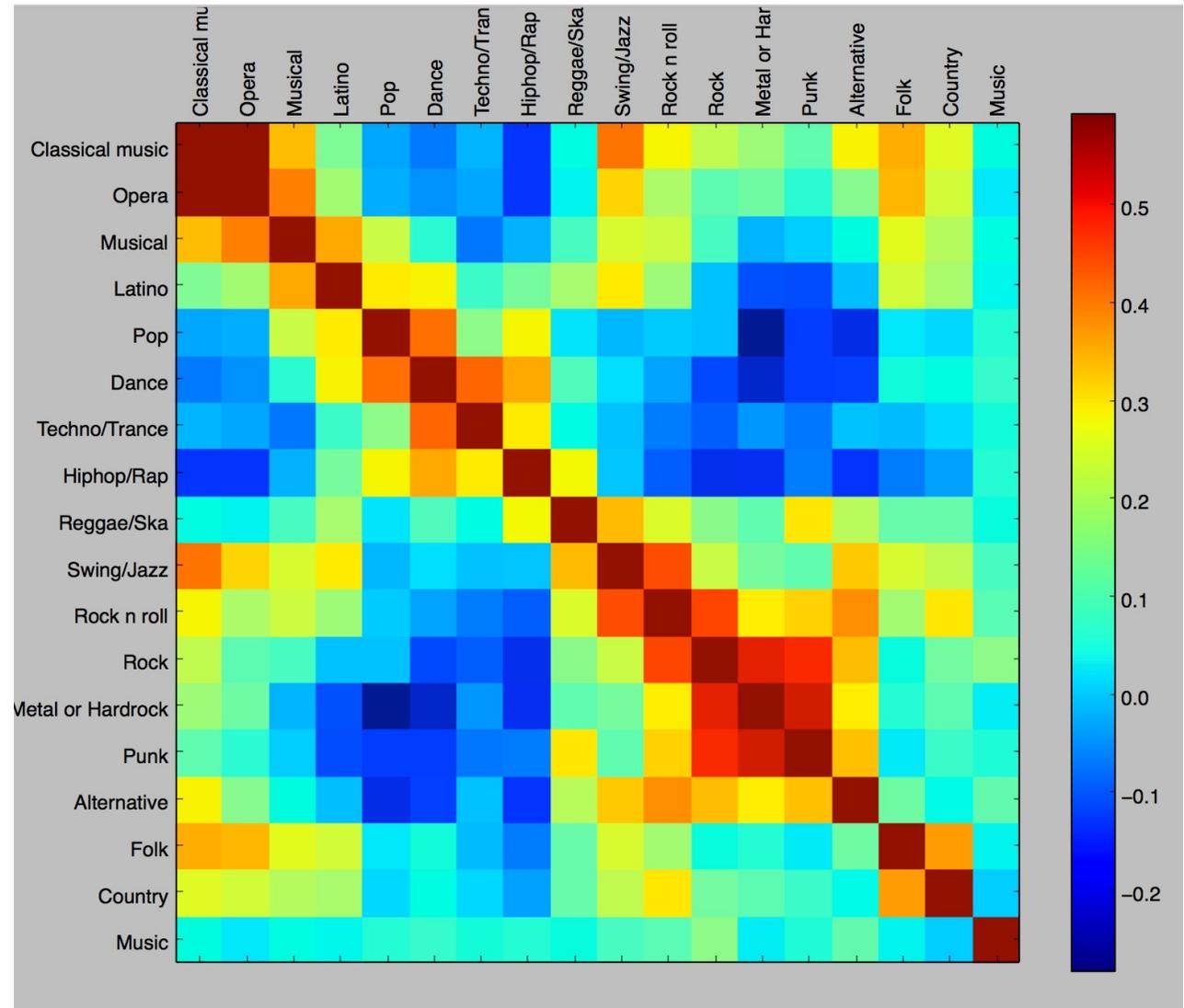
C15 fx 3

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3	4	2	1	1	1	2	3	5	
4	5	2	2	3	4	5	3	5	
5	5	2	1	1	1	1	2	2	
6	5	4	3	2	4	3	5	3	
7	5	2	3	2	3	3	2	5	
8	5	5	3	1	2	2	5	3	
9	5	3	2	1	2	2	4	5	
10	5	3	1	1	2	4	3	5	
11	5	2	5	2	2	5	3	5	
12	5	3	2	1	2	3	4	3	
13	5	1	1	1	4	1	2	5	
14	5	1	2	1	4	3	3	5	
15	5	5	3	2	1	5	5	2	
16	5	2	1	1	2	3	4	5	
17	1	2	2	3	4	3	3	5	
18	5	3	1	1	1	2	4	4	
19	5	3	3	3	2	2	4	4	
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26	5	3	1	1	4	3	3	5	
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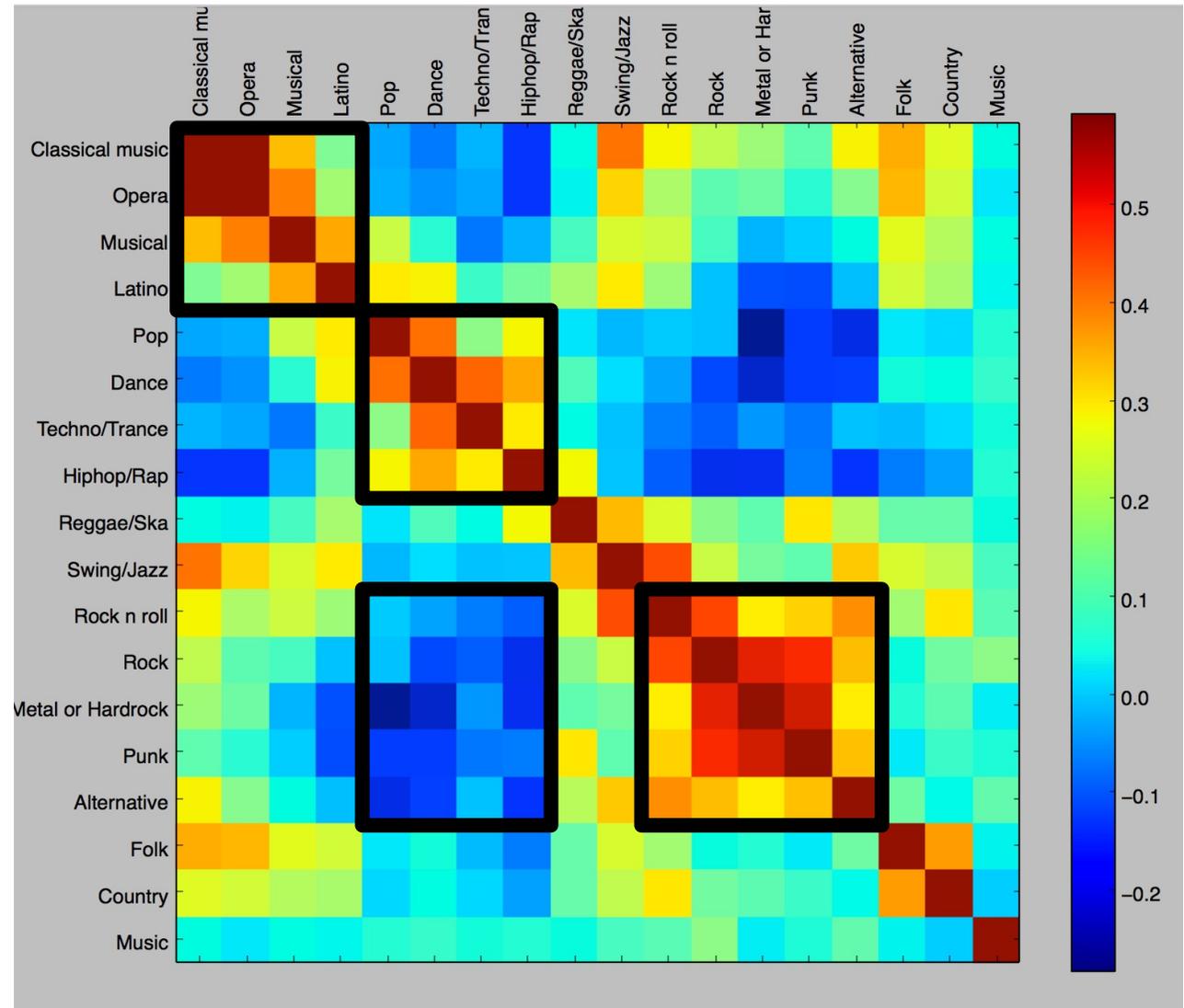
music +

