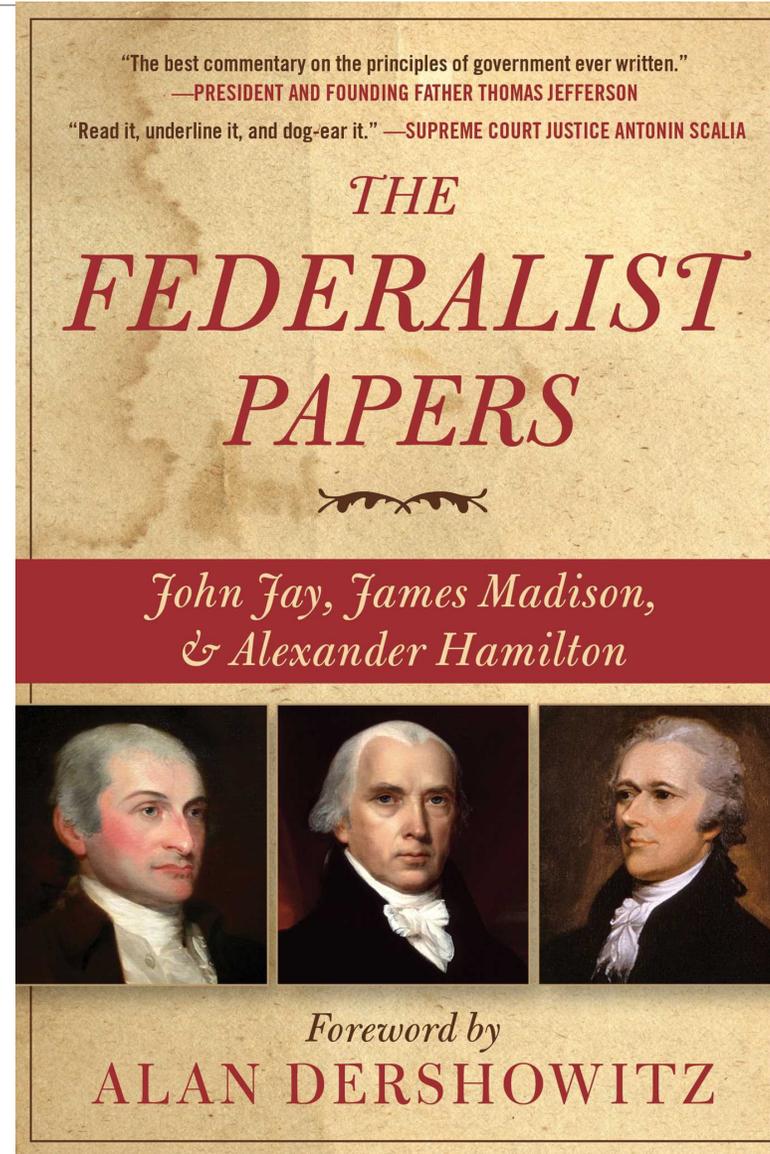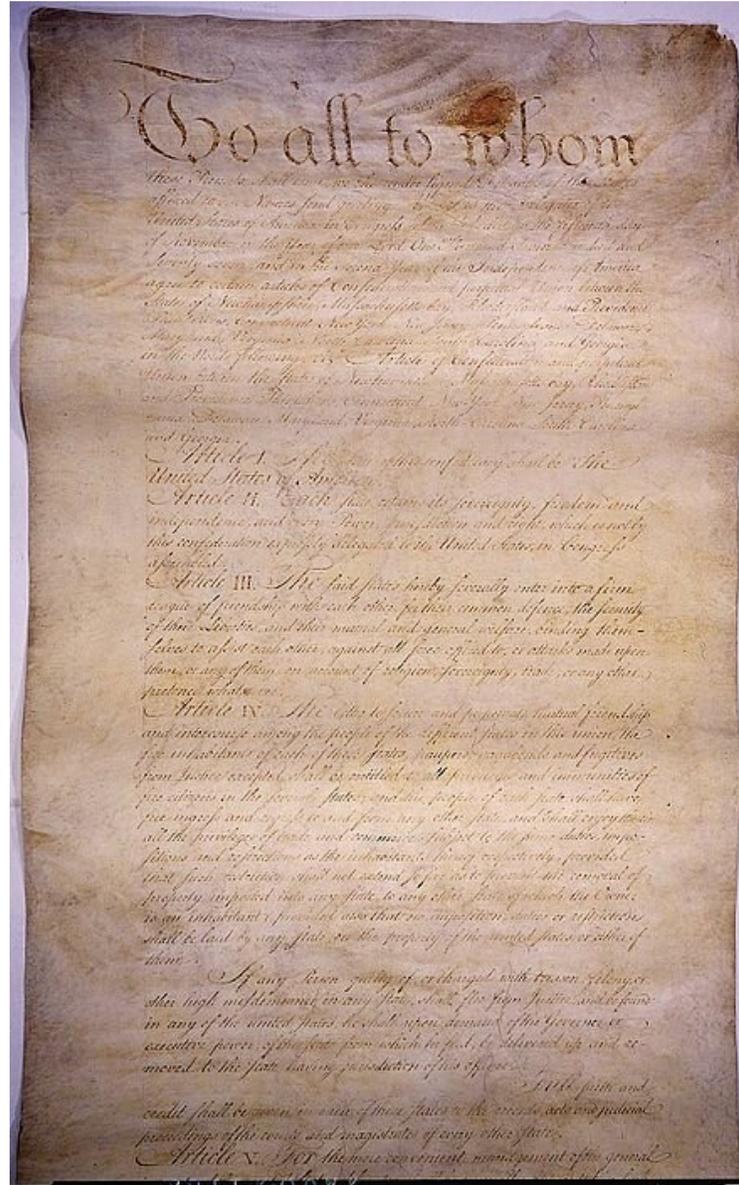# Probabilistic Models

Chris Piech
CS109, Stanford University

Exciting Day!

First, some review

Where are we in CS109?

