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CS109

Section #2
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Section #2: Random Variables

1 Server Downloads

If this problem doesn't convince you that the Poisson and Exponential RVs are coupled, then I'm not sure what will!

A server set up for large file downloads only has enough bandwidth for one download at a time. We can assume there is an infinite queue of downloads requested by users, such that immediately after one download finishes, another download always begins. On average, downloads take 5 minutes to complete, and the time until a download completes is exponentially distributed.

a. Using the random variable X, defined as the length of time a download takes to complete, what is the probability that a download takes longer than 10 mins?

b. Using the random variable Y, defined as the number of downloads that finish over a 10-minute interval, what is the probability that a download takes longer than 10 mins?

2 Continuous Random Variables

Let X be a continuous random variable with the following probability density function:

$$f_X(x) = \begin{cases} c(e^{x-1} + e^{-x}) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the value of c that makes f_X a valid probability distribution.

b. What is $P(X < 0.75)$? What is $P(X < x)$?

3 Better Evaluation of Eye Disease

When a patient has eye inflammation, eye doctors "grade" the inflammation. When "grading" inflammation they randomly look at a single 1 millimeter by 1 millimeter square in the patient's eye and count how many "cells" they see.

There is uncertainty in these counts. If the true average number of cells for a given patient's eye is 6, the doctor could get a different count (say 4, or 5, or 7) just by chance. As of 2021, modern eye medicine did not have a sense of uncertainty for their inflammation grades! In this problem we are going to change that. At the same time we are going to learn about poisson distributions over space.

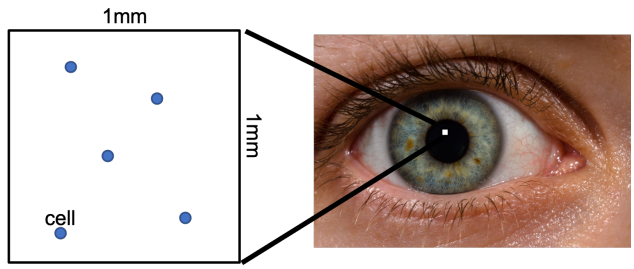

Figure 1: *A 1x1mm sample used for inflammation grading. Inflammation is graded by counting cells in a randomly chosen 1mm by 1mm square. This sample has 5 cells.*

a. Explain, as if teaching, why the number of cells observed in a 1x1 square is governed by a poisson process. Make sure to explain how a binomial distribution could approximate the count of cells. Explain what λ means in this context. Note: for a given person's eye, the presence of a cell in a location is independent of the presence of a cell in another location.

b. For a given patient the true average rate of cells is 5 cells per 1x1 sample. What is the probability that in a single 1x1 sample the doctor counts 4 cells?