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CS109

Section #2
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Section 2: Random Variables

1 Warmups

1. Definitions: Cite Bayes' Theorem. Can you explain to your partner why $P(A|B)$ is different than $P(B|A)$?
2. True or False: $P(AB|C) = P(B|C)P(A|BC)$.

2 Conditional Probabilities: Missing Not at Random

Preamble: We have three big tools for manipulating conditional probabilities:

- Definition of conditional probability: $P(EF) = P(E|F)P(F)$
- Law of Total Probability: $P(E) = P(EF) + P(EF^C) = P(E|F)P(F) + P(E|F^C)P(F^C)$
- Bayes Rule: $P(E|F) = \frac{P(F|E)P(E)}{P(F)} = \frac{P(F|E)P(E)}{P(F|E)P(E)+P(F|E^C)P(E^C)}$

This is a good time to commit these three to memory and start thinking about when each of them is useful.

Problem: You recently tried out a new collaborative note-taking tool in class and want to find out if students like it. You send an email to all 100 people in your class, asking them to reply with whether or not they liked it.

User Response	**Count**
Responded that they liked your tool	40
Responded that they didn't like your tool	45
Did not respond	15

Let L be the event that a person did like your tool. Let M be the event that a person did not respond to your email. We are interested in estimating $P(L)$, however that is hard given the 15 people who did not respond.

a. What is the probability that a user liked your tool and that they responded to the poll $P(L \text{ and } M^C)$?

b. Which formula from class would you use to calculate $P(L)$? Your formula should rely on the context that people who like your tool are in one of two (mutually exclusive) groups: those that did reply, and those that did not.

c. Calculate $P(L)$. You estimate that the probability that someone did not reply, given that they liked the tool is $P(M|L) = \frac{1}{5}$.

3 Taking Expectation: Breaking Vegas

Preamble: When a random variable fits neatly into a family we've seen before (e.g. Binomial), we get its expectation for free. When it does not, we have to use the definition of expectation.

Problem: If you bet on "Red" in Roulette, there is $p = 18/38$ that you with win $Y and a $(1 - p)$ probability that you lose $Y. Consider this algorithm for a series of bets:

Let Y = $1. First you bet Y. If you win, then stop. If you lose, then set Y to be 2Y and repeat.

What are your expected winnings when you stop? It will help to recall that the sum of a geometric series $a^0 + a^1 + a^2 + \cdots = \frac{1}{1-a}$ if $0 < a < 1$. Vegas breaks you: Why doesn't everyone do this?

4 Binomial Babies

Each child in a daycare has a 0.2 probability of having disease A, and has an independent 0.4 probability of having disease B. A child is sick if they have either disease A or disease B.

a. What is the probability that a child is sick?

b. If there are 10 children in a daycare, what is the probability that 3 or more are sick?