Ready 100%

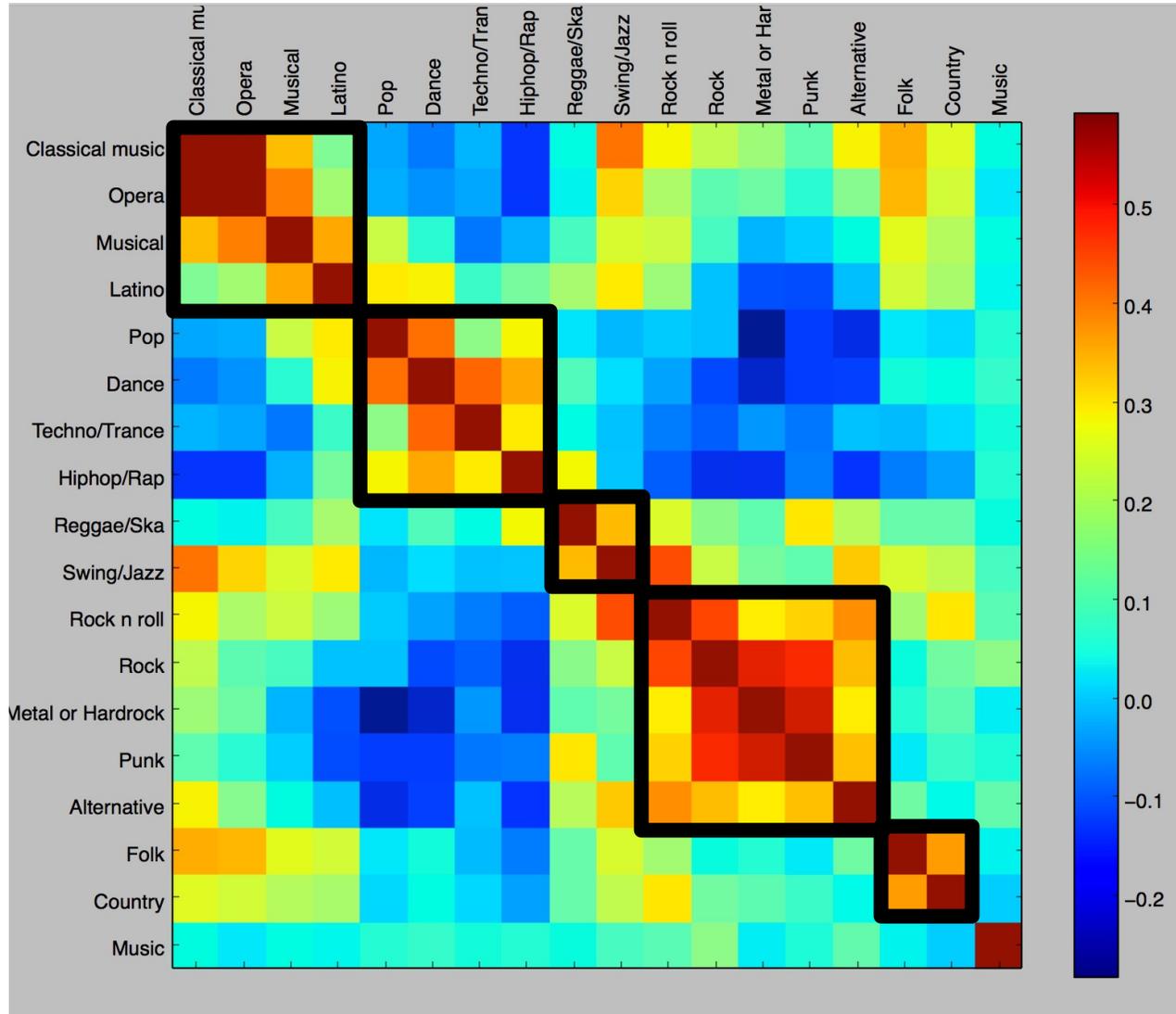
# Correlation of Music Tastes



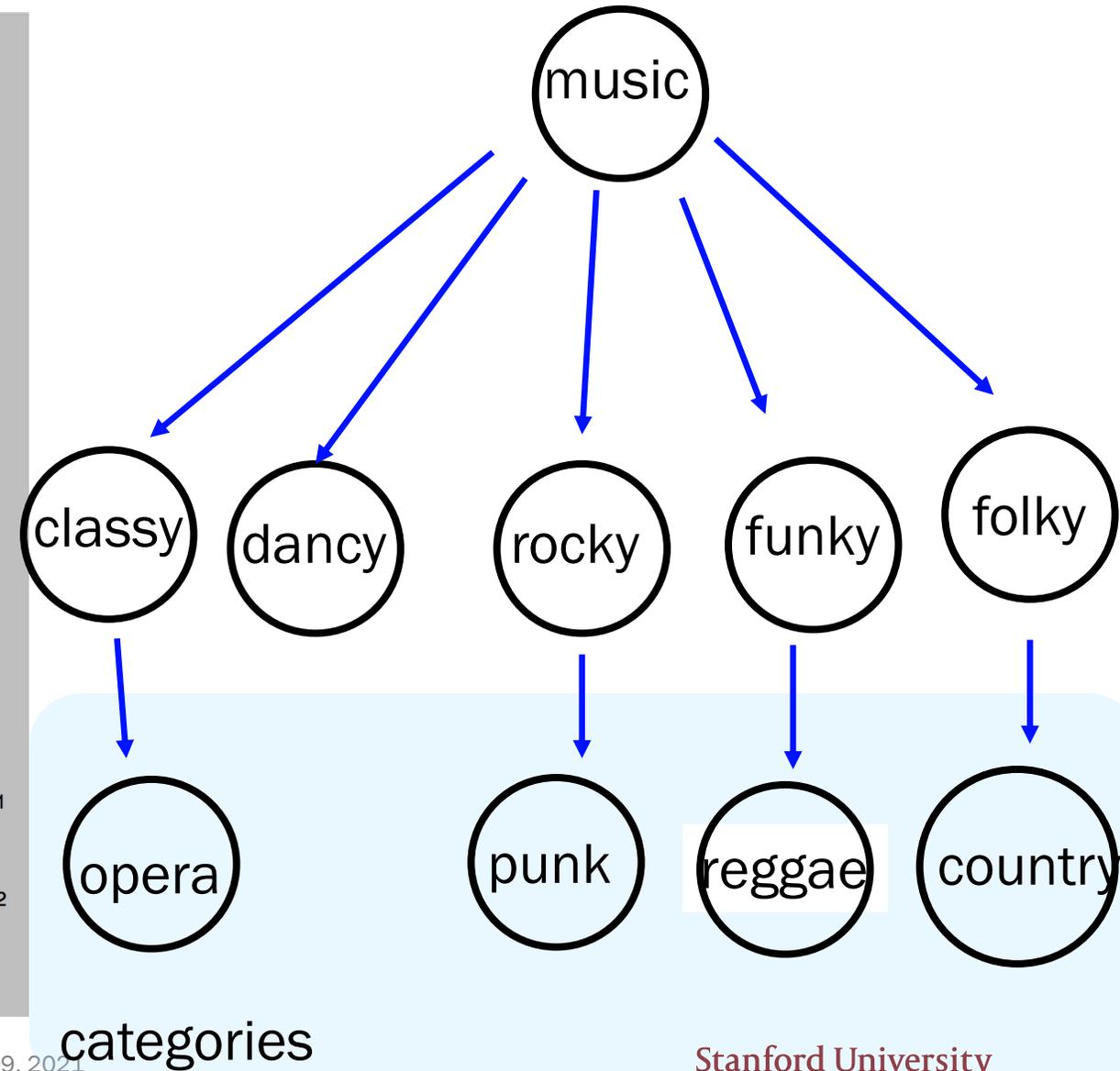
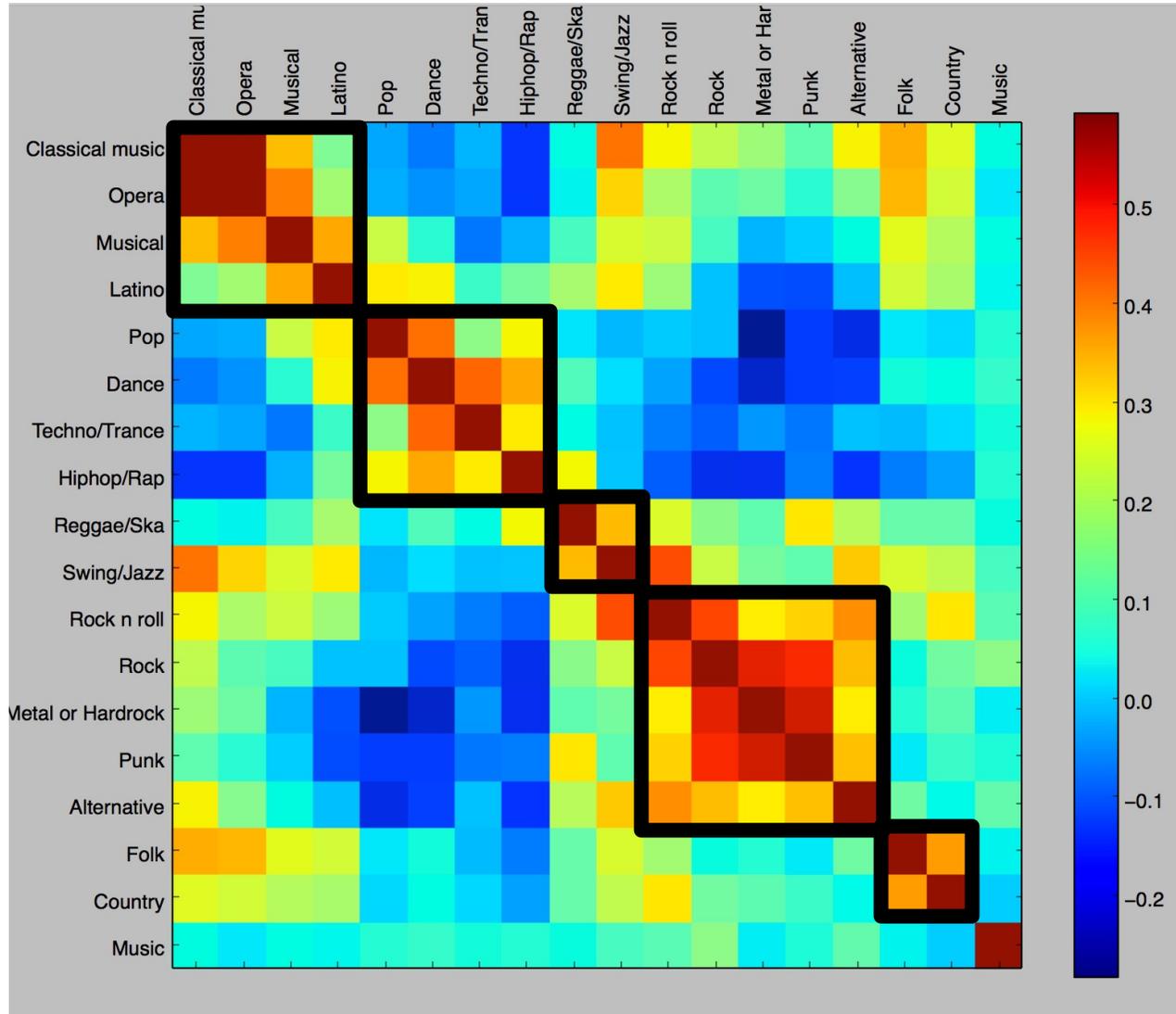
# Correlation of Music Tastes



