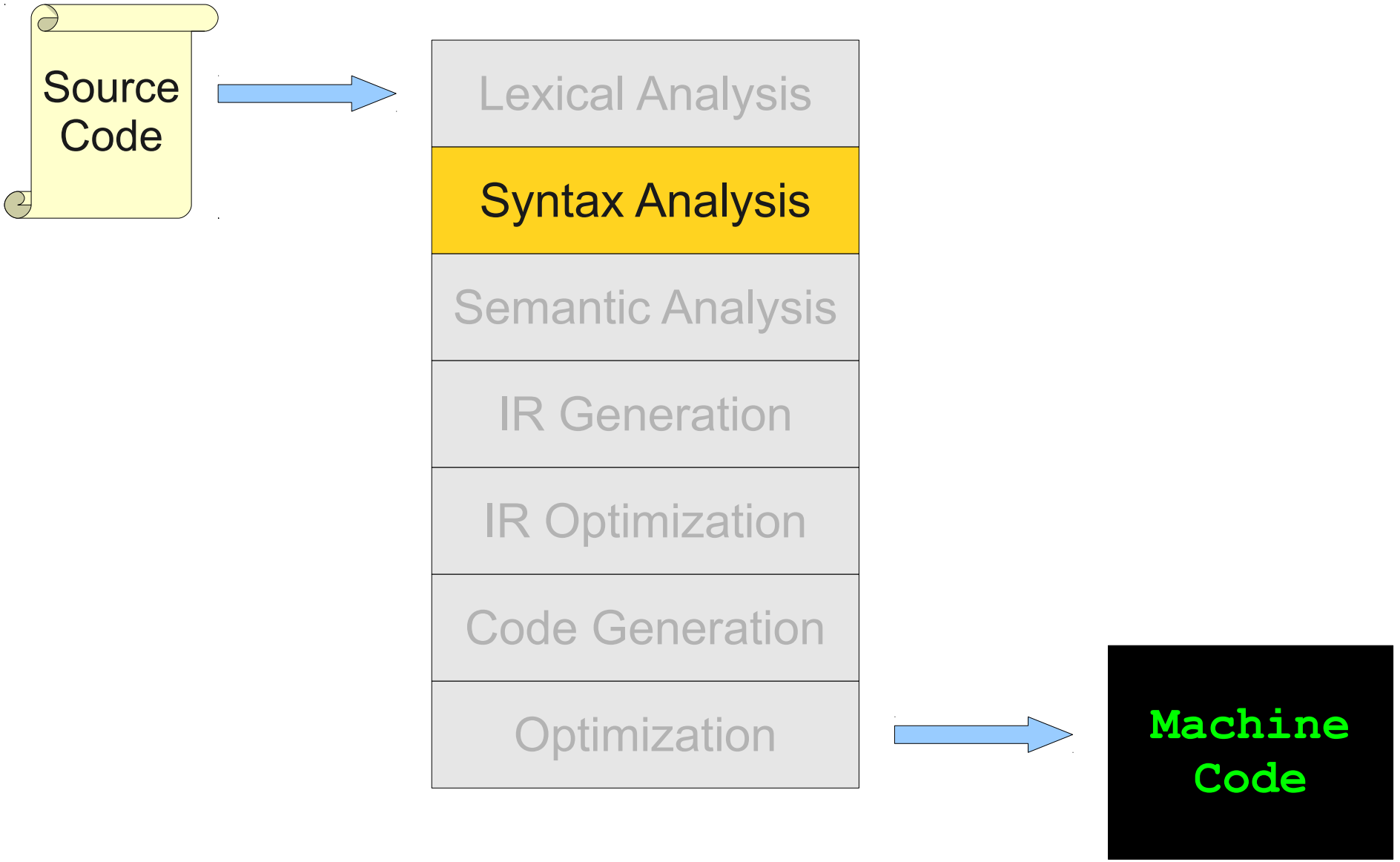
Bottom-Up Parsing III

Announcements

- Written Assignment 1 due **5:00PM today**.
- Submit by email or in the drop-off box in the Gates open area (under the Stanford Venture Fund Laboratories entrance); the filing cabinet should be clearly marked and I'll be nearby.
- Programming Project 2 due one week from today at 11:59PM.

Where We Are


Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code

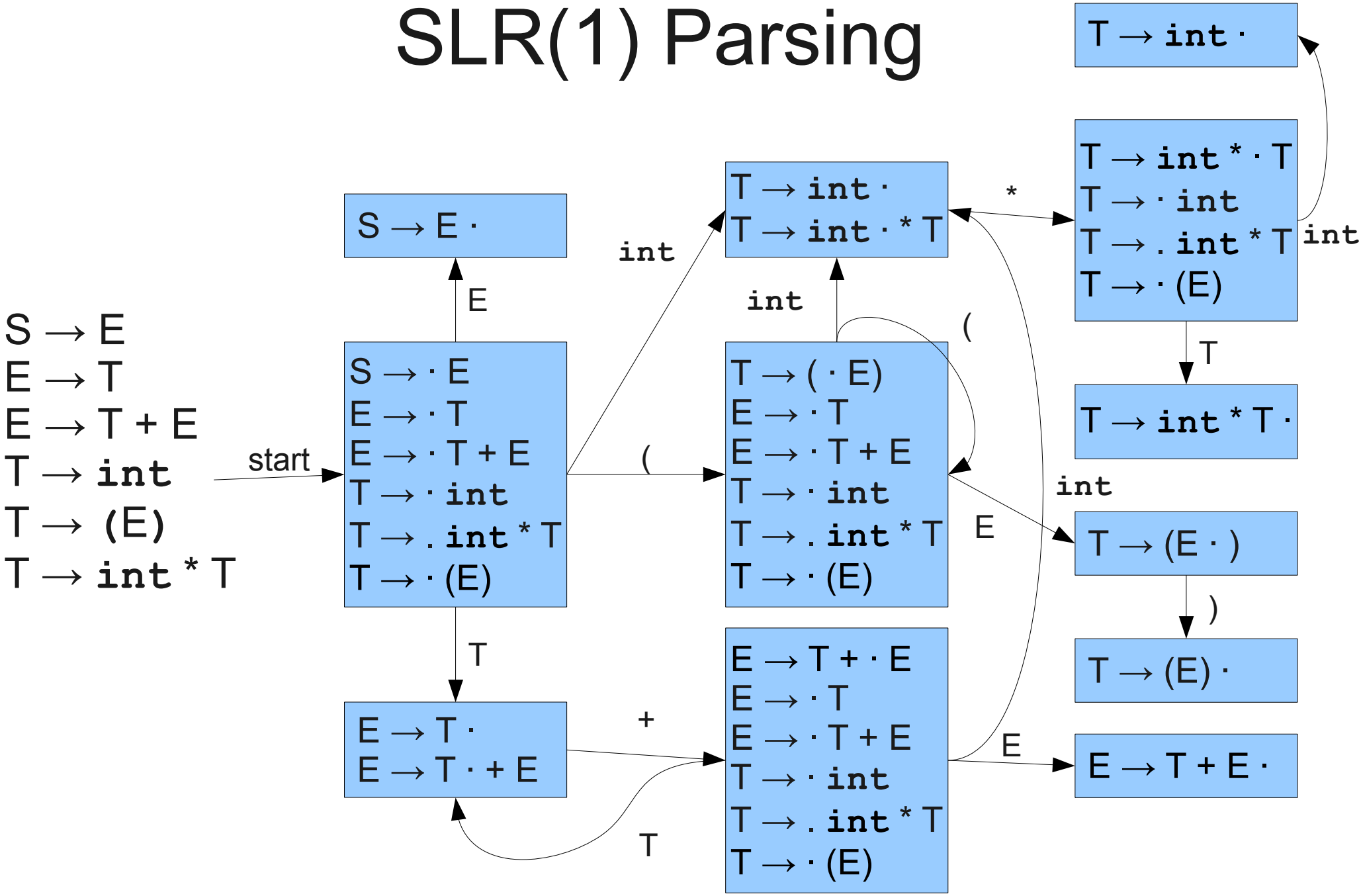
Recap from Last Time

- Bottom-up parsing is often done with a **handle-finding automaton**.
  - Automaton looks for viable prefixes; algorithm determines whether to shift or reduce.
- LR(0) parsing only works on grammars when reduces are unambiguous.
- LR(1) parsing works on a large number of grammars, but requires too large a parse table to be practical.
- SLR(1) parsing augments LR(0) with one lookahead token when a shift/reduce conflict is found.

SLR(1) Parsing

S → E
E → T
E → T + E
T → **int**
T → **(**E**)**
T → **int** * T

SLR(1) Parsing



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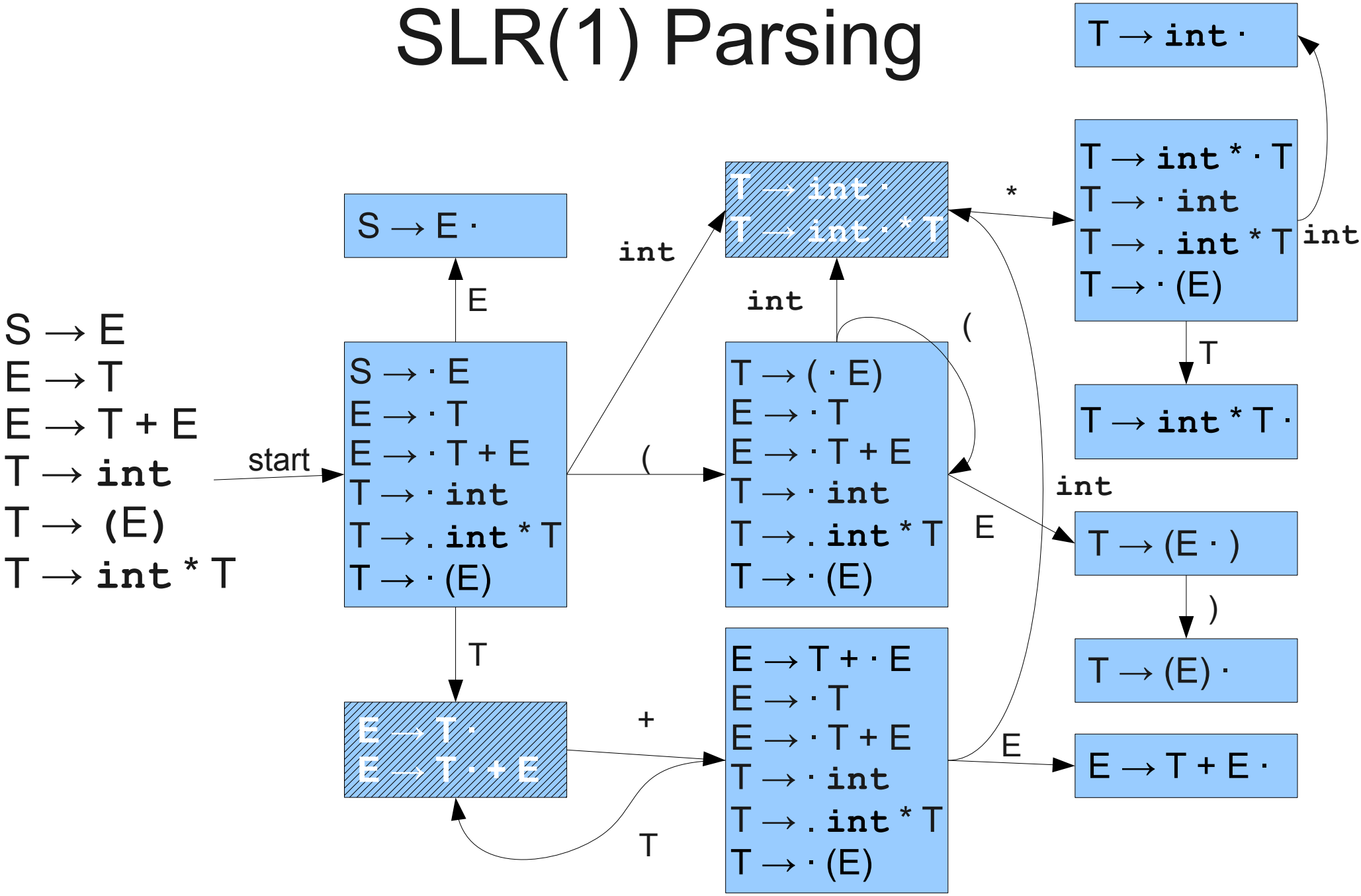

Review of SLR(1)

- Construct an LR(0) automaton.
- Only reduce $A \to v \cdot$ if the next lookahead token is in FOLLOW(A).
- Intuitively, don't reduce when there's no way that the reduction could produce A.

The Limits of SLR(1)

$S \rightarrow E$
$E \rightarrow L = R$
$E \rightarrow R$
$L \rightarrow \texttt{id}$
$L \rightarrow \texttt{*}R$
$R \rightarrow L$

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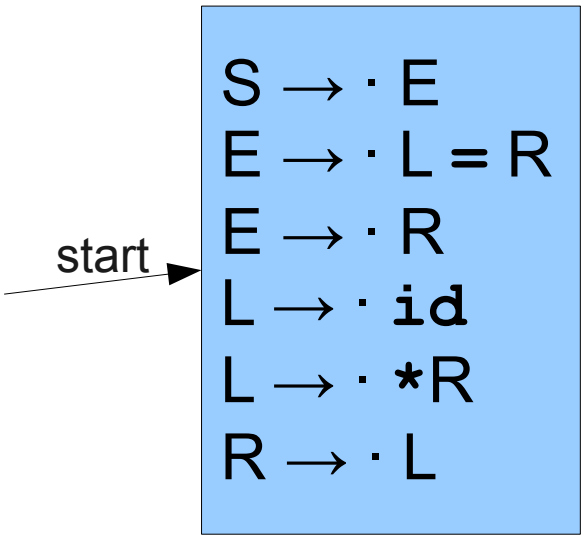

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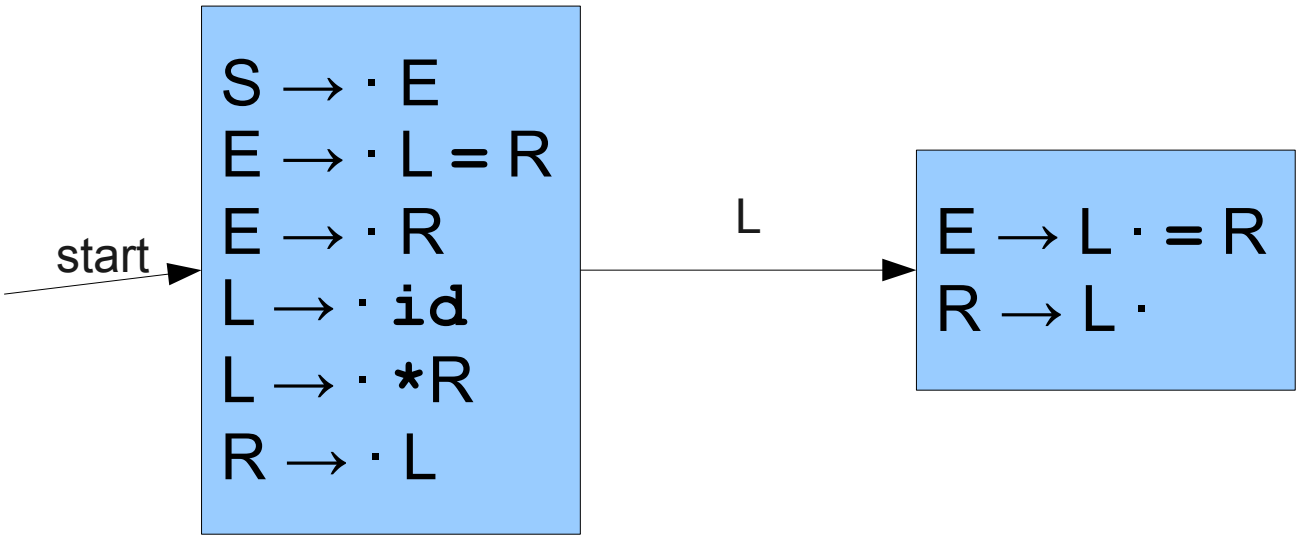

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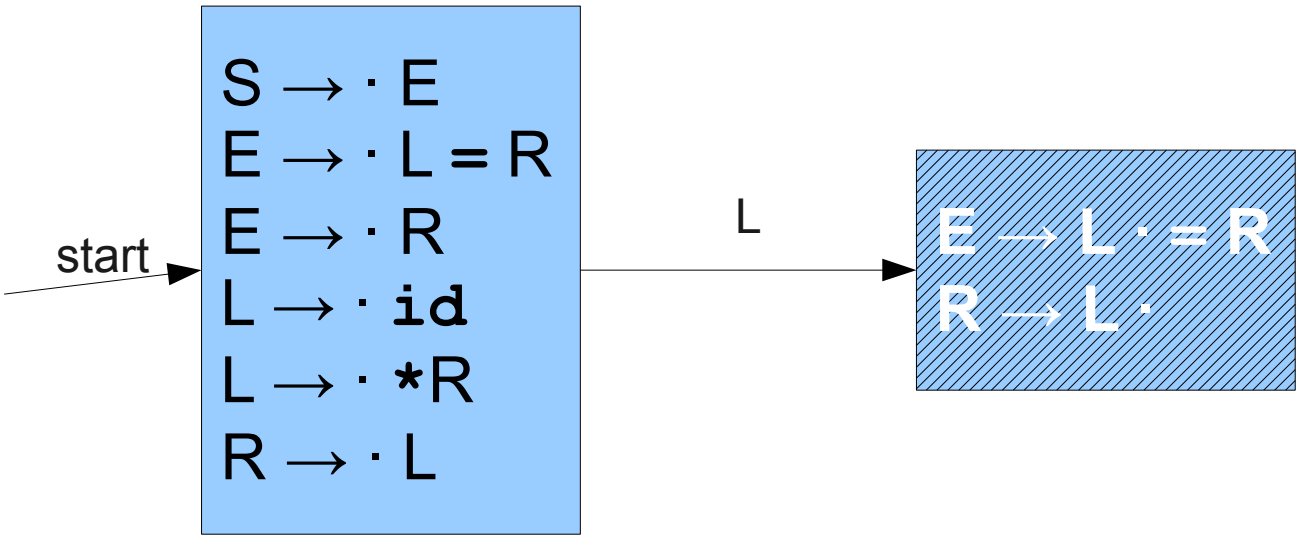

The Limits of SLR(1)

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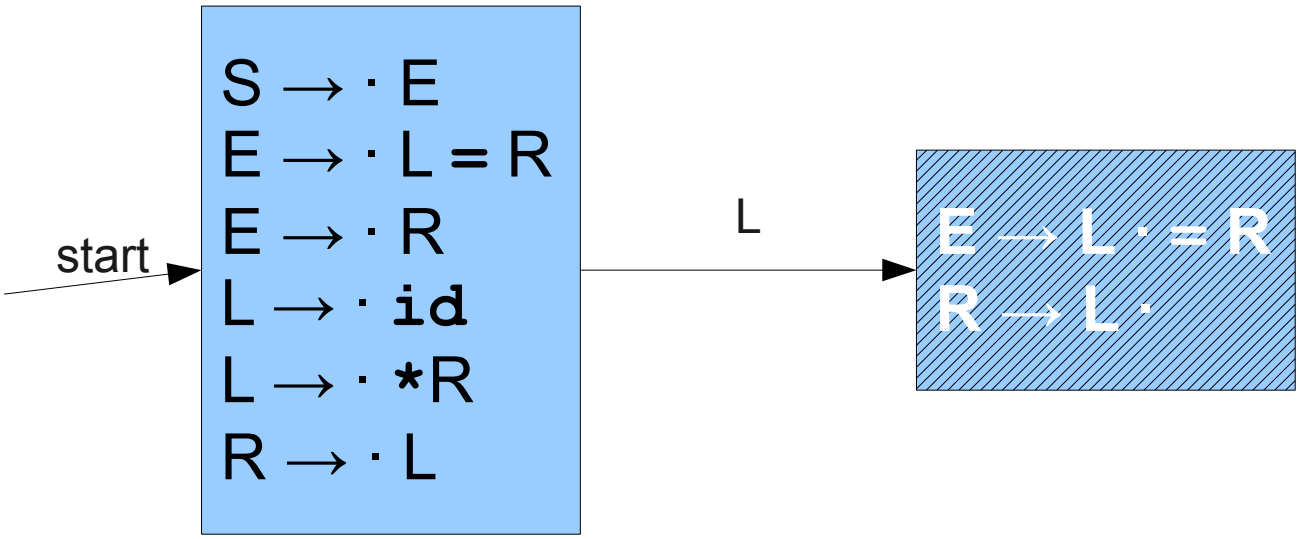
E → R
L → **id**
L → ***R**
R → L



E → L · = R tells us to shift on seeing =
R → L· tells us to reduce on FOLLOW(R).

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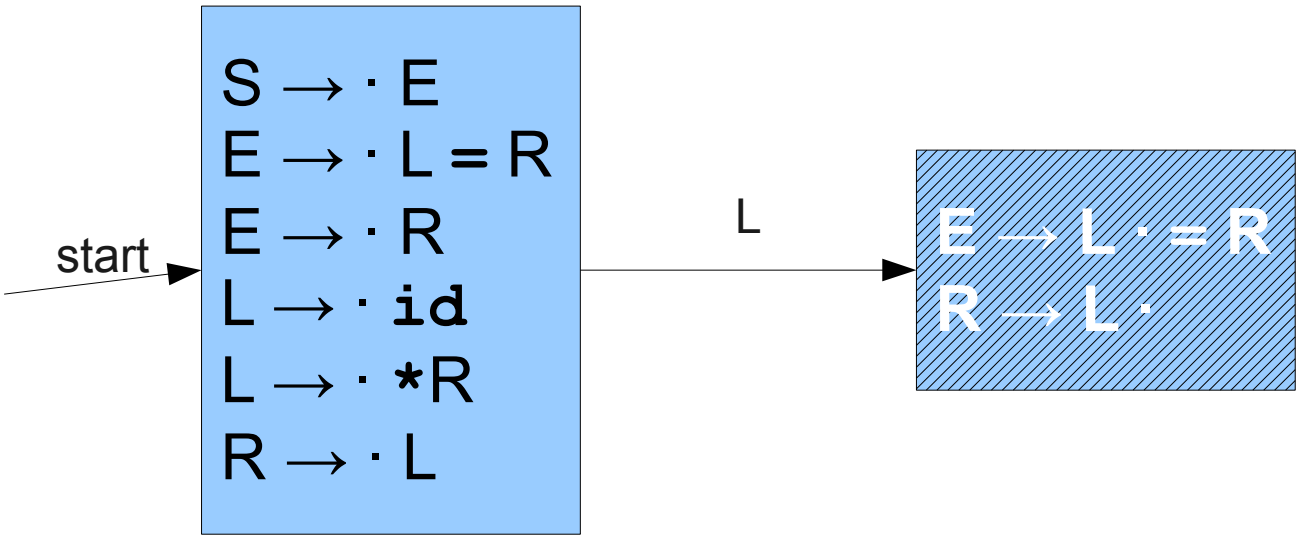

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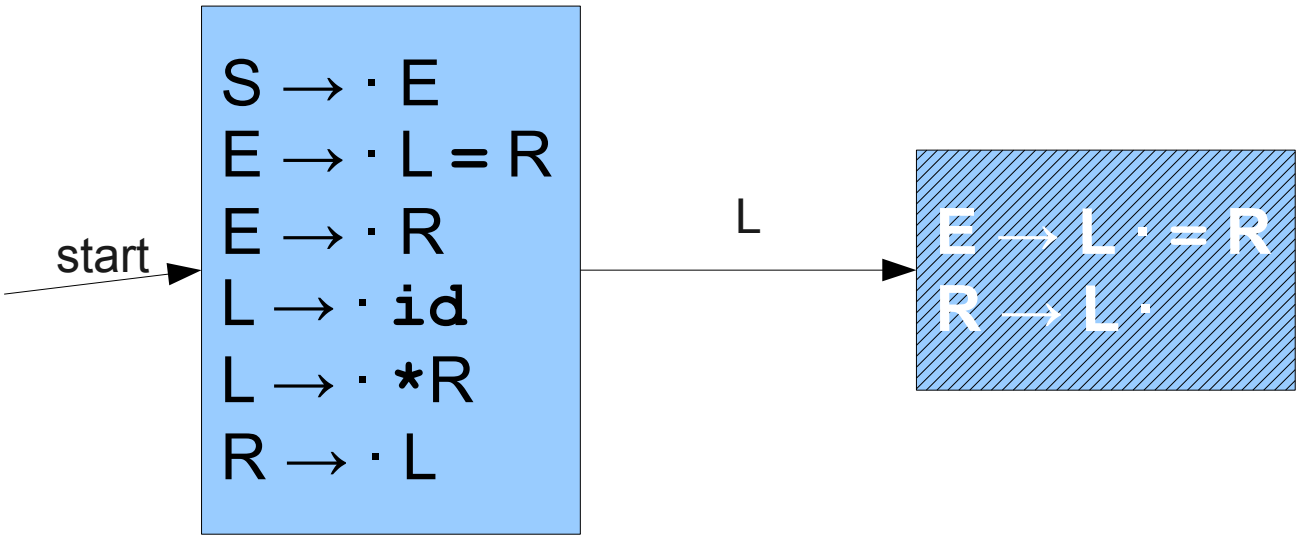

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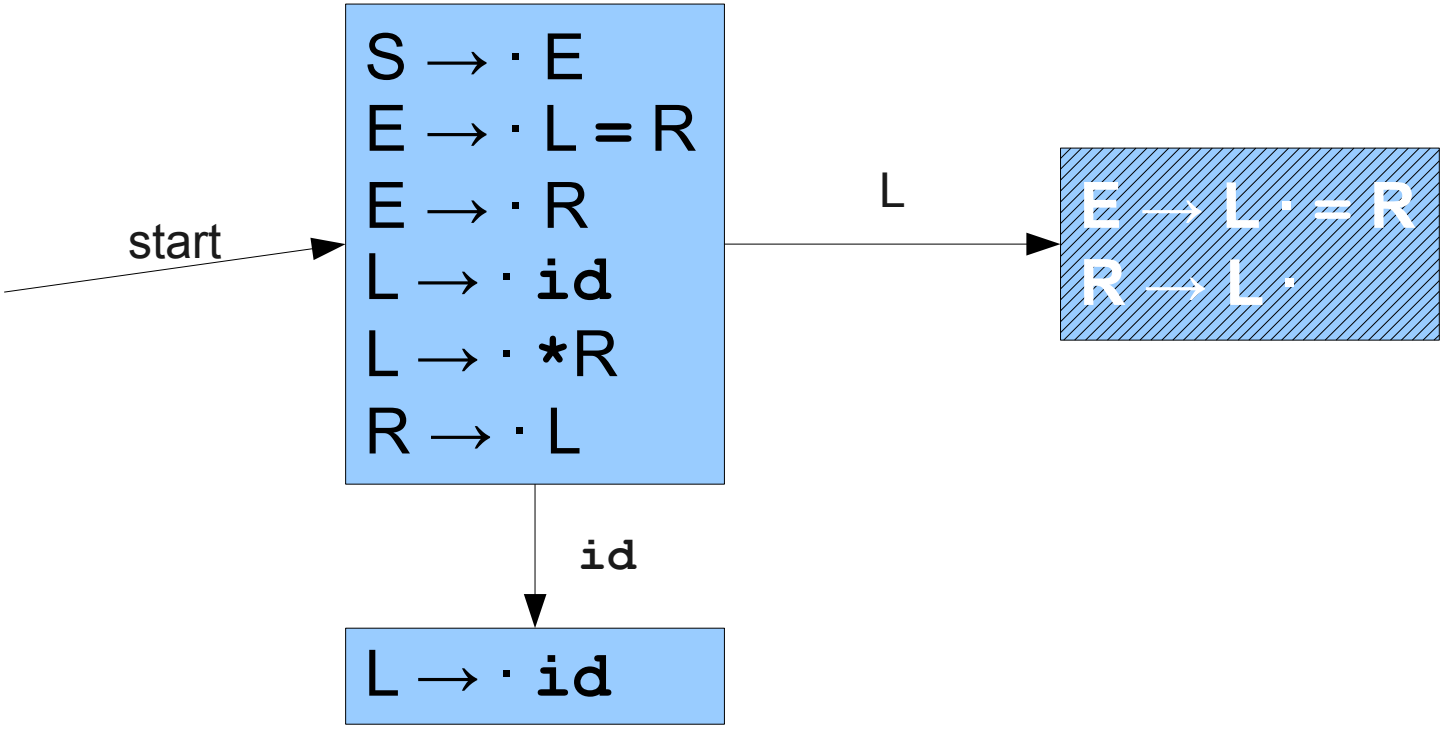
We have a shift/reduce conflict!

