Global Optimization

Lecture 15

Lecture Outline

- Global flow analysis
- Global constant propagation
- Liveness analysis

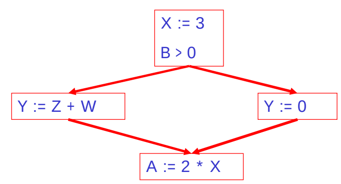
Local Optimization

Recall the simple basic-block optimizations
- Constant propagation
- Dead code elimination

```
X := 3          X := 3          Y := Z * W
Y := Z * W  ➡   Y := Z * W  ➡   Q := 3 + Y
Q := X + Y      Q := 3 + Y
```


Global Optimization

These optimizations can be extended to an entire control-flow graph



Global Optimization

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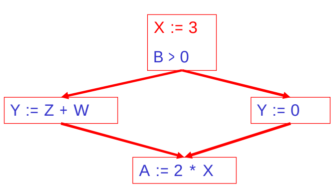

Global Optimization

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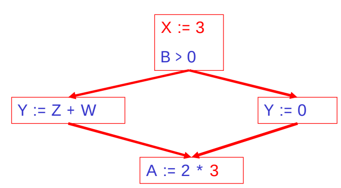
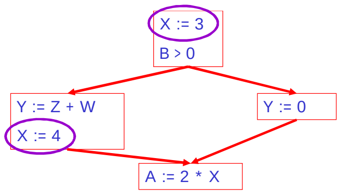


Correctness

- How do we know it is OK to globally propagate constants?
- There are situations where it is incorrect:

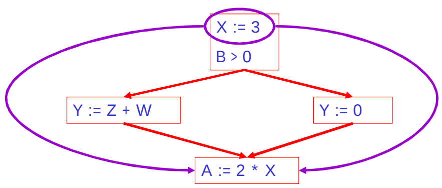


Correctness (Cont.)

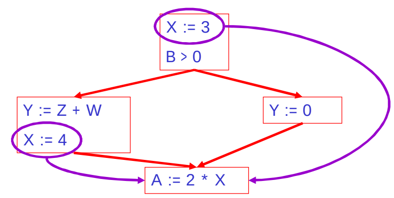
To replace a use of x by a constant k we must know that:

On every path to the use of x, the last assignment to x is x := k **

Example 1 Revisited



Example 2 Revisited



Discussion

- The correctness condition is not trivial to check
- "All paths" includes paths around loops and through branches of conditionals
- Checking the condition requires global analysis
  - An analysis of the entire control-flow graph

Global Analysis

Global optimization tasks share several traits:
- The optimization depends on knowing a property X at a particular point in program execution
- Proving X at any point requires knowledge of the entire program
- It is OK to be conservative. If the optimization requires X to be true, then want to know either
  - X is definitely true
  - Don't know if X is true
- It is always safe to say "don't know"

Global Analysis (Cont.)

- *Global dataflow analysis* is a standard technique for solving problems with these characteristics

- Global constant propagation is one example of an optimization that requires global dataflow analysis

Global Constant Propagation

- Global constant propagation can be performed at any point where ** holds

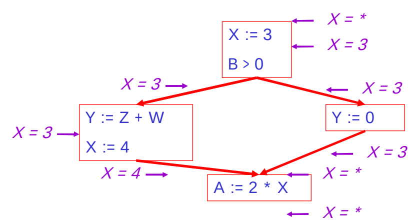
- Consider the case of computing ** for a single variable X at all program points

Global Constant Propagation (Cont.)

- To make the problem precise, we associate one of the following values with X at every program point

value	*interpretation*
#	This statement never executes
c	X = constant c
*	X is not a constant

Example



Using the Information

- Given global constant information, it is easy to perform the optimization
  - Simply inspect the x = ? associated with a statement using x
  - If x is constant at that point replace that use of x by the constant

- But how do we compute the properties x = ?