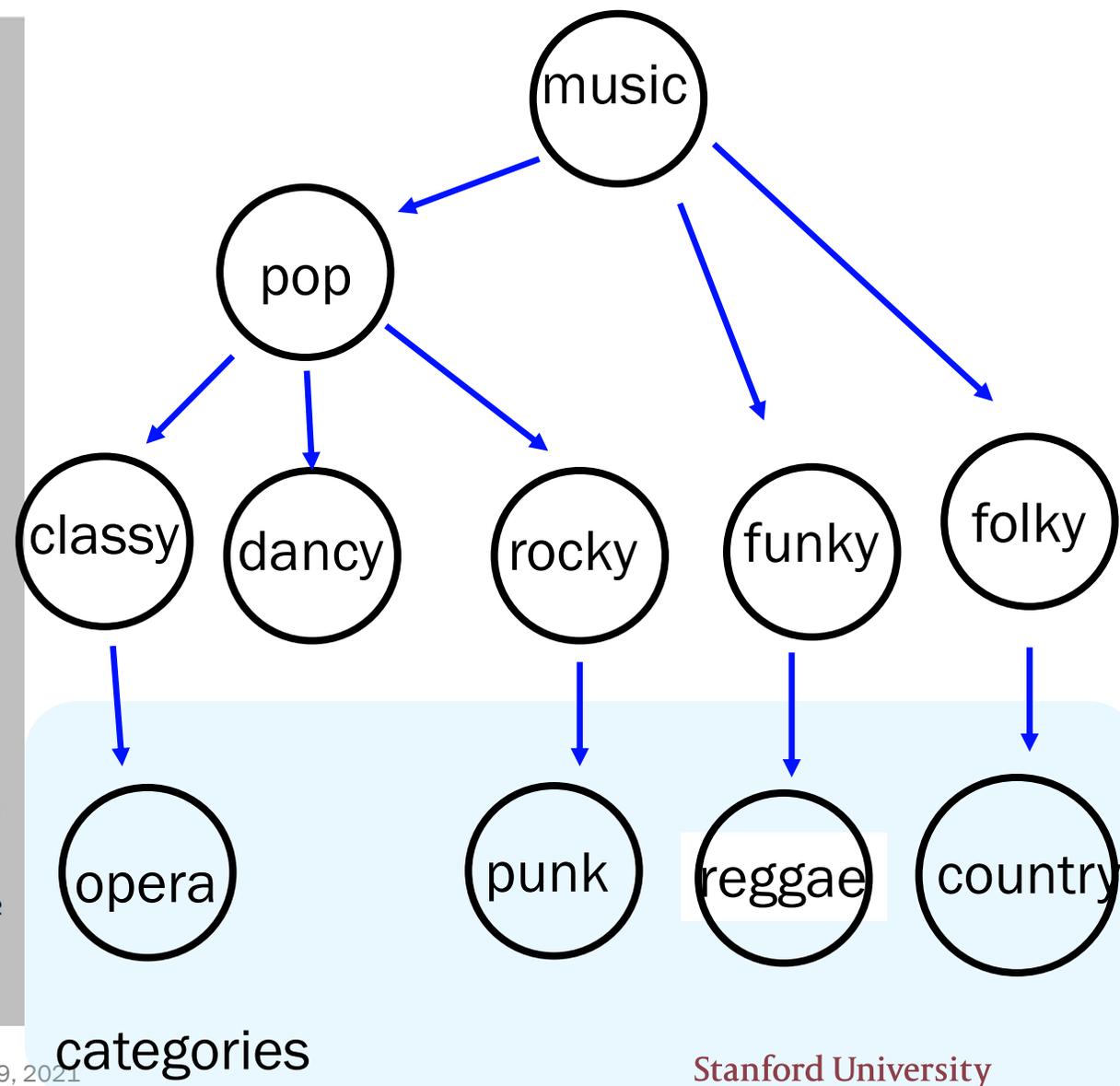
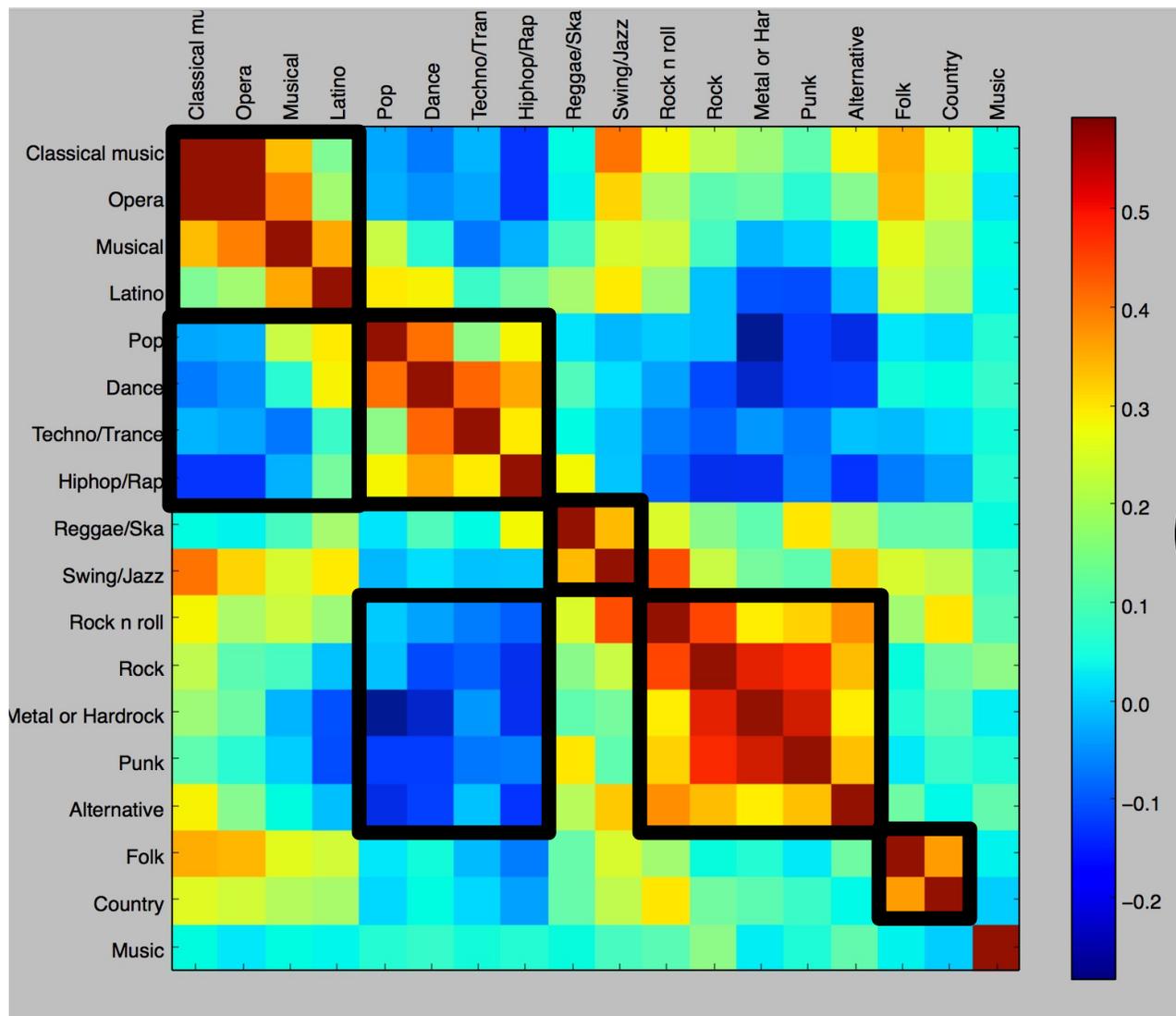
# From Correlation to Bayes Net



