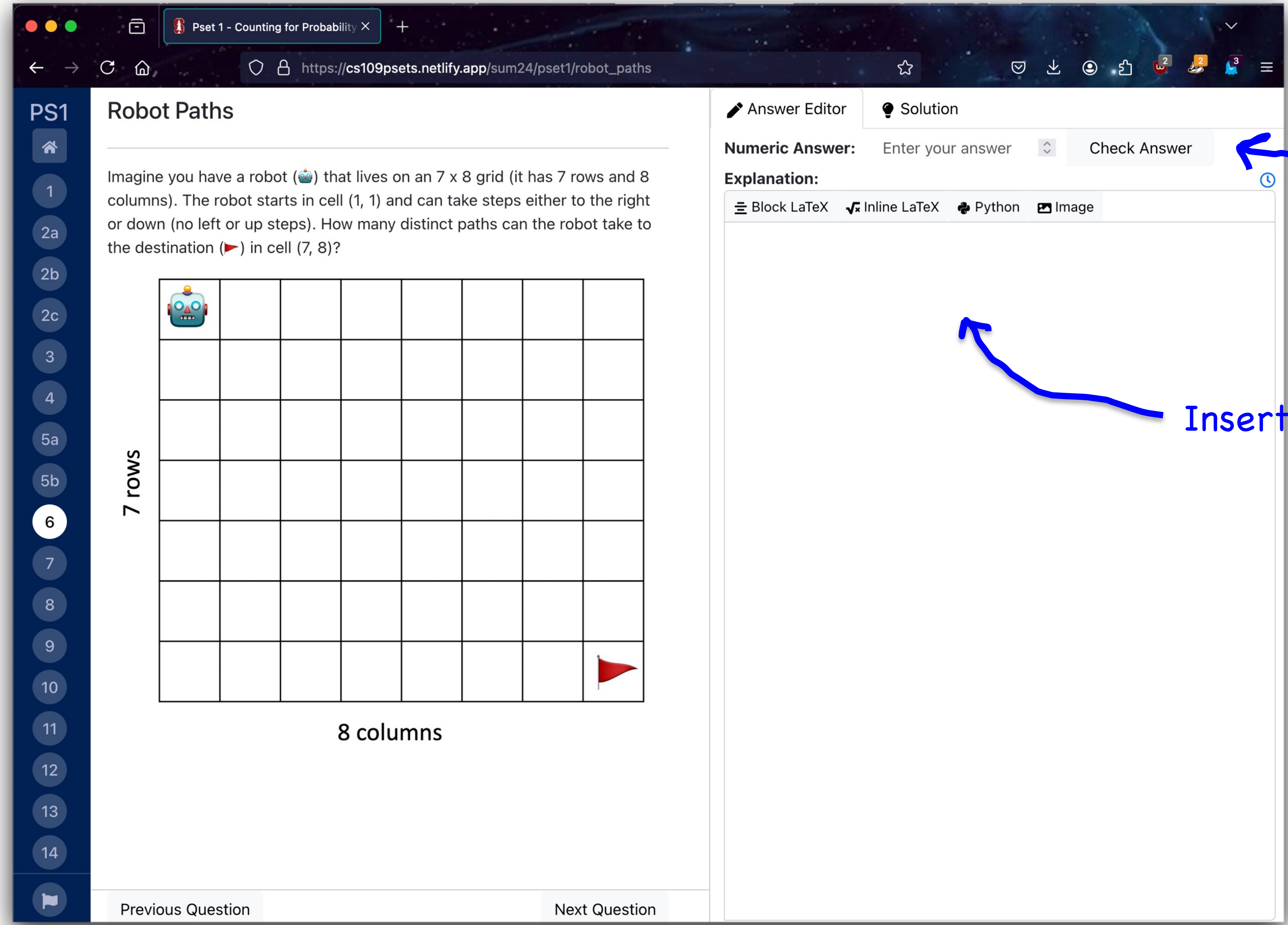
Combinatorics

CS109

Problem Set 1 is out



Check your answer

Insert LaTeX

Submission is automatic!

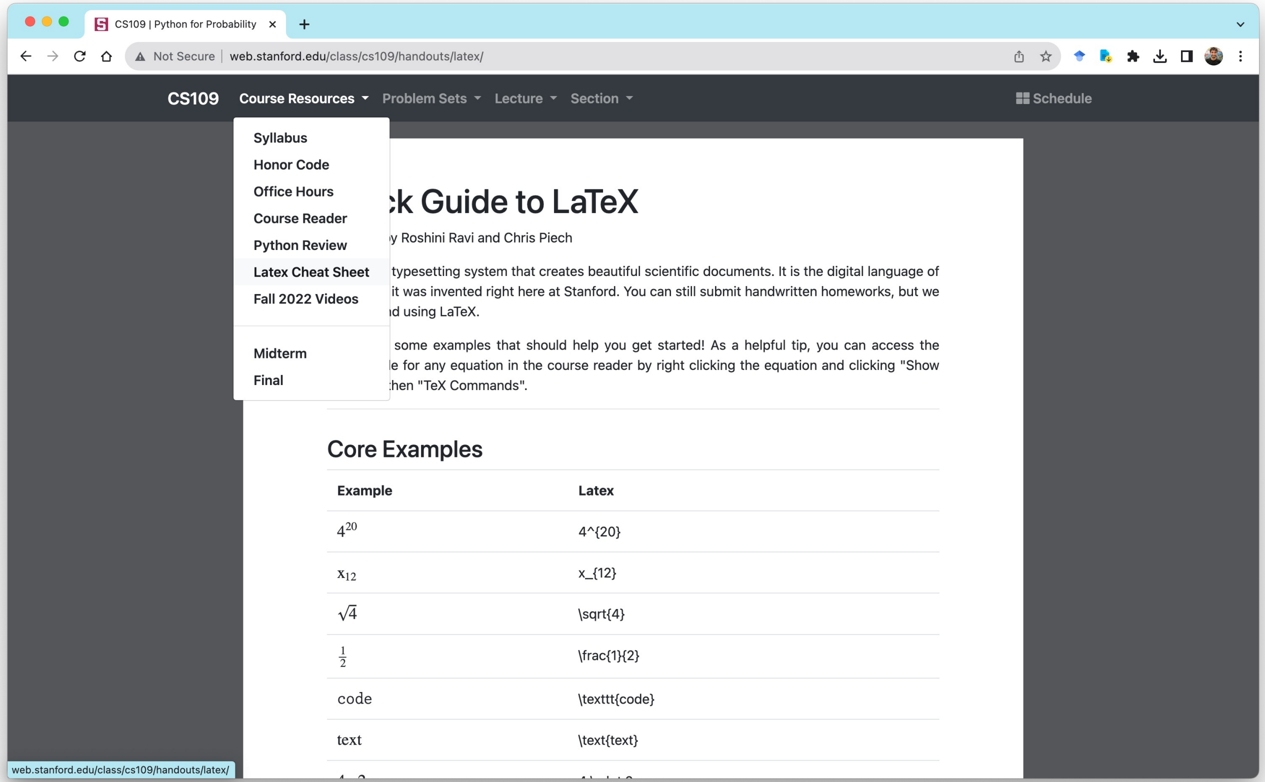
You can use LaTeX to type fancy math

```
\begin{aligned}
P(E) 
&= \sum_{i=0}^n e^{i}\\
&= 0.25
\end{aligned}
```

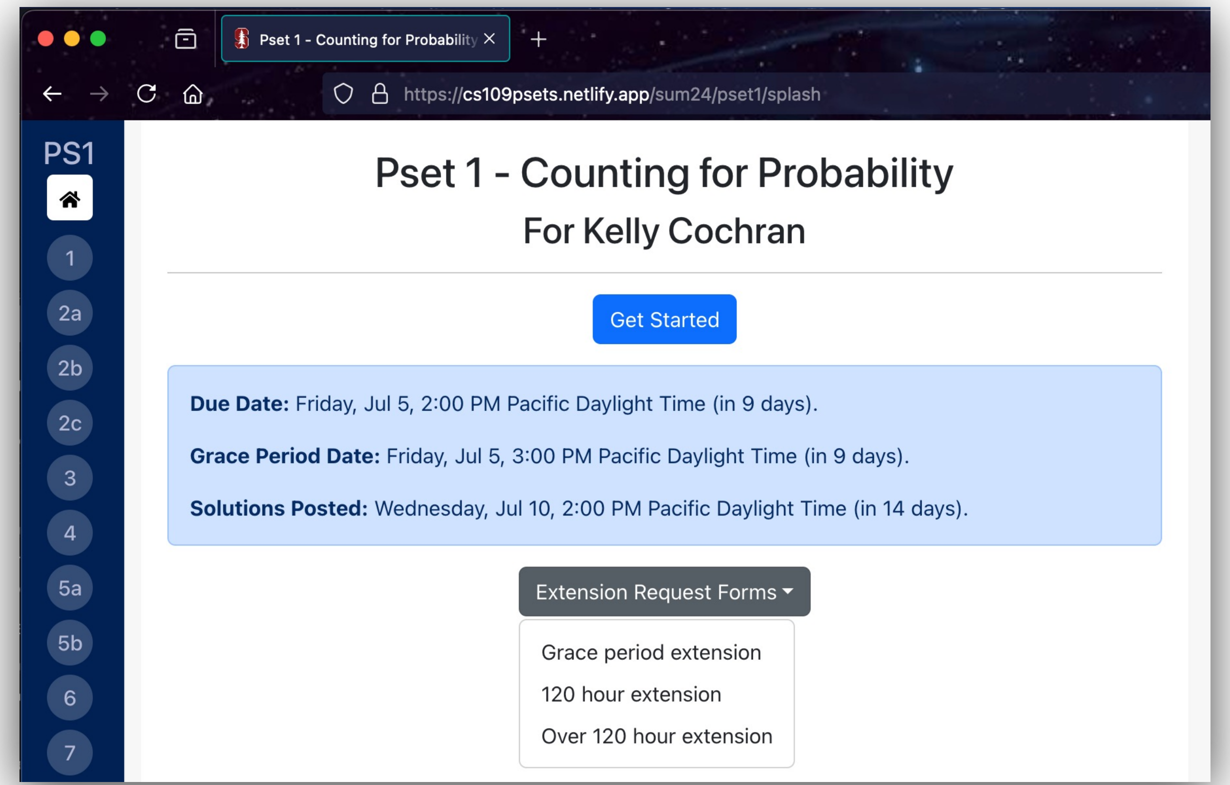
$$
\begin{aligned}
P(E) &= \sum_{i=0}^{n} e^{i} \\
&= 0.25
\end{aligned}
$$

Done

There’s a handout on the course website to help you get started!



Late Policy



Two types of extensions:

1. Grace period (1 hour)
2. Full extension (2 lecture days later)

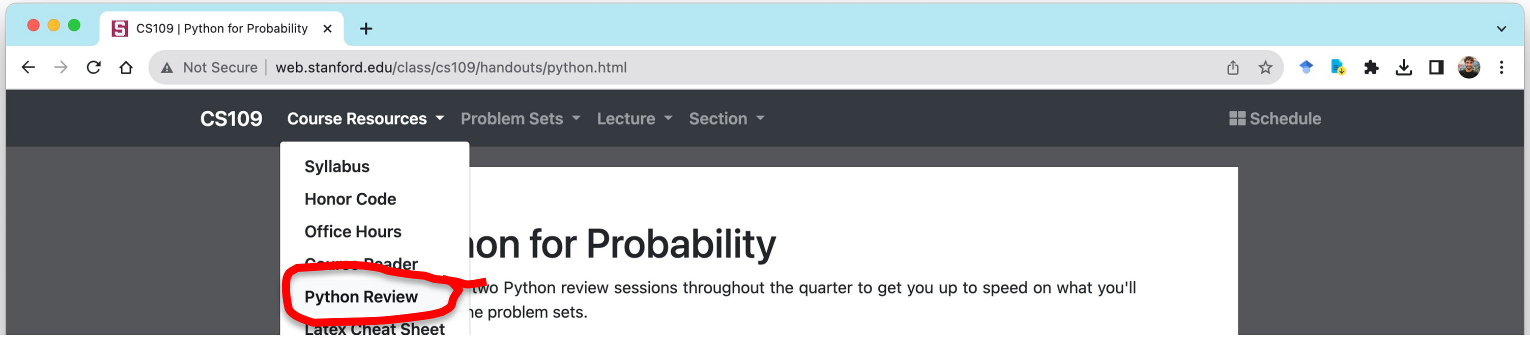
You grant them to yourself in the psetapp.