Why is SLR(1) Weak?

- With LR(1), incredible contextual information.
  - Lookaheads at each state only possible after applying the productions that could get us there.
- With SLR(1), **no** contextual information.
  - FOLLOW(A) means “what could follow A **somewhere** in the grammar?,” even if in a particular state A couldn't possibly have that symbol after it.

A Lack of Context

S → E
E → L = R
E → R
L → **id**
L → ***R**
R → L



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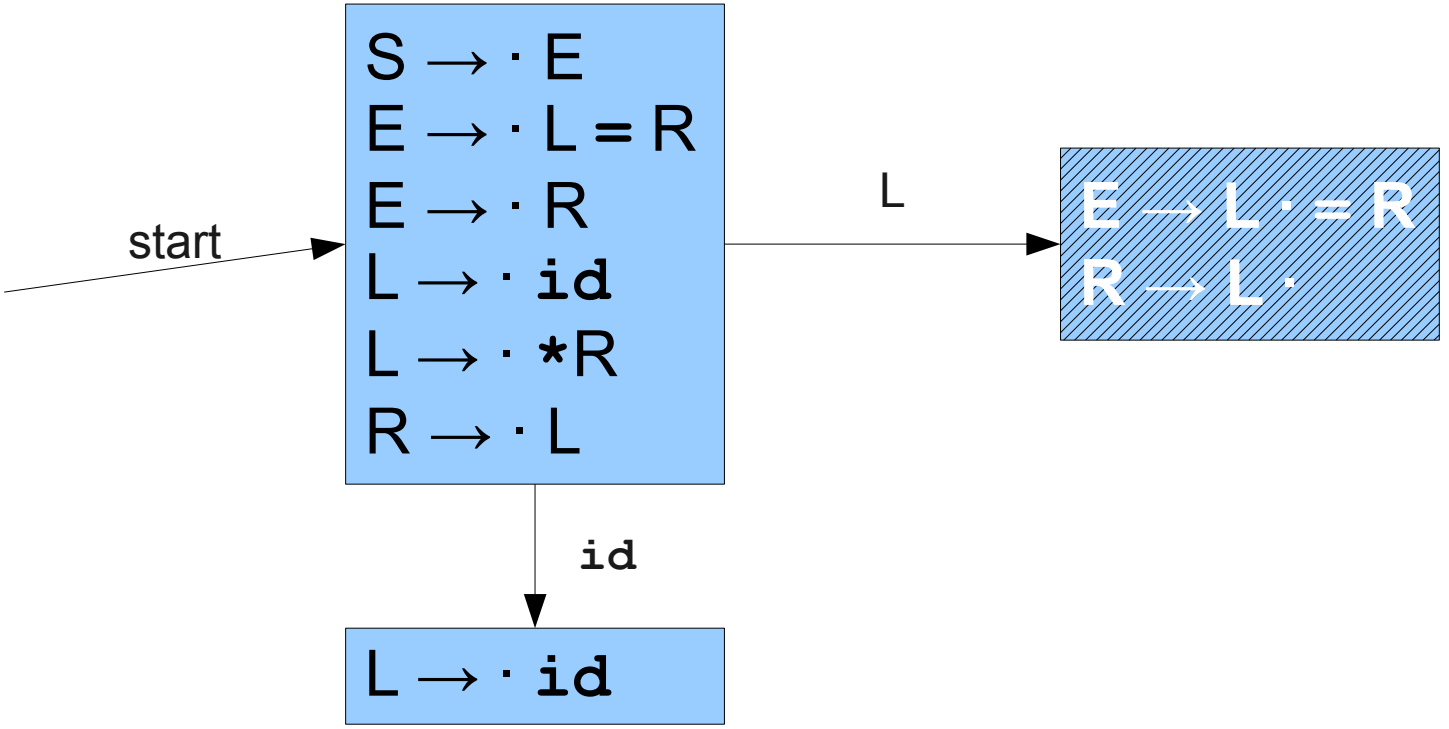

id	=	*	id

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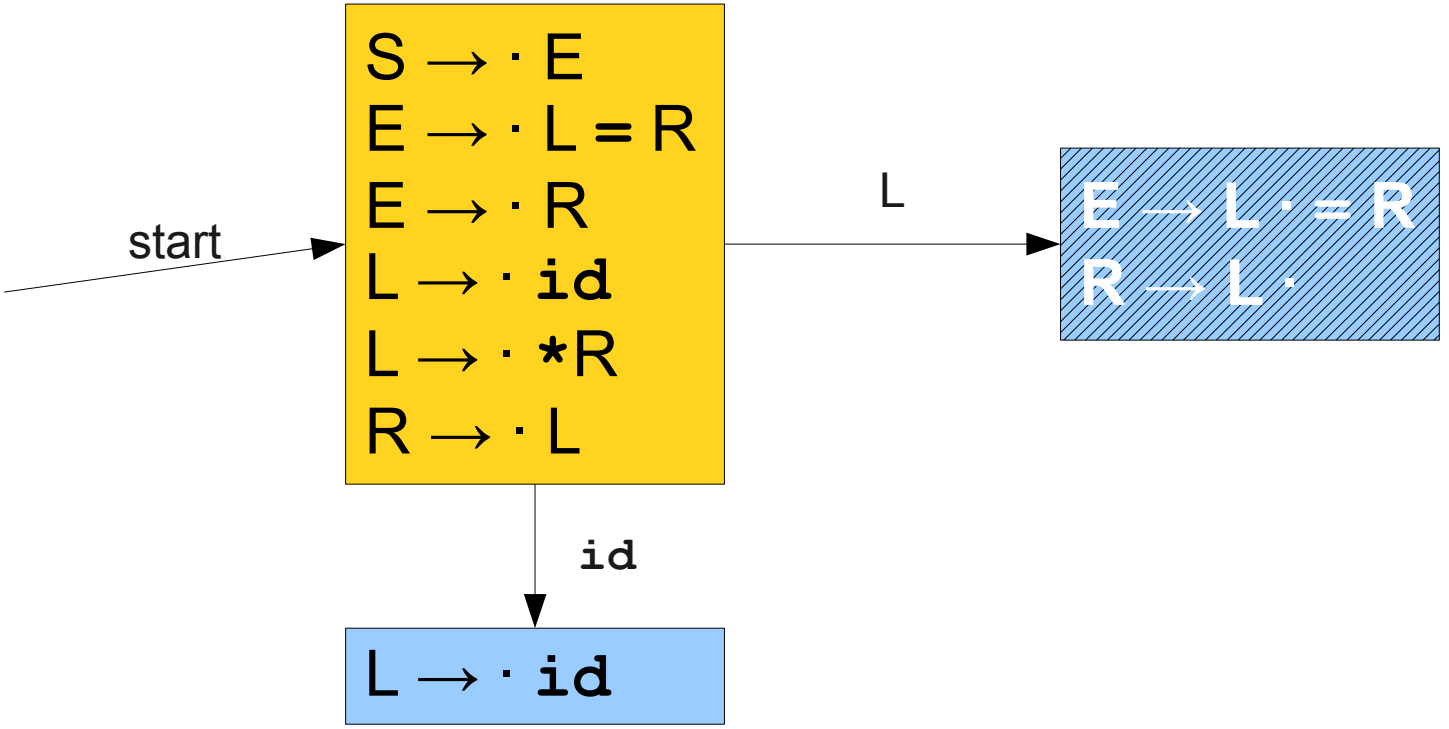

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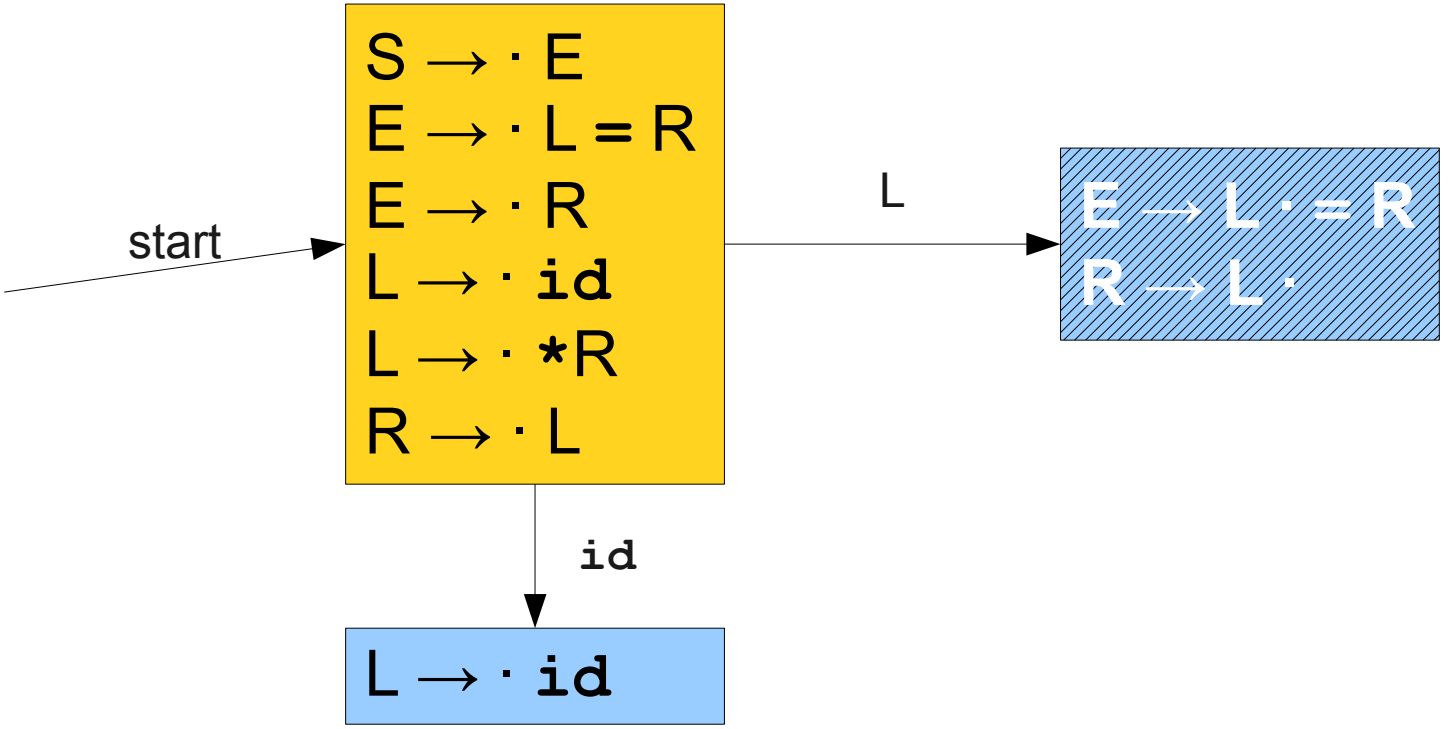

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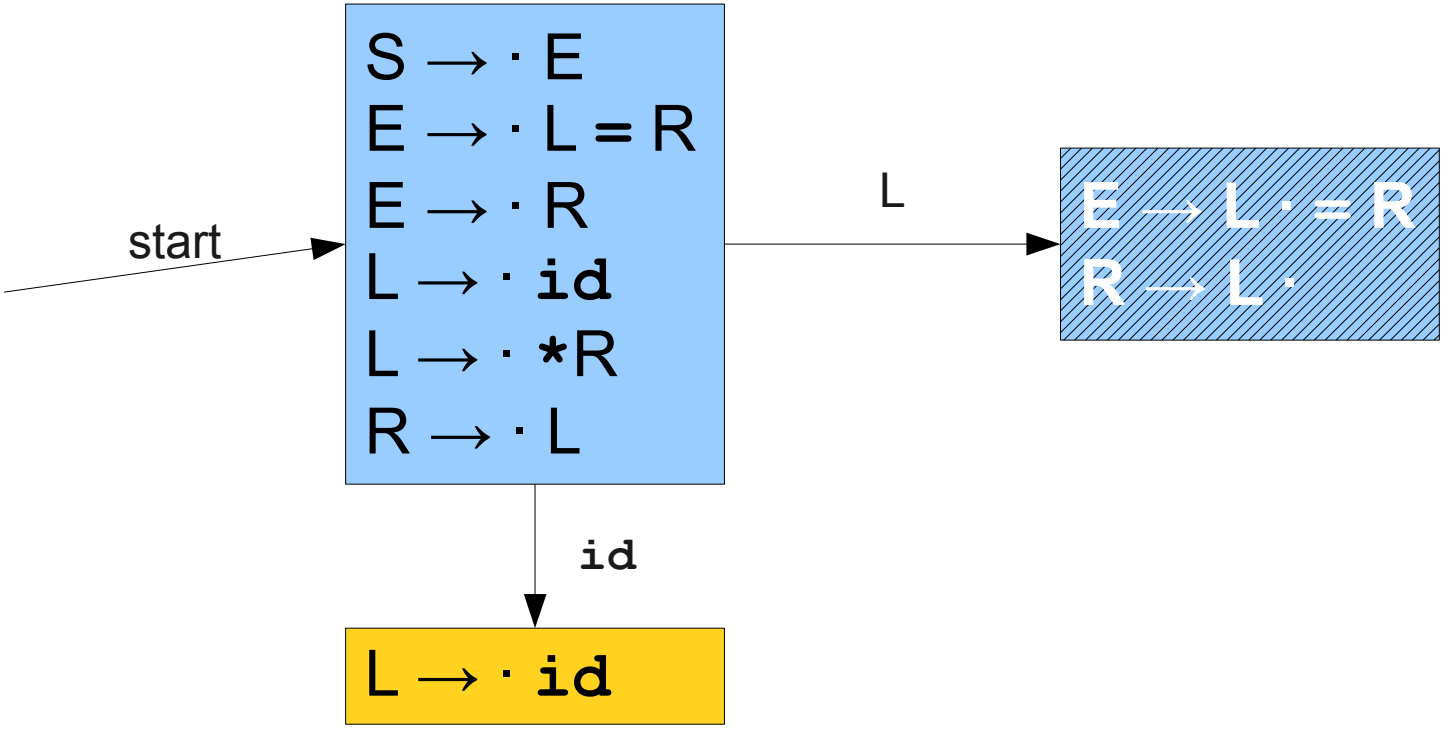

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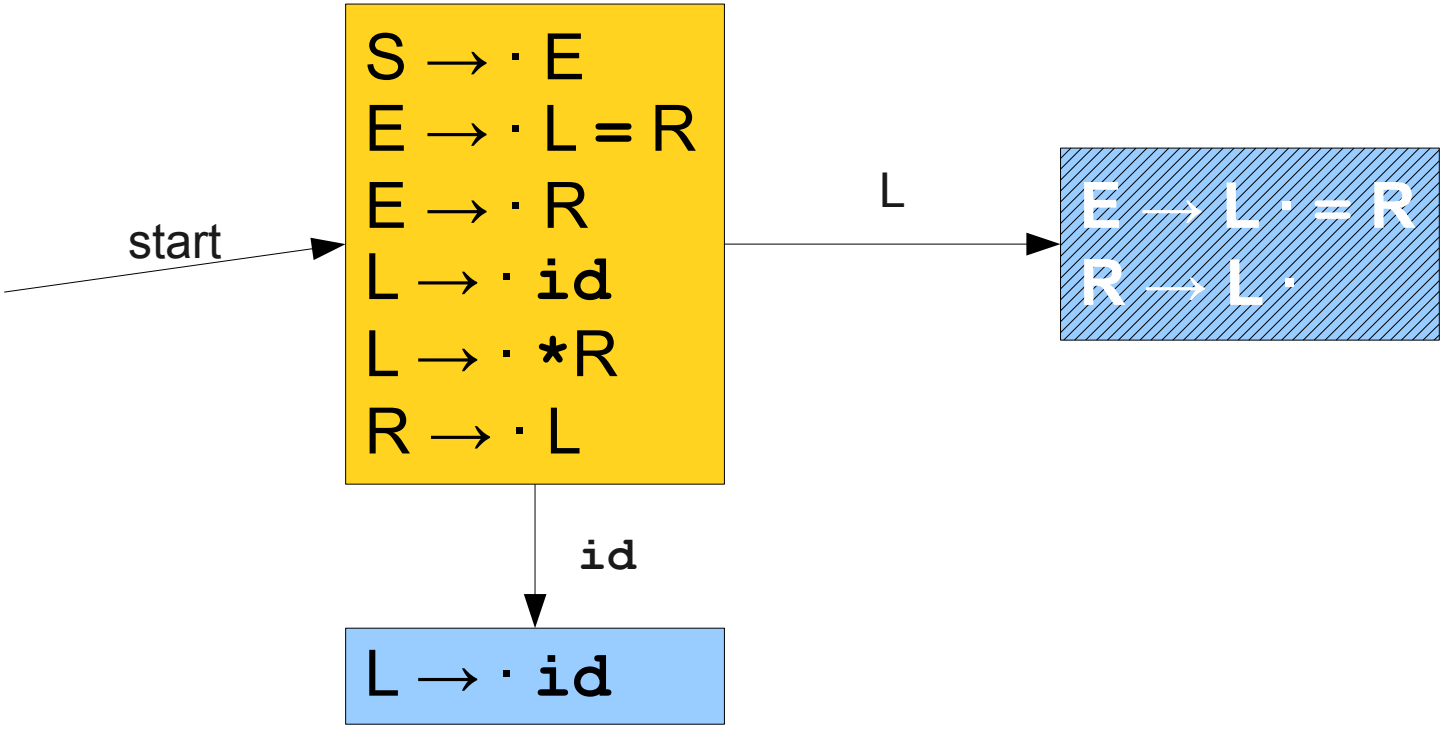

=	*	id

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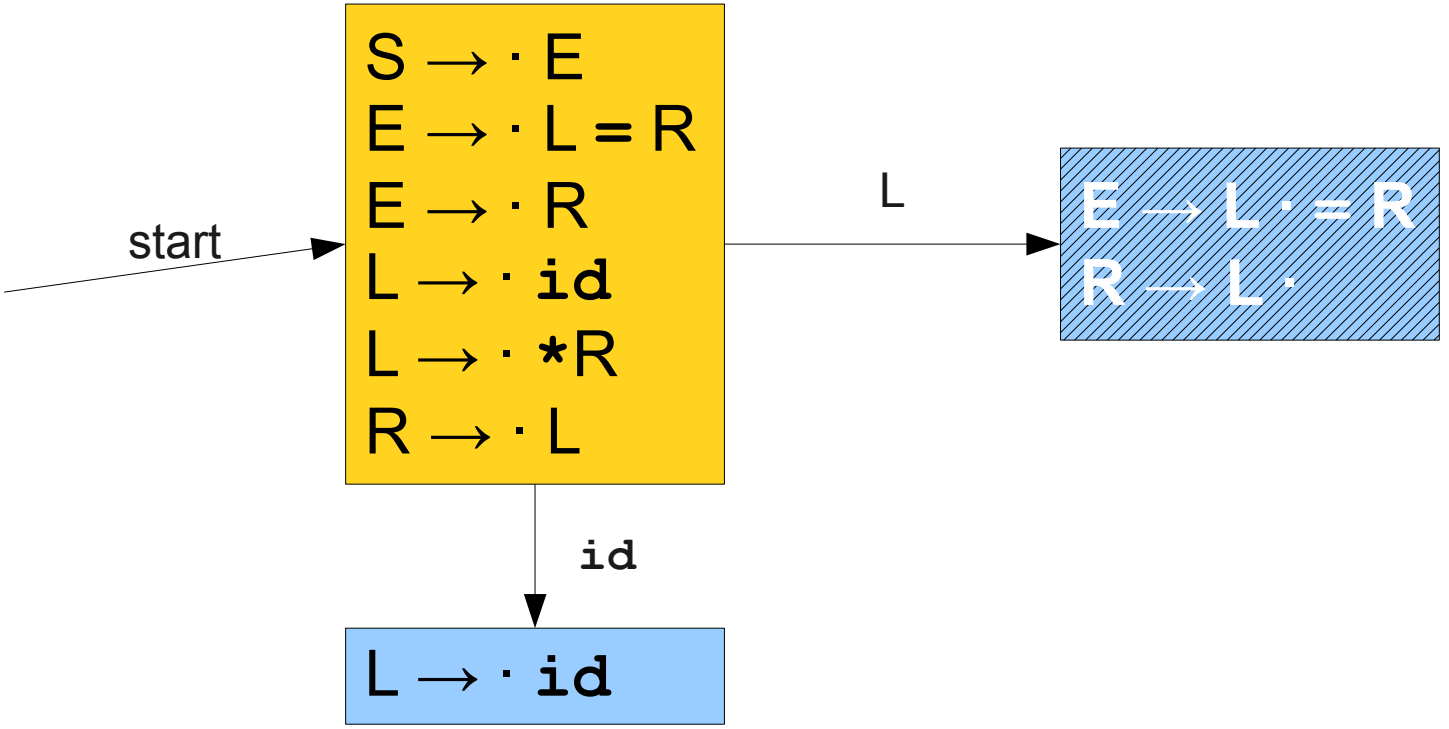

L	=	*	id

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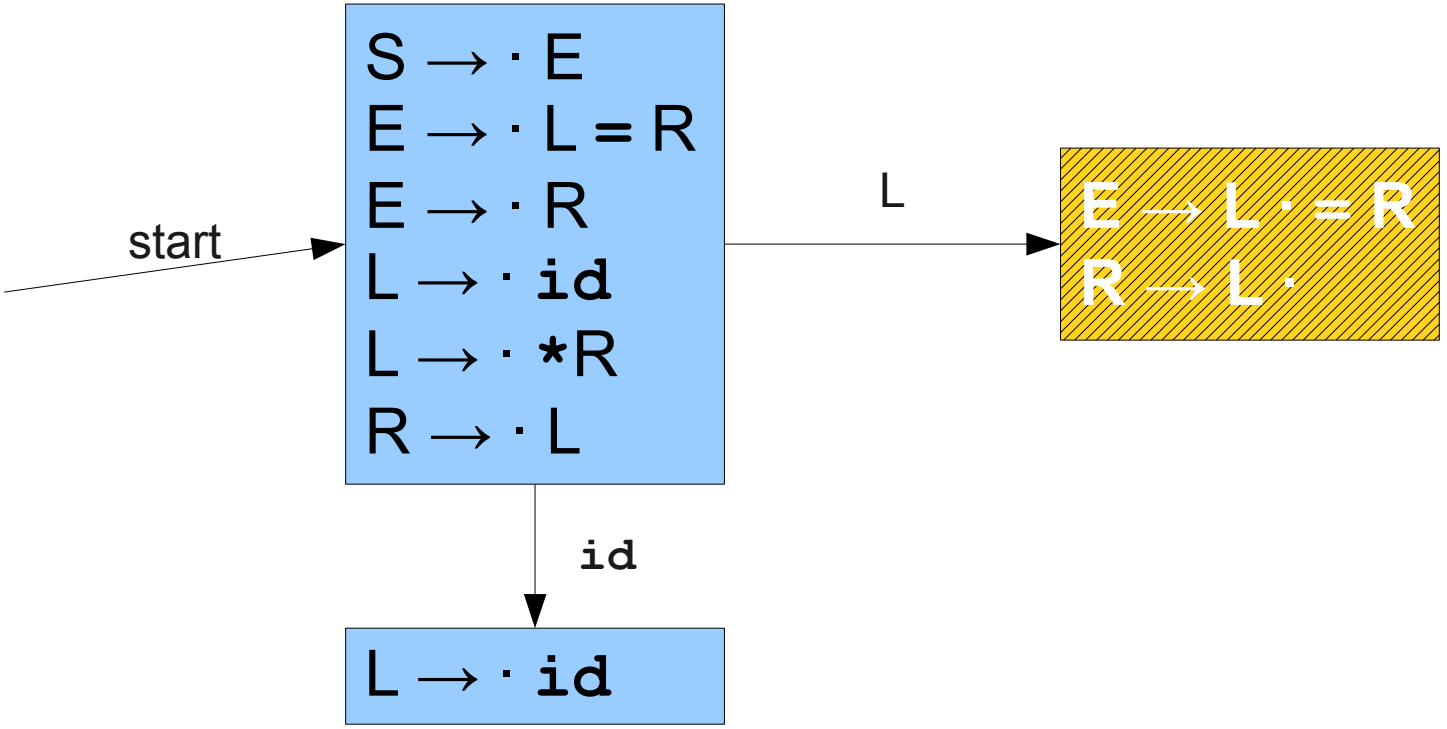

L	=	*	id

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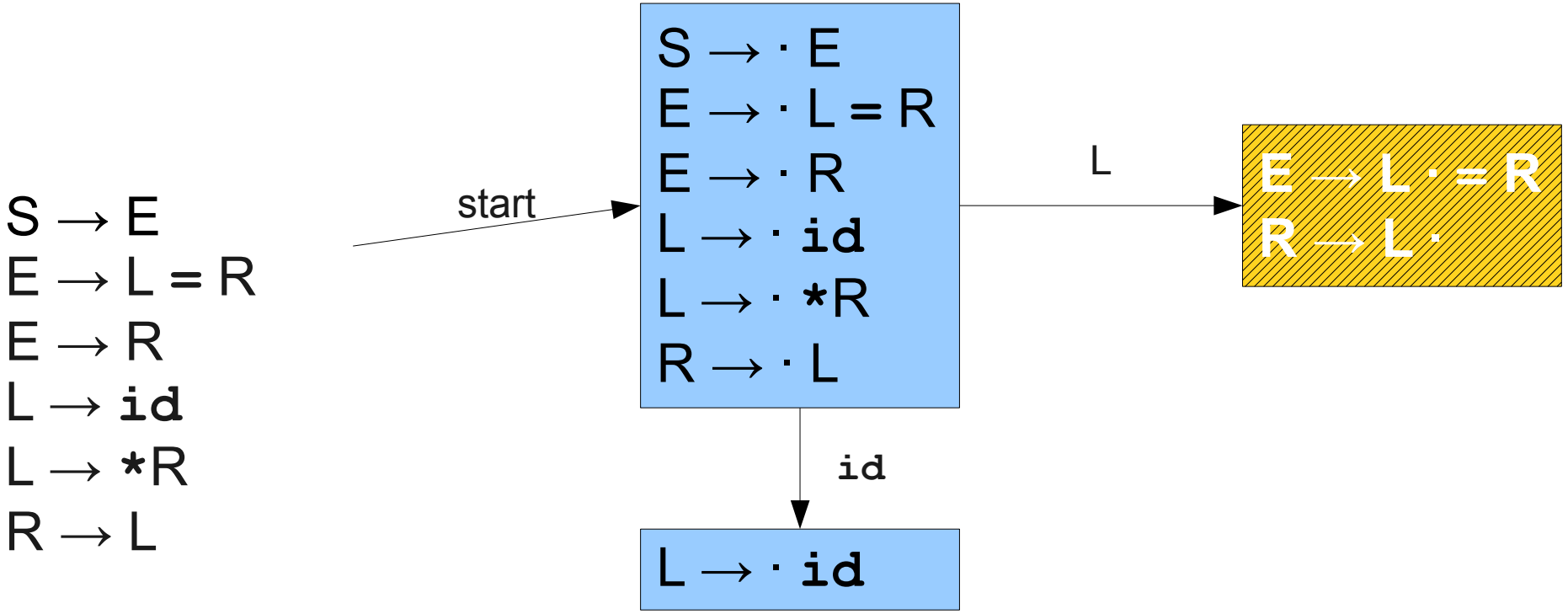