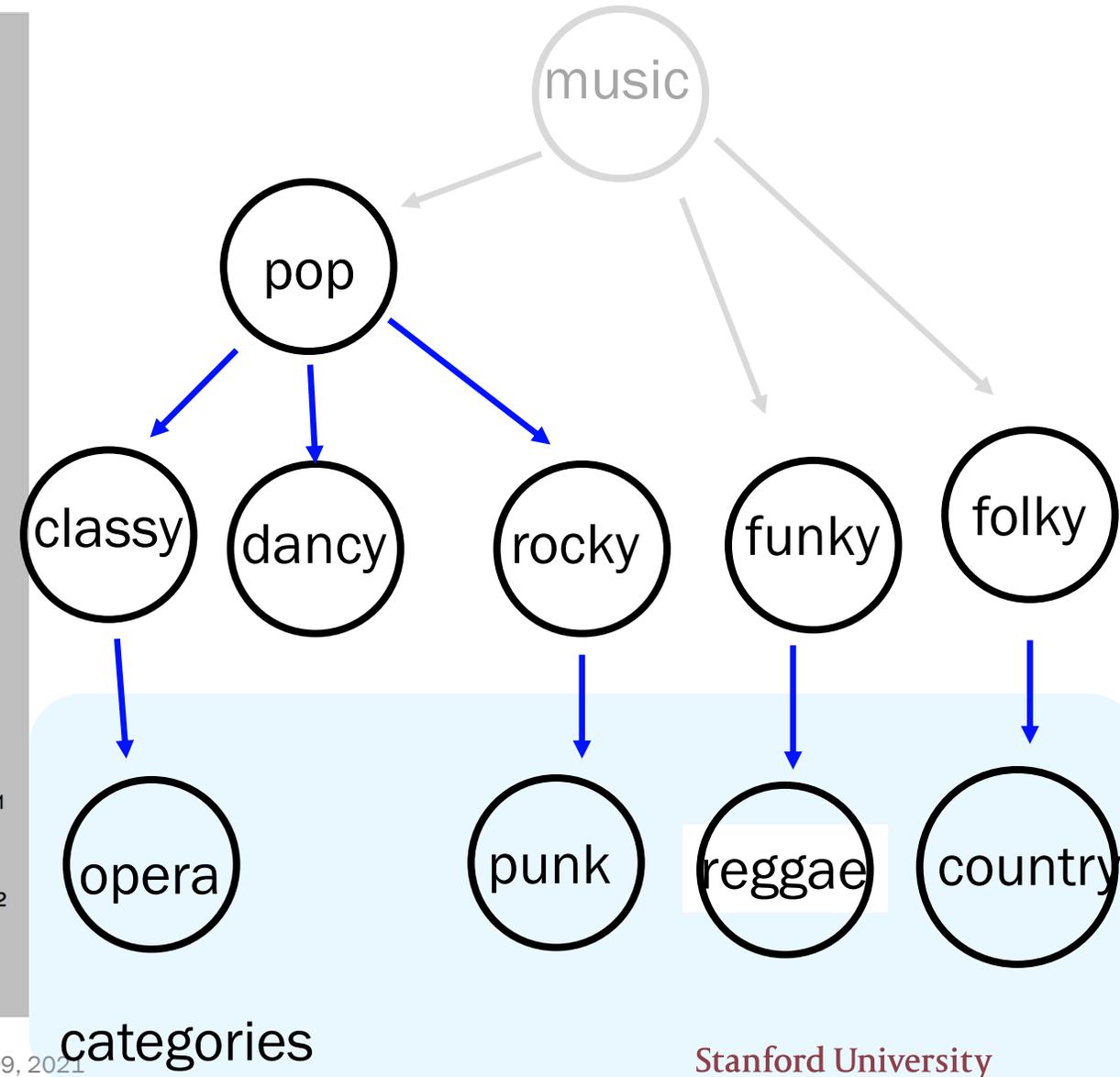
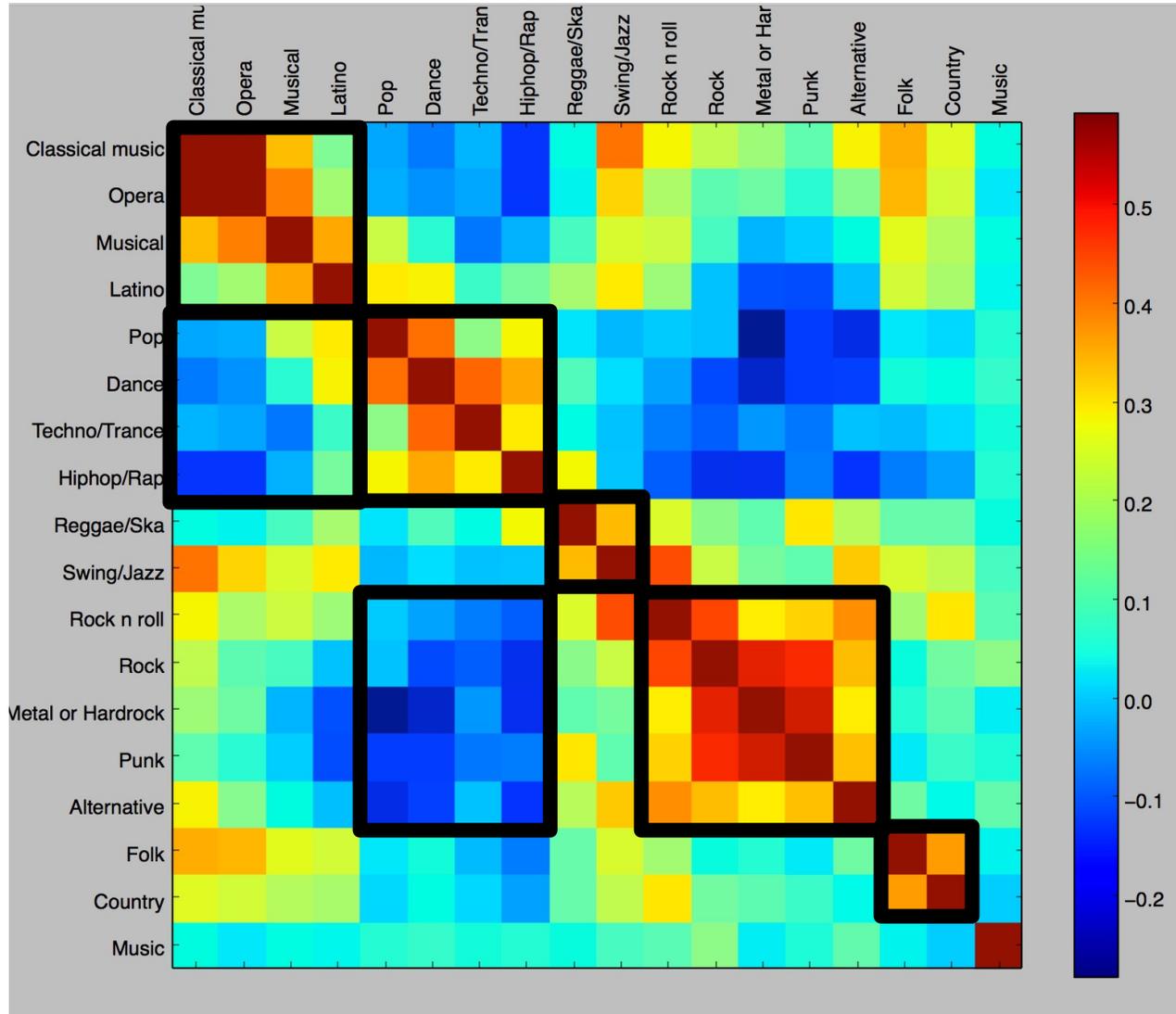
# From Correlation to Bayes Net