But CS109 is a fast class, and we don't want you to fall behind – after additional extensions past two, we start to cap the pset grade.

Python Review Session

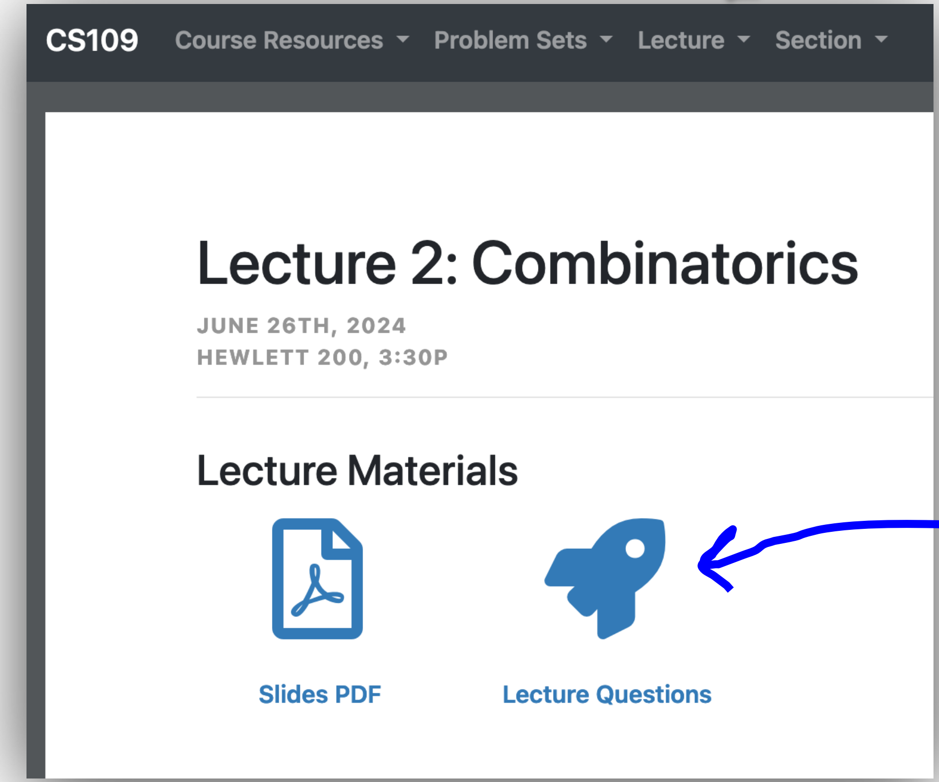
Friday at 5pm PT with Joel
(Zoom)

Find links and recordings on the course website

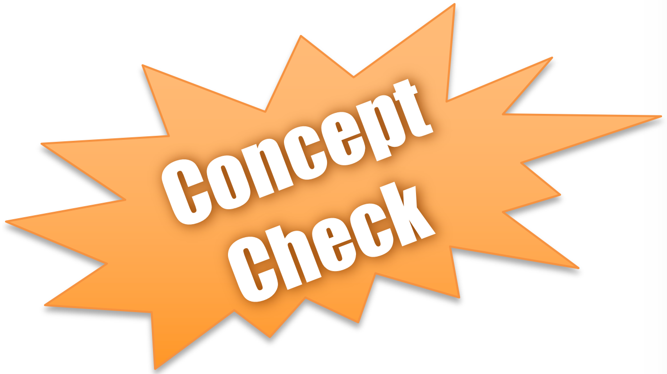


How To Get Attendance Credit

Find today's lecture here



Then click here



Answer each lecture's concept check question in the psetapp – linked on lecture page

Section Signups Are Due Tonight

Back to Learning!


Probabilistic Models
Uncertainty Theory
Machine Learning
Random Variables
Core Probability
Counting
The Journey of CS109:

CS109: From Counting to Machine Learning

Counting Theory

Core Probability

Random Variables

Probabilistic Models

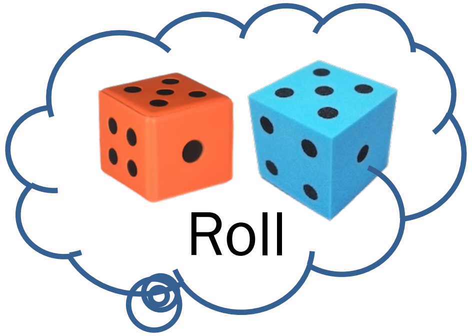
Uncertainty Theory

Machine Learning

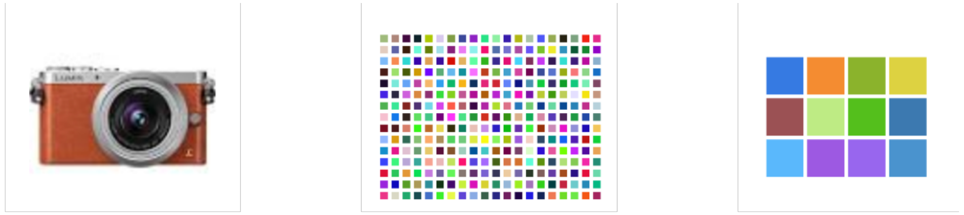
Last Lecture: Step Rule of Counting

Definition: Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two parts, where the first part can result in one of m outcomes and the second part can result in one of n outcomes regardless of the outcome of the first part, then the total number of outcomes for the experiment is $m \cdot n$.



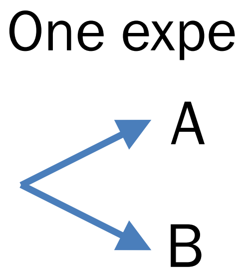
{(1, 1) , (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1) , (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1) , (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1) , (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1) , (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1) , (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}



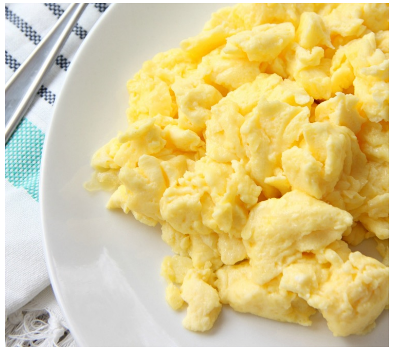
(a) 12 million pixels (b) 300 pixels (c) 12 pixels

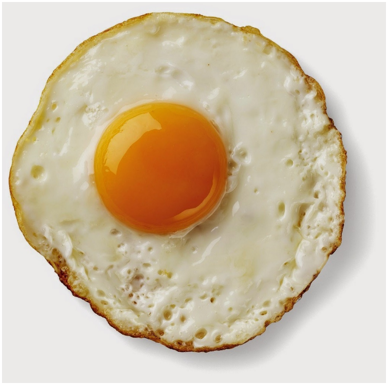
Tip: Associate every rule with example problems that fit the rule

The "Or" Rule, Part 1



- If the outcome of an experiment can be either from
  - Set A, where $|A| = m$,
  - **or** Set B, where $|B| = n$,
  - where $A \cap B = \emptyset$ (no overlap)
- Then the number of outcomes of the experiment is
  - $|A| + |B| = m + n$.





How Many Bit Strings?

Problem: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

How Many Bit Strings?

Problem: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Tip: Whenever you're stuck starting a counting problem, try writing out example outcomes

How Many Bit Strings?

Problem: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Set A	Set B
010000	000010
010001	000110
010010	001010
010011	001110
010100	010010
010101	010110
010110	011010
010111	011110
011000	100010
011001	100110
011010	101010
011011	101110
011100	110010
011101	110110
011110	111010
011111	111110

How Many Bit Strings?

Problem: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

2^4 start with 01	2^4 end with 10
010000	000010
010001	000110
010010	001010
010011	001110
010100	010010
010101	010110
010110	011010
010111	011110
011000	100010
011001	100110
011010	101010
011011	101110
011100	110010
011101	110110
011110	111010
011111	111110
Set A	Set B

How Many Bit Strings?

Problem: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

2^4 start with 01	2^4 end with 10
010000	000010
010001	000110
010010	001010
010011	001110
010100	**010010**
010101	**010110**
010110	**011010**
010111	**011110**
011000	100010
011001	100110
011010	101010
011011	101110
011100	110010
011101	110110
011110	111010
011111	111110
Set A	Set B

How Many Bit Strings?

Problem: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

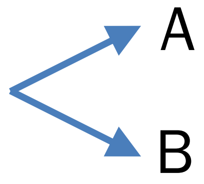
Answer

$$\begin{aligned} N &= |A| + |B| - |A \text{ and } B| \\ &= 16 + 16 - 4 \\ &= 28 \end{aligned}$$

2^4 start with 01	2^4 end with 10
010000	000010
010001	000110
010010	001010
010011	001110
010100	**010010**
010101	**010110**
010110	**011010**
010111	**011110**
011000	100010
011001	100110
011010	101010
011011	101110
011100	110010
011101	110110
011110	111010
011111	111110
Set A	Set B

The Real "Or" Rule (aka Inclusion/Exclusion)

One experiment



- If the outcome of an experiment can be either from
  - Set A,
  - **or** Set B,
  - where $A \cap B$ $might\ not\ be\ empty$,
- Then the number of outcomes of the experiment is
  - N = $|A| + |B| - |A \cap B|$.

The Core Counting Rules

Counting with steps

Definition: Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two parts, where the first part can result in one of m outcomes and the second part can result in one of n outcomes regardless of the outcome of the first part, then the total number of outcomes for the experiment is $m \cdot n$.

Counting with "or"

Definition: Inclusion Exclusion Counting

If the outcome of an experiment can either be drawn from set A or set B, and sets A and B may potentially overlap (i.e., it is not the case that A and B are mutually exclusive), then the number of outcomes of the experiment is $|A \text{ or } B| = |A| + |B| - |A \text{ and } B|$.

Permutations

Counting Possible Orderings

How many letter orderings are possible for the following string?