Overview of Topics

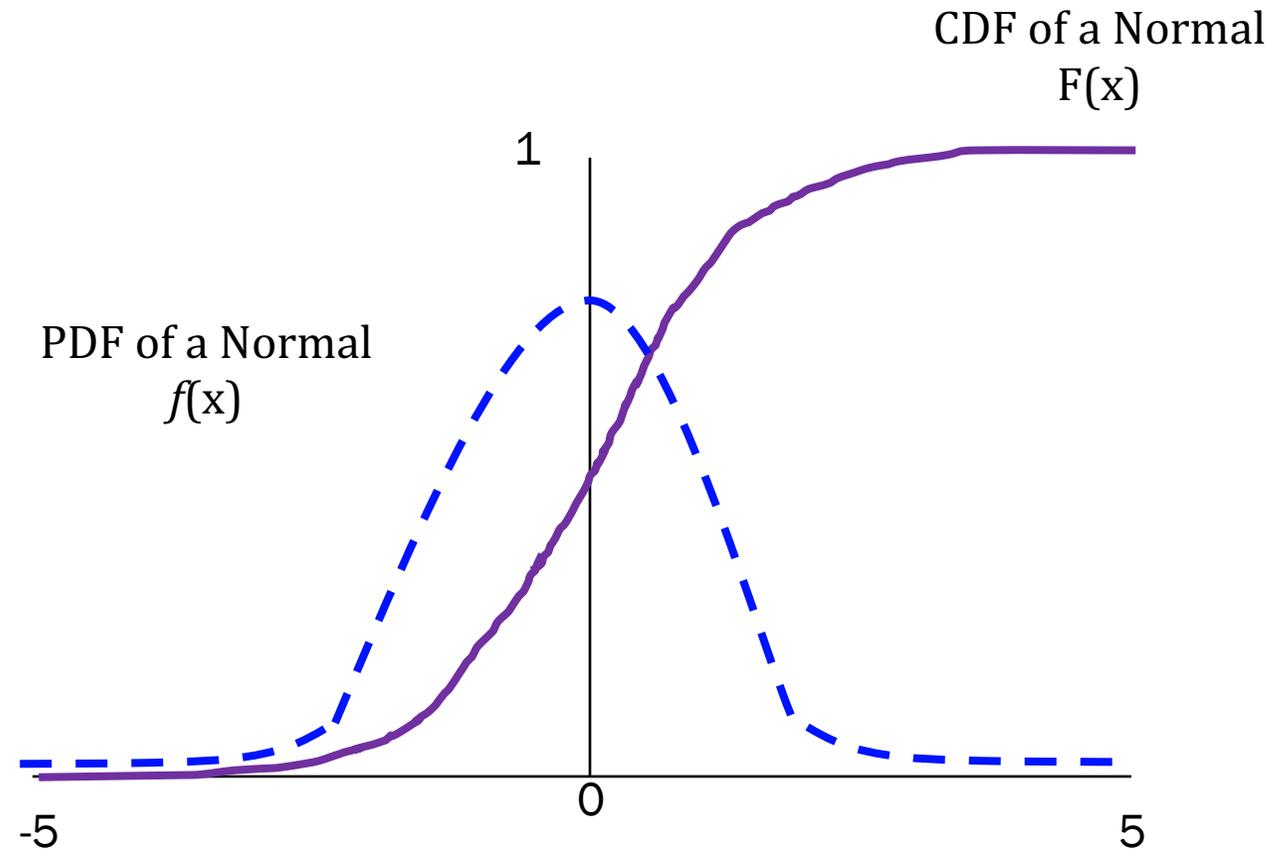Counting Theory — Core Probability — Random Variables — Probabilistic Models — Uncertainty Theory — Machine Learning

Normal Distribution

PDF of a Normal
f(x)

CDF of a Normal
F(x)

1

0

-5

5

f(x) = derivative of probability

F(x) = P(X < x)

Cumulative Density Function

$$\mathcal{N}(\mu, \sigma^2)$$

CDF of Standard Normal: A function that has been solved for numerically

$$F(x) = \Phi\left(\frac{x - \mu}{\sigma}\right)$$

The cumulative density function (CDF) of any normal

Table of $\Phi(Z)$ values in textbook, p. 201 and handout

Practice

Enough Servers?

You are running a website, Alibayes

You want to buy computers based on usage at the busiest minute

You receive $R \sim N(\mu = 10^6, \sigma = 10^4)$ requests at the busiest minute.

You are going to buy N servers

Each server can handle 10,000 requests per min, otherwise you drop requests

What is the smallest value of N such that P(No Drops) > 0.9999

/Practice

Probability Density Function

$$\mathcal{N}(\mu, \sigma^2)$$

"exponential"

the distance to the mean

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

probability density at x

a constant

sigma shows up twice

Does it look less scary like this?

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

This means "e to the power of" and is common function in code math libraries

$$f(x) \propto \frac{1}{\sigma} \cdot \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$$

This means "proportional to". There is a constant but there are many cases where we don't care what it is!

What if you had to take the log of this function?

Log Review

$$e^y = x \qquad \log(x) = y$$

Graph for log(x)