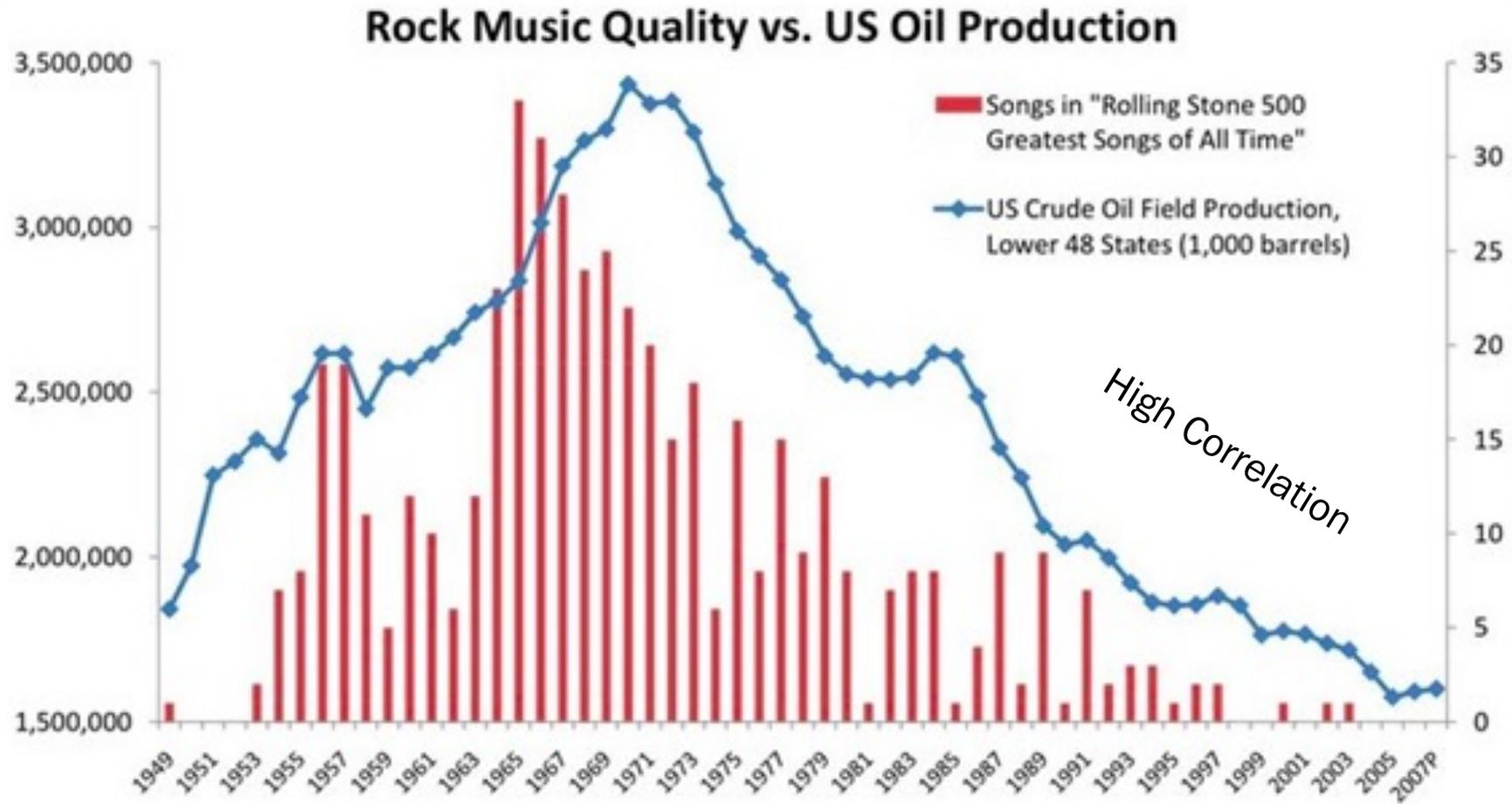
# From Correlation to Bayes Net



# From Correlation to Bayes Net. Alternative!



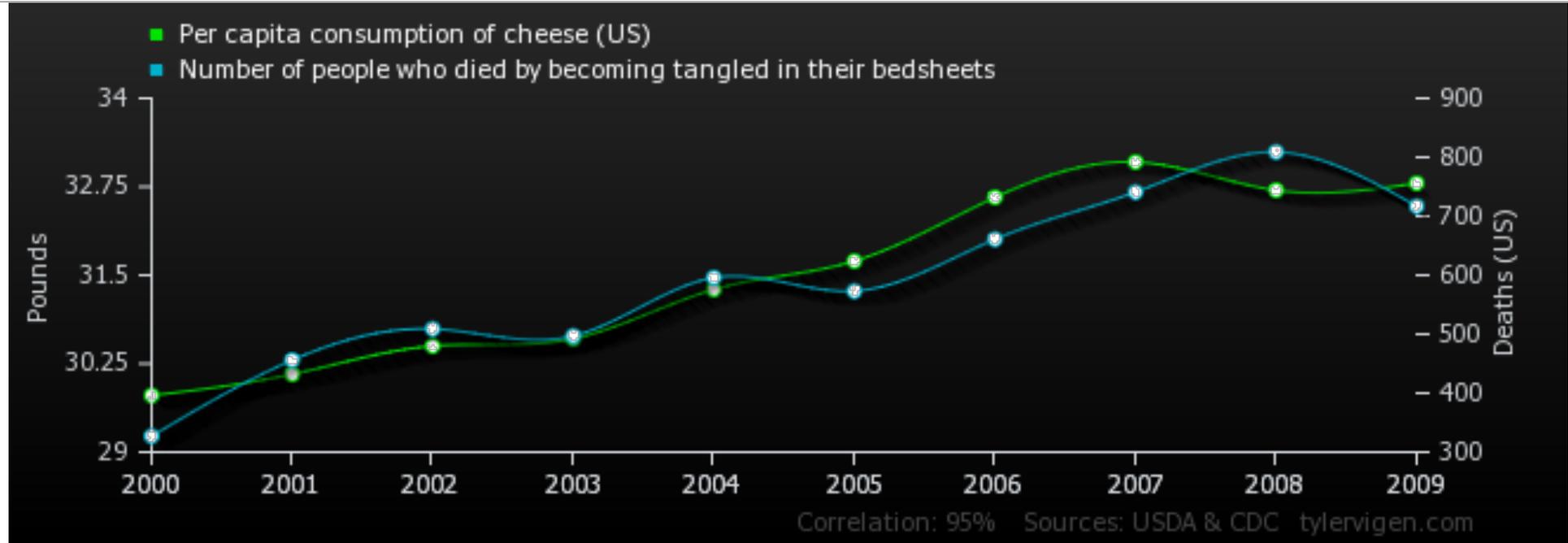
# Rock Music Vs Oil?



Hubbert Peak Theory

<http://www.aei.org/publication/blog/>

# Tell your friends!



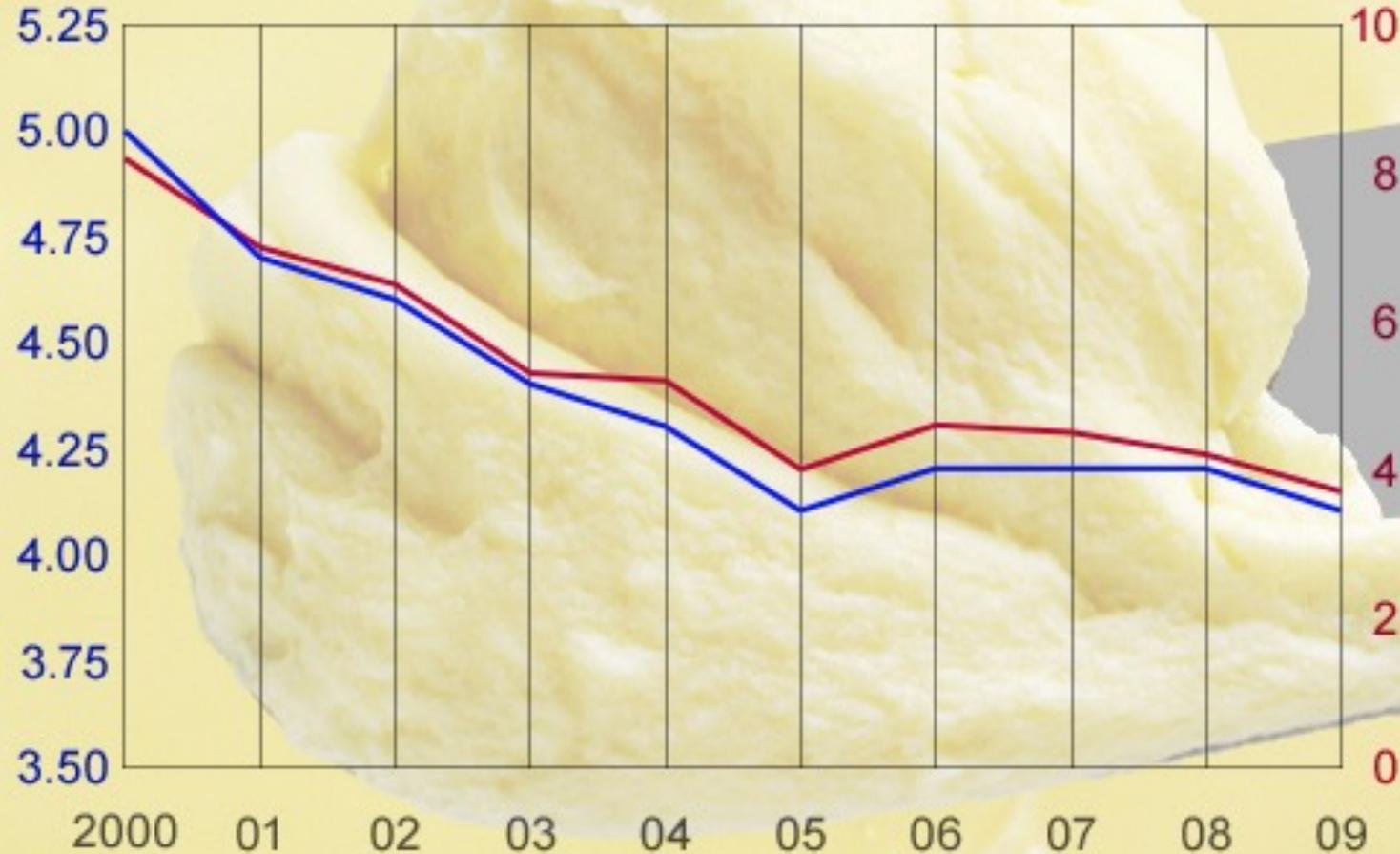
	<u>2000</u>	<u>2001</u>	<u>2002</u>	<u>2003</u>	<u>2004</u>	<u>2005</u>	<u>2006</u>	<u>2007</u>	<u>2008</u>	<u>2009</u>
<i>Per capita consumption of cheese (US) Pounds (USDA)</i>	29.8	30.1	30.5	30.6	31.3	31.7	32.6	33.1	32.7	32.8
<i>Number of people who died by becoming tangled in their bedsheets Deaths (US) (CDC)</i>	327	456	509	497	596	573	661	741	809	717
<b>Correlation: 0.947091</b>										

# Divorce Vs Butter?

Divorce rate  
in Maine per  
1,000 people

Per capita  
consumption of  
margarine (lbs)

Correlation: 99%



Source: US Census, USDA, tylervigen.com

SPL

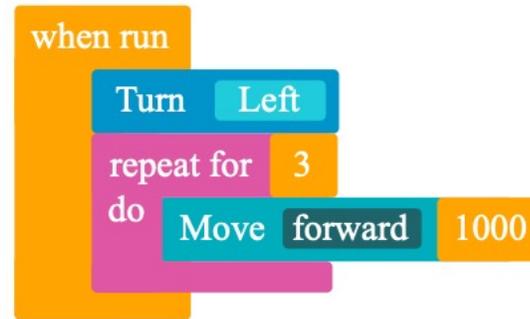
What about Code.org?