CS109

All Possible Orderings Of Characters

CS109	CS190	CS019	CS091	CS910	CS901	C1S09	C1S90
C10S9	C109S	C19S0	C190S	C0S19	C0S91	C01S9	C019S
C09S1	C091S	C9S10	C9S01	C91S0	C910S	C90S1	C901S
SC109	SC190	SC019	SC091	SC910	SC901	S1C09	S1C90
S10C9	S109C	S19C0	S190C	S0C19	S0C91	S01C9	S019C
S09C1	S091C	S9C10	S9C01	S91C0	S910C	S90C1	S901C
1CS09	1CS90	1C0S9	1C09S	1C9S0	1C90S	1SC09	1SC90
1S0C9	1S09C	1S9C0	1S90C	10CS9	10C9S	10SC9	10S9C
109CS	109SC	19CS0	19C0S	19SC0	19S0C	190CS	190SC
0CS19	0CS91	0C1S9	0C19S	0C9S1	0C91S	0SC19	0SC91
0S1C9	0S19C	0S9C1	0S91C	01CS9	01C9S	01SC9	01S9C
019CS	019SC	09CS1	09C1S	09SC1	09S1C	091CS	091SC
9CS10	9CS01	9C1S0	9C10S	9C0S1	9C01S	9SC10	9SC01
9S1C0	9S10C	9S0C1	9S01C	91CS0	91C0S	91SC0	91S0C
910CS	910SC	90CS1	90C1S	90SC1	90S1C	901CS	901SC

Tip: Think about a generative story…

Counting Possible Orderings

Counting Possible Orderings

Step 1:
Chose 1st character

Step 2:
Chose 2nd character

Step 3:
Chose 3rd character

Step 4:
Chose 4th character

Step 5:
Chose 5th character

Counting Possible Orderings

(5 options)

9

Step 1:
Chose 1st character

Step 2:
Chose 2nd character

Step 3:
Chose 3rd character

Step 4:
Chose 4th character

Step 5:
Chose 5th character

Counting Possible Orderings

(5 options) (4 options)

$$\underbrace{\texttt{9}}\ \underbrace{\texttt{C}}\ \underbrace{\quad}\ \underbrace{\quad}\ \underbrace{\quad}$$

Step 1:
Chose 1st character

Step 2:
Chose 2nd character

Step 3:
Chose 3rd character

Step 4:
Chose 4th character

Step 5:
Chose 5th character

Counting Possible Orderings

(5 options)	(4 options)	(3 options)	(2 options)	(1 option)
9	C	0	S	1

Step 1:
Chose 1st character

Step 2:
Chose 2nd character

Step 3:
Chose 3rd character

Step 4:
Chose 4th character

Step 5:
Chose 5th character

Counting Possible Orderings

(5 options) (4 options) (3 options) (2 options) (1 option)

$$\underbrace{\;9\;}\;\underbrace{\;C\;}\;\underbrace{\;0\;}\;\underbrace{\;S\;}\;\underbrace{\;1\;}$$

Step 1:
Chose 1st character

Step 2:
Chose 2nd character

Step 3:
Chose 3rd character

Step 4:
Chose 4th character

Step 5:
Chose 5th character

$\boxed{\text{Answer: } 5! \;\; 5 \times 4 \times 3 \times 2 \times 1}$

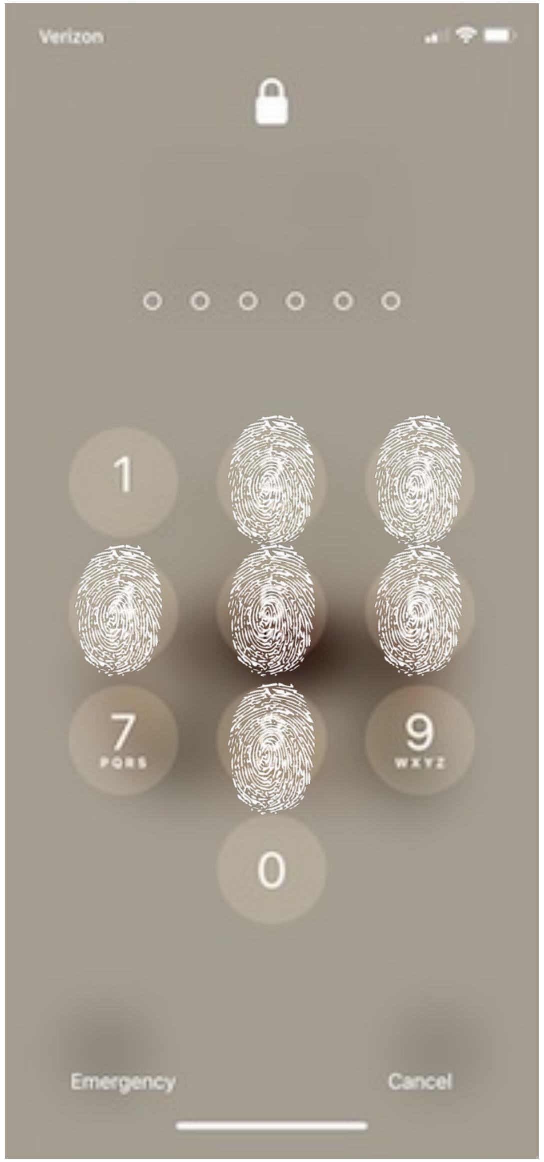
Possible Orderings Are Called Permutations

Permutation: any *ordered* arrangement of objects.

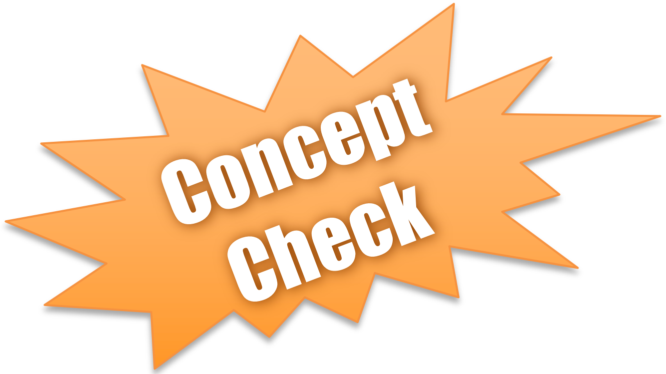
The number of unique orderings (permutations) of n *distinct* objects is:

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

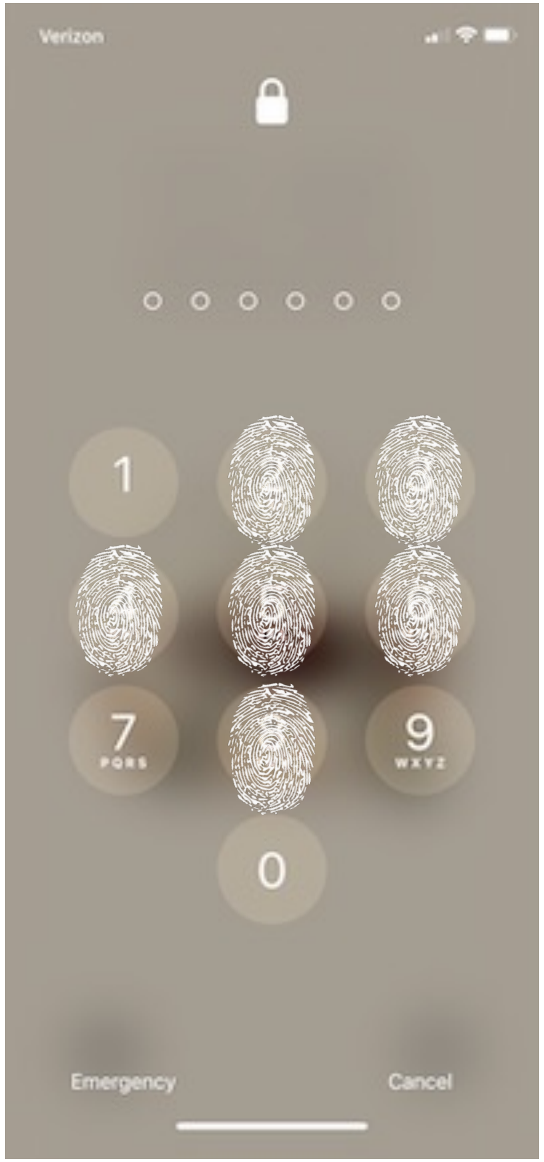
Practice: Unique 6-digit passcodes with six smudges



How many unique 6-digit passcodes are possible if each passcode uses **six** distinct numbers?



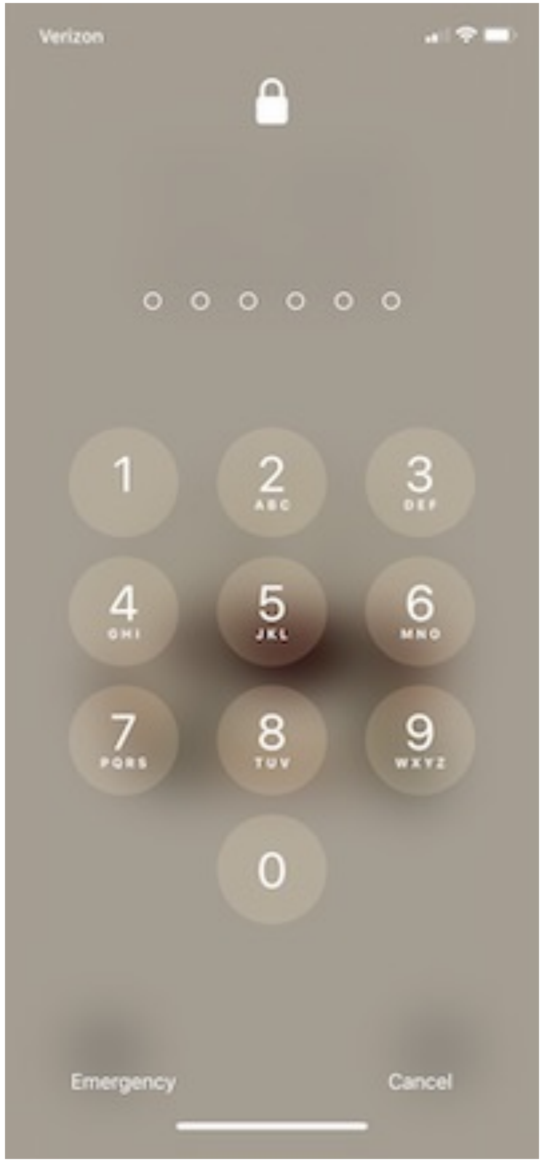
Practice: Unique 6-digit passcodes with six smudges



How many unique 6-digit passcodes are possible if each passcode uses **six** distinct numbers?

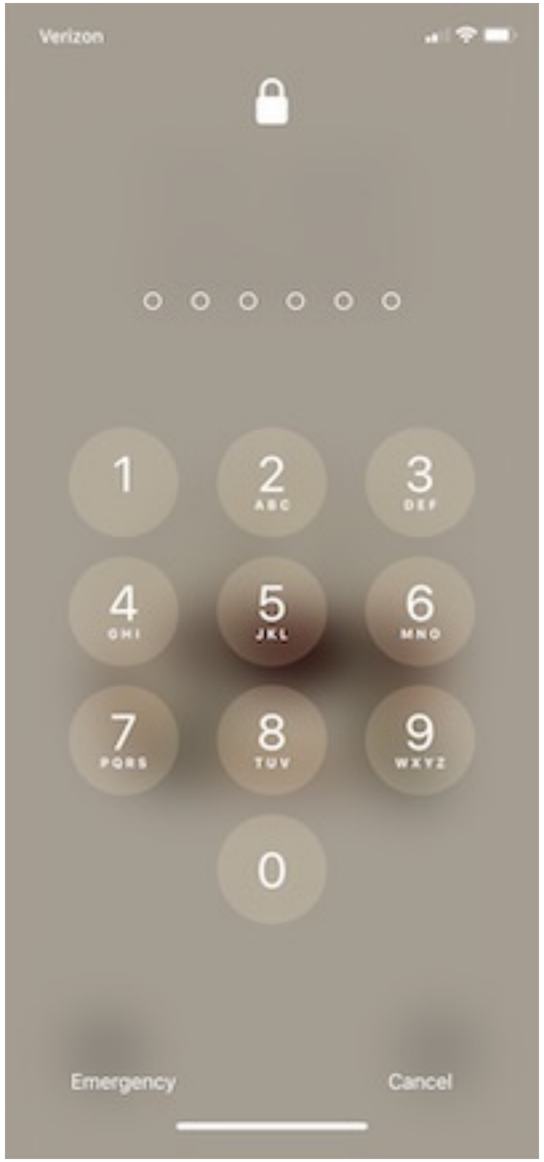
$$6! = 720 \text{ passcodes}$$

Challenge: Unique 6-digit passcodes with six mystery smudges



How many unique passcodes are possible if each passcode is ***any*** 6 unique digits?