x: 0.394607801   y: -0.403834334

More info

anford University
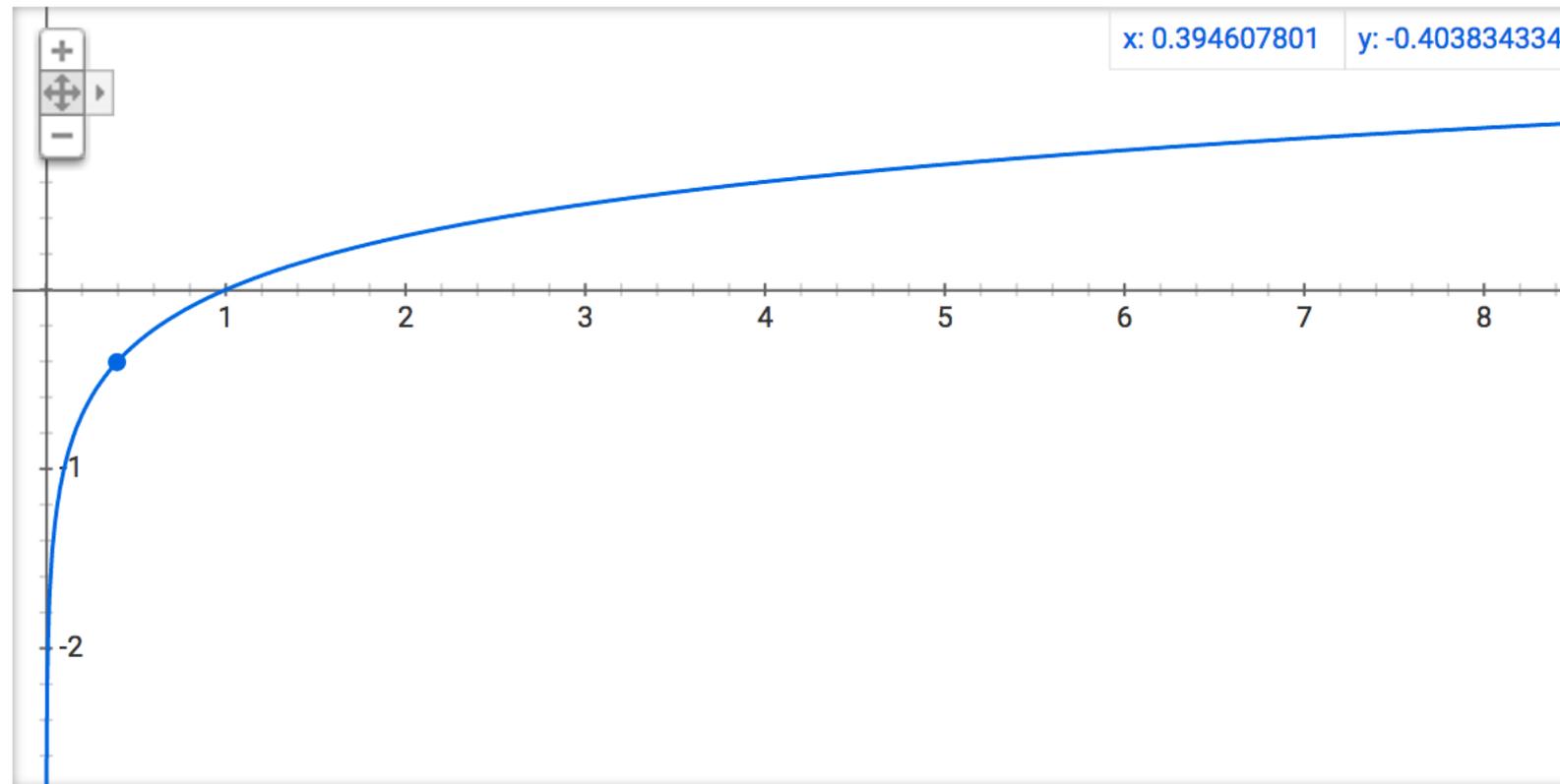
Log Identities

$$\log(a \cdot b) = \log(a) + \log(b)$$

$$\log(a/b) = \log(a) - \log(b)$$
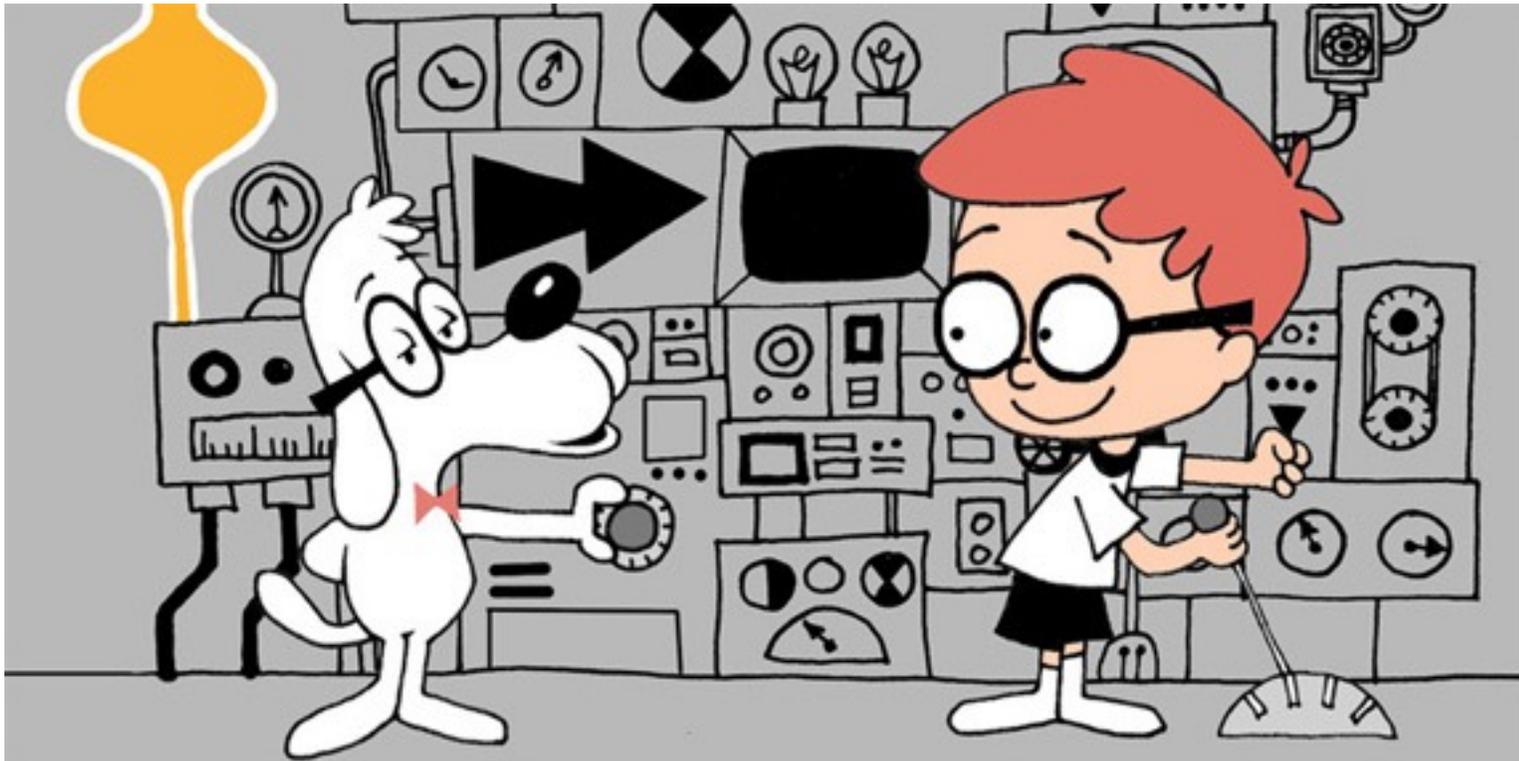
$$\log(a^n) = n \cdot \log(a)$$

Products become sums!

$$\log(a \cdot b) = \log(a) + \log(b)$$

$$\log\left(\prod_i a_i\right) = \sum_i \log(a_i)$$

* Spoiler alert: This is important because the product of many small numbers gets hard for computers to represent.