The Idea

The analysis of a complicated program can be expressed as a combination of simple rules relating the change in information between adjacent statements

Explanation

- The idea is to "push" or "transfer" information from one statement to the next
- For each statement s, we compute information about the value of x immediately before and after s

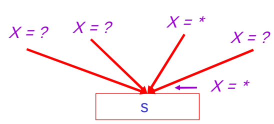
  C(x,s,in) = value of x before s

  C(x,s,out) = value of x after s

Transfer Functions

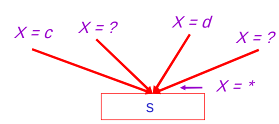
- Define a *transfer* function that transfers information one statement to another
- In the following rules, let statement s have immediate predecessor statements $p_1,\dots,p_n$

Rule 1



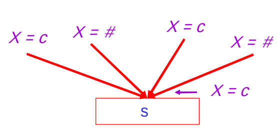
if C(p_i, x, out) = * for any i, then C(s, x, in) = *

Rule 2



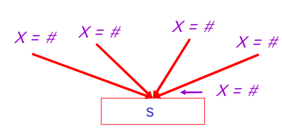
C(p_i, x, out) = c & C(p_j, x, out) = d & d <> c then C(s, x, in) = *

Rule 3



if C(p_i, x, out) = c or # for all i, then C(s, x, in) = c

Rule 4

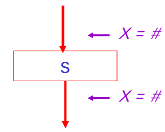


if C(p_i, x, out) = # for all i, then C(s, x, in) = #

The Other Half

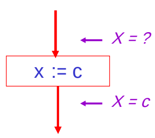
- Rules 1-4 relate the *out* of one statement to the *in* of the next statement
- Now we need rules relating the *in* of a statement to the *out* of the same statement

Rule 5



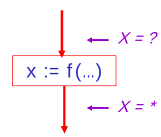
C(s, x, out) = # if C(s, x, in) = #

Rule 6



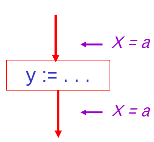
C(x := c, x, out) = c if c is a constant

Rule 7



C(x := f(…), x, out) = *

Rule 8



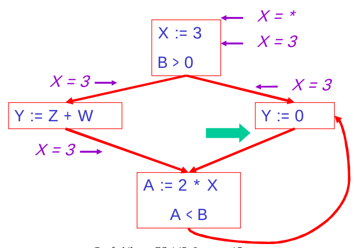
C(y := …, x, out) = C(y := …, x, in) if x <> y

An Algorithm

1. For every entry s to the program, set C(s, x, in) = *
2. Set C(s, x, in) = C(s, x, out) = # everywhere else
3. Repeat until all points satisfy 1-8:
   Pick s not satisfying 1-8 and update using the appropriate rule

The Value

- To understand why we need #, look at a loop



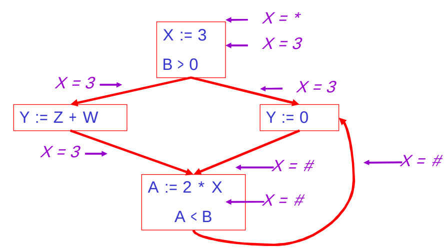
Discussion

- Consider the statement Y := 0
- To compute whether X is constant at this point, we need to know whether X is constant at the two predecessors
  - X := 3
  - A := 2 * X
- But info for A := 2 * X depends on its predecessors, including Y := 0!

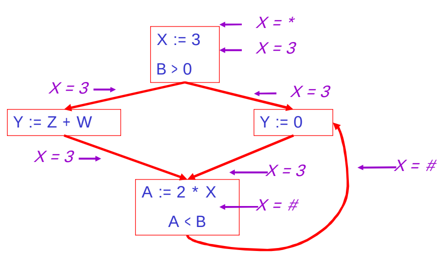
The Value # (Cont.)