Challenge: Unique 6-digit passcodes with six mystery smudges



How many unique passcodes are possible if each passcode is ***any*** 6 unique digits?

$$10 \times 9 \times 8 \times 7 \times 6 \times 5$$

$$= \frac{10!}{4!} = 151200 \text{ passcodes}$$

Combinatorics: Formulas For Common Counting Tasks

Counting tasks on n objects

Sort objects (permutations)

Choose k objects (combinations)

Put objects in r buckets

Distinct (distinguishable)

$n!$

Combinatorics: Formulas For Common Counting Tasks

Counting tasks on n objects

- Sort objects (permutations)
  - Distinct (distinguishable)

    $n!$
  - Some distinct
- Choose k objects (combinations)
- Put objects in r buckets

Permutations With Some Distinct Objects

How many *different* orderings of letters are possible for the string "boba"?

boba, abob, obba…

Permutations With Some Distinct Objects

How many *different* orderings of letters are possible for the string "boba"?

boba	obba	bboa	abob
boab	obab	bbao	abbo
bboa	obba	boba	aobb
bbao	obab	boab	aobb
baob	oabb	babo	abbo
babo	oabb	baob	abob

Are there 4! = 24 unique permutations?

Permutations With Some Distinct Objects

How many *different* orderings of letters are possible for the string "boba"?

boba	obba	bboa	abob
boab	obab	bbao	abbo
bboa	obba	**boba**	aobb
bbao	obab	boab	aobb
baob	oabb	babo	abbo
babo	oabb	baob	abob

Are there 4! = 24 unique permutations?

No!

We're overcounting by a multiple of 2

Permutations With Some Distinct Objects

How many *different* orderings of letters are possible for the string BOBA?

$$\frac{4!}{2} = 12$$

Tip: Starting with a number that you know over- or under-counts, then correcting the difference, is a perfectly valid strategy

Can We Code It? Yes We Can!

baob
bbao
obba
oabb
boab
babo
abbo
aobb
boba
abob
bboa
obab

```python
import itertools

def list_outcomes():
   letters = ['b','o','b','a']
   perms = set(itertools.permutations(letters))
   for perm in perms:
      perm_as_string = "".join(perm)
      print(perm_as_string)


import math

def count_outcomes():
   numerator = math.factorial(4)
   denominator = math.factorial(2)
   print(numerator / denominator)
```

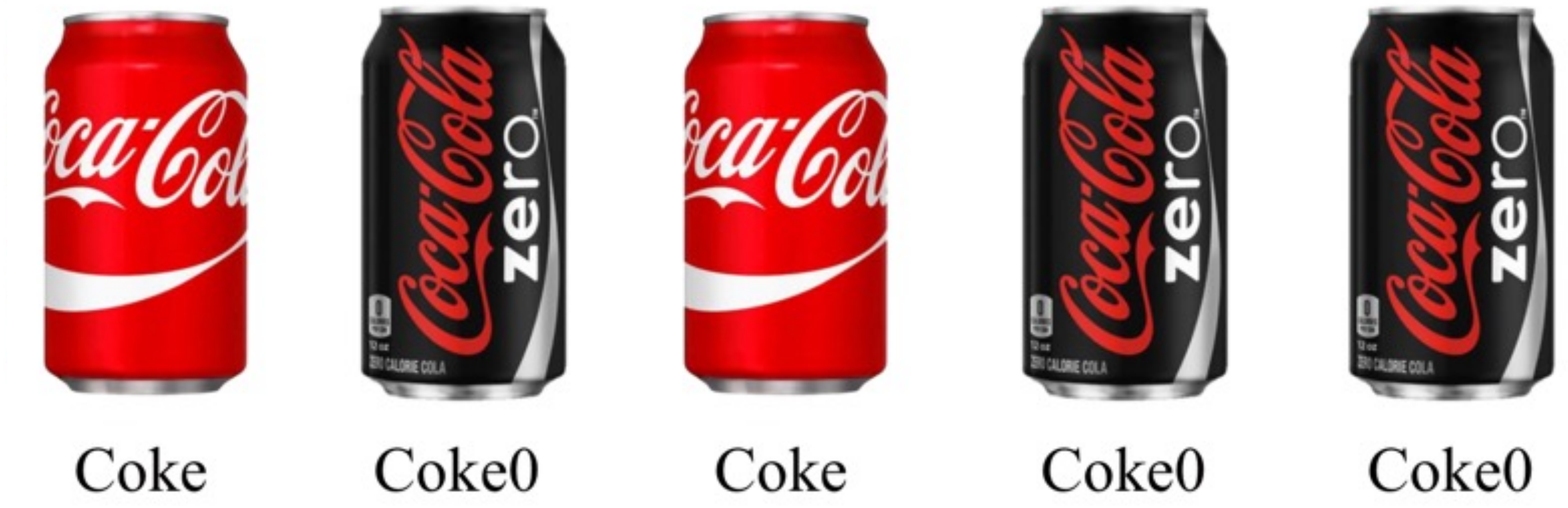
Back To Unique Bit Strings: How Many Permutations?

1 0 1 0 0

Back To Unique Bit Strings: How Many Permutations?

1 0 1 0 0

How many ways can we sort coke cans?



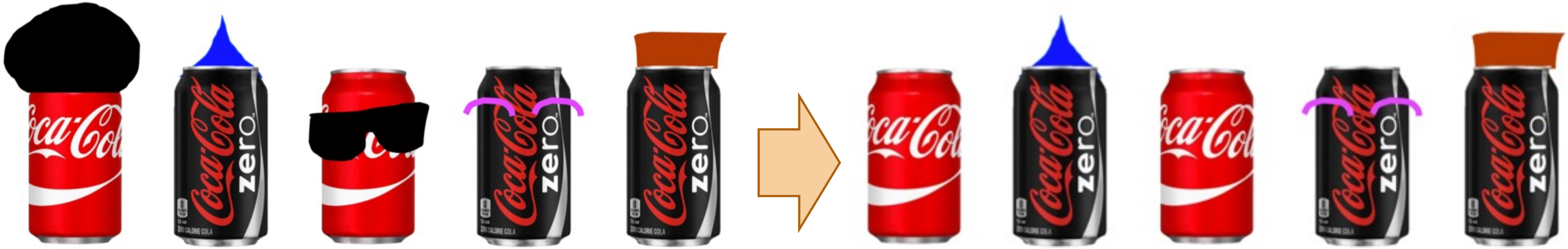
How many ways can we sort n distinct objects?

How many ways can we sort n distinct objects?

# of permutations = 5! 5 x 4 x 3 x 2 x 1

Then, how do we sort semi-distinct objects?

All Distinct | Some Indistinct



5! is an overcount… How do we correct it?

General approach to counting permutations

When there are n objects, and

n_1 are the same (indistinguishable or **in**distinct),

n_2 are the same,

…

n_r are the same,

The number of unique permutations is

$$\frac{n!}{n_1!\,n_2!\cdots n_r!}$$

General approach to counting permutations

When there are n objects, and

n_1 are the same (indistinguishable or **in**distinct),

n_2 are the same,

…

n_r are the same,

The number of unique permutations is

$$\frac{n!}{n_1!\, n_2! \cdots n_r!}$$

For each group of indistinct objects, divide by the overcounted permutations.

Permutations of Semi-Distinct Objects

Order n semi-distinct objects	$\dfrac{n!}{n_1!\,n_2!\cdots n_r!}$

Coke Coke0 Coke Coke0 Coke0

Permutations of Semi-Distinct Objects

Order n semi-distinct objects $\frac{n!}{n_1!\,n_2!\cdots n_r!}$

$$\frac{5!}{2!3!} = 10$$

Boss Battle: String Permutations

Order n semi-distinct objects	$\dfrac{n!}{n_1!\,n_2!\cdots n_r!}$

How many letter orderings are possible for the following string?

MISSISSIPPI

Boss Battle: String Permutations

Order n semi-distinct objects	$\frac{n!}{n_1!\, n_2! \cdots n_r!}$

How many letter orderings are possible for the following string?

MISSISSIPPI

$$\frac{11!}{1!4!4!2!} = 34{,}650$$

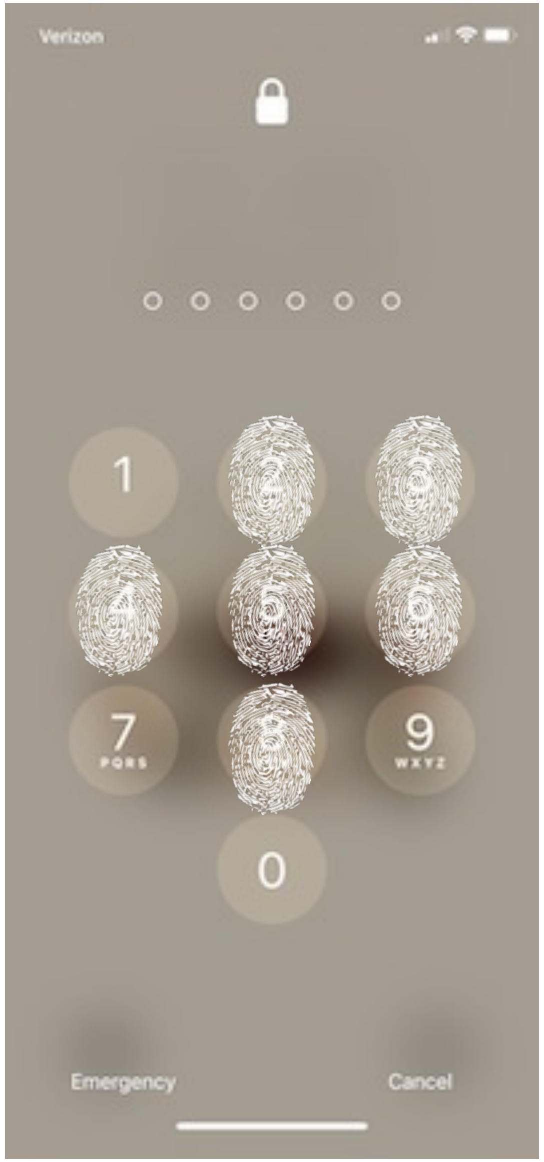
Combinatorics: Formulas For Common Counting Tasks

Counting tasks on n objects

- Sort objects (permutations)
  - Distinct (distinguishable): $n!$
  - Some distinct: $\dfrac{n!}{n_1!\,n_2!\cdots n_r!}$
- Choose k objects (combinations)
- Put objects in r buckets