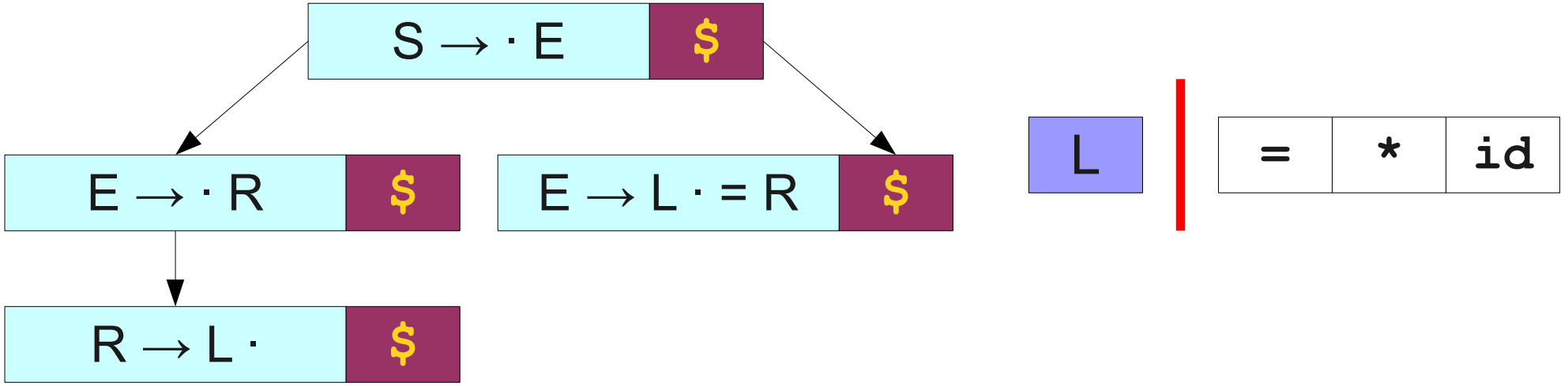

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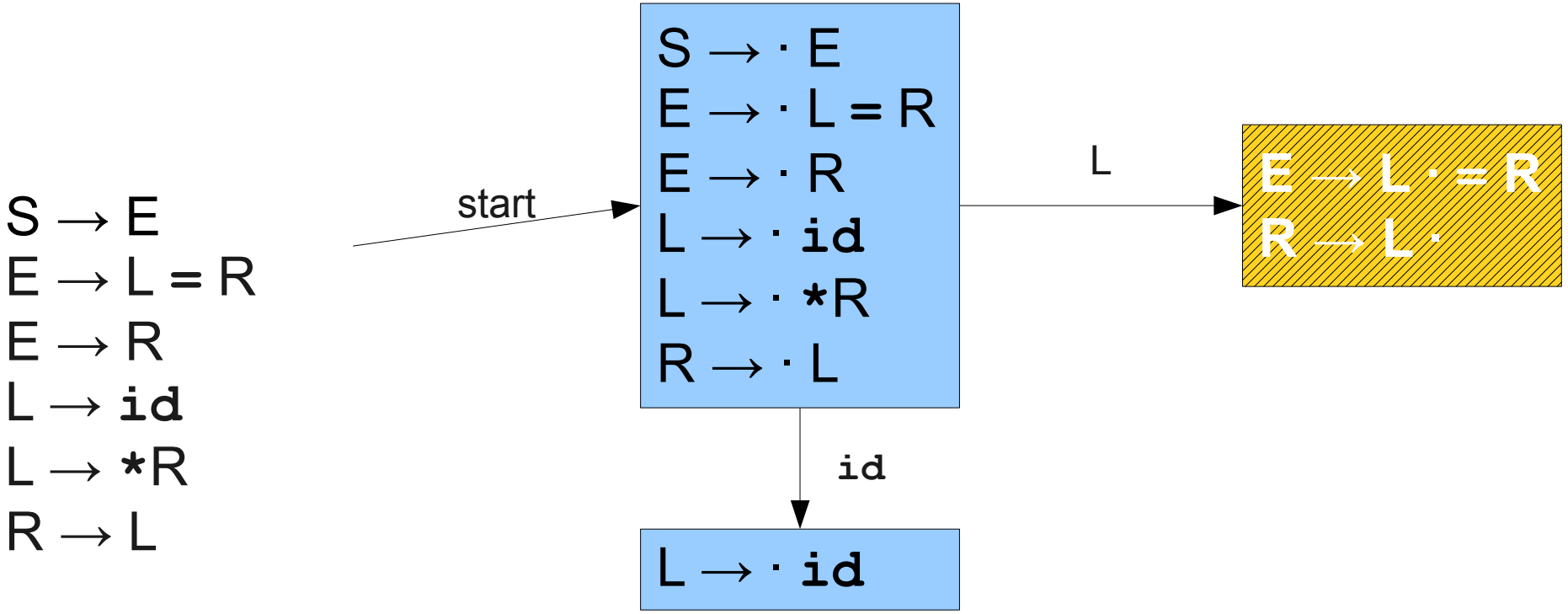

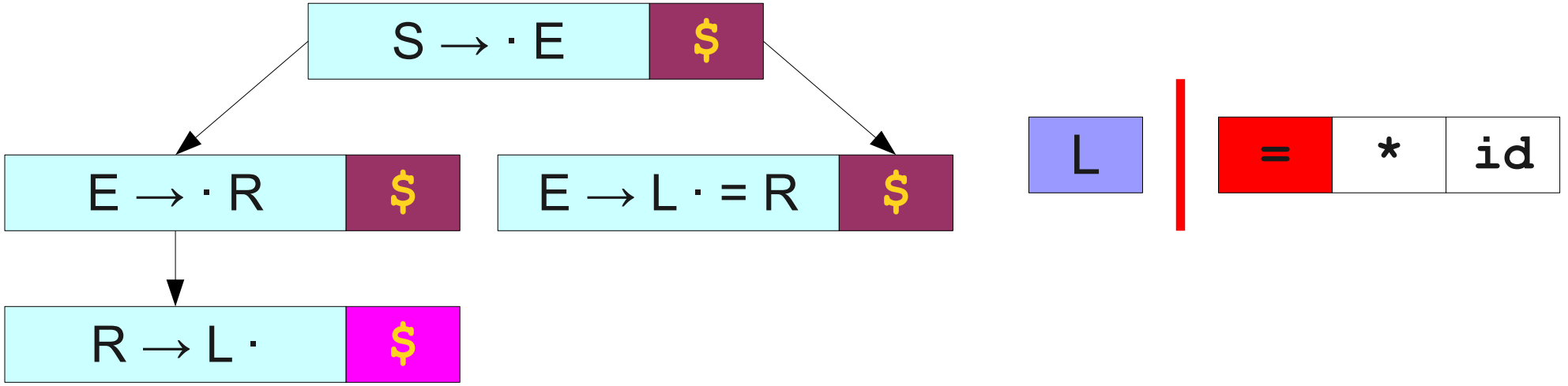

For Reference: LR(1) States

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E → L **=** R
E → R
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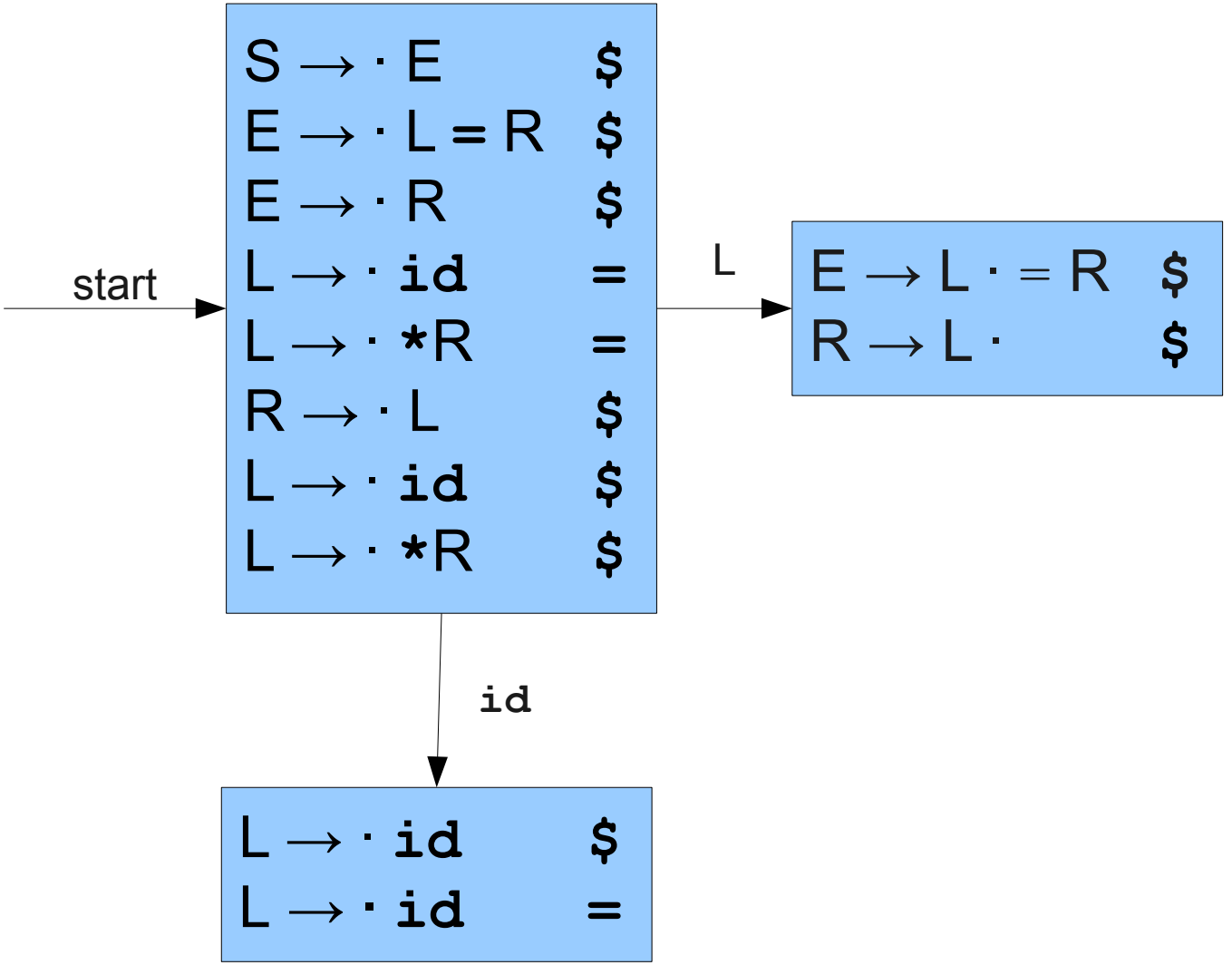

LR(1) and SLR(1)

- SLR(1) is weak because it has no contextual information.
- LR(1) is impractical because its contextual information makes the automaton too big.
- Can we retain the LR(1) automaton's contextual information without all its states?

Review of LR(1)

- Each state in an LR(1) automaton is a combination of an LR(0) state and lookahead information.
- Two LR(1) items have the same **core** if they are identical except for lookahead.

T → (·E)	**$**
E → ·E + T	)
E → ·T	)
T → ·**int**	)
T → ·(E)	)

T → (·E)	)
E → ·E + T	)
E → ·T	)
T → ·**int**	)
T → ·(E)	)

A Surprisingly Powerful Idea

- In an LR(1) automaton, we have multiple states with the same core but different lookahead.
- **What if we merge all these states together?**
- This is called LALR(1)
  - **Lookahead(1) LR(0)**

From LR(1) to LALR(1)

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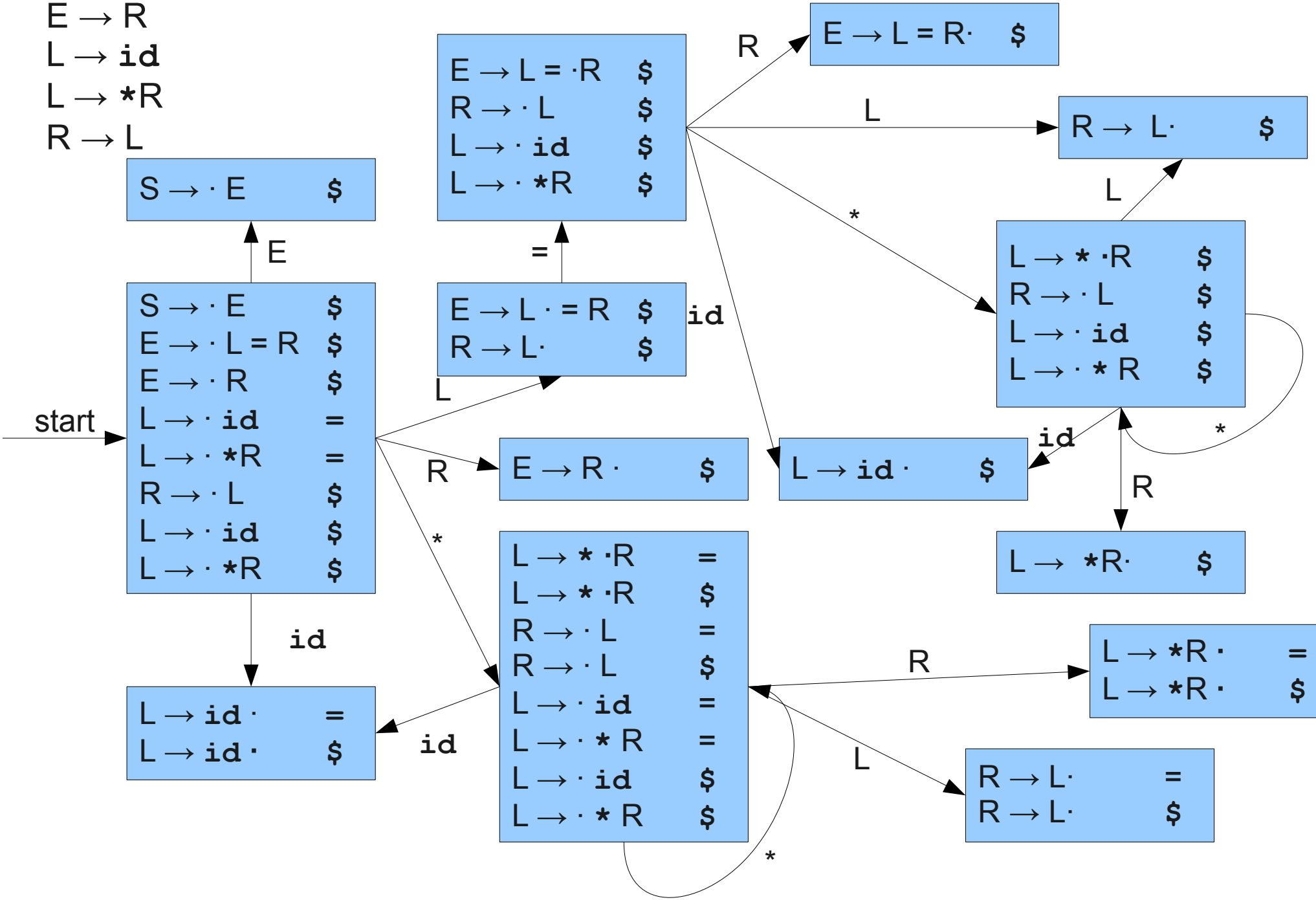

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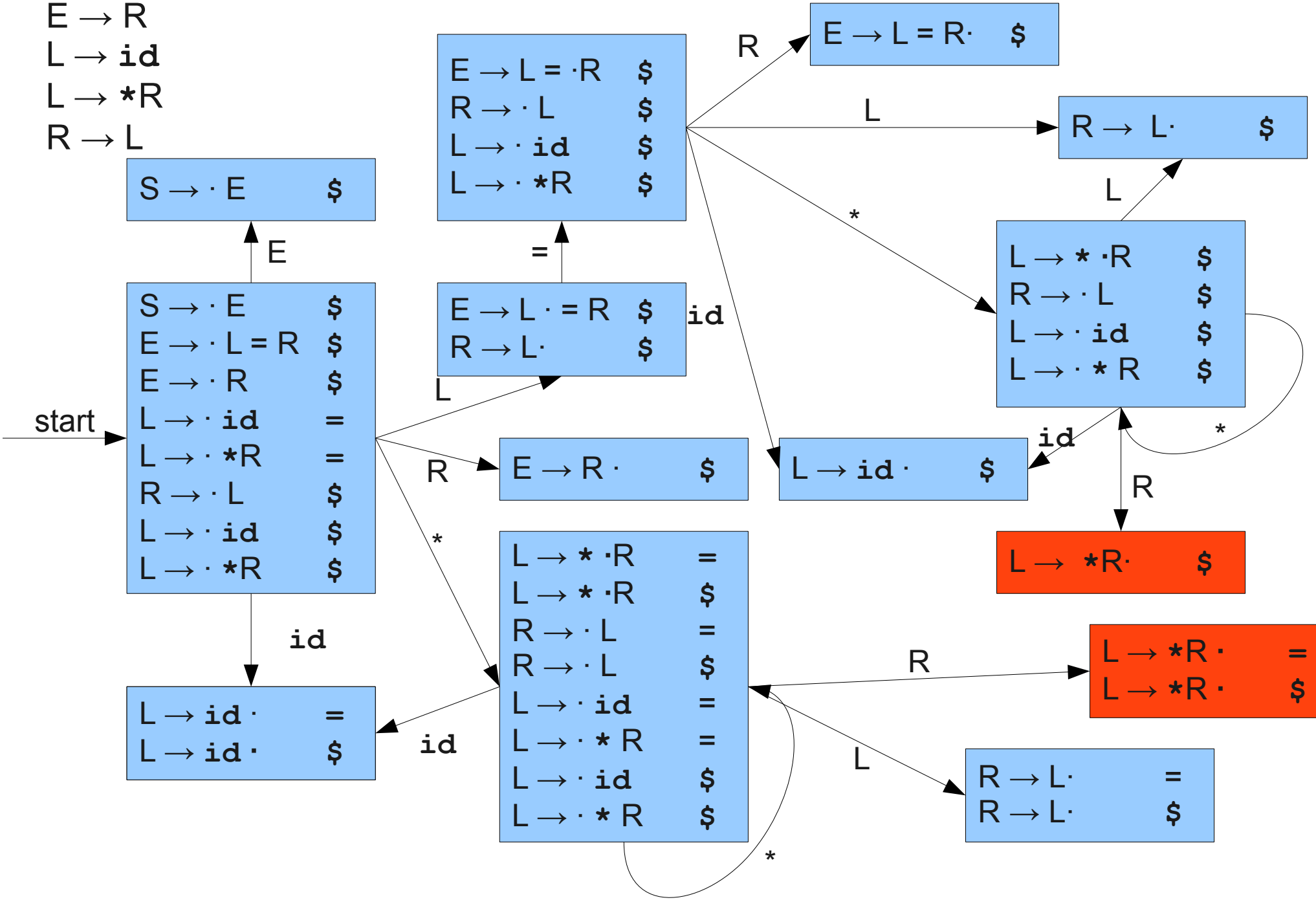

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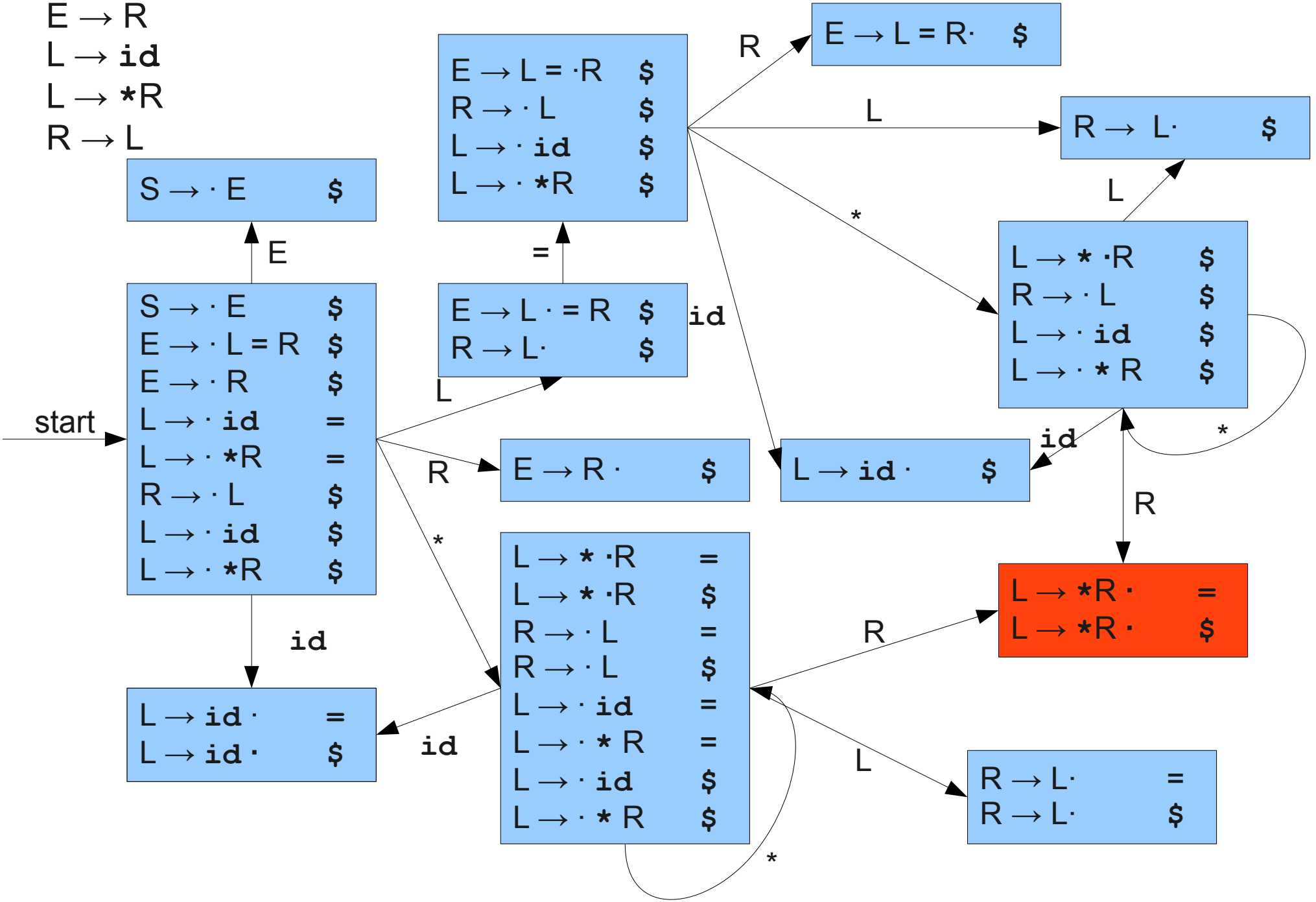

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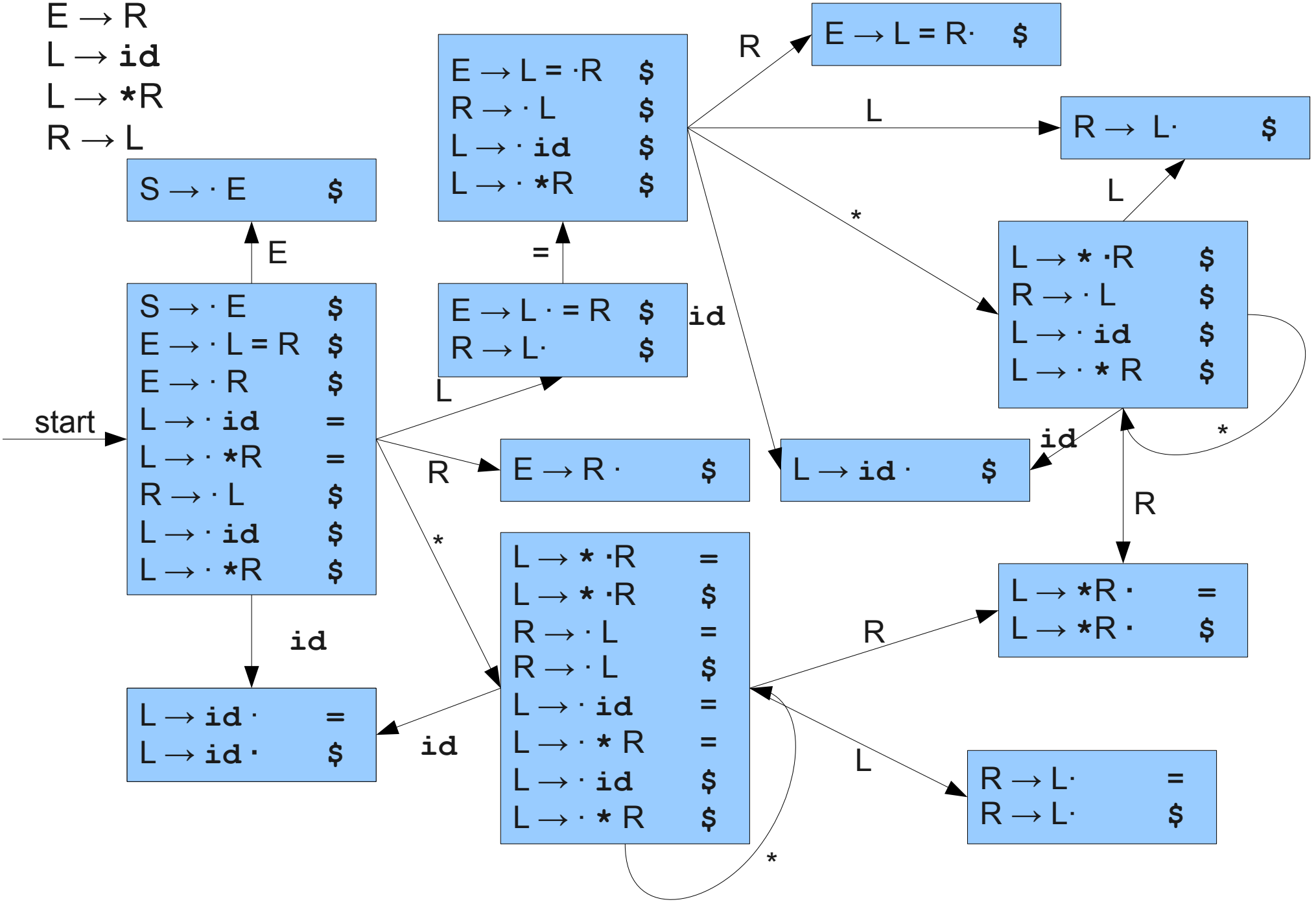

