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CS109

Section #3
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Problems by Chris

1 Conditional Flu (Optional)

If a person has the flu, the distribution of their temperature is Gaussian with mean 101 and variance 1. If a person does not have the flu, the distribution of their temperature is Gaussian with mean 98 and variance 1. All you know about a person is that they have a temperature of 100. What is the probability they have the flu? Historically, 20% of people you analyze have had the flu.

2 Algorithmic Fairness

An AI model makes a binary prediction (G for guess) for whether a person will repay a loan. It is important to show that the model is fair with respect to a binary demographic (D for demographic). But what does fair mean? Let's analyze the historical predictions of the model and compare the predictions to the true outcome (T for truth). Consider the following joint probability table from the model's history:

	D = 0		**D = 1**	
	G = 0	G = 1	G = 0	G = 1
T = 0	0.21	0.32	0.01	0.01
T = 1	0.07	0.28	0.02	0.08

D: is the demographic of an individual (binary).
G: is the "repay" prediction made by the algorithm. 1 means predicted repay.
T: is the true "repay" result. 1 means did repay.

Recall that cell (D = i,G = j,T = k) is the probability P(D = i,G = j,T = k).

a. (4 points) What is $P(D = 1)$?

b. (4 points) What is $P(G = 1|D = 1)$?

c. (6 points) Fairness definition 1: Parity
An algorithm satisfies "parity" if the probability that the algorithm makes a positive prediction ($G = 1$) is the same regardless of the demographic variable. Does this algorithm satisfy parity?

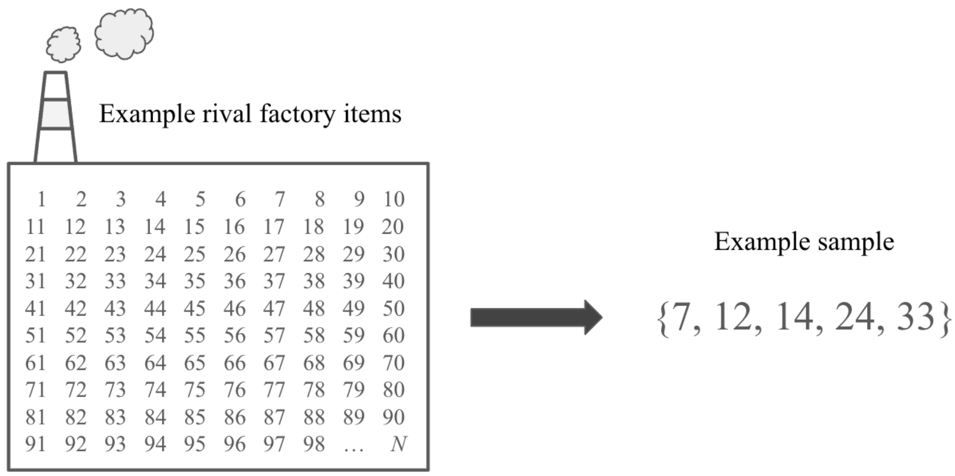
d. (6 points) Fairness definition 2: Calibration (Optional)
An algorithm satisfies "calibration" if the probability that the algorithm is correct ($G = T$) is the same regardless of demographics. In terms of "correct" we need to separately check whether the algorithm is equally likely to be correct for both outcomes ($G = T = 1$ and $G = T = 0$). Does this algorithm satisfy calibration?

e. (6 points) Fairness definition 3: Equality of odds (Optional)
An algorithm satisfies "equality of odds" if the probability that the algorithm predicts a positive outcome given that the true outcome is positive ($G = 1|T = 1$) is the same regardless of demographics. Does this algorithm satisfy equality of odds?

3 Tank Probability

A rival is producing items. We would like to estimate the number of items, N, that they have produced. We notice that each item has a unique serial number and we assume that when we acquire (sample) items each serial number on the item is a positive integer equally likely to be any number from the set $\{1, 2, \ldots, N\}$.

For example, if you randomly acquired (sampled) 5 items produced at the factory, you might see the serial numbers $\{7, 12, 14, 24, 33\}$ which should give you a clue as to what N could be!



a. (7 points) For part (a) only, assume $N = 100$. We sample 5 items. What is the probability that the largest serial number in our sample is 33? Let's solve this with equally likely outcome spaces!

b. (10 points) Your prior belief is that every value of N between 33 and 100 (inclusive) is equally likely. What is your updated probability mass function for N, given that you sampled 5 items and the largest serial number was 33?

c. (3 points) Given that you sampled 5 items and the largest serial number was 33, what is the probability that $N < 50$?

Historical Context: During World War 2, the Allies needed to know how many tanks Nazi Germany was producing. First they sent spies to Germany who estimated Germany produced 1,400 tanks per month. Separately, they noticed that the serial numbers on gear boxes on German tanks were unique and sequential. Using this observation, and a sample of gear box serial codes, mathematicians at the US Statistical Research Group used the math you derived to estimate the amount of tanks produced by Nazi Germany. They estimated expected production was 270 tanks per month. After the war, German records confirmed an actual production rate of 276 tanks per month—the probabilistic method was incredibly accurate!