Unique 6-digit passcodes with six smudges

Order n semi-distinct objects $\dfrac{n!}{n_1!\,n_2!\cdots n_r!}$

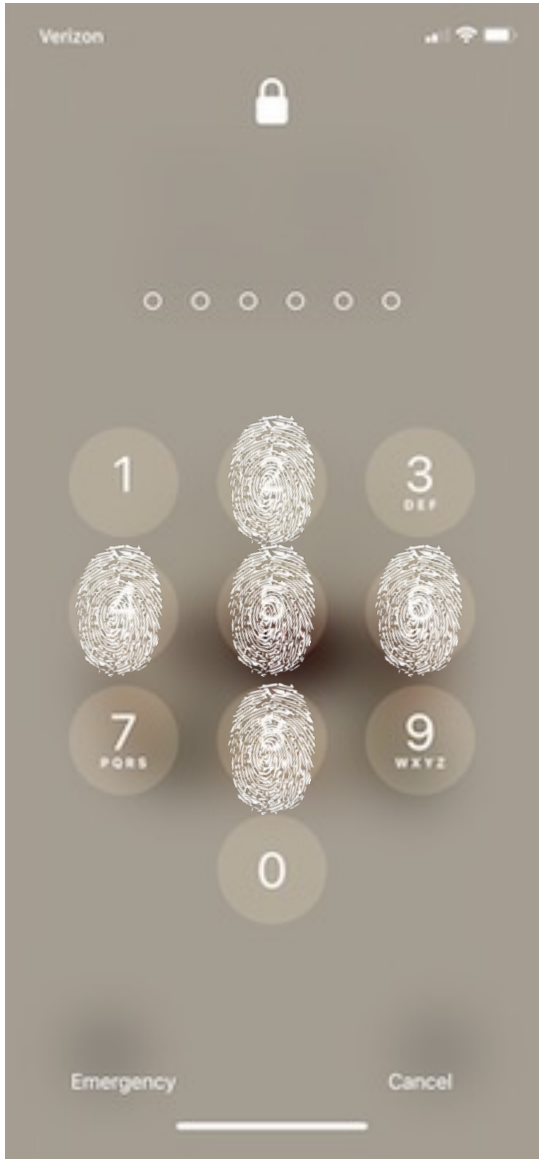


How many unique 6-digit passcodes are possible if each passcode uses **six** distinct numbers?

$$6! = 720 \text{ passcodes}$$

Unique 6-digit passcodes with five smudges

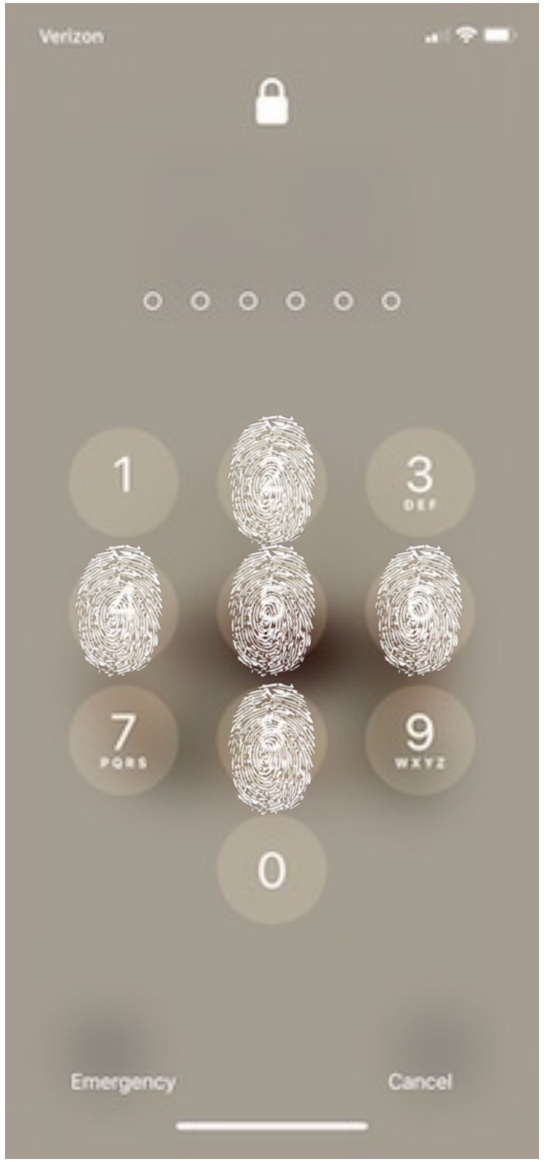
Order n semi-distinct objects $\frac{n!}{n_1!\,n_2!\cdots n_r!}$



How many unique 6-digit passcodes are possible if each passcode uses exactly **five** distinct numbers?

Unique 6-digit passcodes with five smudges

Order n semi-distinct objects $\frac{n!}{n_1! n_2! \cdots n_r!}$



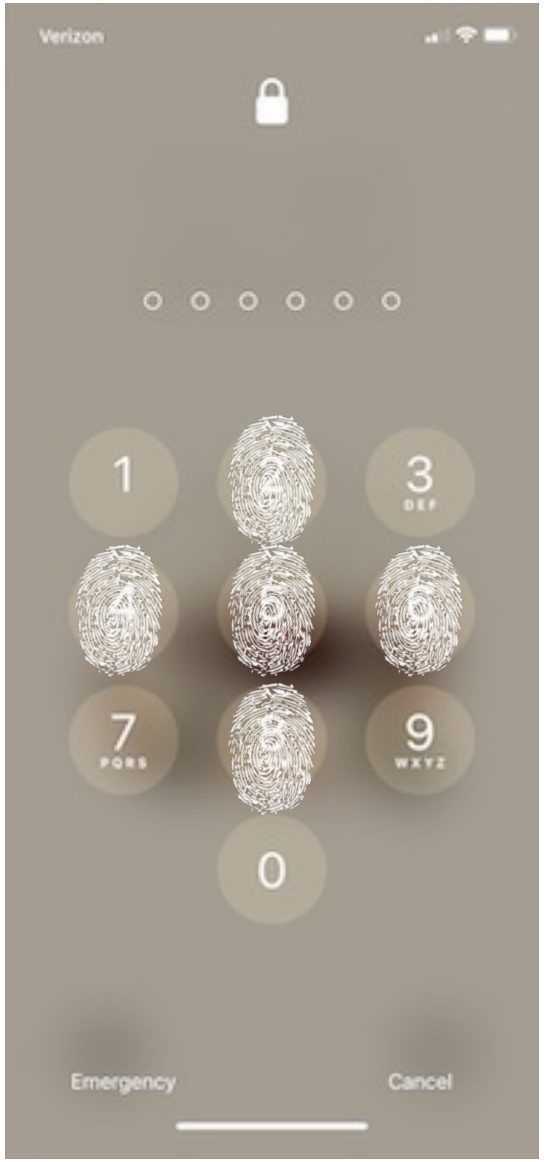
How many unique 6-digit passcodes are possible if each passcode uses exactly **five** distinct numbers?

<u>Steps</u>:

1. Choose digit to repeat — 5 outcomes
2. Create passcode — (sort 6 digits: 4 distinct, 2 indistinct)

Unique 6-digit passcodes with five smudges

Order n semi-distinct objects $\frac{n!}{n_1!\,n_2!\cdots n_r!}$



How many unique 6-digit passcodes are possible if each passcode uses exactly **five** distinct numbers?

<u>Steps</u>:

1. Choose digit to repeat — 5 outcomes
2. Create passcode — (sort 6 digits: 4 distinct, 2 indistinct)

$$5 \times \frac{6!}{2!} = 1{,}800 \text{ passcodes}$$

Combinations

Combinatorics: Formulas For Common Counting Tasks

Counting tasks on n objects

- Sort objects (permutations)
  - Distinct (distinguishable): $n!$
  - Some distinct: $\dfrac{n!}{n_1!\, n_2! \cdots n_r!}$
- Choose k objects (combinations)
  - Distinct
- Put objects in r buckets

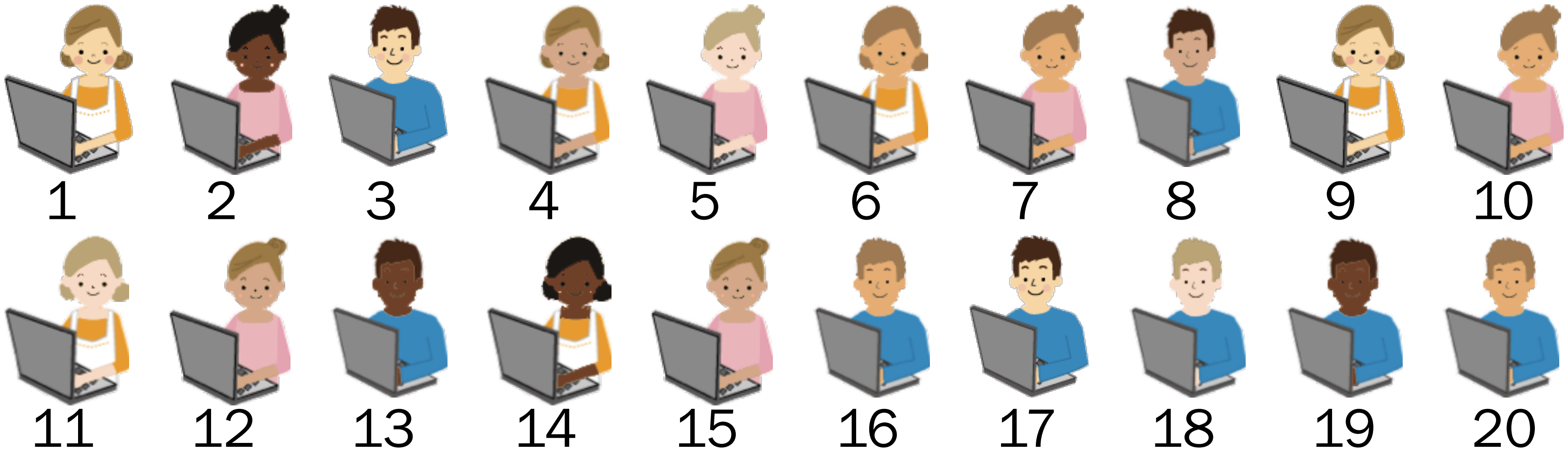
Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?

Think about a generative story…

Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?

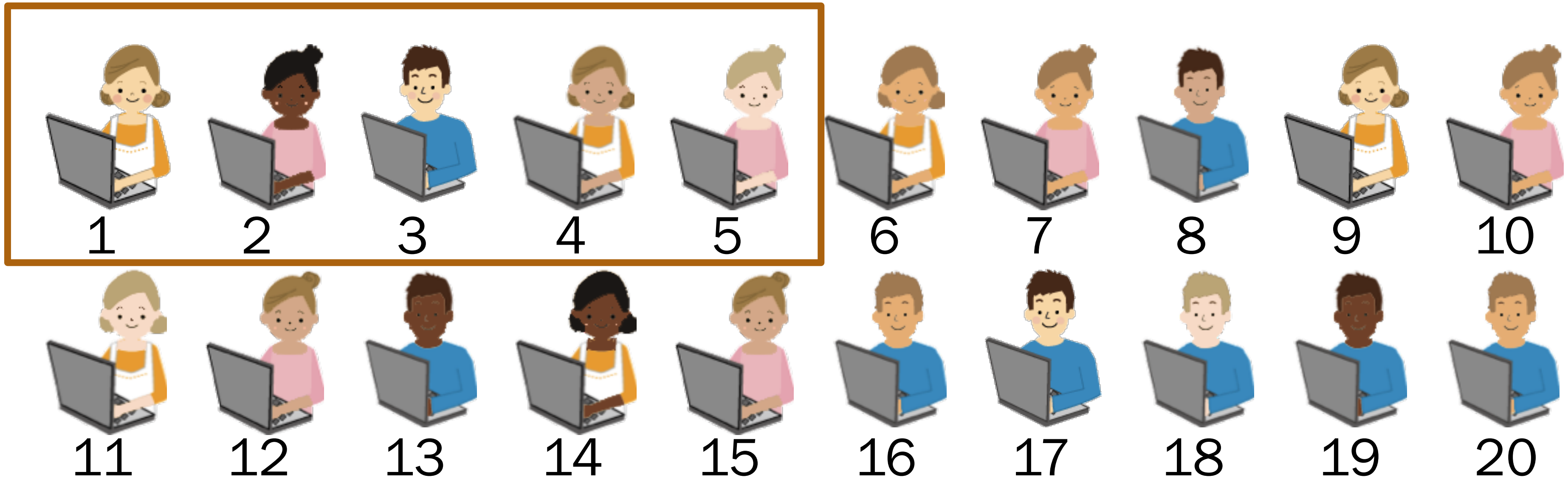


1. n people get in line

$n!$ ways

Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?