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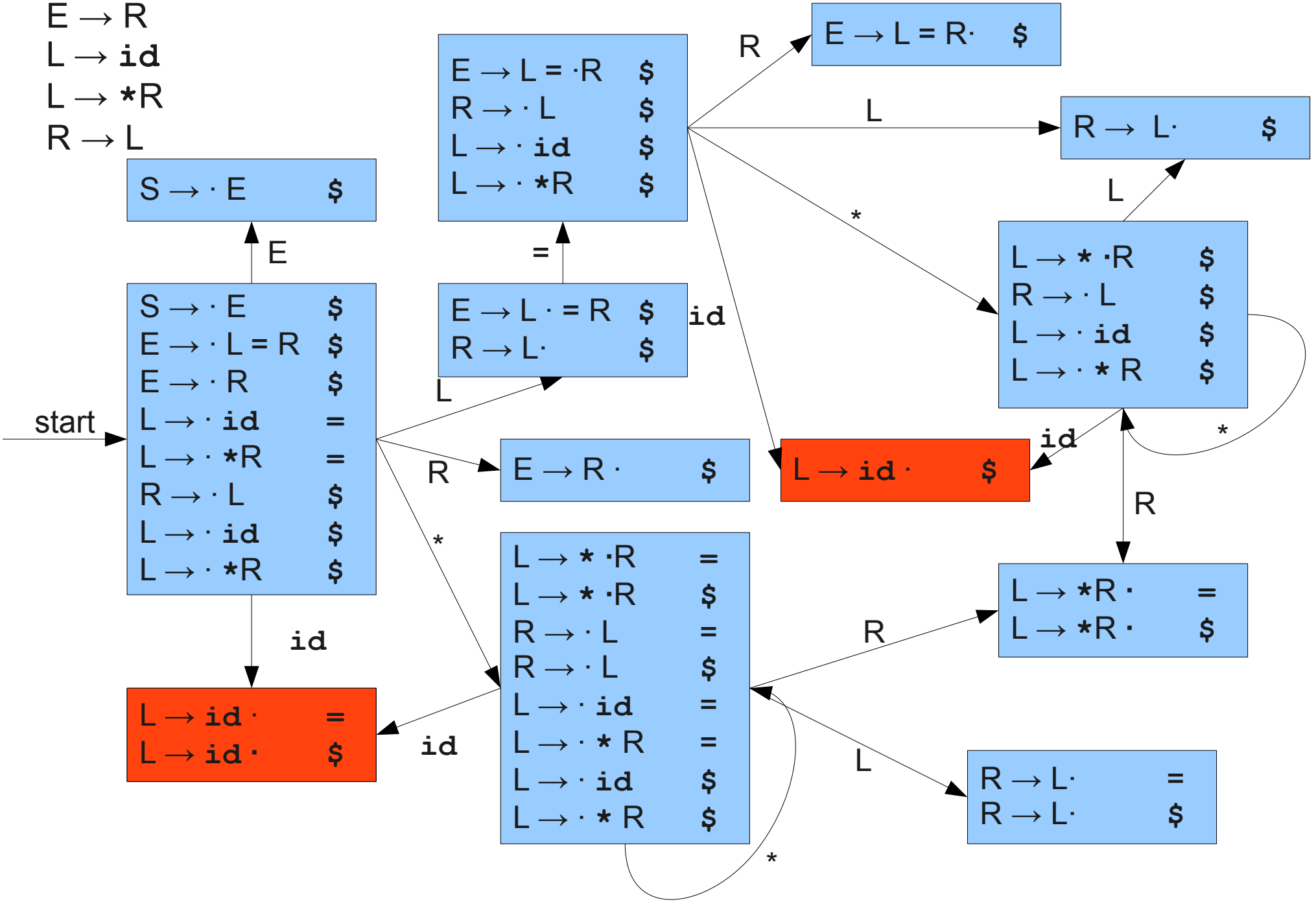

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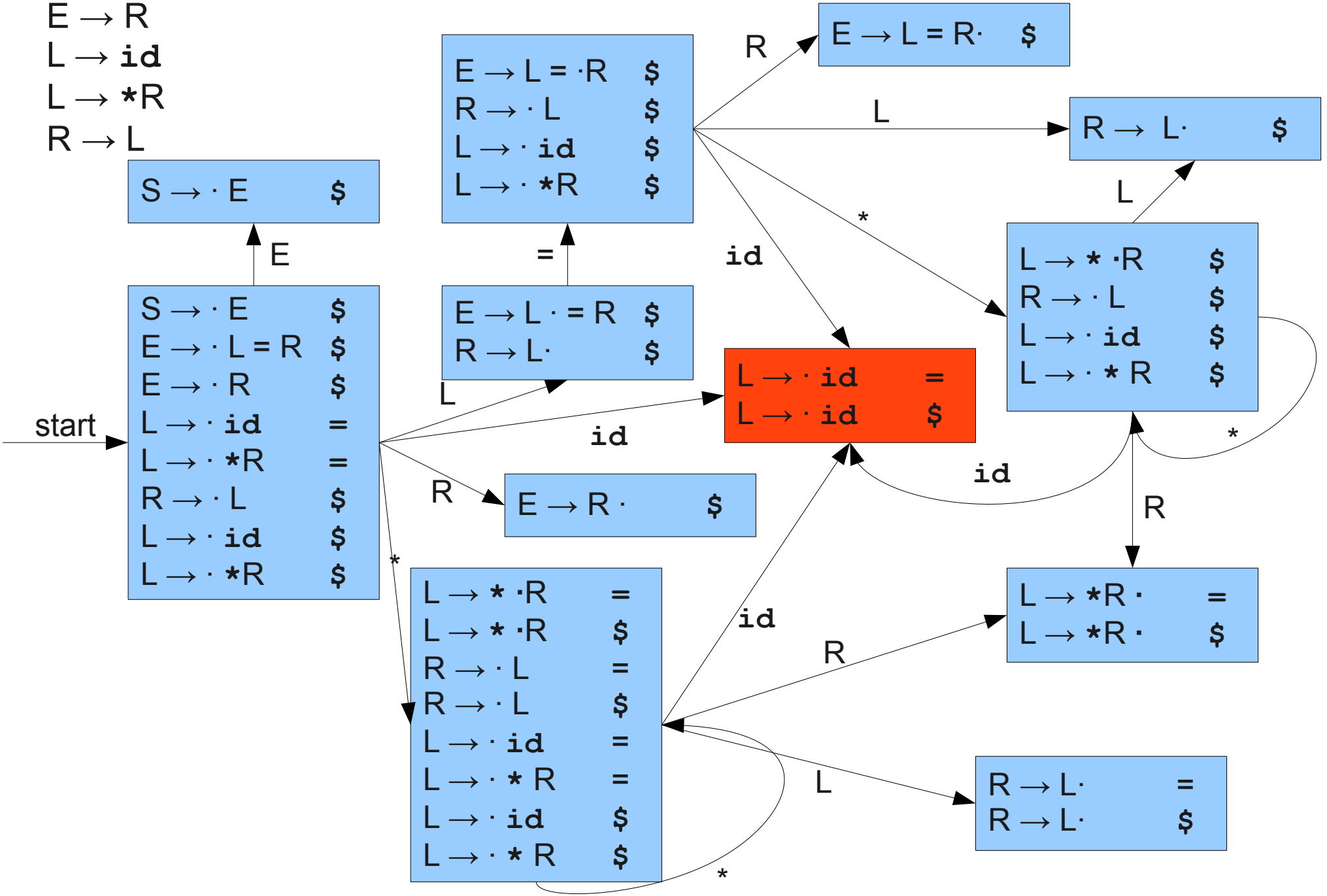

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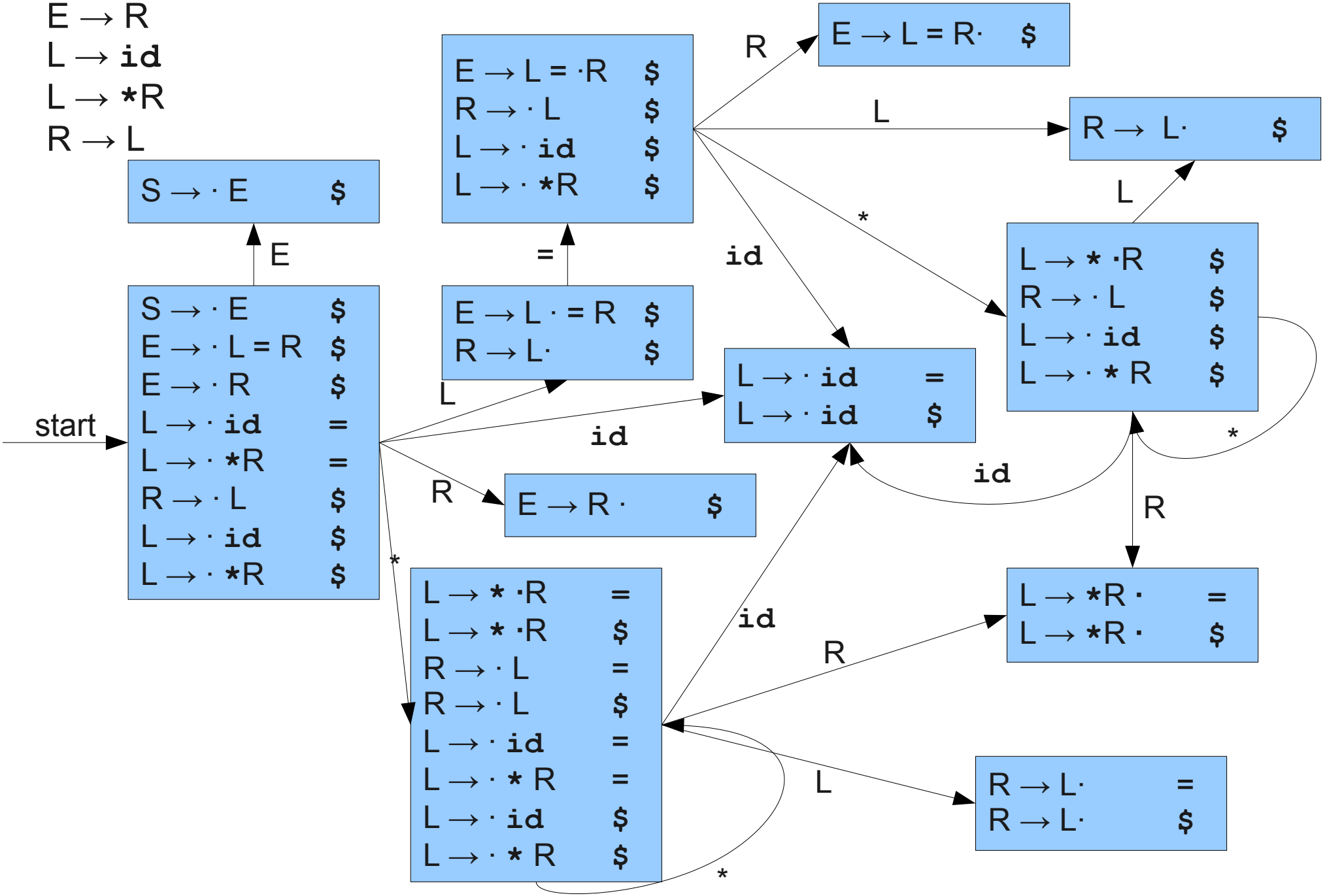

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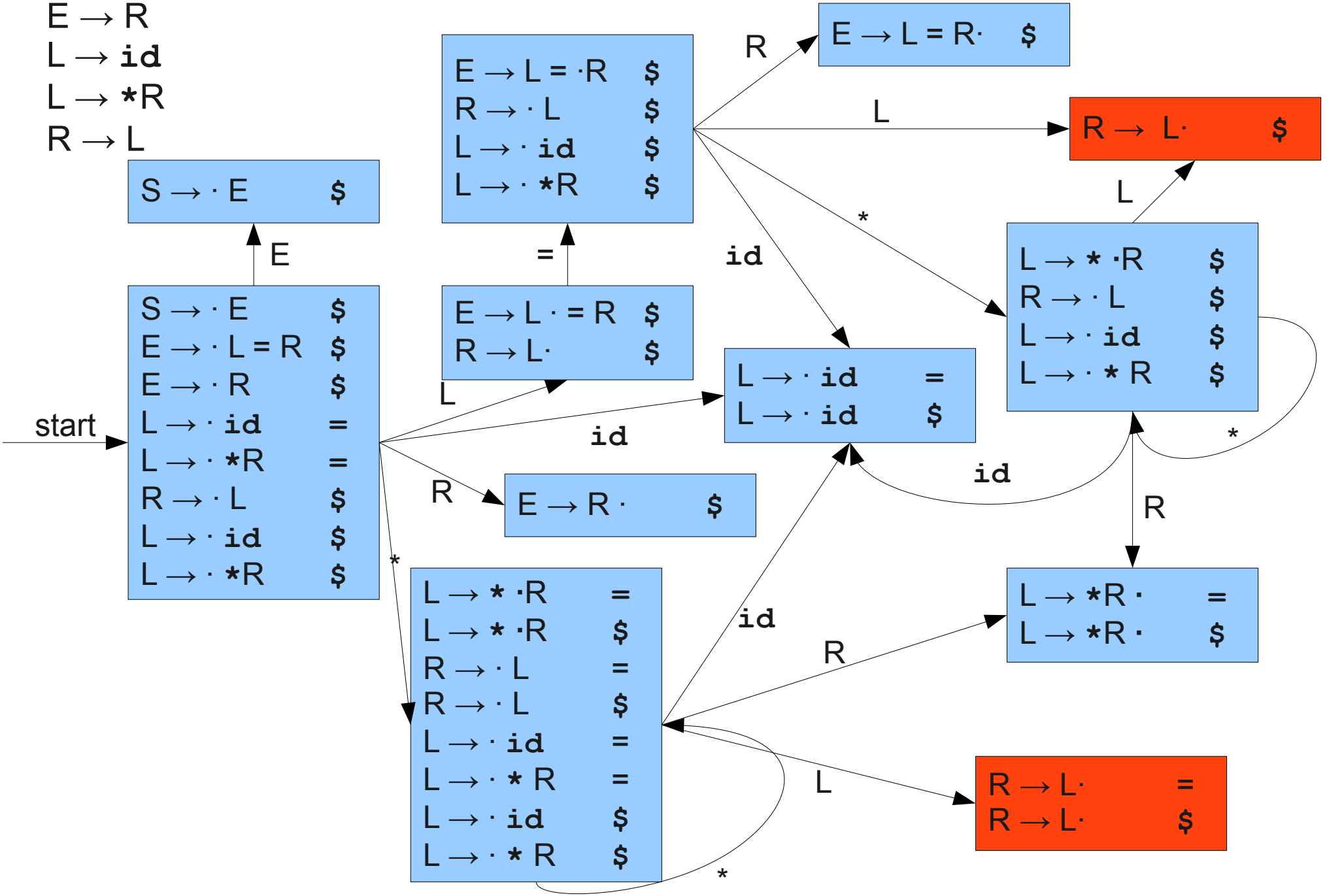

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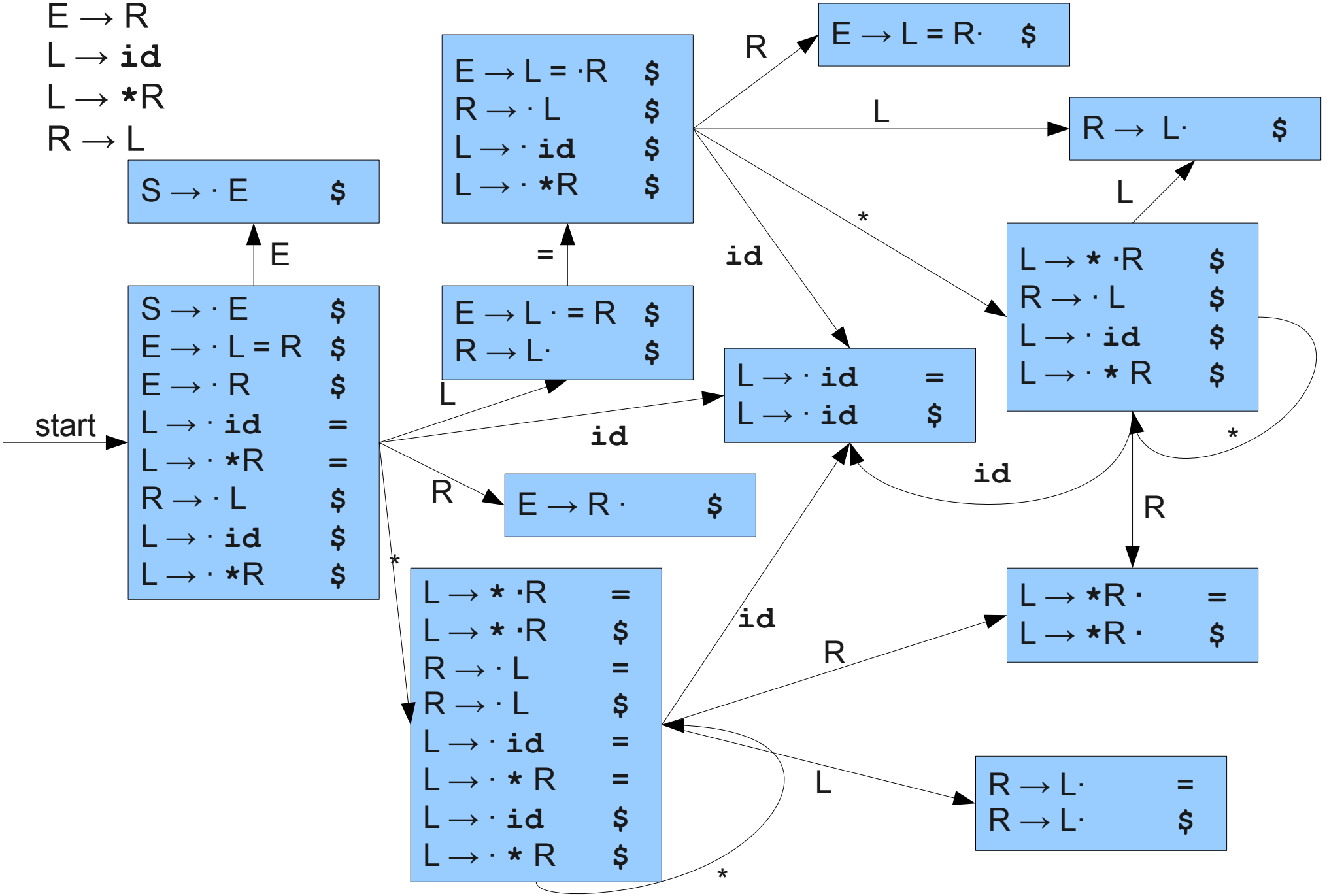

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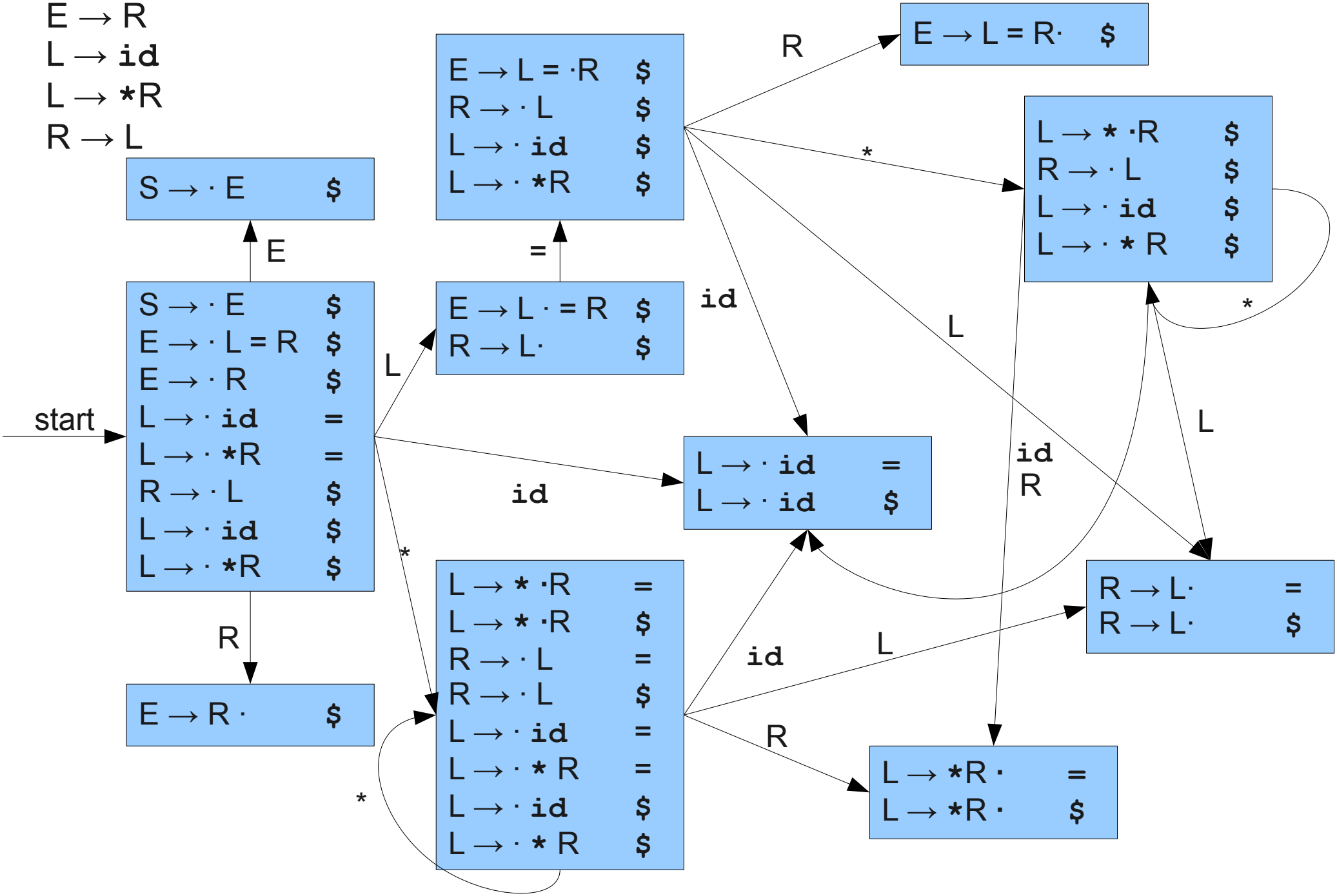

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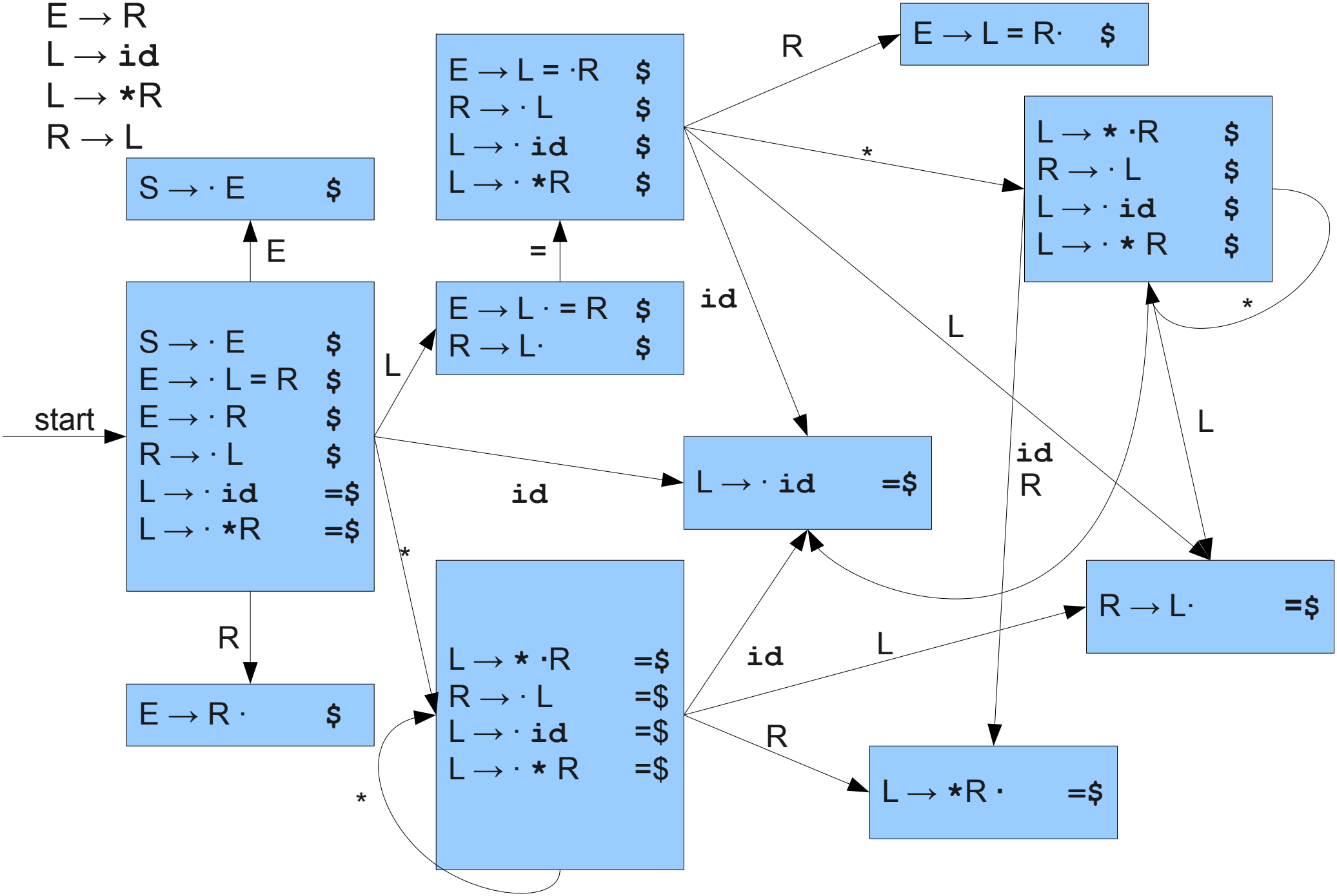

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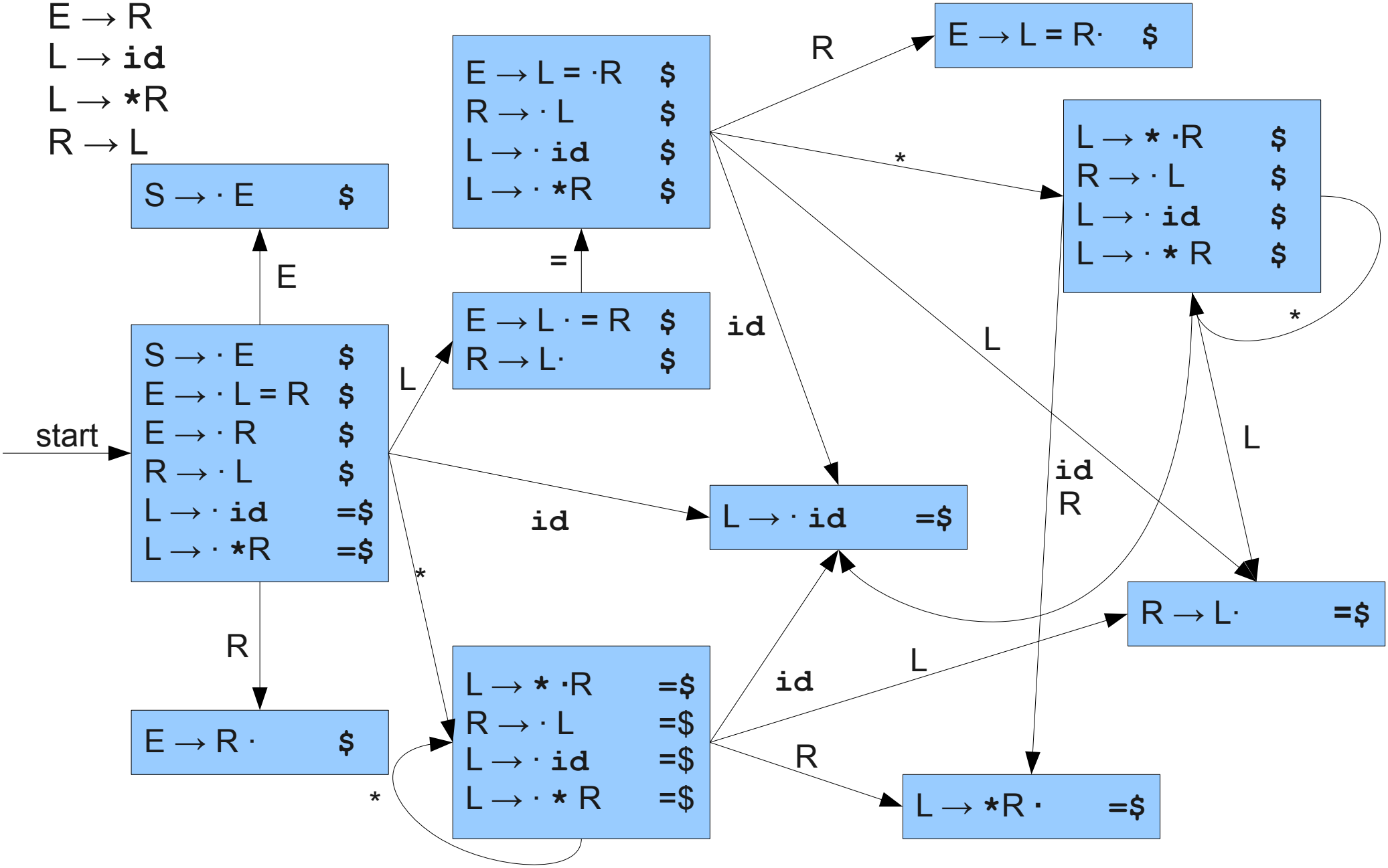

Advantages of LALR(1)

- Maintains **context**.
  - Lookup sets based on the fine-grained LR(1) automaton.
  - Each state's lookup relevant only for that state.
- Keeps automaton **small**.
  - Resulting automaton has same size as LR(0) automaton.