Recall the good times

Permutations
$$n!$$
How many ways are there to order n objects?

Ways to put elements into fixed size containers

How many ways are there to put n objects into r buckets such that:

n_1 go into bucket 1

n_2 go into bucket 2

...

n_r go into bucket r?

$$\frac{n!}{n_1! n_2! \ldots n_r!} = \binom{n}{n_1, n_2, \ldots, n_r}$$

Note: Multinomial > Binomial

Counting unordered objects

Binomial coefficient

How many ways are there
to order n objects such
that k are indistinct and
(n-k) are indistinct

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Called the binomial coefficient
because of something from Algebra

Multinomial coefficient

How many ways are there
to order n objects such that n_1
are indistinct, n_2 are indistinct
etc.

$$\binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1!\, n_2! \cdots n_r!}$$

Multinomials generalize
Binomials for counting.

End of review

[suspense]

Discrete Probabilistic Models

The world is full of interesting probability problems

Have multiple random variables interacting with one another

Multiple Random Variables. Start of Digital Revolution

Multiple Random Variables. Start of Digital Revolution

Conditions that match your symptoms

UNDERSTANDING YOUR RESULTS ⓘ

Migraine headache (adult) ›

Moderate match

Acute Sinusitis ›

Fair match

Stroke ›

Fair match

Gender **Male**   Age **30**   Edit

My Symptoms   Edit

dizziness, one sided headache

Stanford University

Joint probability mass functions

Roll two 6-sided dice, yielding values X and Y.

$$X$$
random variable

$$P(X = 1)$$
probability of
an event
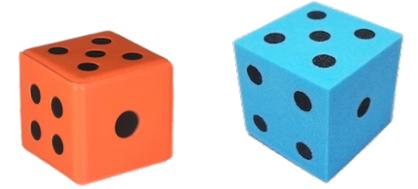
$$P(X = k)$$
probability mass function

Joint probability mass functions

Roll two 6-sided dice, yielding values X and Y.

X
random variable

$P(X = 1)$
probability of an event
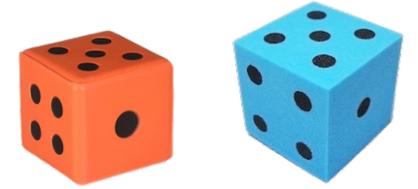
$P(X = k)$
probability mass function

X, Y
random variables

$P(X = 1 \text{ and } Y = 6)$

$P(X = 1, Y = 6)$

recall: the comma

probability of the intersection of two events

$P(X = a, Y = b)$

joint probability mass function

Two dice

Roll two 6-sided dice, yielding values X and Y.

1. What is the joint PMF of X and Y?
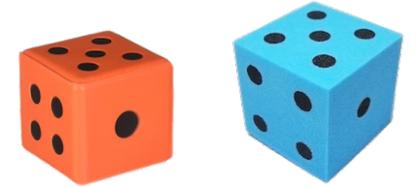
$$P(X = a, Y = b) = 1/36 \qquad (a,b) \in \{(1,1), \dots, (6,6)\}$$

		1	2	3	4	5	6
				X			
	1	1/36	…	…	…	…	1/36
	2	…	…	…	…		
Y	3	…	…	…	…	…	…
	4	…	…	…	…	…	…
	5	…	…	…	…	…	…
	6	1/36	…	…	…	…	1/36

$P(X = 4, Y = 2)$

Probability table
- All possible outcomes for several discrete RVs
- Not parametric (e.g., parameter p in Ber(p))

Dating at Stanford. Data from a few years ago

	Single	In a relationship	It's complicated
Freshman	0.13	0.08	0.02
Sophomore	0.17	0.11	0.02
Junior	0.09	0.10	0.02
Senior	0.02	0.07	0.01
5+	0.06	0.09	0.04

Joint is Complete Information!

	Single	Relationship	Complicated
Frosh	0.13	0.08	0.02
Soph	0.17	0.11	0.02
Junior	0.09	0.10	0.02
Senior	0.02	0.07	0.01
5+	0.06	0.09	0.04

A joint distribution is complete information. It can be used to answer any probability question.

Joint table: mutually exclusive and covers sample space.

	Single	Relationship	Complicated
Frosh	0.13	0.08	0.02
Soph	0.17	0.11	0.02
Junior	0.09	0.10	0.02
Senior	0.02	0.07	0.01
5+	0.06	0.09	0.04

Each combination is mutually exclusive, and they span the sample space

$$\sum_{x \in X} \sum_{y \in Y} P(x, y) = 1$$

X is dating status.
Y is year.

Joint table: mutually exclusive and covers sample space.

	Single	Relationship	Complicated
Frosh	0.13	0.08	0.02
Soph	0.17	0.11	0.02
Junior	0.09	?	0.02
Senior	0.02	0.07	0.01
5+	0.06	0.09	0.04

Each combination is mutually exclusive, and they span the sample space

$$\sum_{x \in X} \sum_{y \in Y} P(x, y) = 1$$

X is dating status.
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Joint table: mutually exclusive and covers sample space.

	Single	Relationship	Complicated
Frosh	0.13	0.08	0.02
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Senior	0.02	0.07	0.01
5+	0.06	0.09	0.04

Each combination is mutually exclusive, and they span the sample space

$$\sum_{x \in X} \sum_{y \in Y} P(x, y) = 1$$

X is dating status.
Y is year.

What is the probability someone is in a relationship?

