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CS 109

Section #7

Section 7 Solution

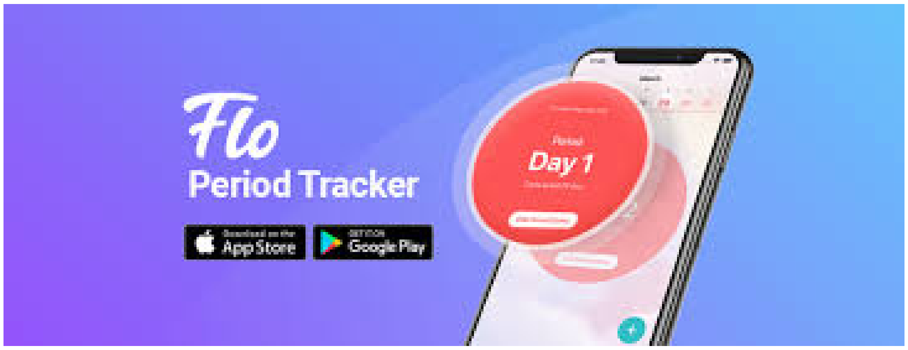
1. **Binary Tree**: Consider the following function for constructing binary trees:

```
def random_binary_tree(p):
   """
   Returns a dictionary representing a random binary tree structure.
   The dictionary can have two keys, "left" and "right".
   """
   if random_bernoulli(p):   # returns true with probability p
      new_node = {}
      new_node["left"] = random_binary_tree(p)
      new_node["right"] = random_binary_tree(p)
      return random_binary_tree
    else:
      return None
```

The `if` branch is taken with probability p (and the `else` branch with probability $1 - p$). A tree with no nodes is represented by `nullptr`; so a tree node with no left child has `nullptr` for the `left` field (and the same for the right child).

Let X be the number of nodes in a tree returned by `randomTree`. You can assume $0 < p < 0.5$. What is $E[X]$, in terms of p?

2. **Flo. Tracking Menstrual Cycles**



Let X represent the length of a menstrual cycle: the number of days, as a continuous value, between the first moment of one period to the first moment of the next, for a given person. X is parameterized by α and β with probability density function:

$$f(X = x) = \beta \cdot (x - \alpha)^{\beta-1} \cdot e^{-(x-\alpha)^2}$$

a. For a particular person, $\alpha = 27$ and $\beta = 2$. Write a simplified version of the PDF of X.

b. For a particular person, $\alpha = 27$ and $\beta = 2$. Write an expression for the probability that they have their period on day 29. In other words, what is the $P(29.0 < X < 30.0)$?

c. For a particular person, $\alpha = 27$ and $\beta = 2$. How many times more likely is their cycle to last **exactly** 28.0 days than exactly 29.0 days? You do not need to give a numeric answer. Simplify your expression.

d. A person has recorded their cycle length for 12 cycles stored in a list: $m = [29.0, 28.5, \ldots, 30.1]$ where m_i is the recorded cycle length for cycle i. Use MLE to estimate the parameter values α and β. Assume that cycle lengths are IID.

   You don't need a closed form solution. Derive any necessary partial derivatives and write up to three sentences describing how a program can use the derivatives in order to chose the most likely parameter values.

Note: Flo is a real "AI based" app that helps people track their period lengths. The real world distribution of periods is thought to be a mixture distribution between a normal and a weibell distribution [1]. This problem only has you estimate parameters for a simplified Weibull [2].