LALR(1) is Powerful

- Every LR(0) grammar is LALR(1).
- Every SLR(1) grammar is LALR(1)
- **Most** (but not all) LR(1) grammars are LALR(1).

LALR(1) isn't LR(1)

- Merging LR(1) states cannot introduce a shift/reduce conflict.
- **Why?**
- Since the items have the same core, a shift/reduce conflict in a LALR(1) state would have to also exist in one of the LR(1) states it was merged from.
- Merging LR(1) states **can** introduce a reduce/reduce conflict.
- Often these conflicts appear without any good reason; this is one limitation of LALR(1).

Constructing LALR(1) Automata

- It's not a good idea to build LALR(1) automata from LR(1) automata.
- **Why?**
- LR(1) automata are impractically large.
- Are there more efficient methods for LALR(1) automata construction?
- **Yes**; we'll see two.

The “Lazy Merging” Technique

- **Idea**: Merge together LR(1) states as they're generated.
- Maintain a worklist of states to process; begin with the initial LR(1) state.
- When adding a new state, if it has the same core as an old state, update the old state and put it back in the worklist.

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LALR(1) Construction

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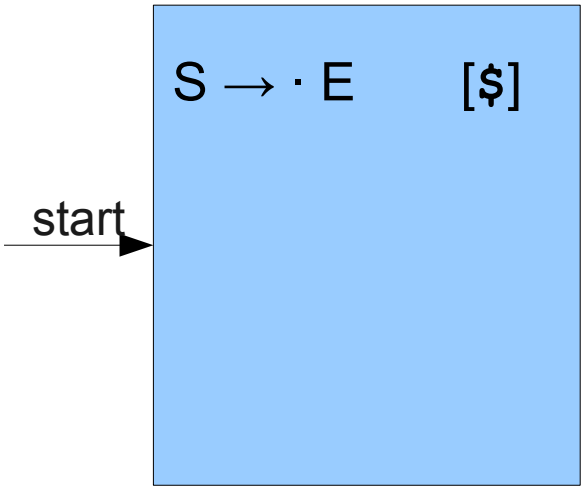
LALR(1) Construction



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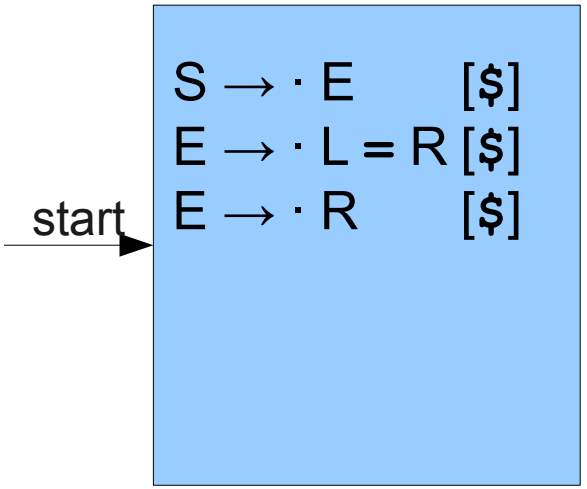

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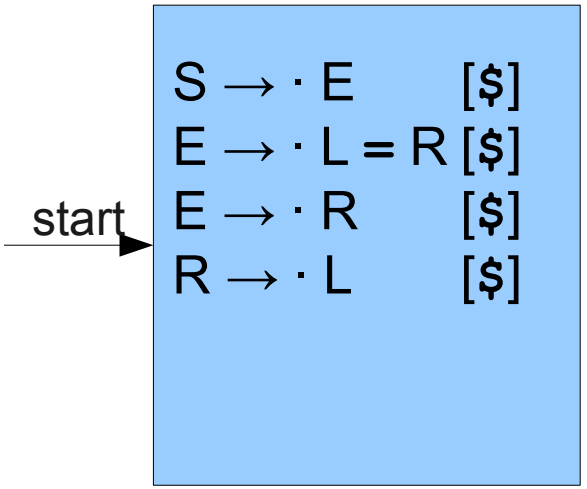

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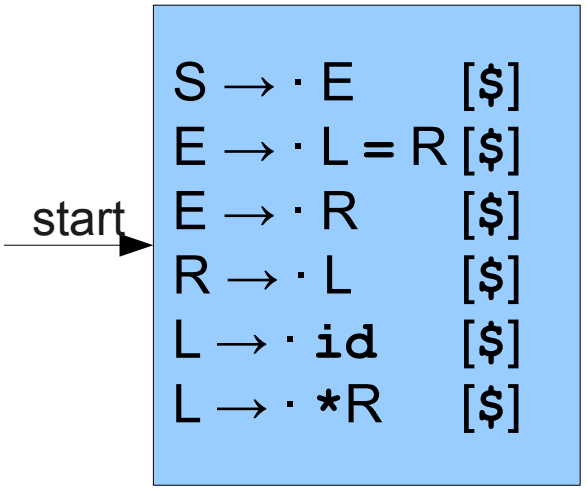

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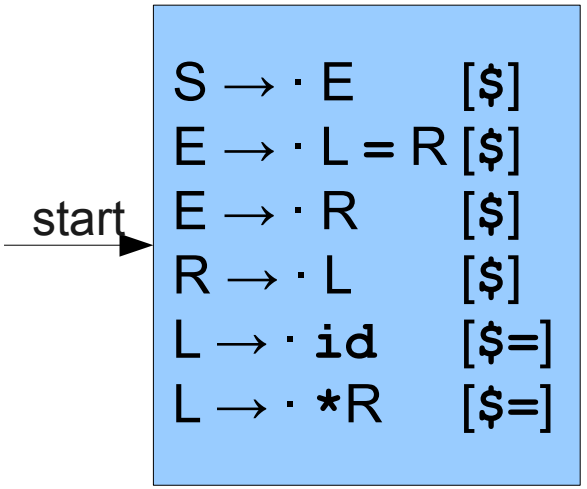
L → ***R**
R → L

LALR(1) Construction



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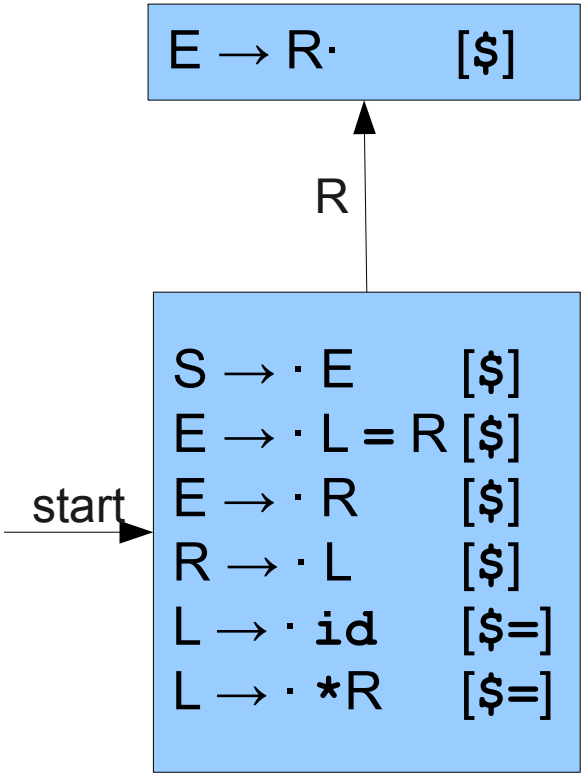

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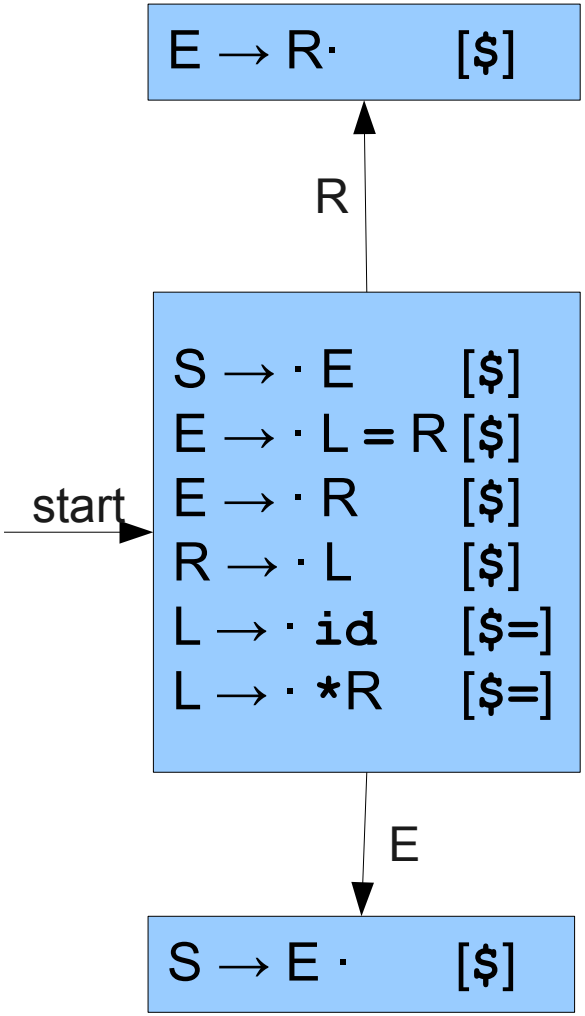

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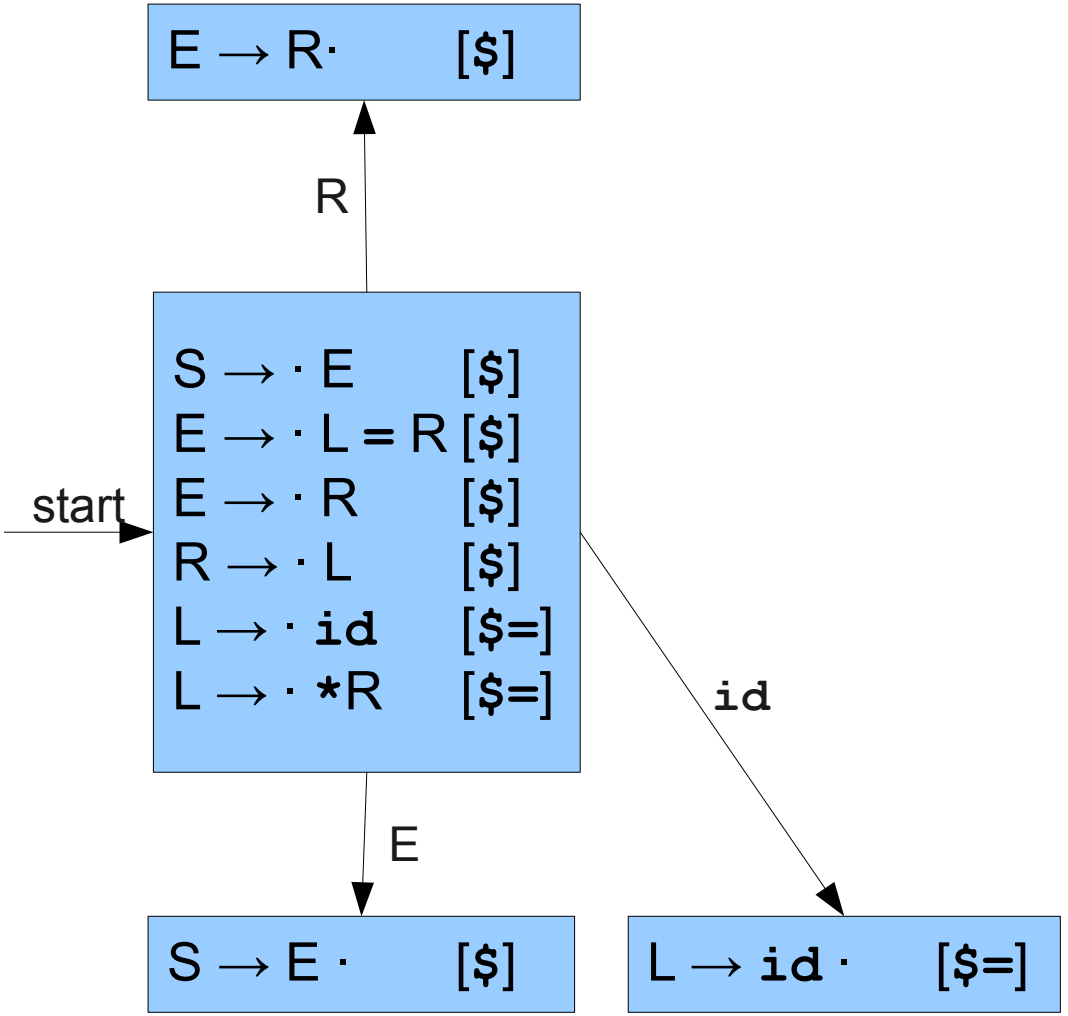

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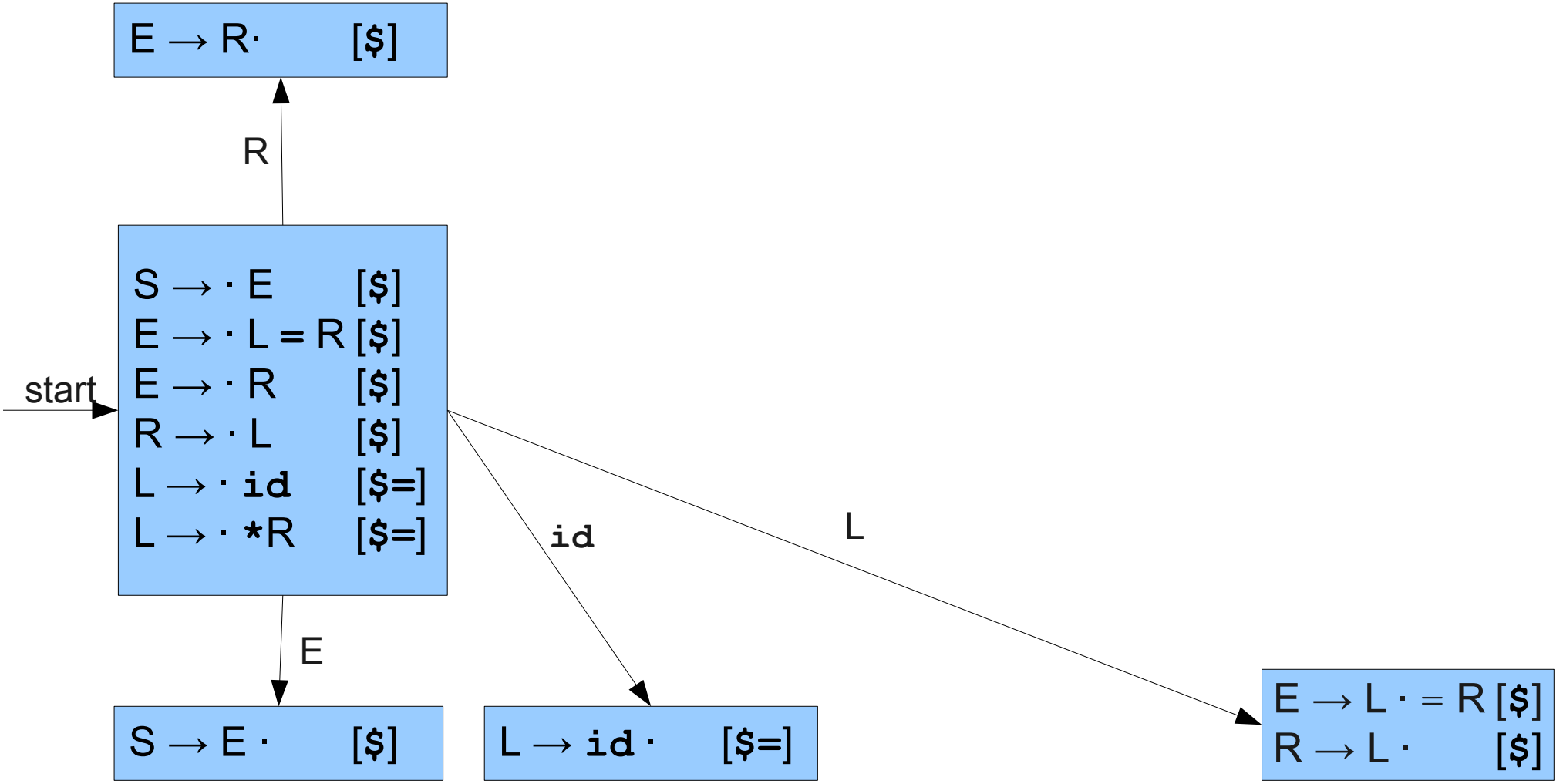

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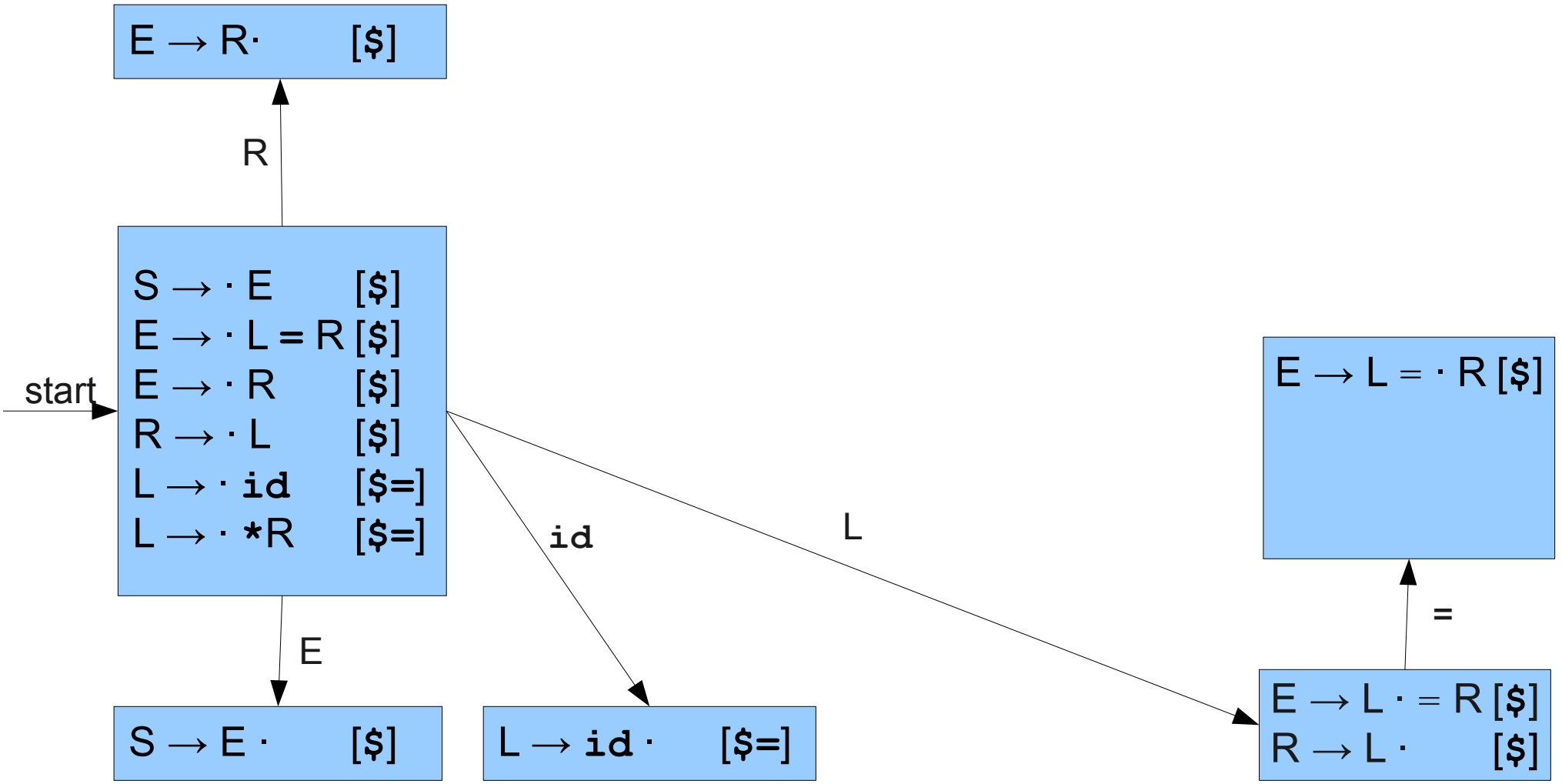

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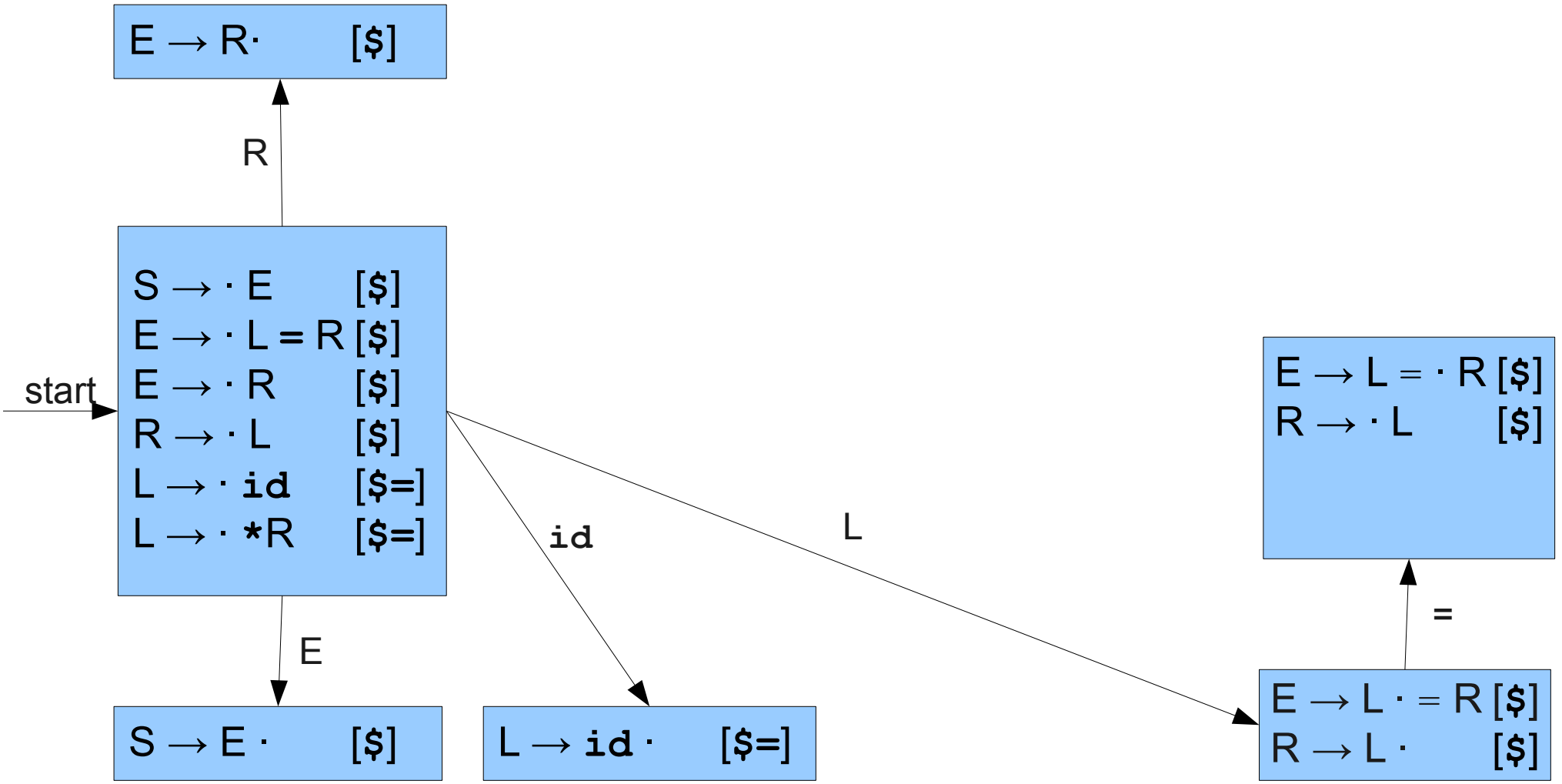

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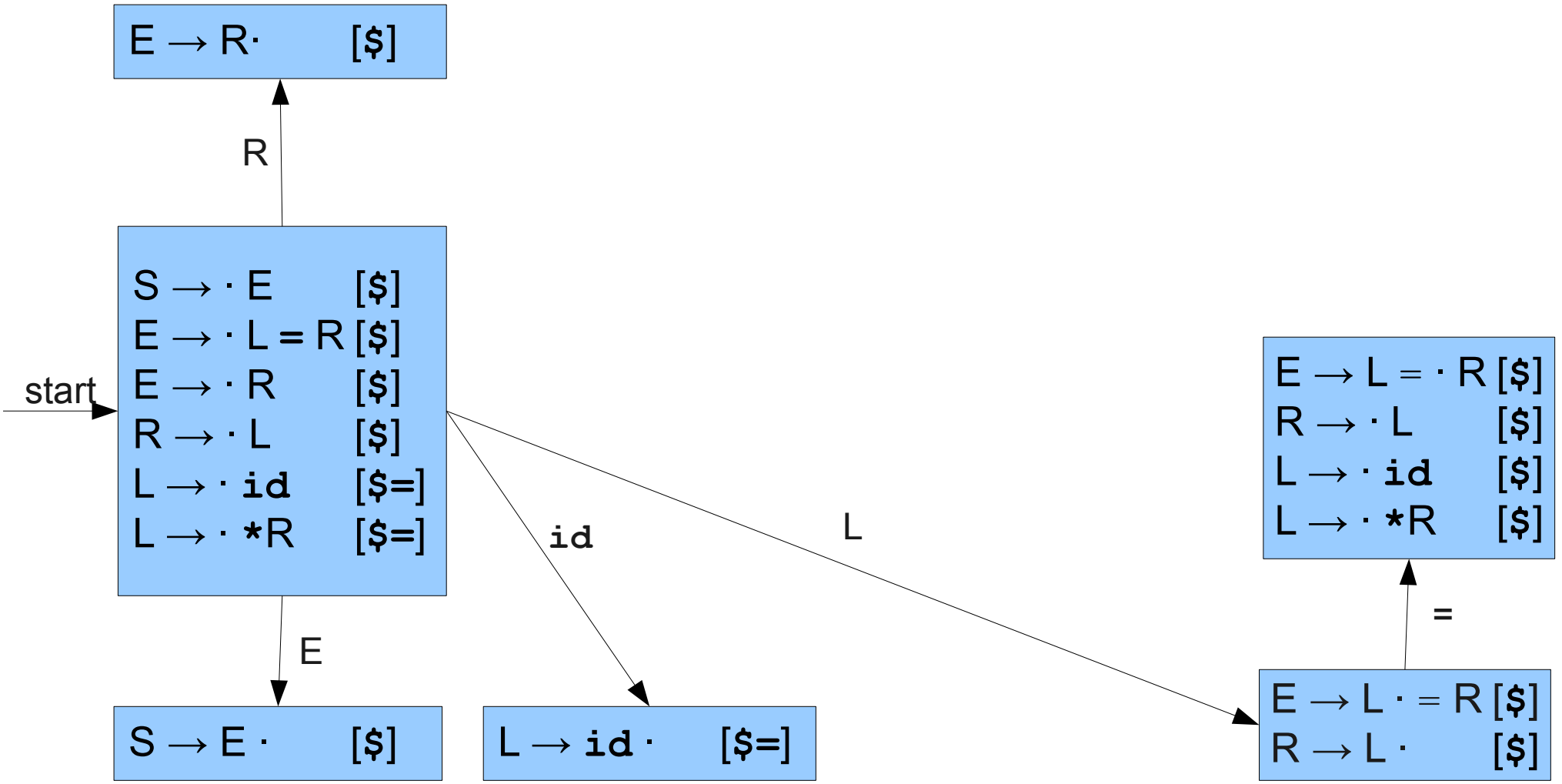