	Single	Relationship	Complicated
Frosh	0.13	0.08	0.02
Soph	0.17	0.11	0.02
Junior	0.09	0.10	0.02
Senior	0.02	0.07	0.01
5+	0.06	0.09	0.04

We can use the law of total probability!
X is dating status. Y is year.

$$P(X = \text{relation}) =$$
$$\sum_{y \in Y} P(X = \text{relation}, Y = y)$$

$$P(X = \text{single}) =$$
$$\sum_{y \in Y} P(X = \text{single}, Y = y)$$

$$P(Y = \text{frosh}) = \sum_{x \in X} P(X = x, Y = \text{frosh}) \qquad P(Y = \text{soph}) = \sum_{x \in X} P(X = x, Y = \text{soph})$$

Welcome the marginal

Marginal Distribution

For two discrete joint random variables X and Y,
the joint probability mass function is defined as:

$$P(X = a, Y = b)$$

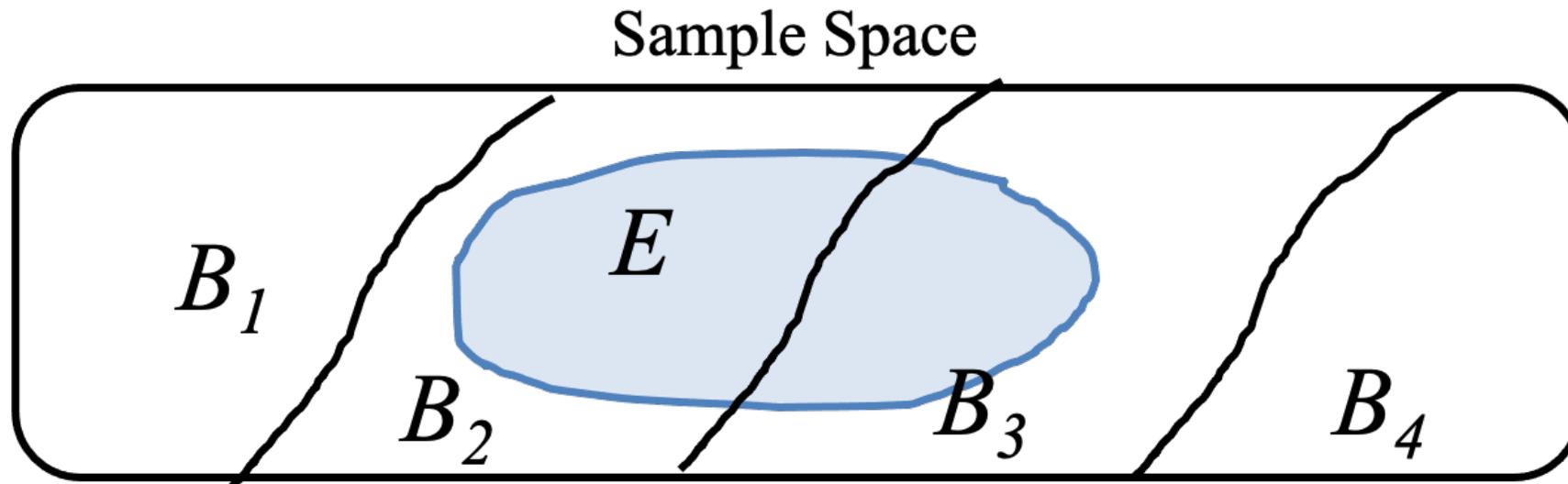The marginal distributions of the joint PMF are defined as:

$$P(X = a) = \sum_y P(X = a, Y = y)$$

$$P(Y = b) = \sum_x P(X = x, Y = b)$$

Use marginal distributions to get a 1-D RV from a joint PMF.

Marginal Distribution. Law of Total Probability for RVs

$$P(X = a) = \sum_y P(X = a, Y = y)$$

Sample Space

E

B_1

B_2

B_3

B_4

Why is that called the marginal?

Key limitation of the joint: it is too big

What about 3 Random Variables?

D is disease, S is can smell, F is fever status

$D = 0$

	$S = 0$	$S = 1$
$F = \text{none}$	0.024	0.783
$F = \text{low}$	0.003	0.092
$F = \text{high}$	0.001	0.046

$D = 1$

	$S = 0$	$S = 1$
$F = \text{none}$	0.006	0.014
$F = \text{low}$	0.005	0.011
$F = \text{high}$	0.004	0.011

$$P(D = 1) = \sum_{f} \sum_{s} P(D = 1, F = f, S = s)$$

What about 10 Random Variables?

Imagine you have **10 discrete** RVs which can each take on **5 values**

$$\text{\# unique assignments} = 5^{10}$$

10 million entries in your joint table.

So, we are going to need models …

… **probabilistic models** …

Sometimes the structure of the variables suggests a more efficient representation

Roll 100 dice.
X_1 = How many 1s?
X_2 = How many 2s?
X_3 = How many 3s?
X_4 = How many 4s?
X_5 = How many 5s?
X_6 = How many 6s?

How big is the joint table?

Multinomial RV

Probability

Binomial RV

What is the probability
of getting k successes
and $n - k$ failures
in n trials?

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Binomial # of ways of
ordering the successes

Probability of each ordering
of k successes is equal +
mutually exclusive

Multinomial RV

What is the probability of
getting c_1 of outcome 1,
c_2 of outcome 2, ..., and
c_m of outcome m
in n trials?

Multinomial RVs also generalize
Binomial RVs for probability!

Multinomial Random Variable?

Consider an experiment of n independent trials:
- Each trial results in one of m outcomes. $P(\text{outcome } i) = p_i,\ \sum_{i=1}^{m} p_i = 1$
- Let X_i = # trials with outcome i

Joint PMF

$$P(X_1 = c_1, X_2 = c_2, \ldots, X_m = c_m) = \qquad p_1^{c_1} p_2^{c_2} \cdots p_m^{c_m}$$

where $\displaystyle\sum_{i=1}^{m} c_i = n$ and $\displaystyle\sum_{i=1}^{m} p_i = 1$

Probability of each ordering is equal + mutually exclusive

Multinomial Random Variable

Consider an experiment of n independent trials:
- Each trial results in one of m outcomes. $P(\text{outcome } i) = p_i$, $\sum_{i=1}^{m} p_i = 1$
- Let X_i = # trials with outcome i

Joint PMF

$$P(X_1 = c_1, X_2 = c_2, \ldots, X_m = c_m) = \binom{n}{c_1, c_2, \ldots, c_m} p_1^{c_1} p_2^{c_2} \cdots p_m^{c_m}$$

where $\sum_{i=1}^{m} c_i = n$ and $\sum_{i=1}^{m} p_i = 1$

Multinomial # of ways of ordering the outcomes

Probability of each ordering is equal + mutually exclusive

Hello dice rolls, my old friends

A 6-sided die is rolled 7 times.

