1 Product Rule of Counting

1. Suppose a university ID uses the following scheme: “The first two digits of your 7-digit student number are the year you were admitted. The third digit indicates the quarter you were admitted: 1 for summer quarter; 2 and 3 for autumn, 4 for winter, and 5 for spring. The rest of the digits are random.” E.g. A student admitted in the fall of 2020 could have a student number of “2021234”. What is the maximum number of new VALID and distinct student numbers from winter 2021 - fall 2022 (spanning two years)?
Answer: 100000

2. The 9 course staff compete in a race. In how many distinct ways can the 3 medals (gold, silver, bronze) be awarded if Mitchell wins either a silver or a gold?
Hint: Think about the two cases separately.
Answer: 112

2 Permutations

1. How many distinguishable ways can the letters of “doge” be arranged?
Answer: 24

2. How many distinguishable ways can the letters of “doggy” be arranged?
Hint: A letter shows up twice in this string. How can we account for this?
Answer: 105
Explanation: We get it by first treating all 7 letters as all DIFFERENT, i.e. 7! ways to rearrange them. Then, we notice that the letter ‘o’ is repeated four times along with ‘g’ repeating twice. So we account for these un-unique permutations by dividing 7! by 4! and 2! respectively, which gives us the final answer 105.

3 Combinations

1. A typical starting line-up for a soccer team has 1 goalkeeper, 4 defenders, 4 midfielders and 2 forwards. The 2019 Men’s soccer team roster has 4 goalkeepers, 8 defenders, 18 midfielders and 3 forwards. In how many ways can the head coach choose the starting line-up?
Hint: Product rule of counting
Answer: 2570400

4 Complementary Counting

1. How many ways are there for the 9 staff members be seated in a row, if Alex cannot be at either end?
Answer: 282240

2. How many ways are there for the 9 staff members be seated in a row, if Alex and Mitchell cannot both be at the ends?
Answer: 352800

5 Factorial Algebra

Key Takeaway: \((n + 1)! = (n + 1) \cdot n!\)

1. Write 8 · 7 · 6 · 5 · 4 as a ratio of two factorials
Format: [Number]!/[Number]!
Answer: 8!/3!

2. Solve for \(n\): \(\frac{n+2}{4} = 60\).
Answer: \(n = 3\)

\(^1\) from UW
6 Multinomial Coefficients

1. How many ways are there to distribute 15 people into 4 different Zoom breakout rooms such that the first three rooms have 4 people, and the last room has 3 people?
   Answer: 15765750

7 Divider Method

1. A hypothetical Meme Awards Committee is picking the “best meme” of 2020. How many ways can they give 10 *indistinguishable* nominations to 3 different people, assuming they each get at least one vote?
   
   Hint: It’s a variation of Divider Method (aka Stars and Bars) in that it “forces a star between each bar”. Because of the condition that “they each get at least one vote”, consider giving each one vote first before distributing the remaining 7 votes.
   
   Answer: 36

   Explanation: Because of the condition that “they each get at least one vote”, we cannot have a ”bar” at either side of the arrangement (i.e. a “star” must be at both ends). Therefore we have ”9 slots” to place 2 (3-1) bars: \( \binom{9}{2} = 36 \)

8 Inclusion Exclusion Principle

1. How many rearrangements of “doggy” are there such that it either begins with the letter “d” OR ends with the letter “y”?  
   
   Hint: Use the Inclusion Exclusion Principle
   
   Answer: 21