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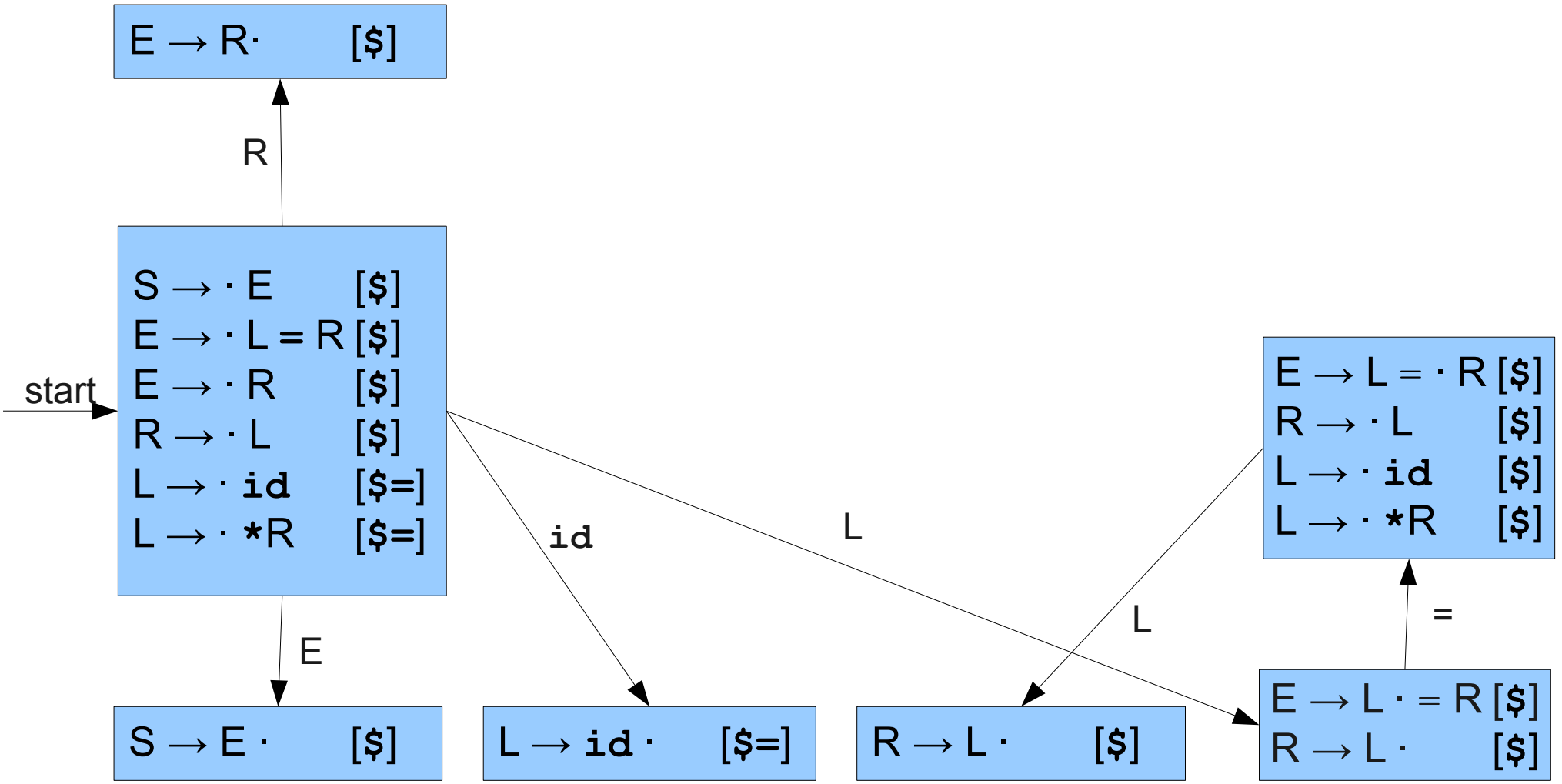

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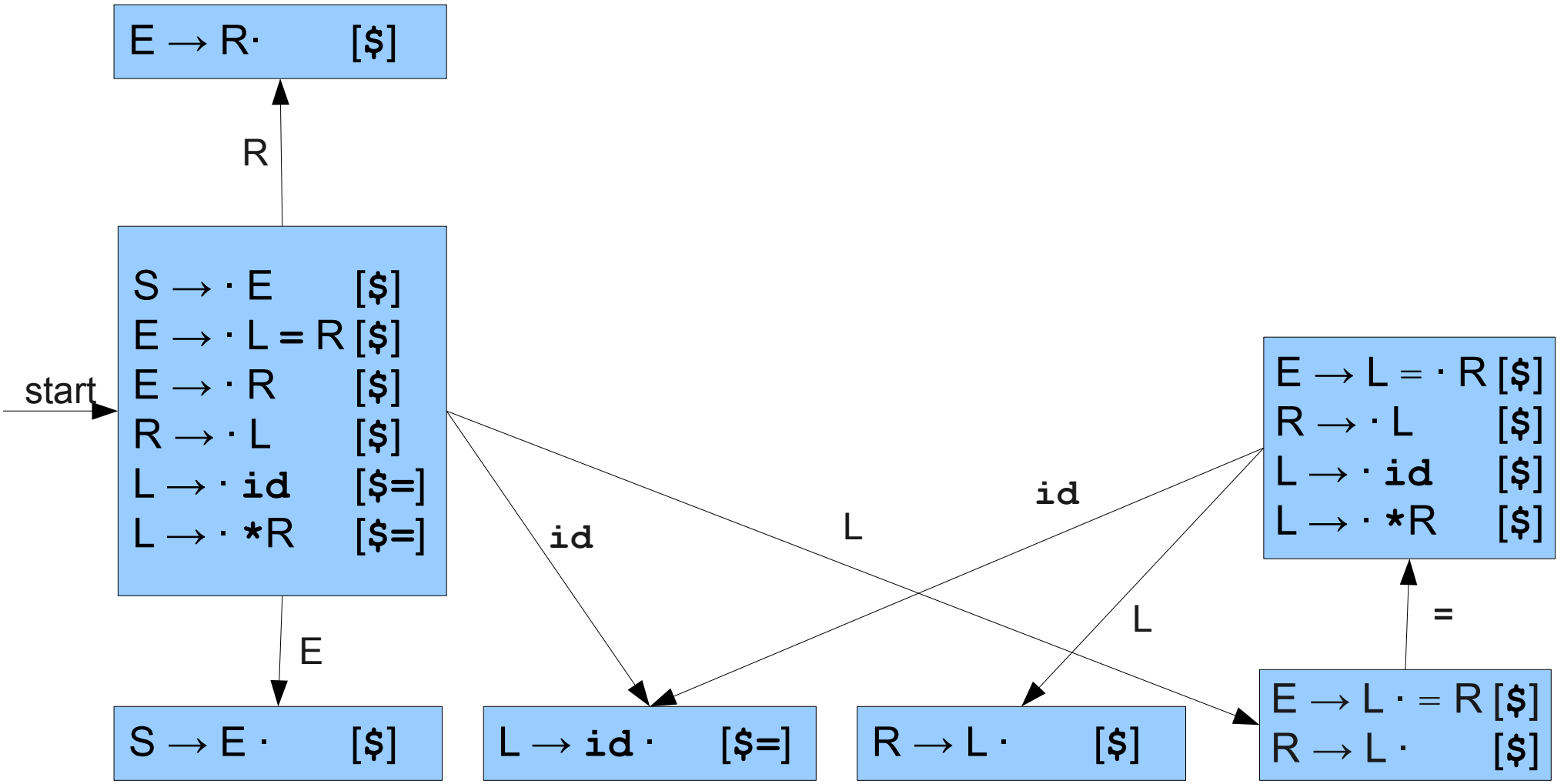

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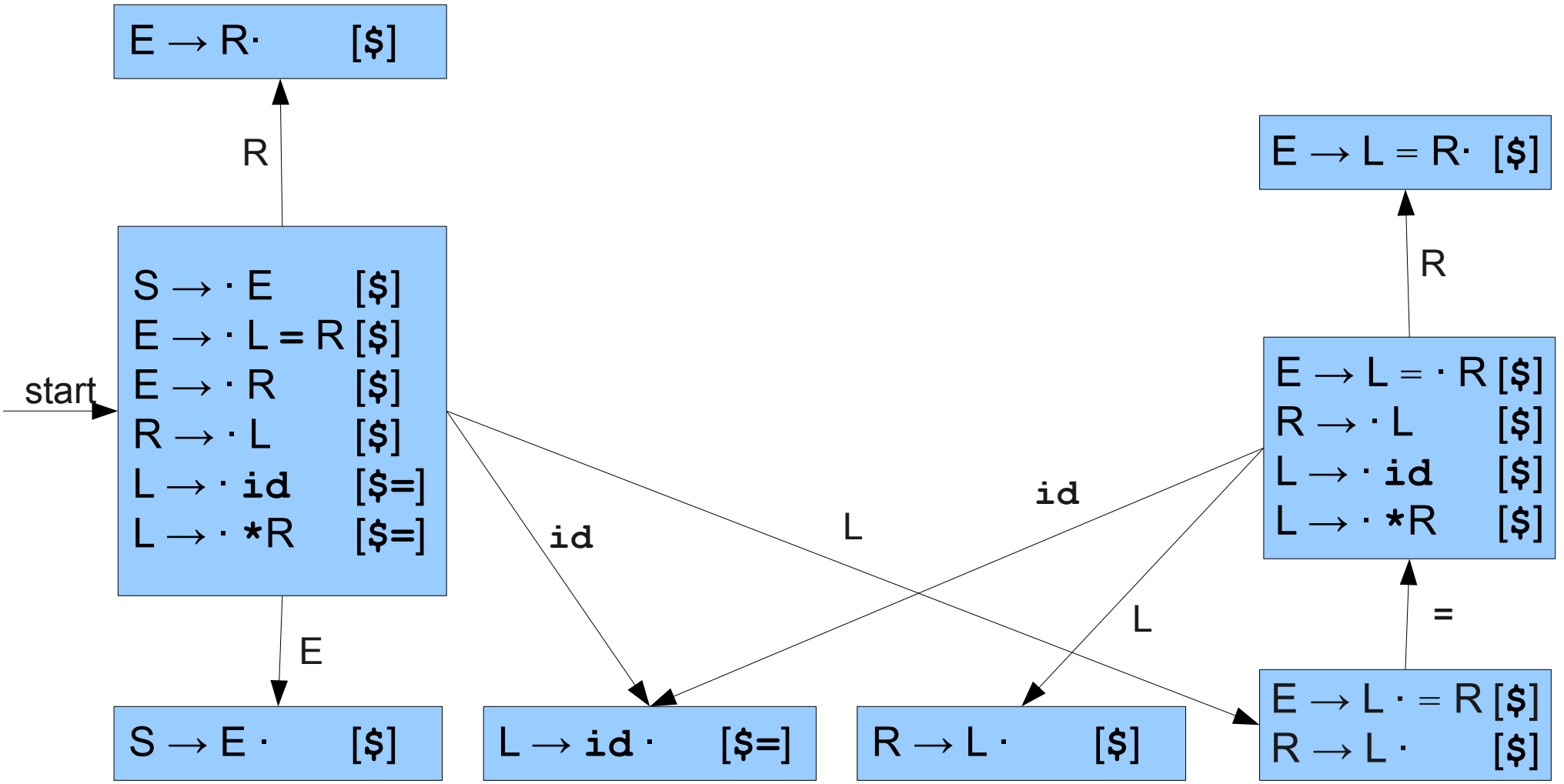

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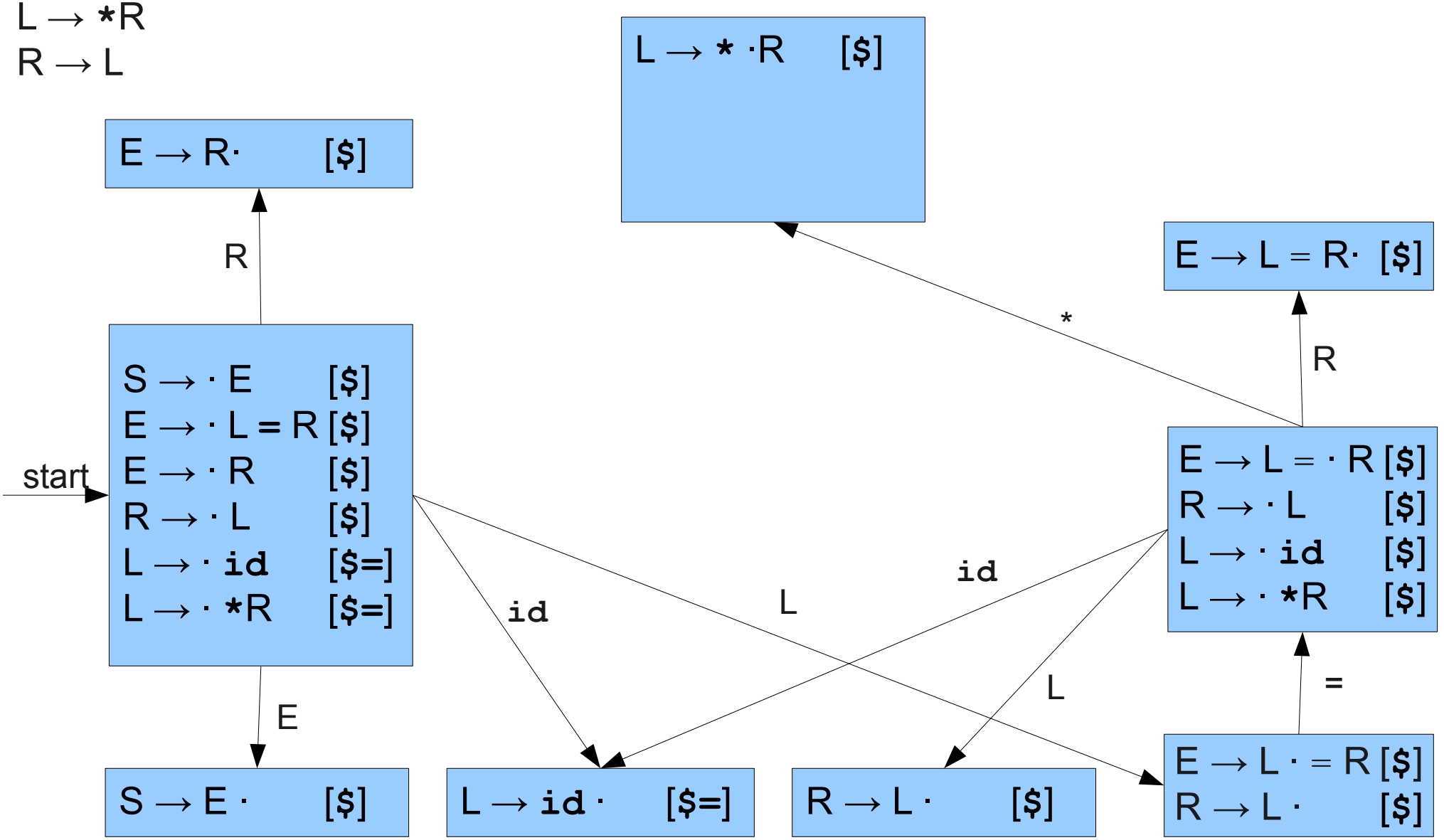

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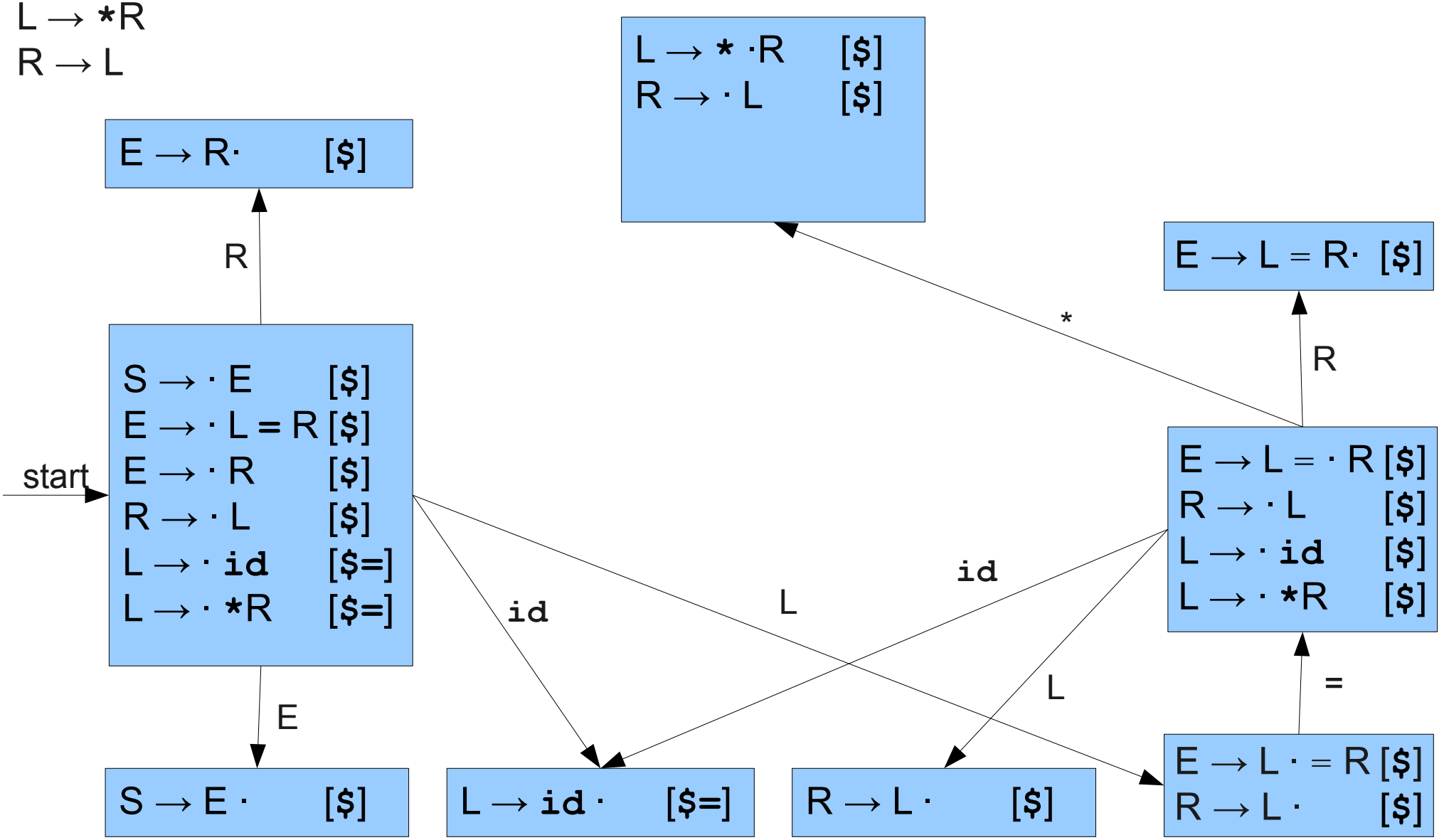

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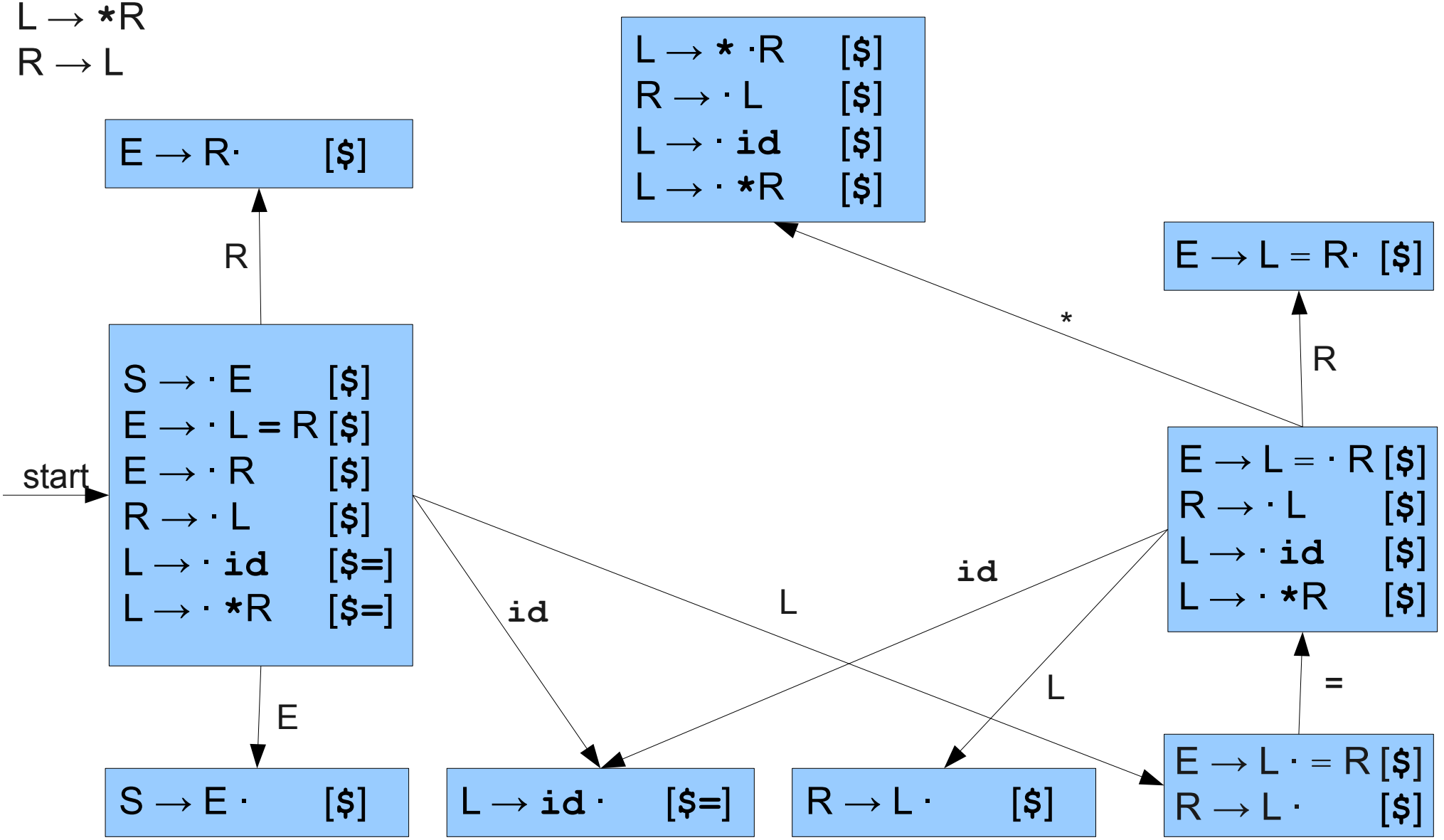

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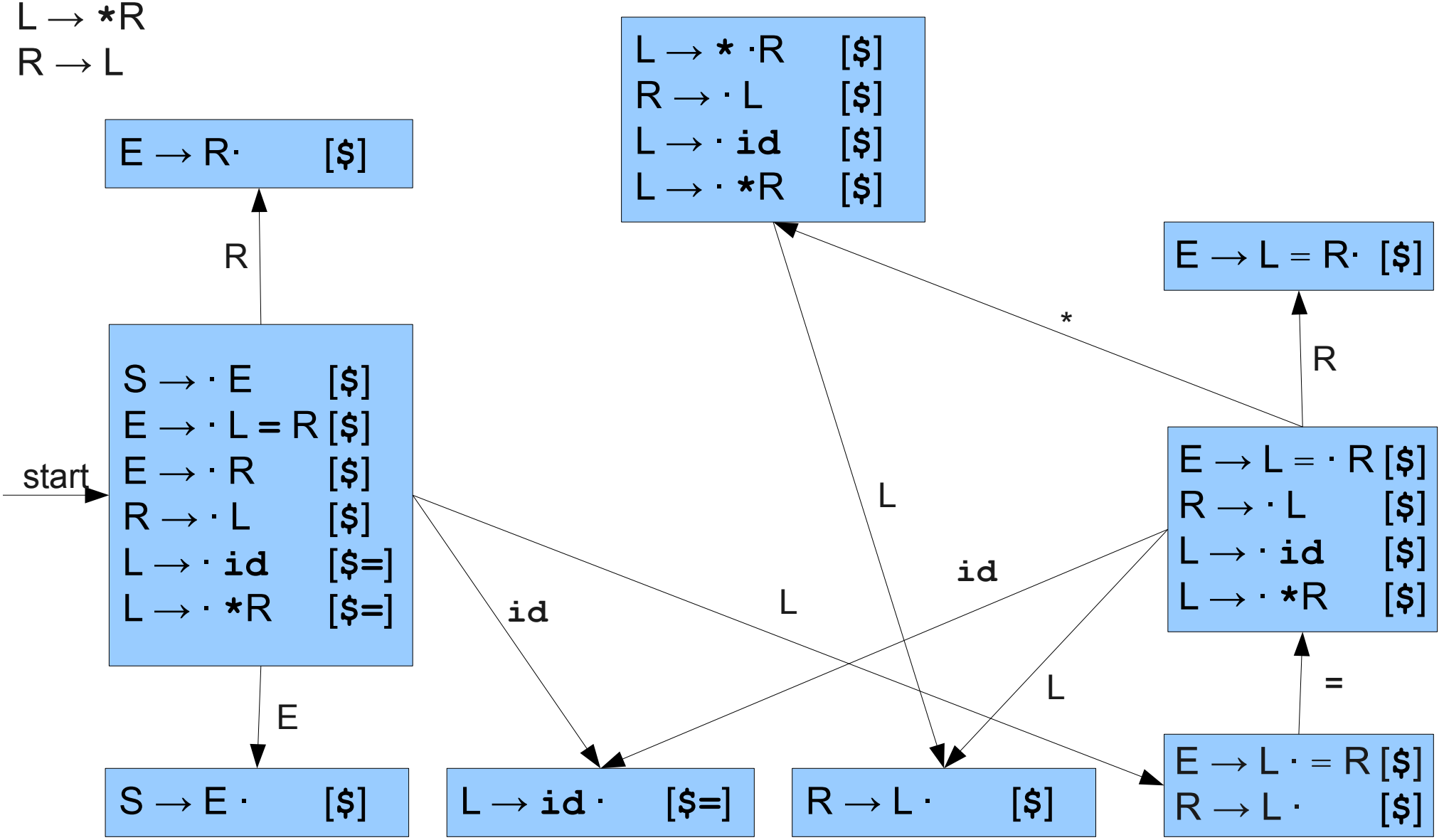