What is the probability of getting:

- 1 one
- 1 two
- 0 threes
- 2 fours
- 0 fives
- 3 sixes

Hello dice rolls, my old friends

A 6-sided die is rolled 7 times.

What is the probability of getting:

- 1 one
- 1 two
- 0 threes
- 2 fours
- 0 fives
- 3 sixes

$$P(X_1 = 1, X_2 = 1, X_3 = 0, X_4 = 2, X_5 = 0, X_6 = 3)$$

$$= \binom{7}{1,1,0,2,0,3} \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^0 \left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^0 \left(\frac{1}{6}\right)^3 = 420 \left(\frac{1}{6}\right)^7$$

Hello dice rolls, my old friends

A 6-sided die is rolled 7 times.

What is the probability of getting:

- 1 one
- 1 two
- 0 threes
- 2 fours
- 0 fives
- 3 sixes

# of times
a six appears

$$P(X_1 = 1, X_2 = 1, X_3 = 0, X_4 = 2, X_5 = 0, X_6 = 3)$$

$$= \binom{7}{1,1,0,2,0,3}\left(\frac{1}{6}\right)^1\left(\frac{1}{6}\right)^1\left(\frac{1}{6}\right)^0\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)^0\left(\frac{1}{6}\right)^3 = 420\left(\frac{1}{6}\right)^7$$

choose where
the sixes appear

probability
of rolling a six

this many times

Multinomial Random Variable

Consider an experiment of n independent trials:
- Each trial results in one of m outcomes. $P(\text{outcome } i) = p_i$, $\sum_{i=1}^{m} p_i = 1$
- Let X_i = # trials with outcome i

Joint PMF

$$P(X_1 = c_1, X_2 = c_2, \ldots, X_m = c_m) = \binom{n}{c_1, c_2, \ldots, c_m} p_1^{c_1} p_2^{c_2} \cdots p_m^{c_m}$$

where $\sum_{i=1}^{m} c_i = n$ and $\sum_{i=1}^{m} p_i = 1$

Example:
- Rolling 2 twos, 3 threes, and 5 fives on 10 rolls of a fair-sided die
- Generating a random 5-word phrase with 1 "the", 2 "bacon", 1 "put", 1 "on"

Hello dice rolls, my old friends

A 6-sided die is rolled 7 times.
What is the probability of getting:

- 1 one
- 1 two
- 0 threes
- 2 fours
- 0 fives
- 3 sixes

# of times
a six appears

$$P(X_1 = 1, X_2 = 1, X_3 = 0, X_4 = 2, X_5 = 0, X_6 = 3)$$

$$= \binom{7}{1,1,0,2,0,3} \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^0 \left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^0 \left(\frac{1}{6}\right)^3 = 420 \left(\frac{1}{6}\right)^7$$

choose where
the sixes appear

probability
of rolling a six

this many times

Parameters of a Multinomial RV?

$X \sim \text{Bin}(n, p)$ has parameters n, p…

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

p: probability of success outcome on a single trial

A Multinomial RV has parameters $n, p_1, p_2, \ldots, p_m$ (Note $p_m = 1 - \sum_{i=1}^{m-1} p_i$)

$$P(X_1 = c_1, X_2 = c_2, \ldots, X_m = c_m) = \binom{n}{c_1, c_2, \ldots, c_m} p_1^{c_1} p_2^{c_2} \cdots p_m^{c_m}$$

p_i: probability of outcome i on a single trial

Where do we get p_i from?

Pedagogic pause

The Federalist Papers

Intro to Natural Language Processing

Probabilistic text analysis

Ignoring the order of words...

What is the probability of any given word that you write in English?

- $P(\text{word} = \text{"the"}) > P(\text{word} = \text{"pokemon"})$
- $P(\text{word} = \text{"Stanford"}) > P(\text{word} = \text{"Cal"})$

Probabilities of *counts* of words = Multinomial distribution 👉

A document is a large multinomial.

(according to the Global Language Monitor, there are 988,968 words in the English language used on the internet.)

Model text as a multinomial

Example document:
"Pay for Viagra with a credit-card. Viagra is great. So are credit-cards. Risk free Viagra. Click for free."
n = 18

It's a Multinomial!

$$P \left(\begin{matrix} \text{Viagra = 2} \\ \text{Free = 2} \\ \text{Risk = 1} \\ \text{Credit-card: 2} \\ ... \\ \text{For = 2} \end{matrix} \Bigg| \text{spam} \right) = \frac{n!}{2!2! \dots 2!} p_{\text{viagra}}^2 p_{\text{free}}^2 \dots p_{\text{for}}^2$$

Probability of seeing this document | spam

The probability of a word in spam email being viagra
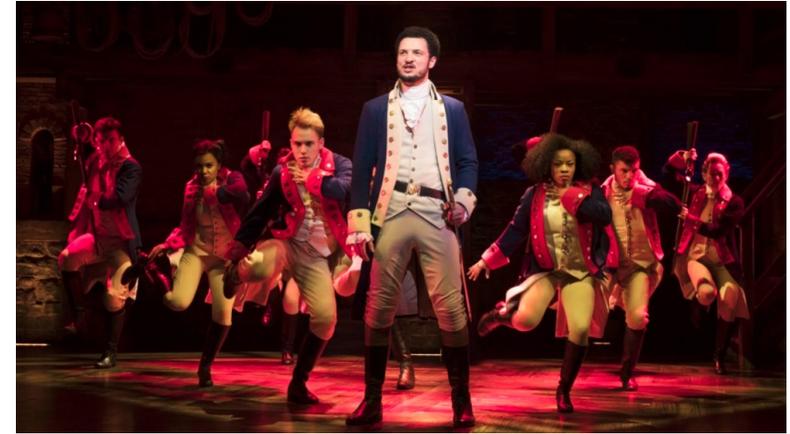
Stanford University

Who wrote the federalist papers?