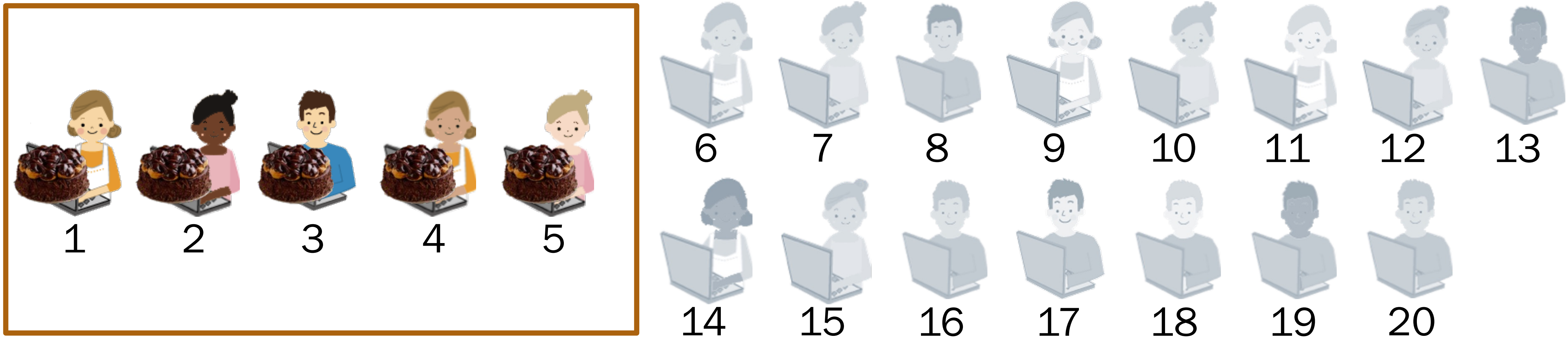
1. n people get in line

   $n!$ ways

2. Put first k in cake group

Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?



1. n people get in line

   $n!$ ways

2. Put first k in cake group

Combinations with cake

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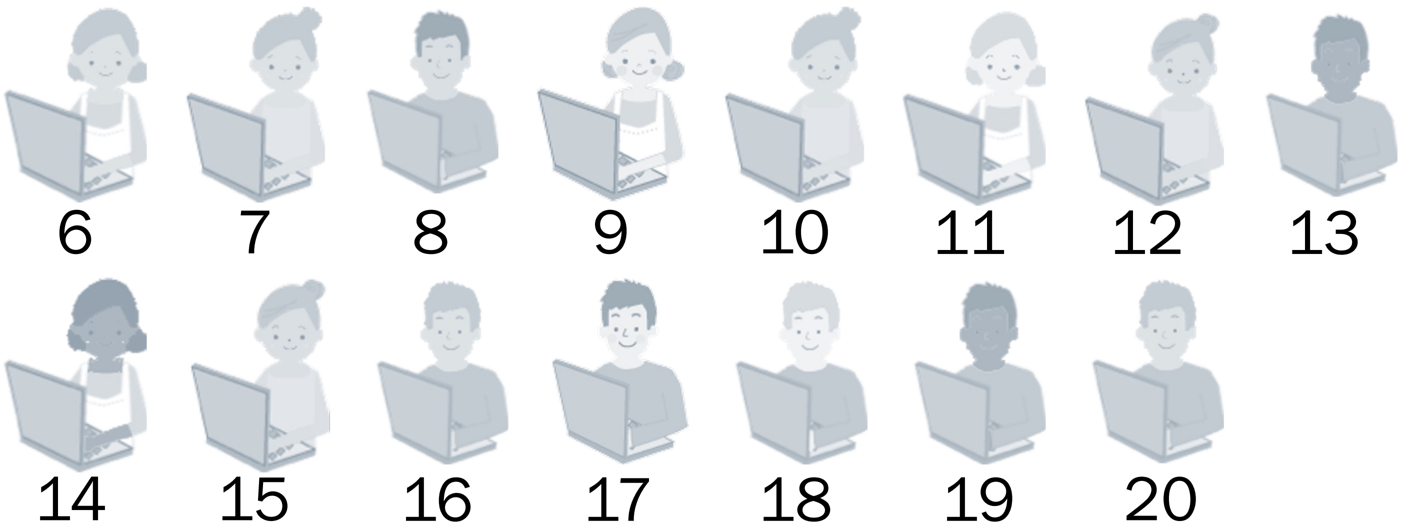

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   $k!$ different permutations

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   $n!$ ways

2. Put first k in cake group

3. Un-order the cake group

   $k!$ different permutations

4. Un-order the non-cake group

   $(n-k)!$ different permutations

Combinations

A combination is an **unordered** selection of k objects from a set of n distinct objects.

The number of ways of making this selection is

$$\frac{n!}{k!\,(n-k)!} = n! \times \frac{1}{k!} \times \frac{1}{(n-k)!}$$

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1. Order n distinct objects
2. Correct for overcounting: any ordering of the chosen group is the same
3. Correct for overcounting: any ordering of the not-chosen group is the same

Combinations

A combination is an **unordered** selection of k objects from a set of n distinct objects.

The number of ways of making this selection is

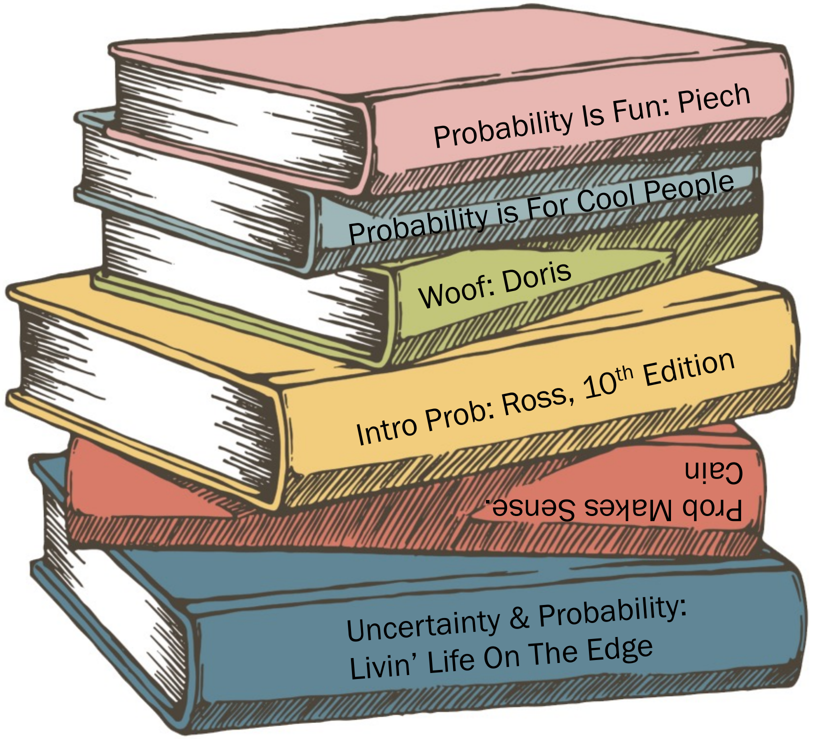
$$\frac{n!}{k!\,(n-k)!} = n! \times \frac{1}{k!} \times \frac{1}{(n-k)!} = \binom{n}{k}$$

Binomial Coefficient

Practice: Combinations of Books

Choose k of n distinct objects	$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$

How many ways are there to choose 3 books from a set of 6 distinct books?

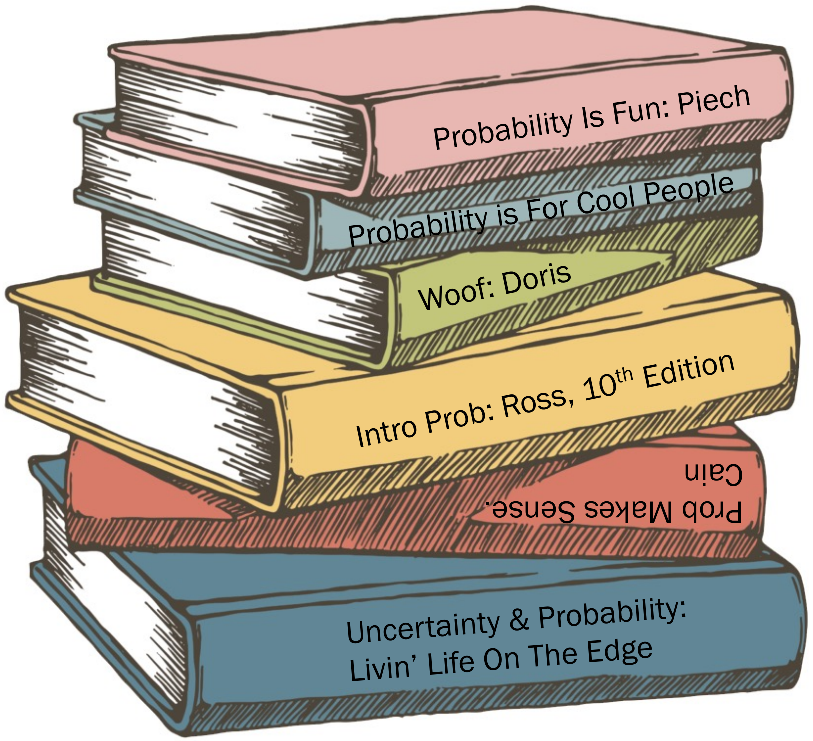


Practice: Combinations of Books

Choose k of n distinct objects $\binom{n}{k} = \dfrac{n!}{k!\,(n-k)!}$

How many ways are there to choose 3 books from a set of 6 distinct books?

$$\binom{6}{3} = \frac{6!}{3!\,3!} = 20 \text{ ways}$$



Can We Code It? Yes We Can

How many unique hands of 5 cards are there in a 52 card deck?

Can We Code It? Yes We Can

How many unique hands of 5 cards are there in a 52 card deck?

$$\binom{52}{5}$$

Can We Code It? Yes We Can

How many unique hands of 5 cards are there in a 52 card deck?


```
def main():
    total = math.comb(52, 5)
    print(total)
```

```
def main():
    cards = make_deck()
    all_hands = itertools.combinations(cards, 5)
    for hand in all_hands:
        print(hand)
```

Balls In Bins

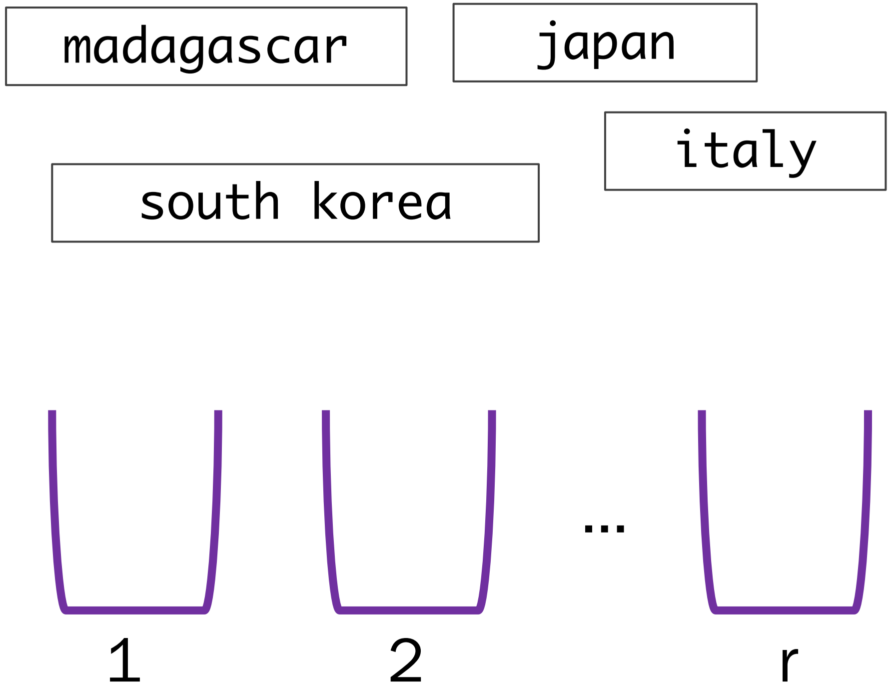
Combinatorics: Formulas For Common Counting Tasks

Counting tasks on n objects

- Sort objects (permutations)
  - Distinct (distinguishable): $n!$
  - Some distinct: $\dfrac{n!}{n_1!\,n_2!\cdots n_r!}$
- Choose k objects (combinations)
  - Distinct: $\binom{n}{k}$
- Put objects in r buckets
  - Distinct
  - Indistinct

~~Balls in bins~~ Hash tables and distinct strings

How many ways are there to hash n distinct strings to r buckets?