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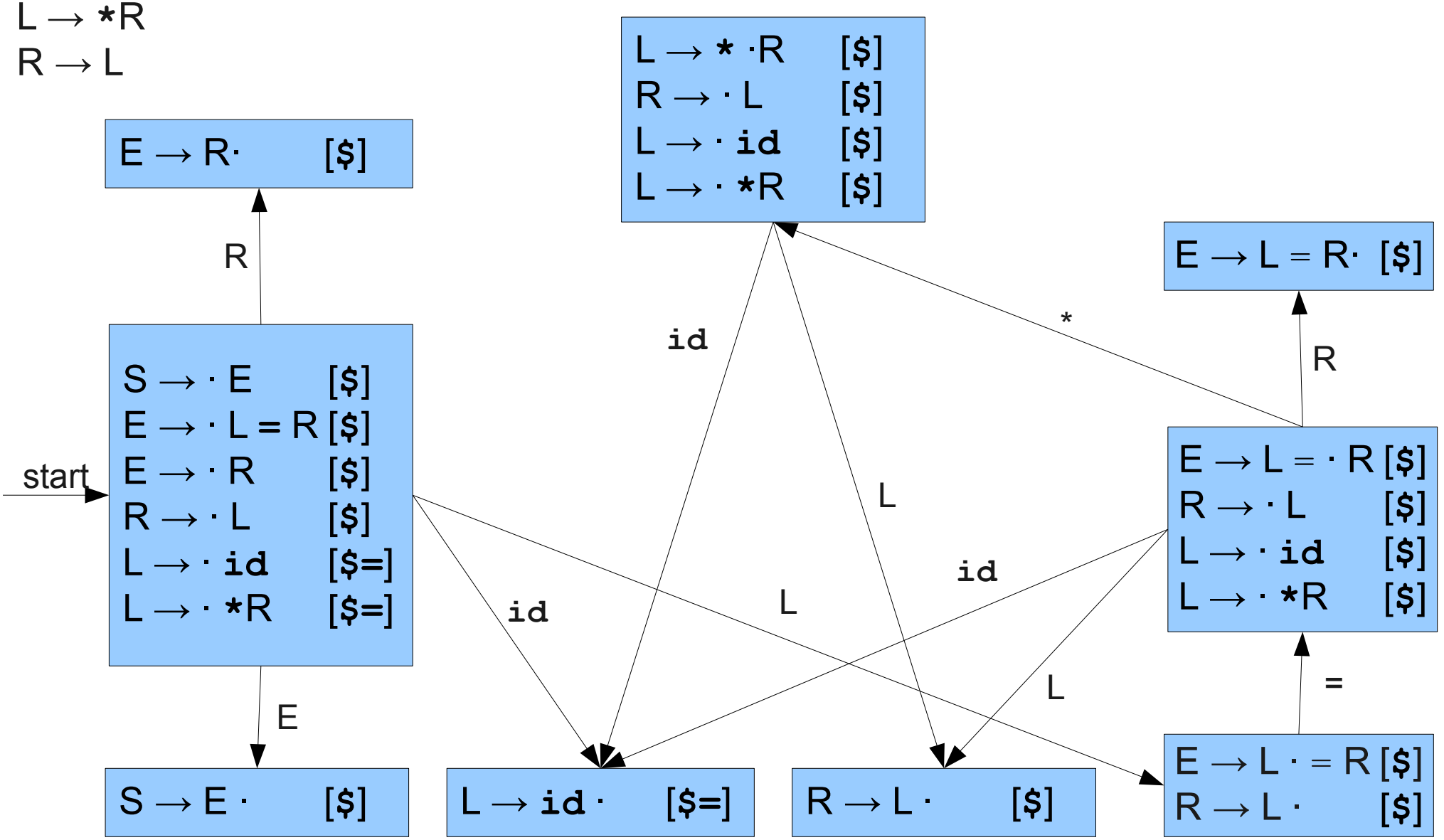

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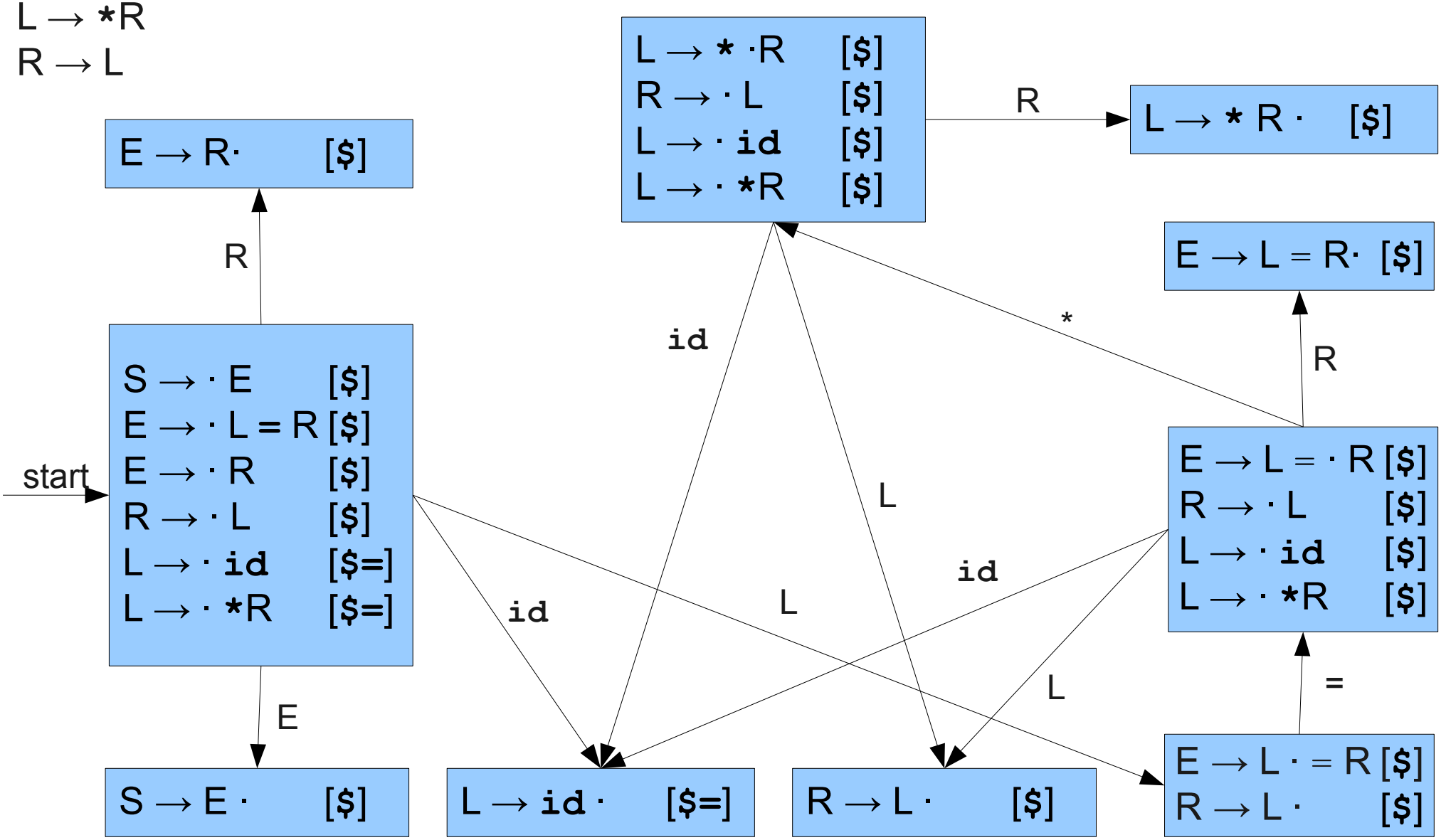

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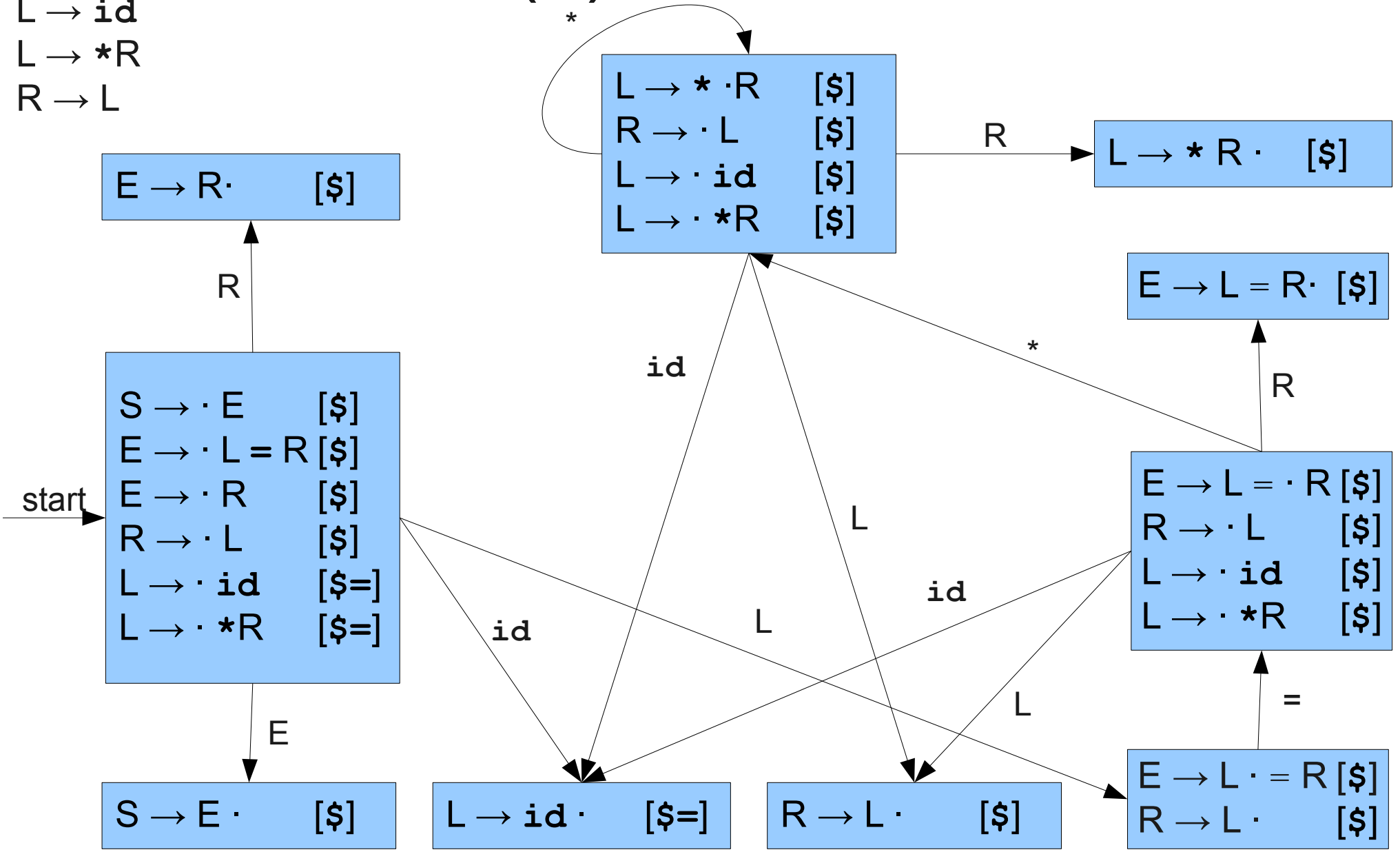

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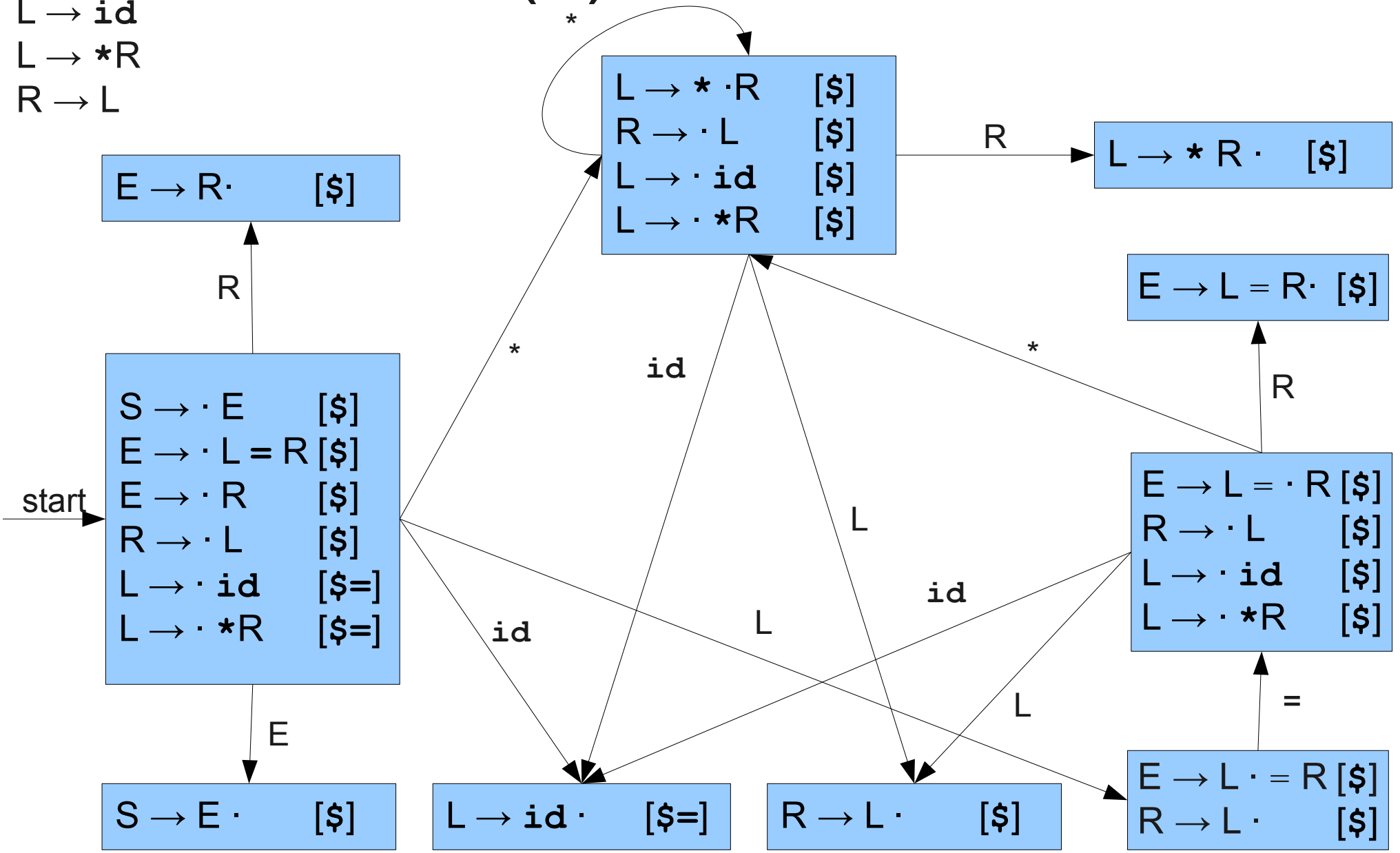

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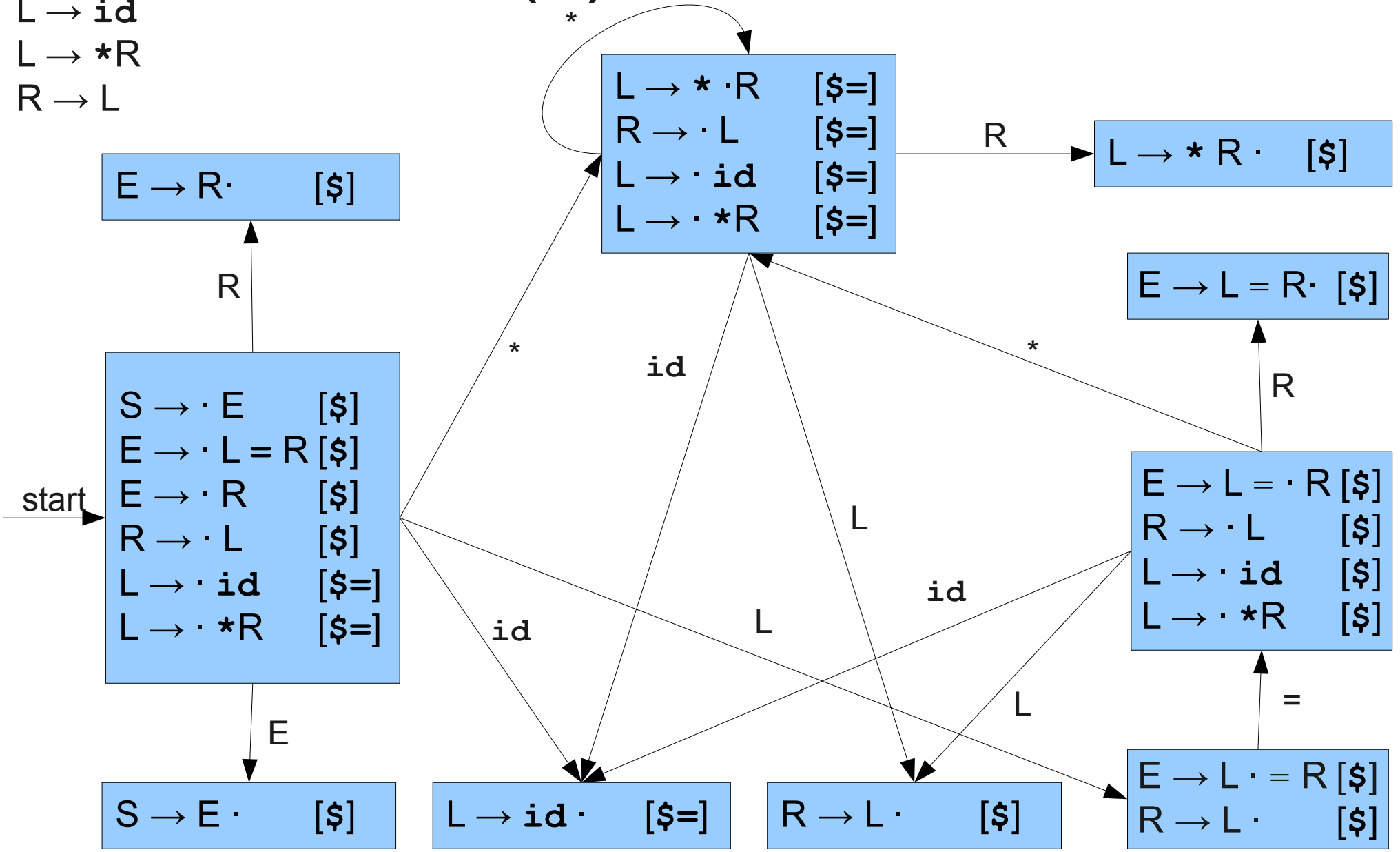

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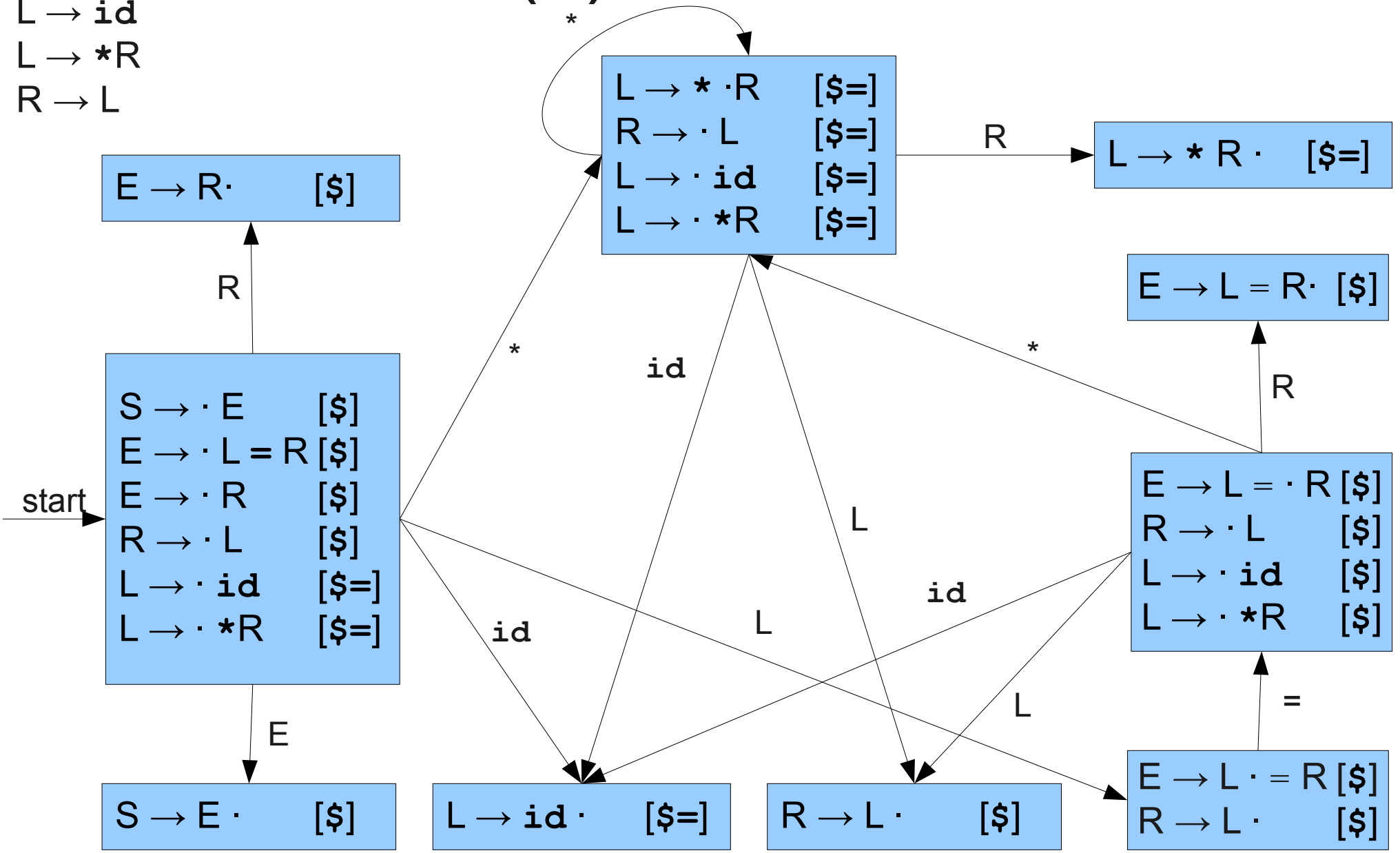

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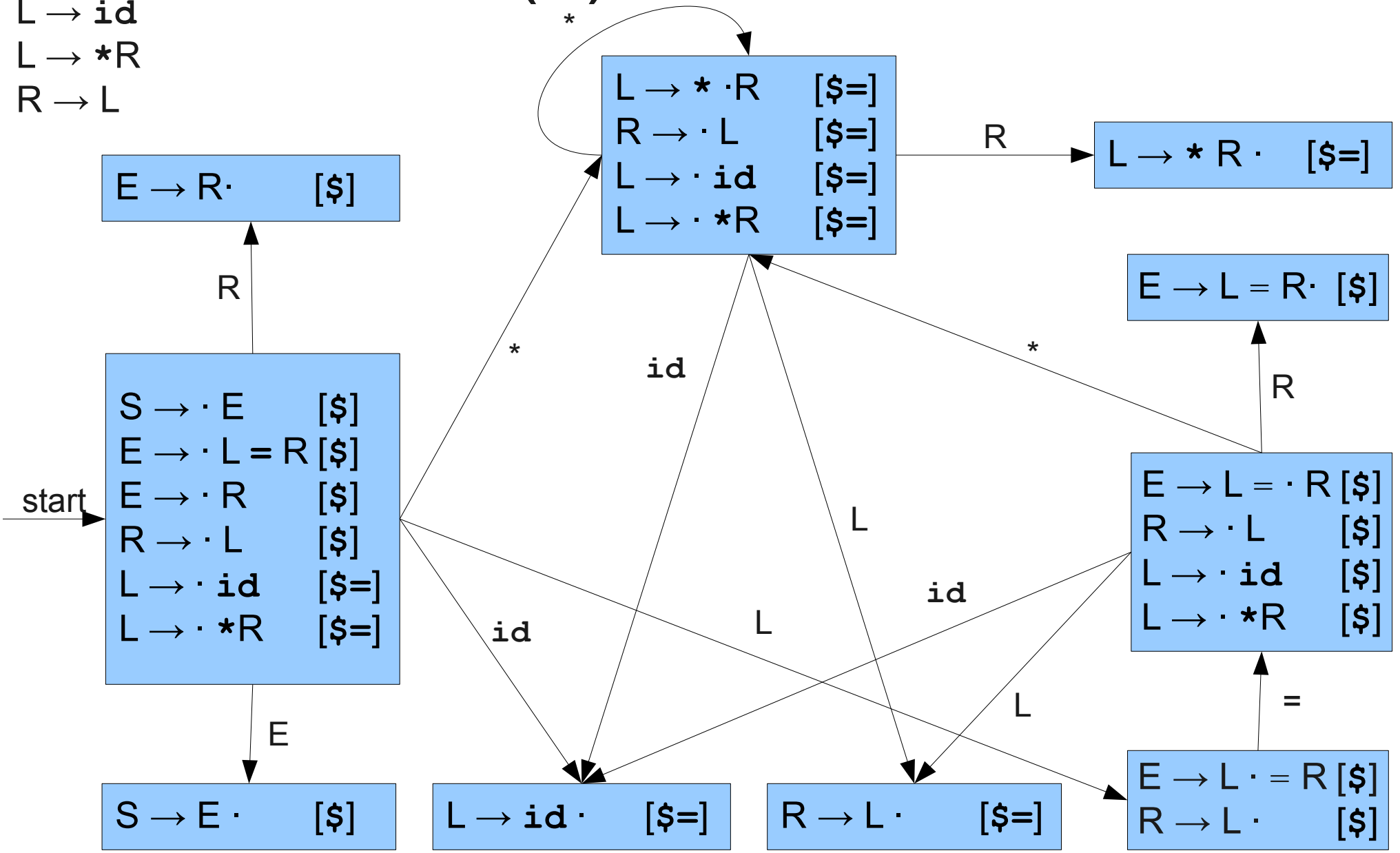

Analysis of our Algorithm

- Since we merge as we go, size of the partial automaton never exceeds size of overall automaton.
- However, this algorithm could be very slow in practice.
- **Why?**

Analysis of our Algorithm

- Since we merge as we go, size of the partial automaton never exceeds size of overall automaton.
- However, this algorithm could be very slow in practice.
- **Why?**
- We might still have to generate all the LR(1) states, even if they immediately get merged.
- This can be very slow.

