

CS109A Combinatorics Extra Practice

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1 Product Rule of Counting

1. Suppose a university ID uses the following scheme¹: “The first two digits of your 7-digit student number are the year you were admitted. The third digit indicates the quarter you were admitted: 1 for summer quarter; 2 and 3 for autumn, 4 for winter, and 5 for spring. The rest of the digits are random.” E.g. A student admitted in the fall of 2020 could have a student number of “2021234”.

What is the maximum number of new VALID and distinct student numbers from winter 2021 - fall 2022 (spanning two years)?

Answer: 100000

2. The 9 course staff compete in a race. In how many distinct ways can the 3 medals (gold, silver, bronze) be awarded if Mitchell wins either a silver or a gold?

Hint: Think about the two cases separately.

Answer: 112

2 Permutations

1. How many distinguishable ways can the letters of “doge” be arranged?

Answer: 24

2. How many distinguishable ways can the letters of “doggy” be arranged?

Hint Hint: A letter shows up twice in this string. How can we account for this?

Answer: 105

Explanation: We get it by first treating all 7 letters as all DIFFERENT, i.e. $7!$ ways to rearrange them. Then, we notice that the letter 'o' is repeated four times along with 'g' repeating twice. So we account for these un-unique permutations by dividing $7!$ by $4!$ and $2!$ respectively, which gives us the final answer 105.

3 Combinations

1. A typical starting line-up for a soccer team has 1 goalkeeper, 4 defenders, 4 midfielders and 2 forwards. The 2019 Men's soccer team roster has 4 goalkeepers, 8 defenders, 18 midfielders and 3 forwards. In how many ways can the head coach choose the starting line-up?

Hint: Product rule of counting

Answer: 2570400

4 Complementary Counting

1. How many ways are there for the 9 staff members be seated in a row, if Alex cannot be at either end?

Answer: 282240

2. How many ways are there for the 9 staff members be seated in a row, if Alex and Mitchell cannot both be at the ends?

Answer: 352800

5 Factorial Algebra

Key Takeaway: $(n + 1)! = (n + 1) \cdot n!$

1. Write $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$ as a ratio of two factorials

Format: [Number]!/[Number]!

Answer: $8!/3!$

2. Solve for n : $\frac{n+2}{n-1} = 60$.

Answer: $n = 3$

¹from UW

6 Multinomial Coefficients

1. How many ways are there to distribute 15 people into 4 different Zoom breakout rooms such that the first three rooms have 4 people, and the last room has 3 people?

Answer: 15765750

7 Divider Method

1. A hypothetical Meme Awards Committee is picking the “best meme” of 2020. How many ways can they give 10 *indistinguishable* nominations to 3 different people, assuming they each get at least one vote?

Hint: It’s a variation of Divider Method (aka Stars and Bars) in that it “forces a star between each bar”. Because of the condition that “they each get at least one vote”, consider giving each one vote first before distributing the remaining 7 votes.

Answer: 36

Explanation: Because of the condition that “they each get at least one vote”, we cannot have a “bar” at either side of the arrangement (i.e. a “star” must be at both ends). Therefore we have “9 slots” to place 2 (3-1) bars: $C(9,2) = 36$

8 Inclusion Exclusion Principle

1. How many rearrangements of “doggy” are there such that it either begins with the letter “d” OR ends with the letter “y”?

Hint: Use the Inclusion Exclusion Principle

Answer: 21