# Computers Couldn't Understand Code

60,000 students attempted this problem  
37,000 unique solutions



Challenge

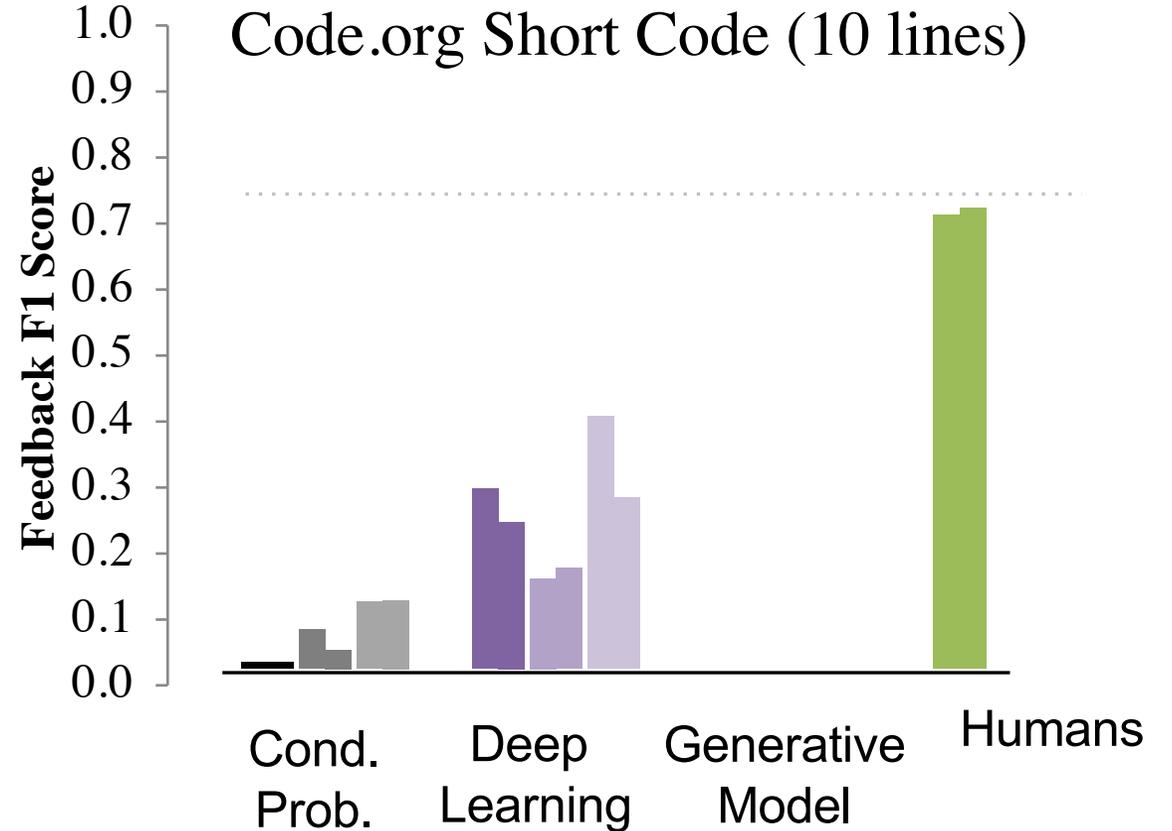


Student Code

You need to  
move and  
turn in your  
loop

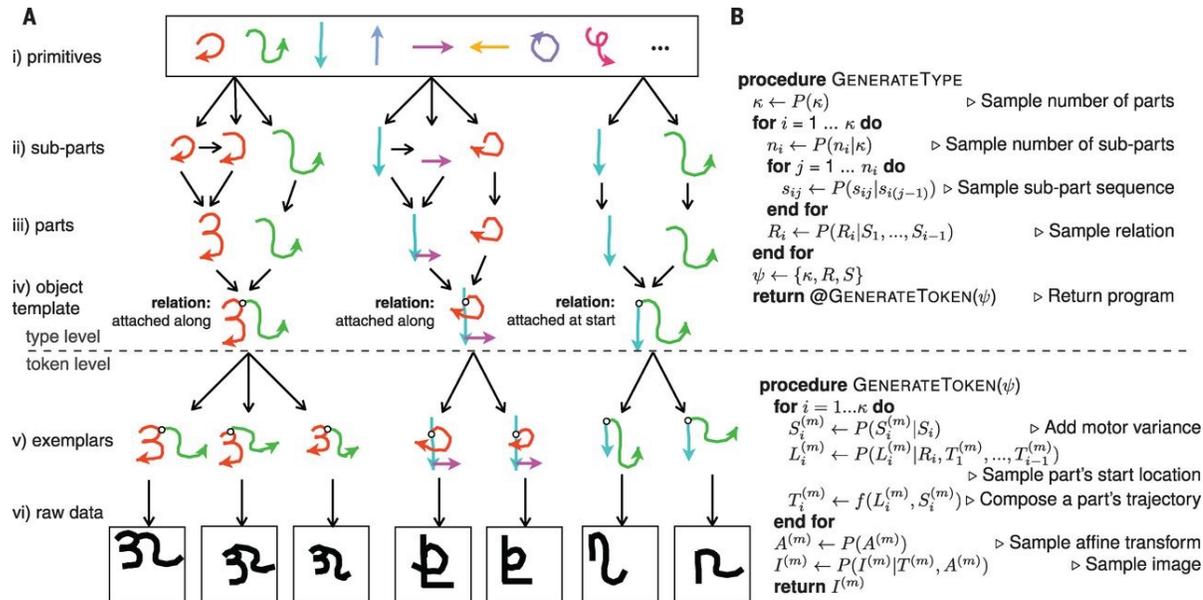
Insight

# Computers Couldn't Understand Code

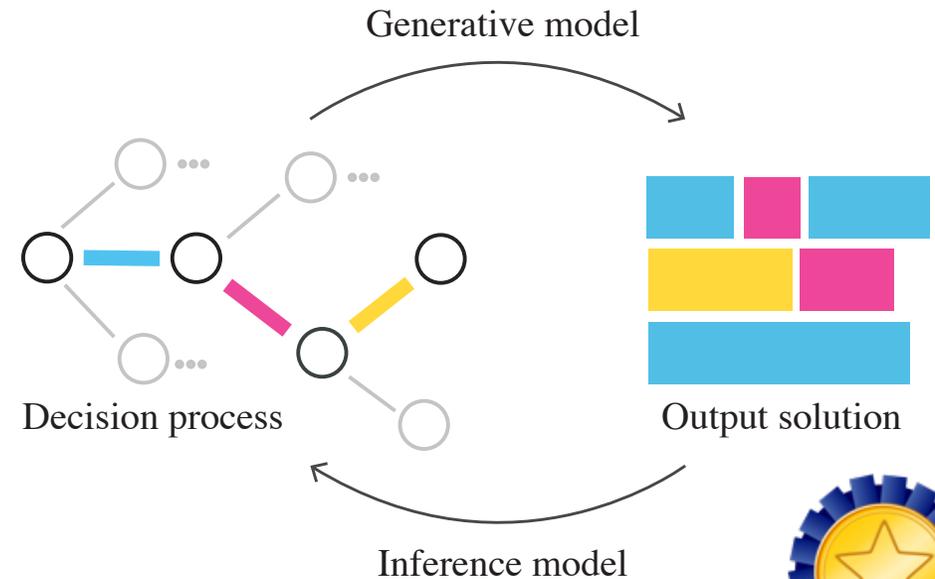


