04: Intro to Probability

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Weekend (Week?) Updates
Announcements

• Section starts tomorrow and assignments are out!
• OH start this week – we’re trying something new called QueueMeIn to host our OH queue
• Pset 1 went out last Friday - on-time due date is upcoming Sunday
Quick review of sets

$E$ and $F$ are events in $S$.

Experiment:

Die roll

$S = \{1, 2, 3, 4, 5, 6\}$

Let $E = \{1, 2\}$, and $F = \{2, 3\}$
Quick review of sets

$E$ and $F$ are events in $S$.

Experiment:

Die roll

$S = \{1, 2, 3, 4, 5, 6\}$

Let $E = \{1, 2\}$, and $F = \{2, 3\}$

**def** **Union** of events, $E \cup F$

The event containing all outcomes in $E \text{ or } F$.

$E \cup F = \{1, 2, 3\}$
Quick review of sets

$E$ and $F$ are events in $S$.

Experiment:
Die roll
$S = \{1, 2, 3, 4, 5, 6\}$
Let $E = \{1, 2\}$, and $F = \{2, 3\}$
Let $G = \{5\}$

**def** Intersection of events, $E \cap F$
The event containing all outcomes in $E$ *and* $F$.

**def** Mutually exclusive events $F$ and $G$ means that $F \cap G = \emptyset$

$E \cap F = EF = \{2\}$
Quick review of sets

$E$ and $F$ are events in $S$.

Experiment:
Die roll
$S = \{1, 2, 3, 4, 5, 6\}$
Let $E = \{1, 2\}$, and $F = \{2, 3\}$

• **def** Complement of event $E$, $E^C$
  • The event containing all outcomes in that are **not** in $E$.

$$E^C = \{3, 4, 5, 6\}$$
Roll two 6-sided fair dice. What is $P(\text{sum} = 7)$?

$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$
Roll two dice

Roll two 6-sided fair dice. What is $P(\text{sum} = 7)$?

$$S = \{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \}$$

$$E = \text{in red}$$

$$P(E) = \frac{|E|}{|S|} = \frac{6}{36} = 0.16\overline{6}$$
Roll two dice

Roll two 6-sided fair dice. What is P(sum = 7)?

\[ S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \]

\[ E = \text{in red} \]

\[ P(E) = \frac{|E|}{|S|} = \frac{1}{11} = 0.083 \]
Roll two indistinct dice

Roll two 6-sided fair dice. What is $P(\text{sum} = 7)$?

$S = \{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 3), (3, 4), (3, 5), (3, 6),
(4, 4), (4, 5), (4, 6),
(5, 5), (5, 6),
(6, 6)\}$

$E = \text{in red}$

$P(E) = \frac{|E|}{|S|} = \frac{3}{21} = 0.143$
Cats and sharks

We have 4 cats and 3 sharks in a bag. We draw 3 items from the bag. What is \( P(1 \text{ cat and 2 sharks drawn}) \)?
Make indistinct items distinct to get equally likely sample spaces outcomes.
Poker Straights and Computer Chips

1. Consider equally likely 5-card poker hands.
   Define "poker straight" as 5 consecutive rank cards of any suit
   What is \( P(\text{Poker straight}) \)?

2. Computer chips: \( n \) chips are manufactured, 1 of which is defective. \( k \) chips are randomly selected from \( n \) for testing.
   What is \( P(\text{defective chip is in } k \text{ selected chips?}) \)?
Serendipity

Let it find you.

SERENDIPITY
the effect by which one accidentally stumbles upon something truely wonderful, especially while looking for something entirely unrelated.

WHEN YOU MEET YOUR BEST FRIEND
Somewhere you didn't expect to.
Serendipity

• The population of Stanford is $n = 17,000$ people.
• You are friends with $r = 54$ people.
• Walk into a room, see $k = 250$ random people.
• Assume each group of $k$ Stanford people is equally likely to be in the room.

What is the probability that you see someone you know in the room?

http://web.stanford.edu/class/cs109/demos/serendipity.html
Serendipity

• The population of Stanford is \( n = 17,000 \) people.
• You are friends with \( r = 54 \) people.
• Walk into a room, see \( k = 250 \) random people.
• Assume each group of \( k \) Stanford people is equally likely to be in the room.

What is the probability that you see at least one friend in the room?

Define

• \( S \) (unordered)
• \( E: \geq 1 \) friend in the room

What strategy would you use?

A. \( P(\text{exactly } 1) + P(\text{exactly } 2) \\
P(\text{exactly } 3) + \cdots \)

B. \( 1 - P(\text{see no friends}) \)
Many times, it is easier to compute $P(E^C)$. 