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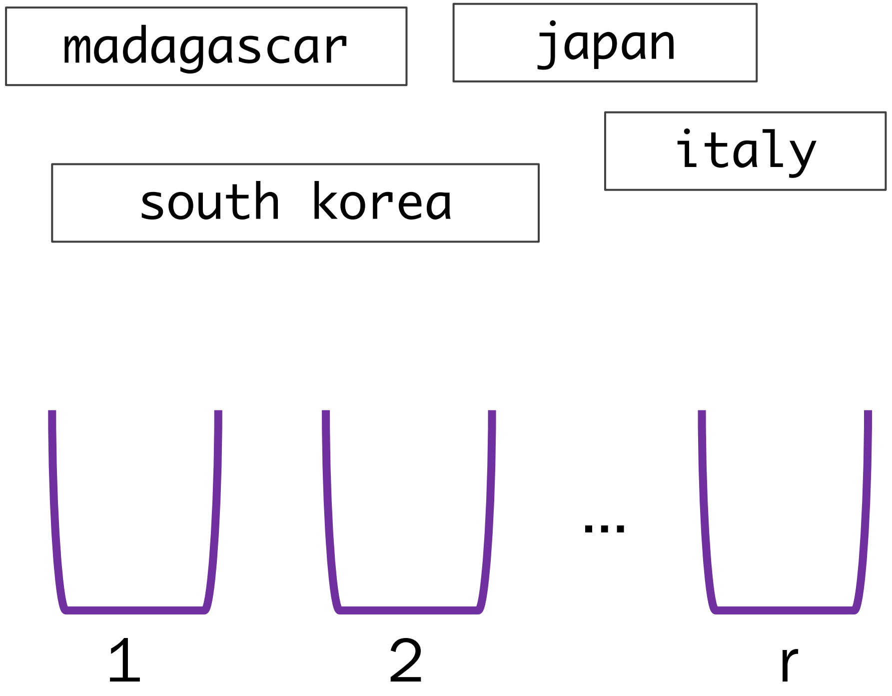

Steps:

1. Bucket 1st string – r choices
2. Bucket 2nd string – r choices

…

n. Bucket n^{th} string – r choices

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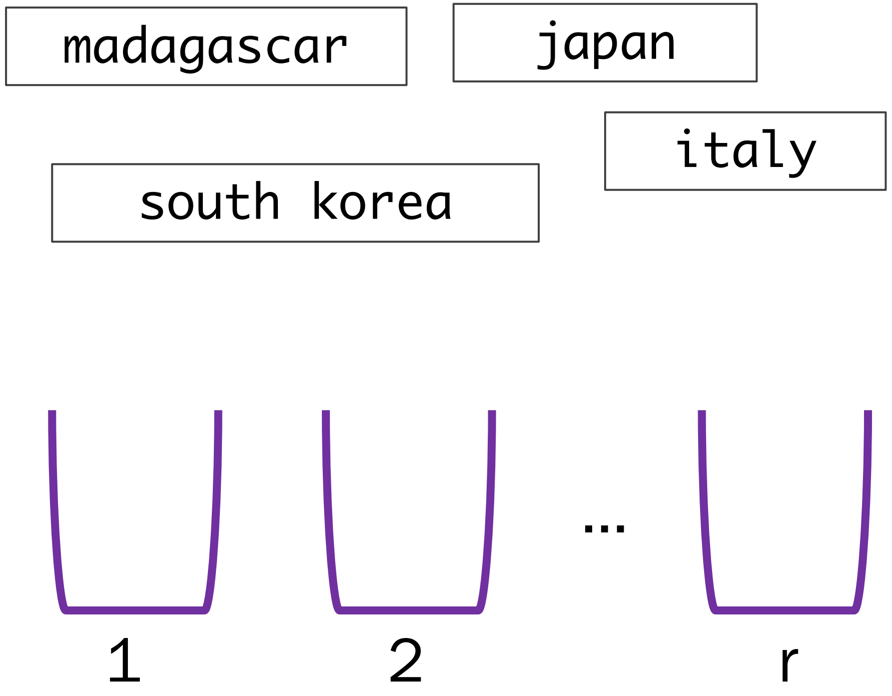

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r^n outcomes

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Choose k objects (combinations)

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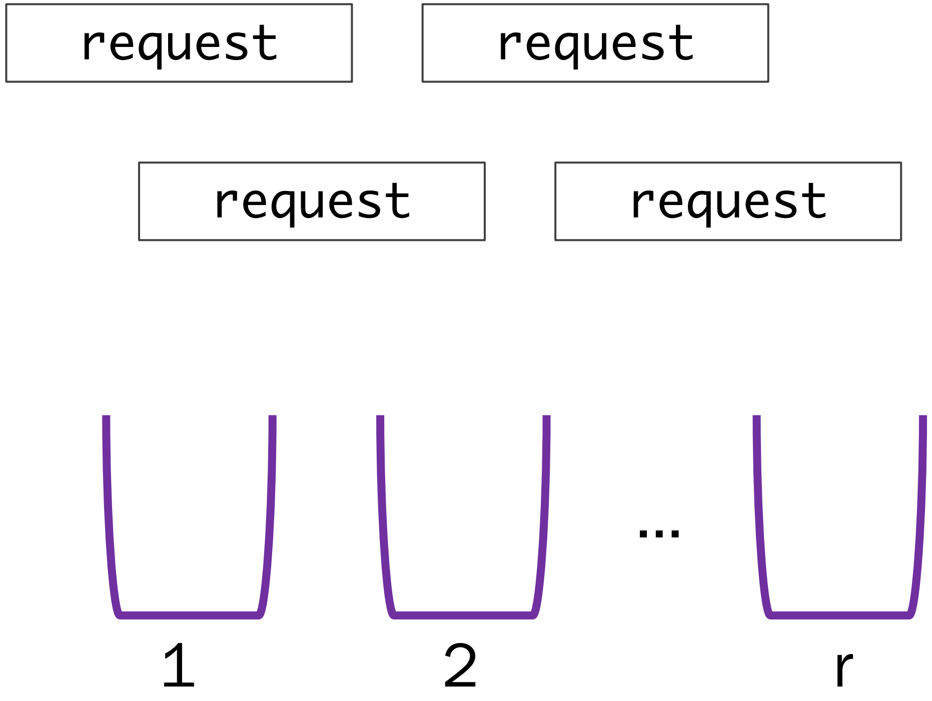
Put objects in r buckets

- Distinct: r^n
- Indistinct

Servers and indistinct requests

How many ways are there to distribute n indistinct web requests to r servers?



Servers and indistinct requests

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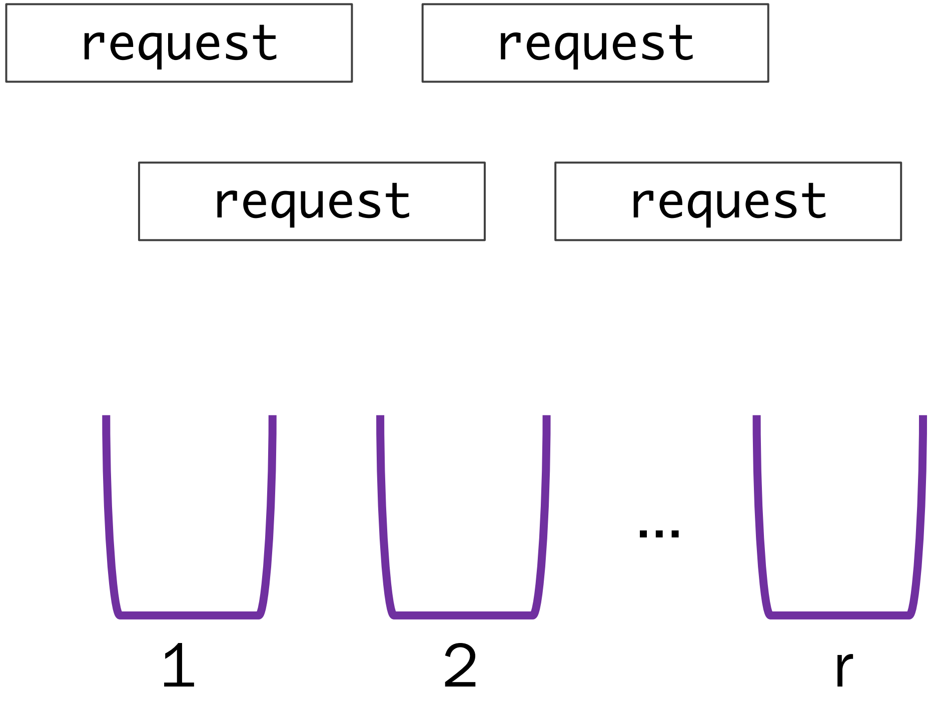


What does one outcome look like?

- Server 1 has x_1 requests,
- Server 2 has x_2 requests,

  ...
- Server r has x_r requests

constraint: $\sum_{i=1}^{r} x_i = n$

Flowers In Vases

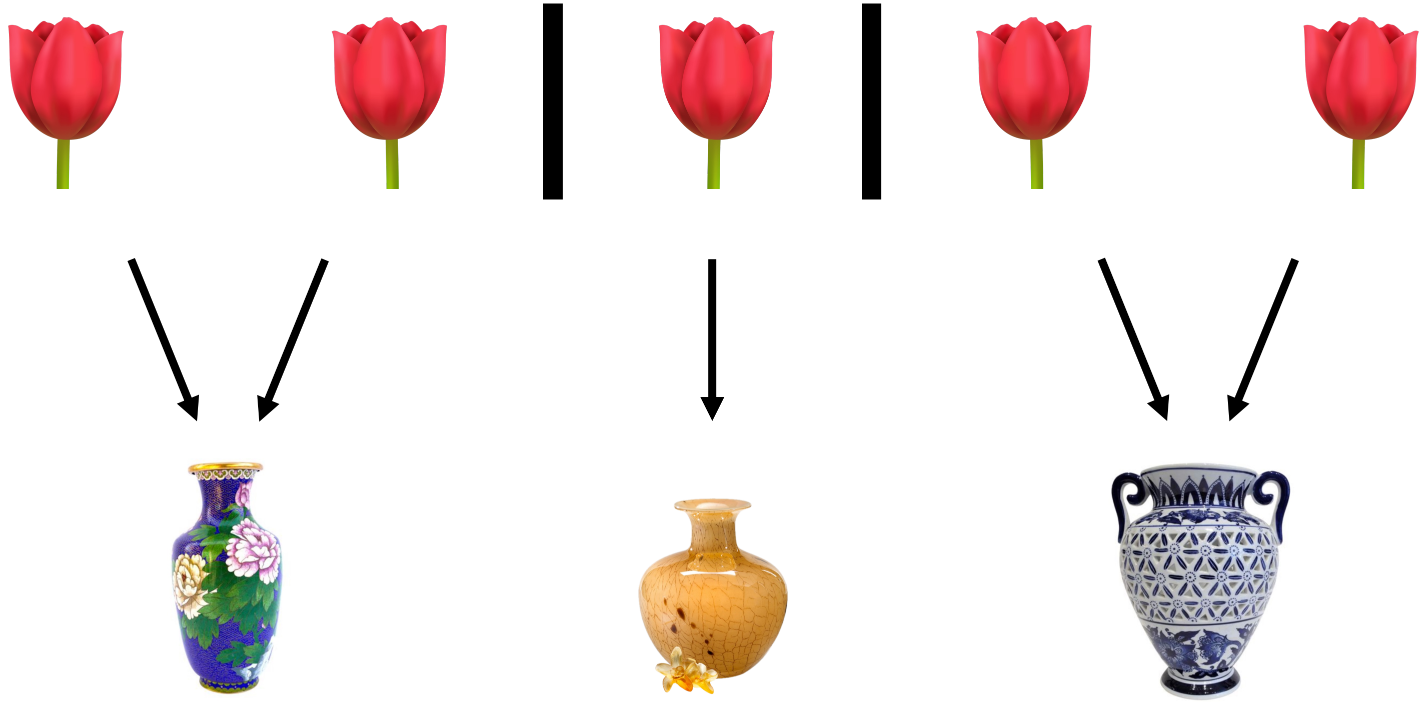
How many ways can we put $n = 5$ indistinct flowers in $r = 3$ distinct vases?