- Because of cycles, all points must have values at all times
- Intuitively, assigning some initial value allows the analysis to break cycles
- The initial value # means "So far as we know, control never reaches this point"

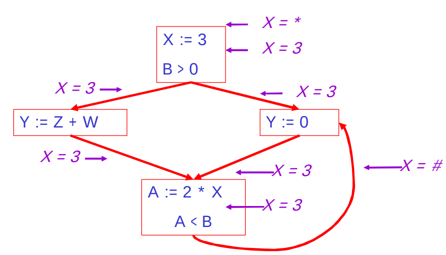
Example



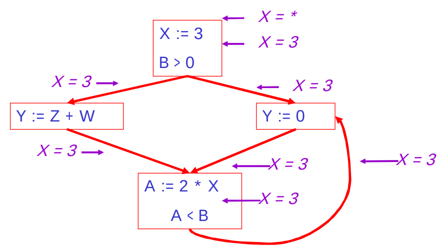
Example



Example



Example



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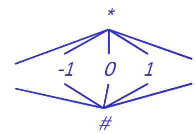
Orderings

- We can simplify the presentation of the analysis by ordering the values

  # < c < *

- Drawing a picture with "lower" values drawn lower, we get



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Orderings (Cont.)

- * is the greatest value, # is the least
  - All constants are in between and incomparable
- Let *lub* be the least-upper bound in this ordering
- Rules 1-4 can be written using lub:

  C(s, x, in) = lub { C(p, x, out) | p is a predecessor of s }

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Termination

- Simply saying "repeat until nothing changes" doesn't guarantee that eventually nothing changes
- The use of lub explains why the algorithm terminates
  - Values start as # and only *increase*
  - # can change to a constant, and a constant to *
  - Thus, C(s, x, _) can change at most twice

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Termination (Cont.)

Thus the algorithm is linear in program size

Number of steps =

Number of C(….) value computed * 2 =

Number of program statements * 4

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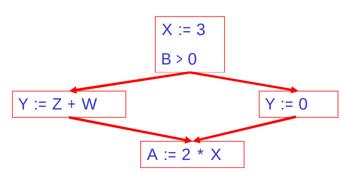
Liveness Analysis

Once constants have been globally propagated, we would like to eliminate dead code



After constant propagation, X := 3 is dead (assuming X not used elsewhere)

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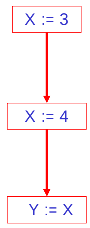
Live and Dead

- The first value of x is *dead* (never used)
- The second value of x is *live* (may be used)
- Liveness is an important concept



Liveness

A variable x is live at statement s if

- There exists a statement s' that uses x
- There is a path from s to s'
- That path has no intervening assignment to x

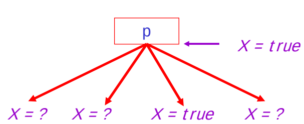
Global Dead Code Elimination

- A statement x := … is dead code if x is dead after the assignment
- Dead statements can be deleted from the program
- But we need liveness information first . . .

Computing Liveness

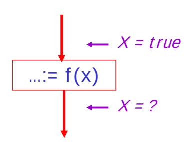
- We can express liveness in terms of information transferred between adjacent statements, just as in copy propagation
- Liveness is simpler than constant propagation, since it is a boolean property (true or false)

Liveness Rule 1



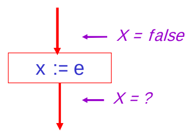
L(p, x, out) = ∨ { L(s, x, in) | s a successor of p }

Liveness Rule 2



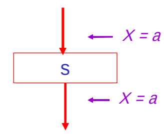
L(s, x, in) = true if s refers to x on the rhs

Liveness Rule 3



L(x := e, x, in) = false if e does not refer to x

Liveness Rule 4



L(s, x, in) = L(s, x, out) if s does not refer to x

Algorithm

1. Let all L(…) = false initially
2. Repeat until all statements s satisfy rules 1-4

   Pick s where one of 1-4 does not hold and update using the appropriate rule

Termination

- A value can change from false to true, but not the other way around
- Each value can change only once, so termination is guaranteed
- Once the analysis is computed, it is simple to eliminate dead code

Forward vs. Backward Analysis

We've seen two kinds of analysis:

Constant propagation is a *forwards* analysis: information is pushed from inputs to outputs

Liveness is a *backwards* analysis: information is pushed from outputs back towards inputs

Analysis

- There are many other global flow analyses
- Most can be classified as either forward or backward
- Most also follow the methodology of local rules relating information between adjacent program points