# Generative Model of Grading

Lake et al, 2015

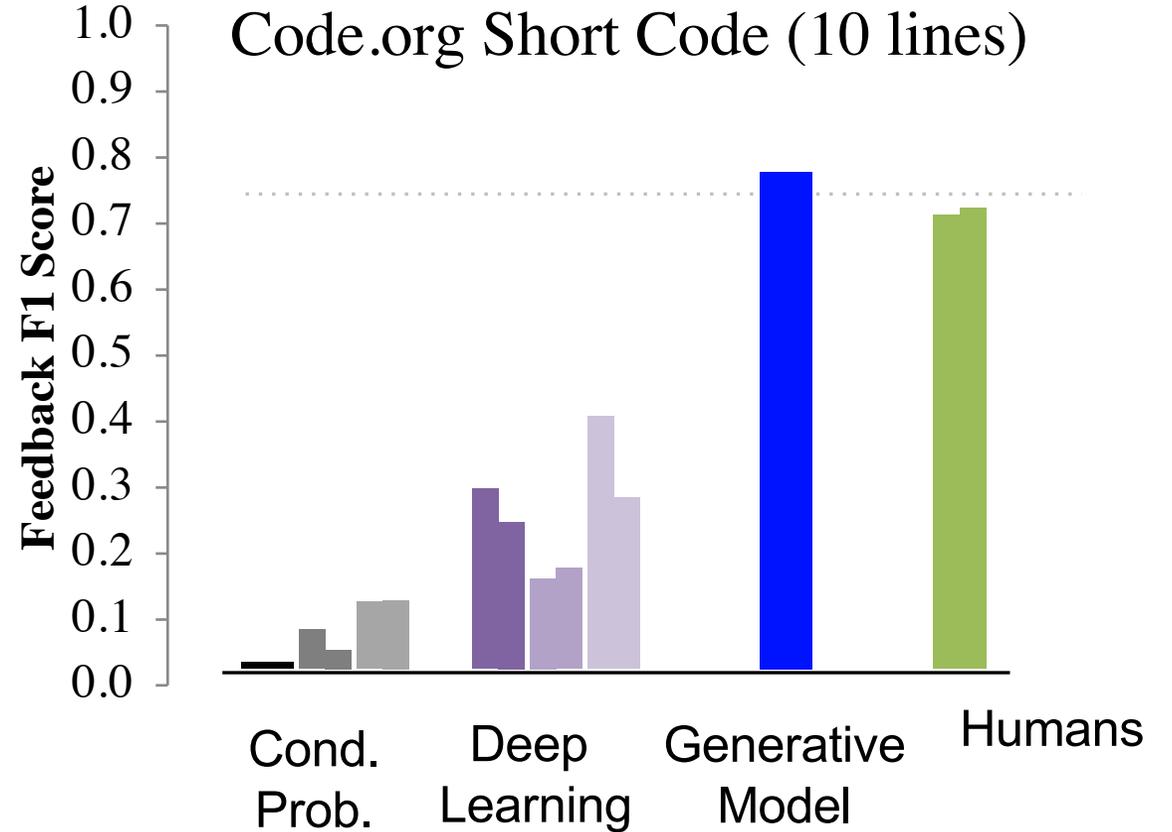


Muke Wu, Ali Malik, Noah Goodman, Chris Piech, 2019

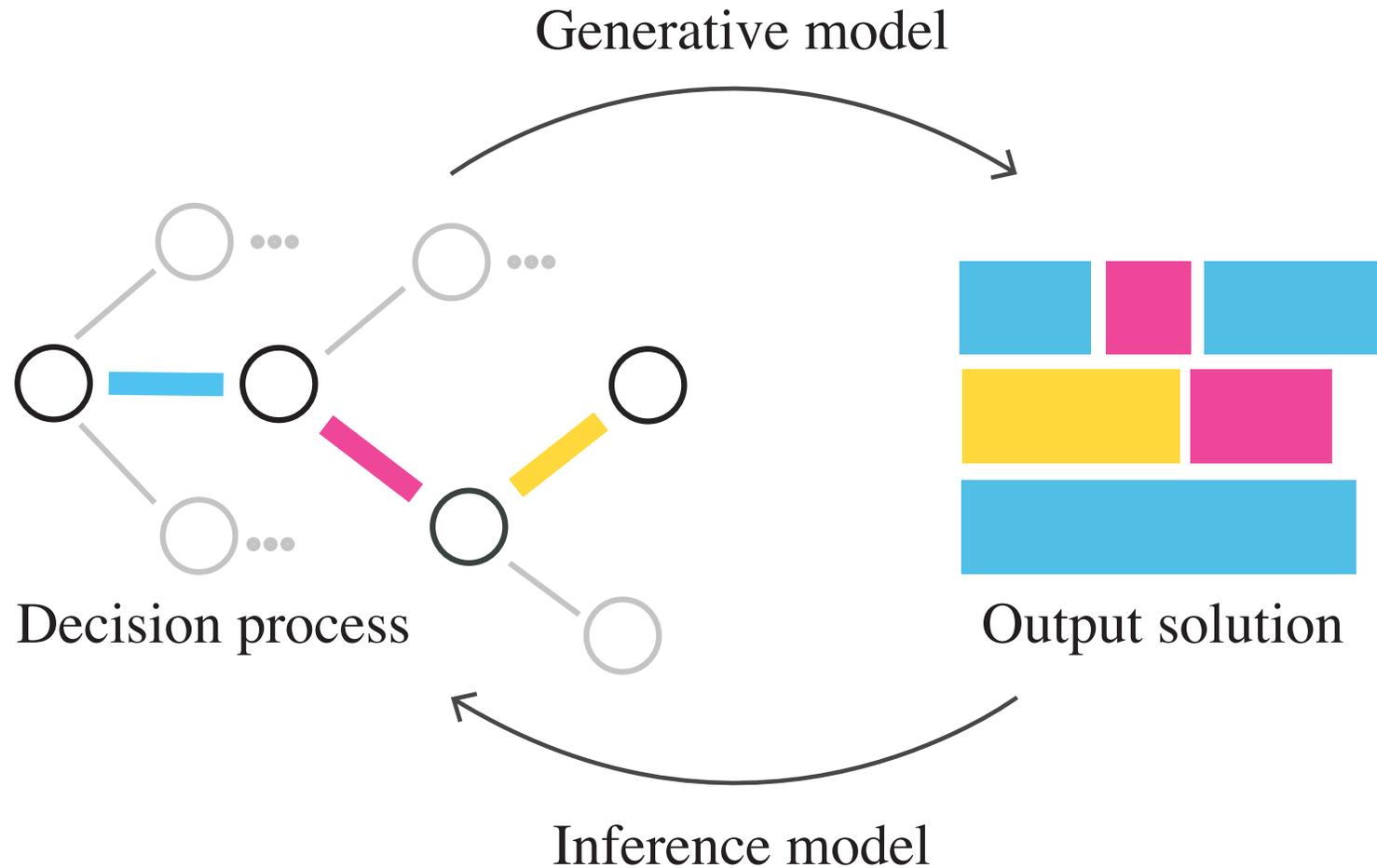


*Outstanding Student  
paper award, AAI 2019*

# Computers Couldn't Understand Code



# Friday: General Inference



See you Friday.

Que te vayas bien