Old and New Analysis

Authorship of the Federalist Papers

- 85 essays advocating ratification of the US constitution

- Written under the pseudonym "Publius" (really, Alexander **Hamilton**, James **Madison**, John **Jay**)

Who wrote which essays?

- Analyze probability of words in each essay and compare against word distributions from known writings of three authors

Who wrote Federalist Paper 53?

madison.txt

```
1  To the People of the State of New York:
2
3  AMONG the numerous advantages promised by a
   wellconstructed Union, none deserves to be more
   accurately developed than its tendency to break
   and control the violence of faction. The friend
   of popular governments never finds himself so
   much alarmed for their character and fate, as
   when he contemplates their propensity to this
   dangerous vice. He will not fail, therefore, to
   set a due value on any plan which, without
   violating the principles to which he is attached,
   provides a proper cure for it. The instability,
   injustice, and confusion introduced into the
   public councils, have, in truth, been the mortal
   diseases under which popular governments have
   everywhere perished; as they continue to be the
   favorite and fruitful topics from which the
   adversaries to liberty derive their most specious
   declamations. The valuable improvements made by
   the American constitutions on the popular models,
   both ancient and modern, cannot certainly be too
   much admired; but it would be an unwarrantable
   partiality, to contend that they have as
   effectually obviated the danger on this side, as
   was wished and expected. Complaints are
   everywhere heard from our most considerate and
   virtuous citizens, equally the friends of public
   and private faith, and of public and personal
   liberty, that our governments are too unstable,
   that the public good is disregarded in the
   conflicts of rival parties, and that measures are
   too often decided, not according to the rules of
   justice and the rights of the minor party, but by
   the superior force of an interested and
   overbearing majority. However anxiously we may
   wish that these complaints had no foundation, the
   evidence, of known facts will not permit us to
   deny that they are in some degree true. It will
   be found, indeed, on a candid review of our
   situation, that some of the distresses under
   which we labor have been erroneously charged on
   the operation of our governments; but it will be
   found, at the same time, that other causes will
   not alone account for many of our heaviest
   misfortunes; and, particularly, for that
   prevailing and increasing distrust of public
```

Line 3, Column 154   Tab Size: 4   Plain Text

hamilton.txt

```
1  The Utility of the Union in Respect to Commercial
   Relations and a Navy
2  Hamilton for the Independent Journal.
3
4  To the People of the State of New York:
5  THE importance of the Union, in a commercial
   light, is one of those points about which there
   is least room to entertain a difference of
   opinion, and which has, in fact, commanded the
   most general assent of men who have any
   acquaintance with the subject. This applies as
   well to our intercourse with foreign countries as
   with each other.
6
7  There are appearances to authorize a supposition
   that the adventurous spirit, which distinguishes
   the commercial character of America, has already
   excited uneasy sensations in several of the
   maritime powers of Europe. They seem to be
   apprehensive of our too great interference in
   that carrying trade, which is the support of
   their navigation and the foundation of their
   naval strength. Those of them which have colonies
   in America look forward to what this country is
   capable of becoming, with painful solicitude.
   They foresee the dangers that may threaten their
   American dominions from the neighborhood of
   States, which have all the dispositions, and
   would possess all the means, requisite to the
   creation of a powerful marine. Impressions of
   this kind will naturally indicate the policy of
   fostering divisions among us, and of depriving
   us, as far as possible, of an active commerce in
   our own bottoms. This would answer the threefold
   purpose of preventing our interference in their
   navigation, of monopolizing the profits of our
   trade, and of clipping the wings by which we
   might soar to a dangerous greatness. Did not
   prudence forbid the detail, it would not be
   difficult to trace, by facts, the workings of
   this policy to the cabinets of ministers.
8
9  If we continue united, we may counteract a policy
   so unfriendly to our prosperity in a variety of
   ways. By prohibitory regulations, extending, at
   the same time, throughout the States, we may
   oblige foreign countries to bid against each
```

Line 5, Column 249   Tab Size: 4   Plain Text

unknown.txt

```
1  To the People of the State of New York:
2  I SHALL here, perhaps, be reminded of a current
   observation, ``that where annual elections end,
   tyranny begins.'' If it be true, as has often
   been remarked, that sayings which become
   proverbial are generally founded in reason, it is
   not less true, that when once established, they
   are often applied to cases to which the reason of
   them does not extend. I need not look for a proof
   beyond the case before us. What is the reason on
   which this proverbial observation is founded? No
   man will subject himself to the ridicule of
   pretending that any natural connection subsists
   between the sun or the seasons, and the period
   within which human virtue can bear the temptations
   of power. Happily for mankind, liberty is not, in
   this respect, confined to any single point of
   time; but lies within extremes, which afford
   sufficient latitude for all the variations which
   may be required by the various situations and
   circumstances of civil society. The election of
   magistrates might be, if it were found expedient,
   as in some instances it actually has been, daily,
   weekly, or monthly, as well as annual; and if
   circumstances may require a deviation from the
   rule on one side, why not also on the other side?
   Turning our attention to the periods established
   among ourselves, for the election of the most
   numerous branches of the State legislatures, we
   find them by no means coinciding any more in this
   instance, than in the elections of other civil
   magistrates. In Connecticut and Rhode Island, the
   periods are half-yearly. In the other States,
   South Carolina excepted, they are annual. In South
   Carolina they are biennial as is proposed in the
   federal government. Here is a difference, as four
   to one, between the longest and shortest periods;
   and yet it would be not easy to show, that
   Connecticut or Rhode Island is better governed, or
   enjoys a greater share of rational liberty, than
   South Carolina; or that either the one or the
   other of these States is distinguished in these
   respects, and by these causes, from the States
   whose elections are different from both. In
   searching for the grounds of this doctrine, I can
   discover but one, and that is wholly inapplicable
   to our case. The important distinction so well
```

Line 2, Column 519   Tab Size: 4   Plain Text

Where to start?

We have words, we want to know probability of authorship. We also know probability of words given author...

Well hello again...