Flowers In Vases

What does one possible outcome look like? Can we build a generative story?

Flowers In Vases

What does one possible outcome look like? Can we build a generative story?



Goal: order n indistinct objects and $r - 1$ indistinct dividers.

Flowers In Vases

What does one possible outcome look like? Can we build a generative story?

Start by treating objects and dividers as distinct.

Goal: order n indistinct objects and $r - 1$ indistinct dividers.

Flowers In Vases

What does one possible outcome look like? Can we build a generative story?

Start by treating objects and dividers as distinct.

1. Order n distinct objects and $r-1$ distinct dividers

$$(n+r-1)!$$

Goal: order n indistinct objects and $r-1$ indistinct dividers.

Flowers In Vases

What does one possible outcome look like? Can we build a generative story?

Start by treating objects and dividers as distinct.

1. Order n distinct objects and $r-1$ distinct dividers

$$(n+r-1)!$$

2. Un-order n objects

$$\frac{1}{n!}$$

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Flowers In Vases

What does one possible outcome look like? Can we build a generative story?

Start by treating objects and dividers as distinct.

1. Order n distinct objects and $r-1$ distinct dividers

$$(n+r-1)!$$

2. Un-order n objects

$$\frac{1}{n!}$$

3. Un-order $r-1$ dividers

$$\frac{1}{(r-1)!}$$

Goal: order n indistinct objects and $r-1$ indistinct dividers.

The Divider Method: Indistinct Balls In Bins

The number of ways to distribute n indistinct objects into r buckets is

$$(n+r-1)! \times \frac{1}{n!} \times \frac{1}{(r-1)!} = \boxed{\binom{n+r-1}{r-1}}$$

This is equivalent to the number of ways to permute $n+r-1$ objects,

such that n are indistinct objects, and $r-1$ are indistinct dividers.

Balls in Bins (n indistinct objects, r buckets) $\binom{n+r-1}{r-1}$

Integer solutions to equations

How many integer solutions are there to the following equation:

$$x_1 + x_2 + \cdots + x_r = n,$$

where for all i, x_i is an integer such that $0 \leq x_i \leq n$?

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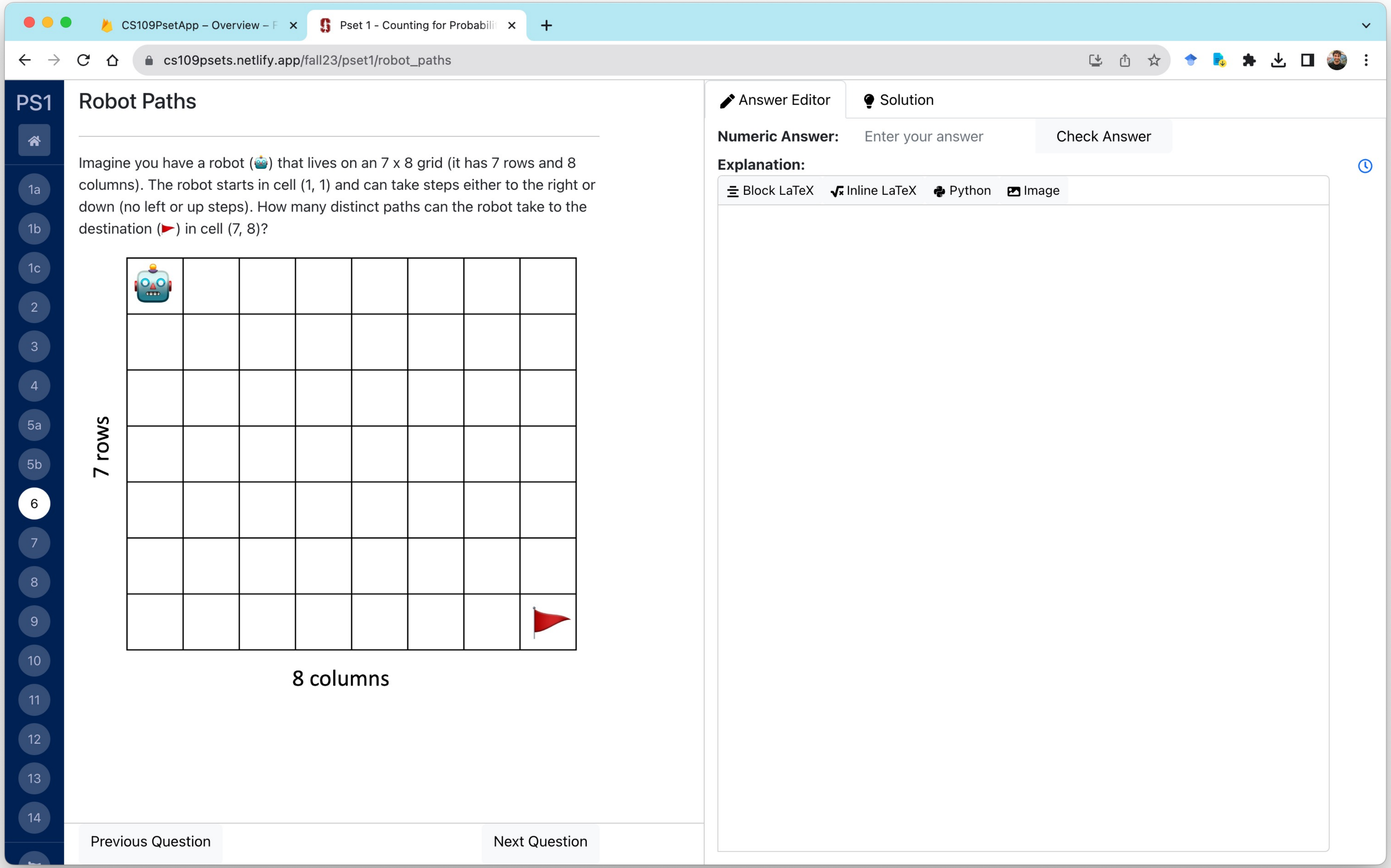
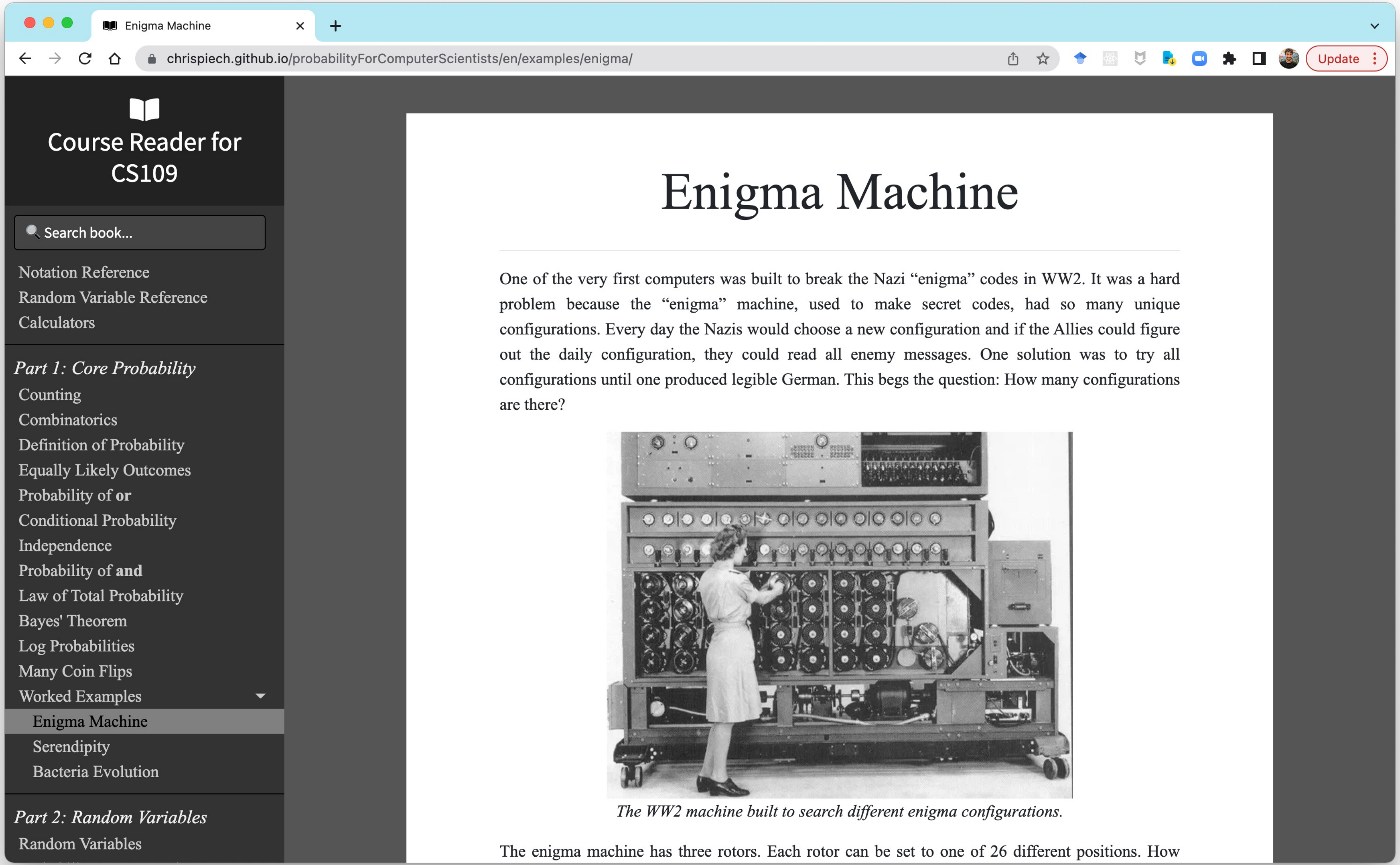
- **Choose k objects (combinations)**
  - Distinct: $\binom{n}{k}$
- **Put objects in r buckets**
  - Distinct: r^n
  - Indistinct: $\binom{n+r-1}{r-1}$

You’re ready for the first 6 problems now!



Want to go deeper? Cool examples in the course reader



Up next: going from counts to probabilities…