Who wrote Federalist Paper 53?

Prob Document
given Hamilton

Prior belief it was
Hamilton

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

Prob Hamilton given
Document

Prob of the
document???

Who wrote Federalist Paper 53?

Model document as a
multinomial where we care
about count of words

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

Who wrote Federalist Paper 53?

Loop over unique words

Prob hamilton would write word i

Prior belief it was Hamilton

Number of times word i is in the doc

$$P(H|D) = \frac{\binom{n}{c_1 \ldots c_k} \cdot \prod_i h_i^{c_i} \cdot P(H)}{P(D)}$$

Prob Hamilton given Document

Prob of the document???

Who wrote Federalist Paper 53?

Prob that Hamilton wrote it

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

$$= \frac{P(H) \cdot \binom{n}{c_1 \dots c_m} \cdot \prod_i h_i^{c_i}}{P(D)}$$

Prob that Madison wrote it

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

$$= \frac{P(M) \cdot \binom{n}{c_1 \dots c_m} \cdot \prod_i m_i^{c_i}}{P(D)}$$

$$\frac{P(H|D)}{P(M|D)} = \frac{P(M) \cdot \binom{n}{c_1 \dots c_k} \cdot \prod_i h_i^{c_i}}{P(D)} \bigg/ \frac{P(H) \cdot \binom{n}{c_1 \dots c_k} \cdot \prod_i m_i^{c_i}}{P(D)}$$

$$= \frac{\prod_i h_i^{c_i}}{\prod_i m_i^{c_i}}$$

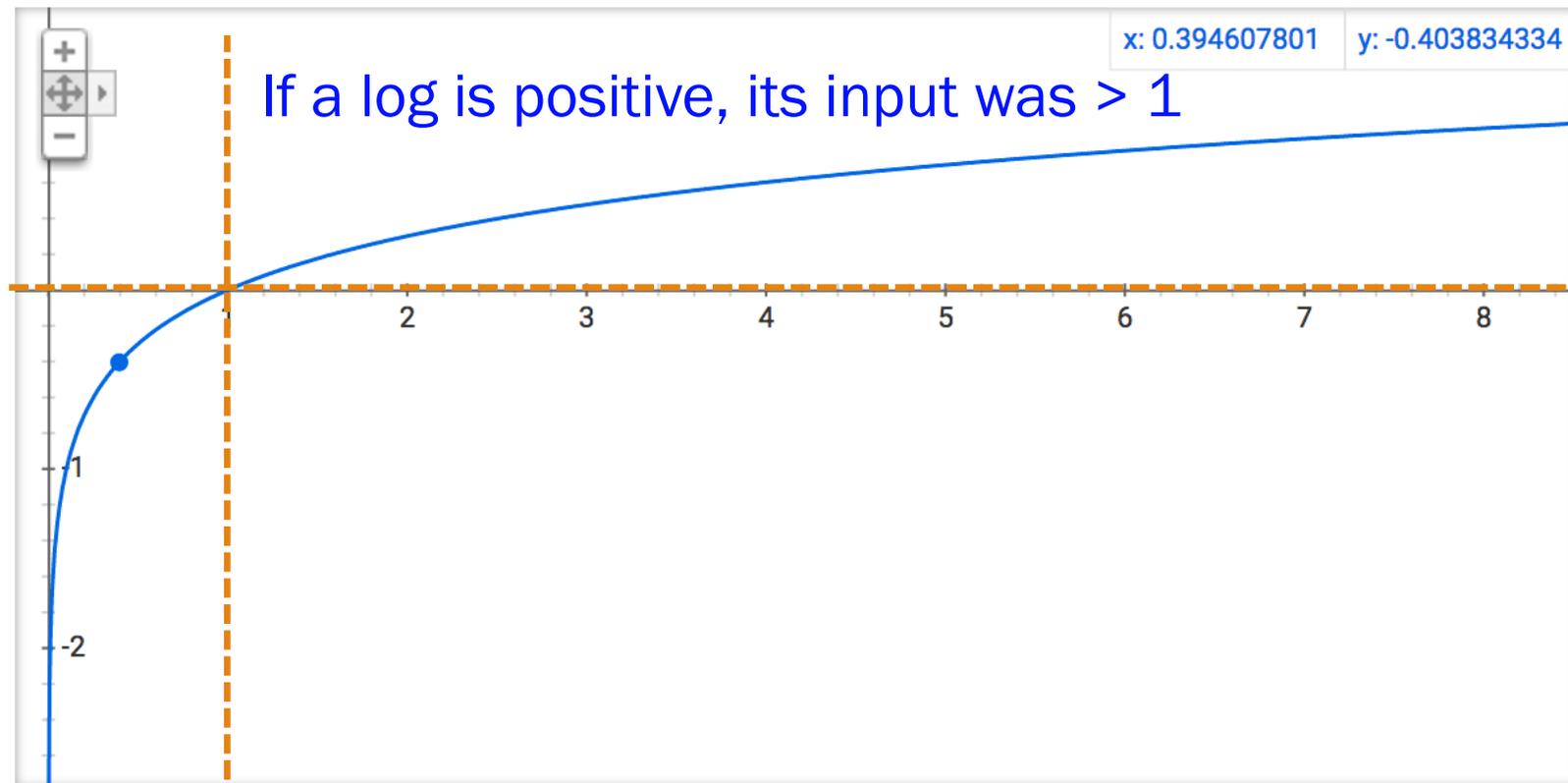# To the code

What happened?

All our probabilities are zero...

Use logs when probabilities become too small!

$$\frac{P(H|D)}{P(M|D)} = \frac{\prod_i m_i^{c_i}}{\prod_i h_i^{c_i}}$$

$$\log \frac{P(H|D)}{P(M|D)} = \log \frac{\prod_i h_i^{c_i}}{\prod_i m_i^{c_i}}$$

$$= \sum_i \log h_i^{c_i} - \sum_i \log m_i^{c_i}$$

$$= \sum_i c_i \cdot \log h_i - \sum_i c_i \log m_i$$

What does it mean if a log value is positive / negative

Graph for log(x)

x: 0.394607801   y: -0.403834334

If a log is positive, its input was > 1

If a log is negative, its input was between 0 and 1

More info

To be continued…