SLR uses FOLLOW sets

- Recall: FOLLOW(A) is the set of terminals that can follow A in a production:

  $$\text{FOLLOW}(A) = \{ t \mid S \rightarrow^* uAtv \}$$

- SLR is LR(0), with reductions augmented using FOLLOW sets.
- This is too weak for two reasons:
  - It ignores **context** (what state we're in).
  - It ignores **which reduction** we're doing.
- A stronger concept is the **lookahead set**.

LALR uses LA sets

- The **lookahead set** LA(q, A → *w*) is defined as

  LA(q, A → *w*) = { t | S →* *u*A*t*v and *uw* reaches q }

- Here, "*uw* reaches q" means that the LR(0) automaton, when run on *uw* reaches state q.
- Intuitively, if we're in some state q and are going to reduce A to *w*, LA(q, A → *w*) is the set of terminals that could actually follow A at this point.
- Much more precise than FOLLOW sets.

LA and FOLLOW

- The **lookahead set** LA(q, A → *w*) is defined as

  LA(q, A → *w*) = { t | S →* *u*At*v* and *uw* reaches q }

- The **follow set** FOLLOW(A) is defined as

  FOLLOW(A) = { t | S →* *u*At*v* }

- These definitions are very similar to one another.
- If we can compute LA from FOLLOW, we can construct a LALR(1) parser efficiently.

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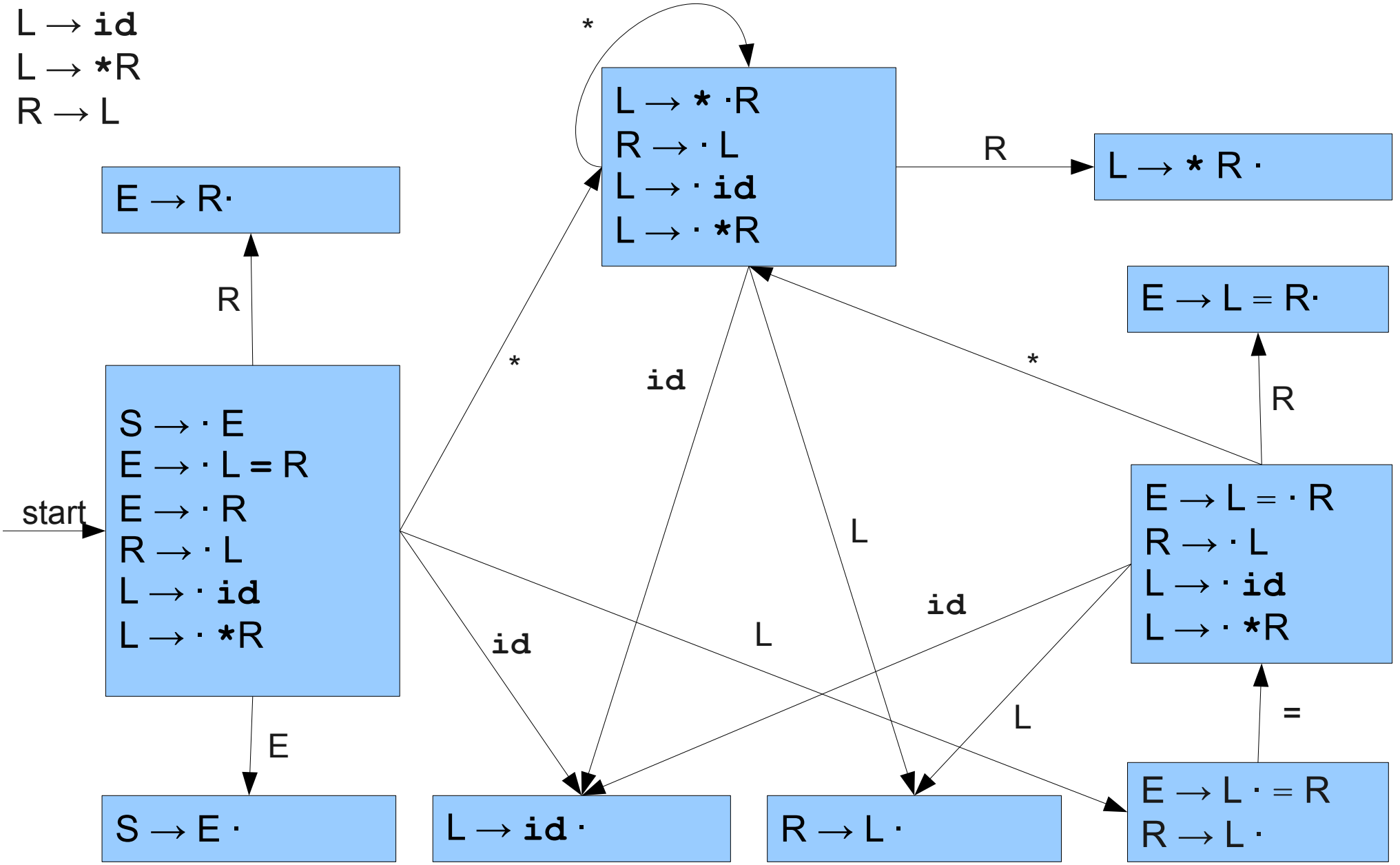
An LR(0) Automaton



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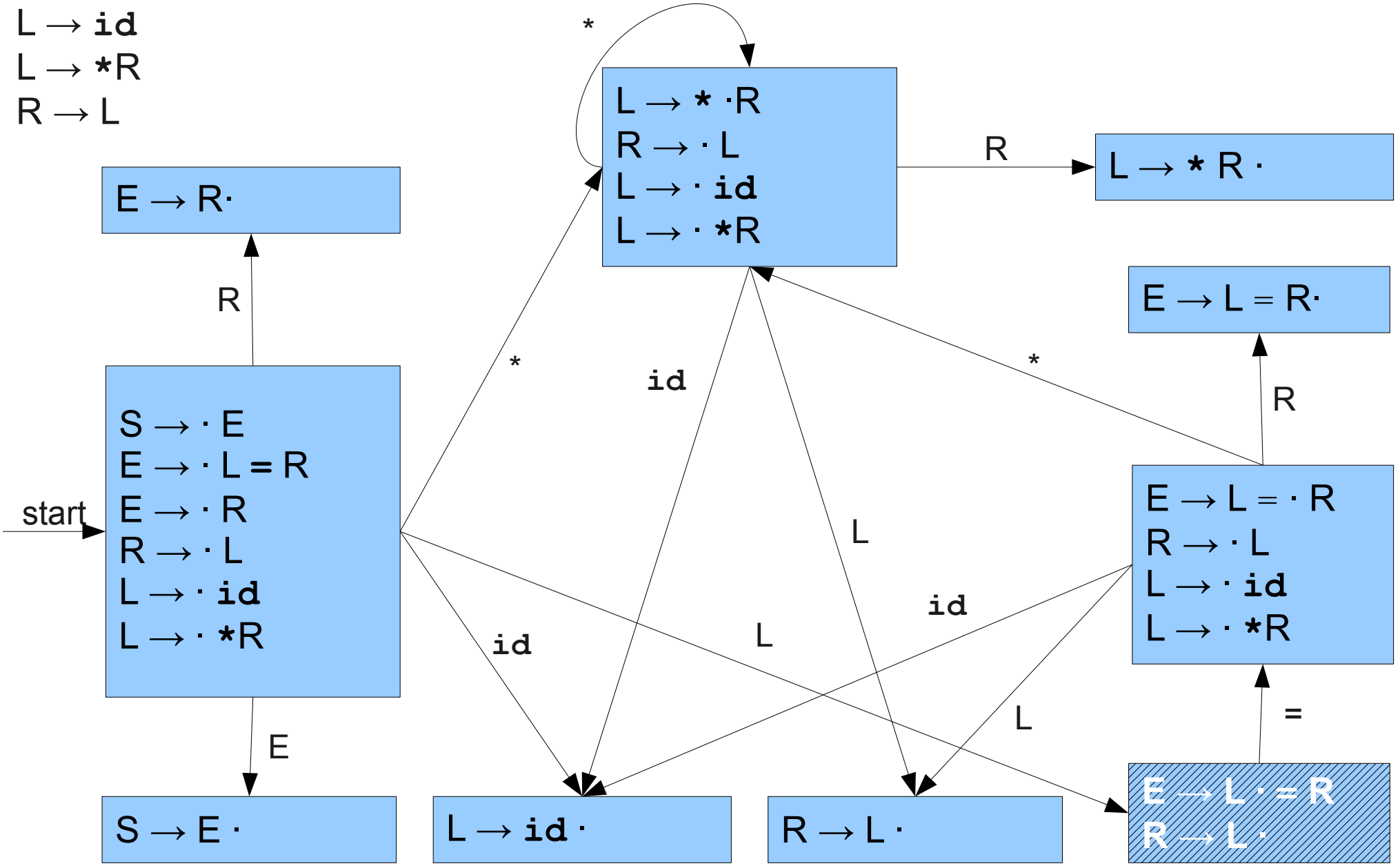
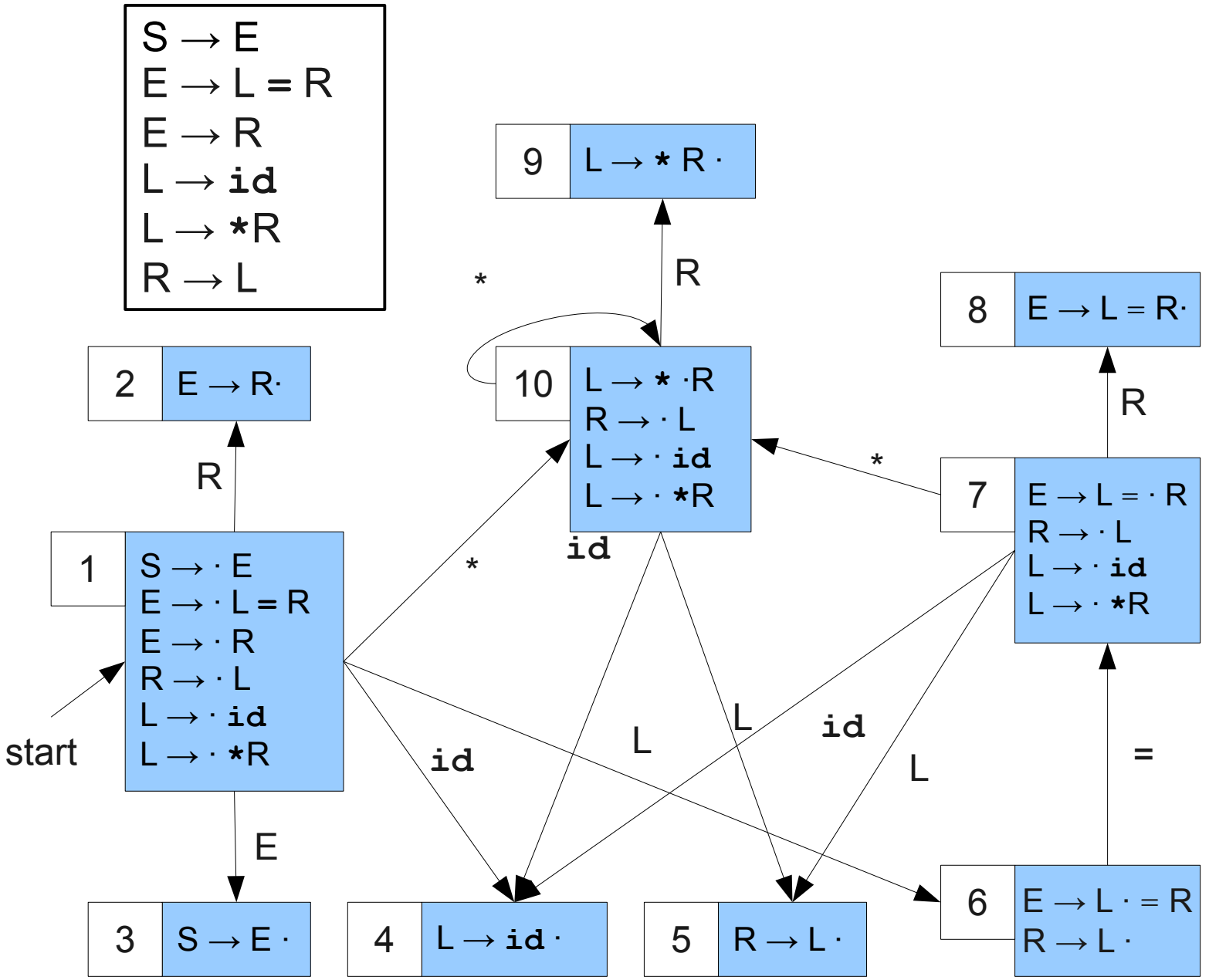


What if we used the LR(0) automaton to **change the grammar** to retain more context?

Prepare for one of the most beautiful constructions of the quarter...

Augmenting the Grammar



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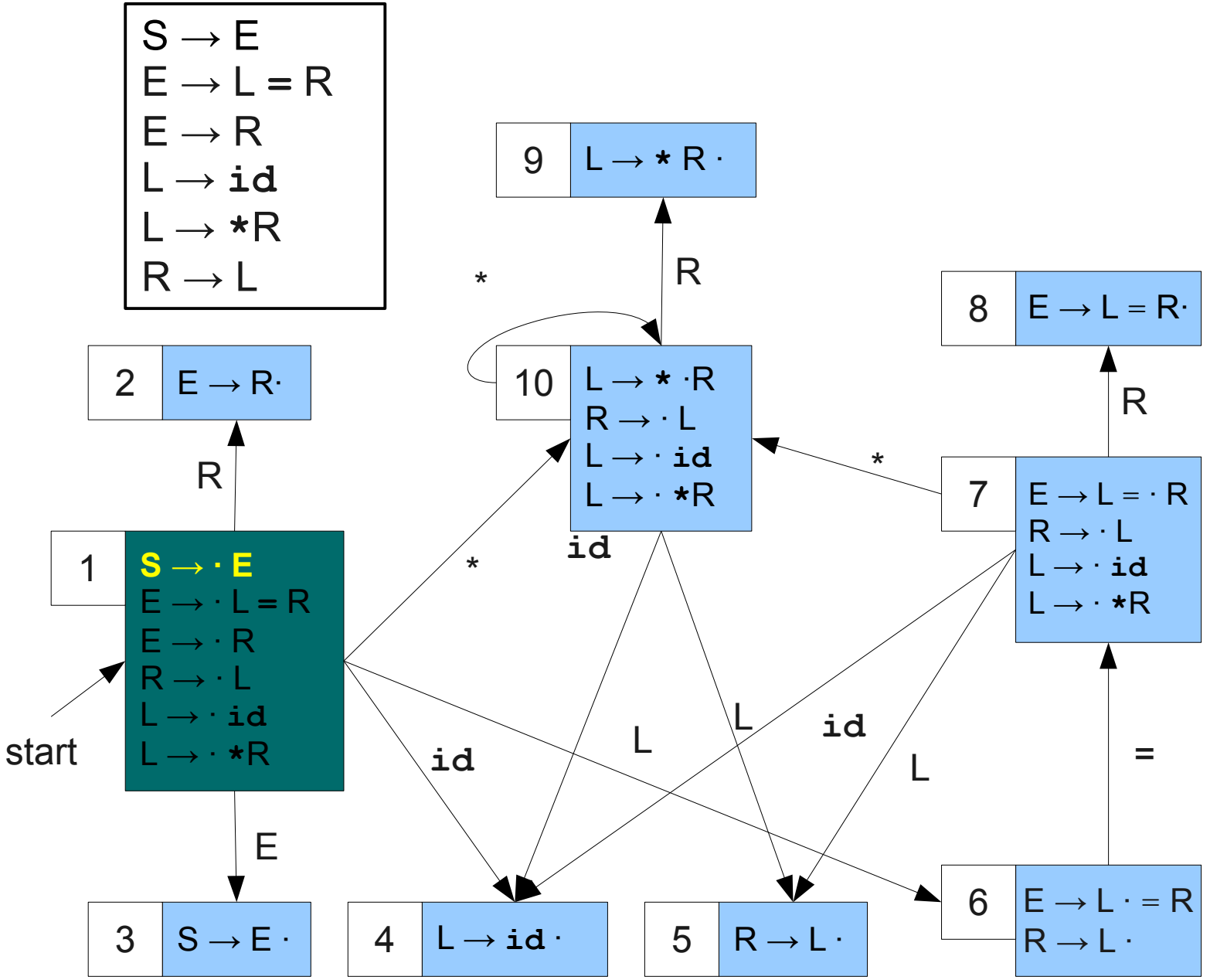
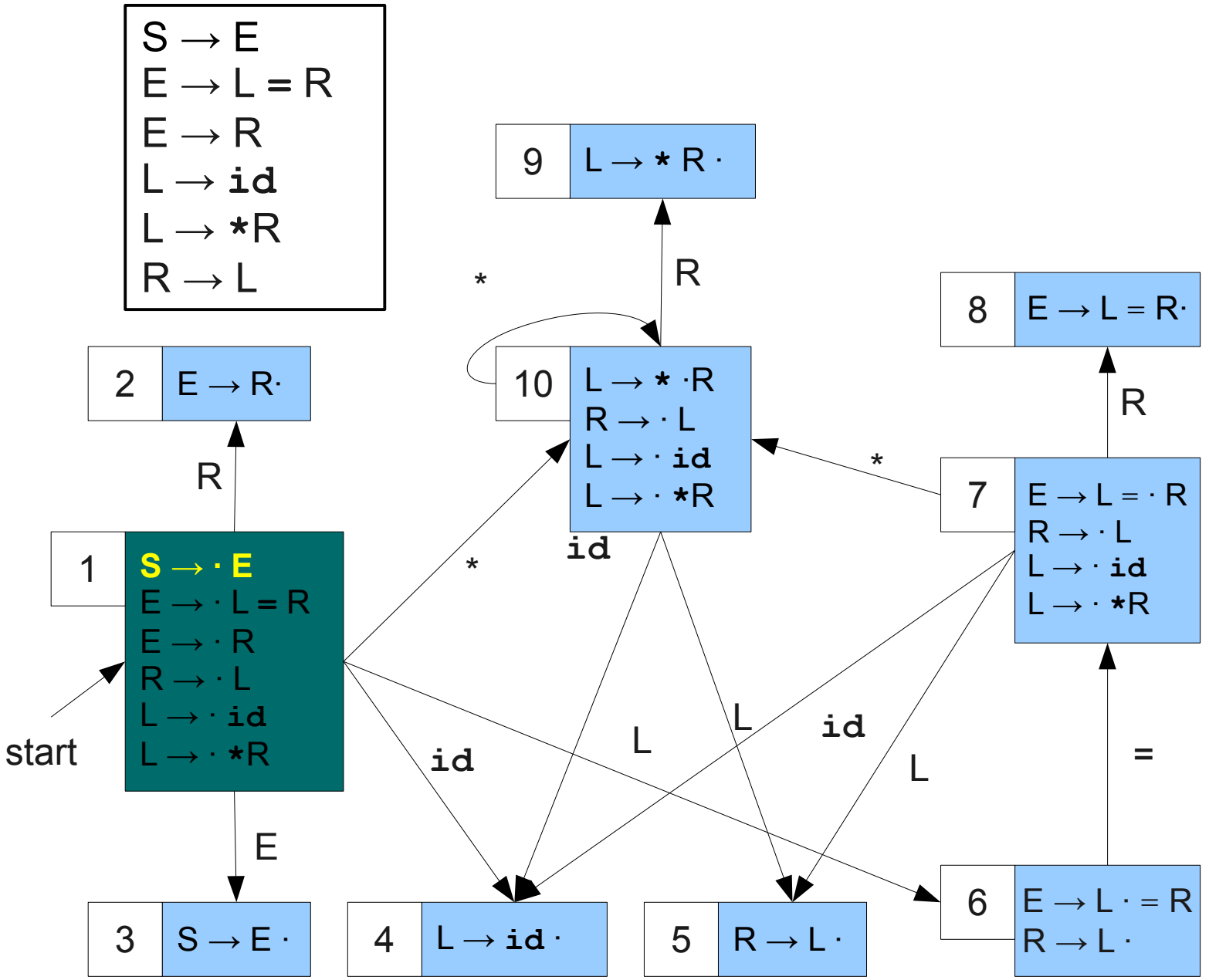


Augmenting the Grammar

$S_1 \rightarrow E_{1\text{-}3}$



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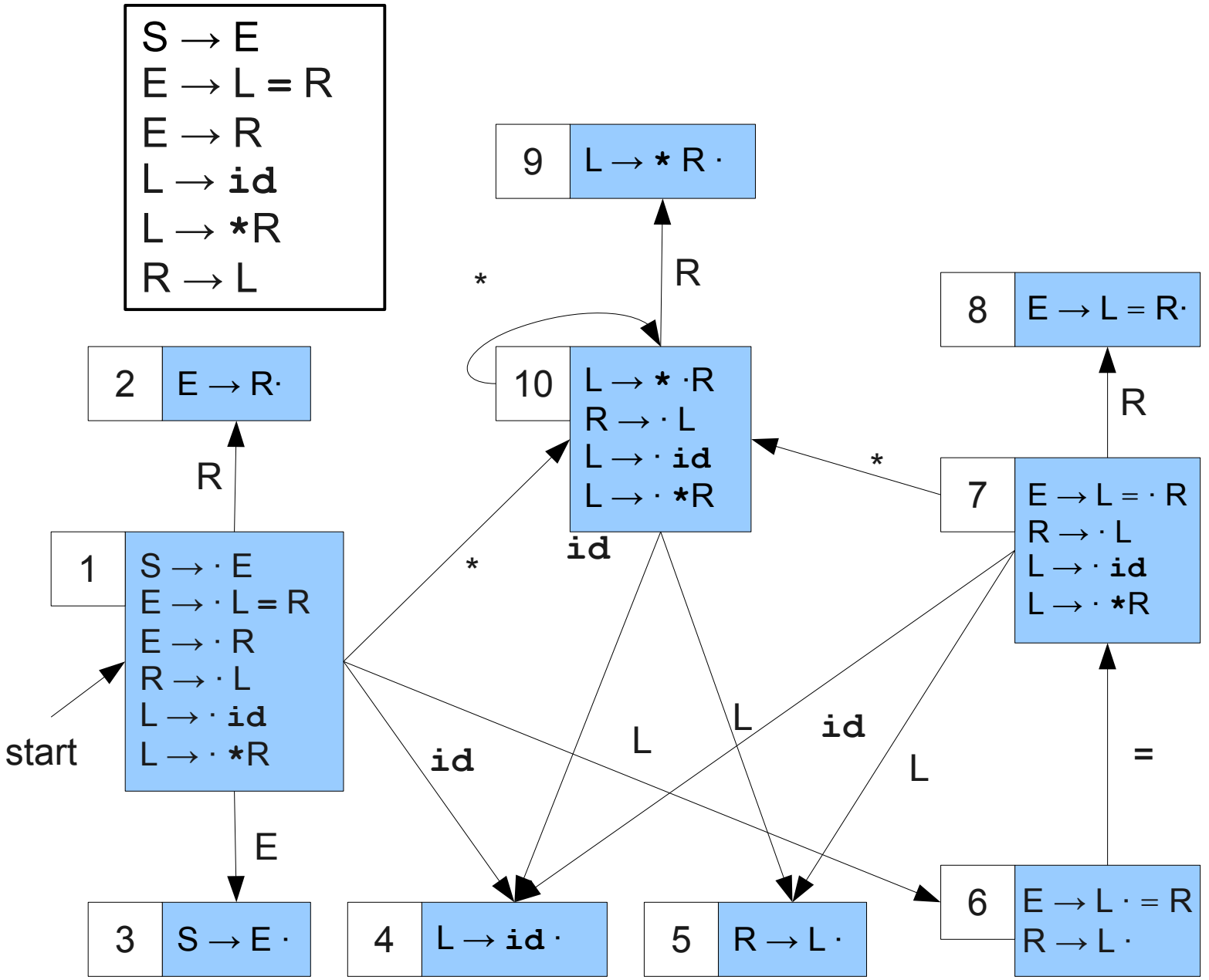

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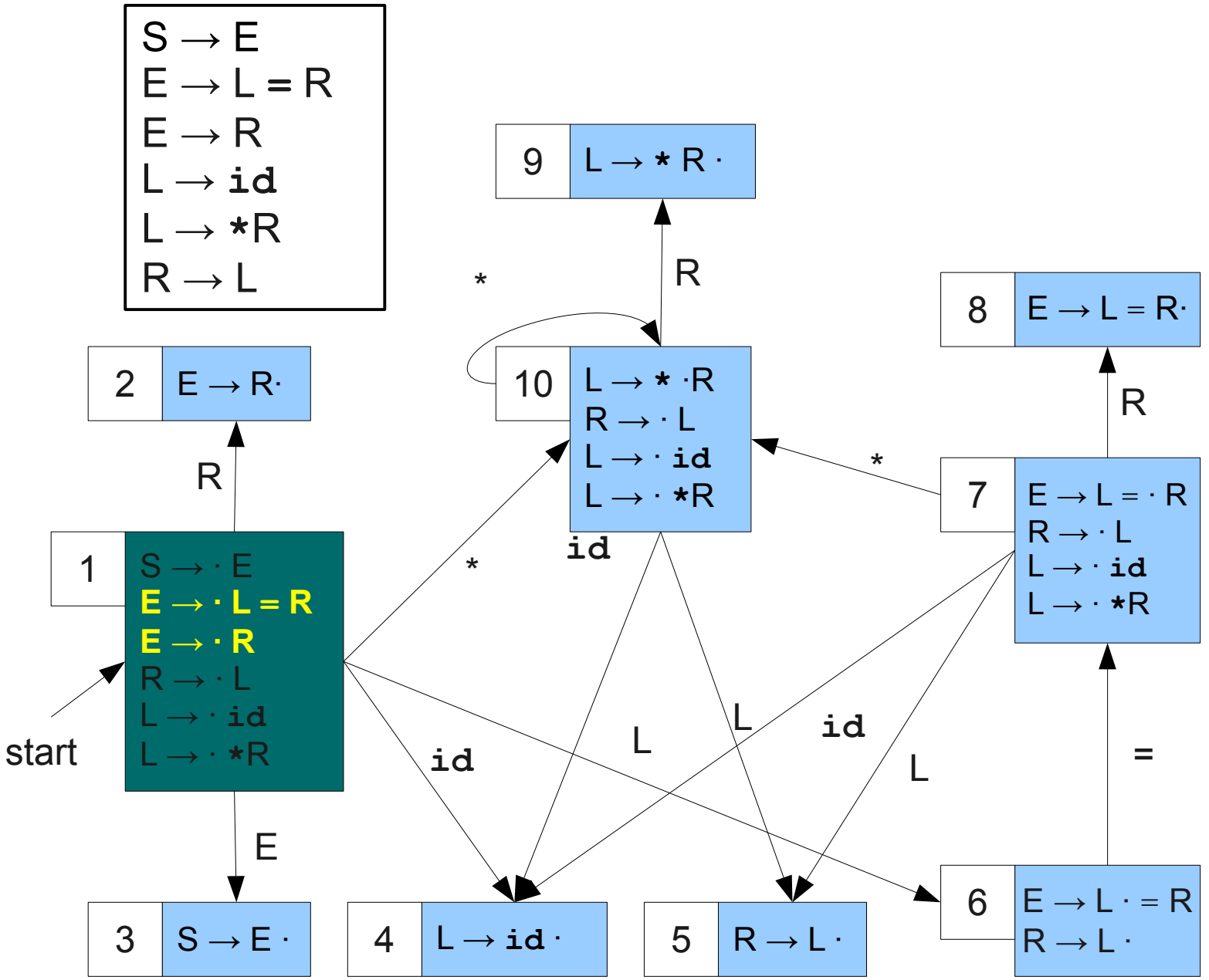

Augmenting the Grammar

$S_1 \rightarrow E_{1\text{-}3}$

$E_{1\text{-}3} \rightarrow L_{1\text{-}6} = R_{7\text{-}8}$

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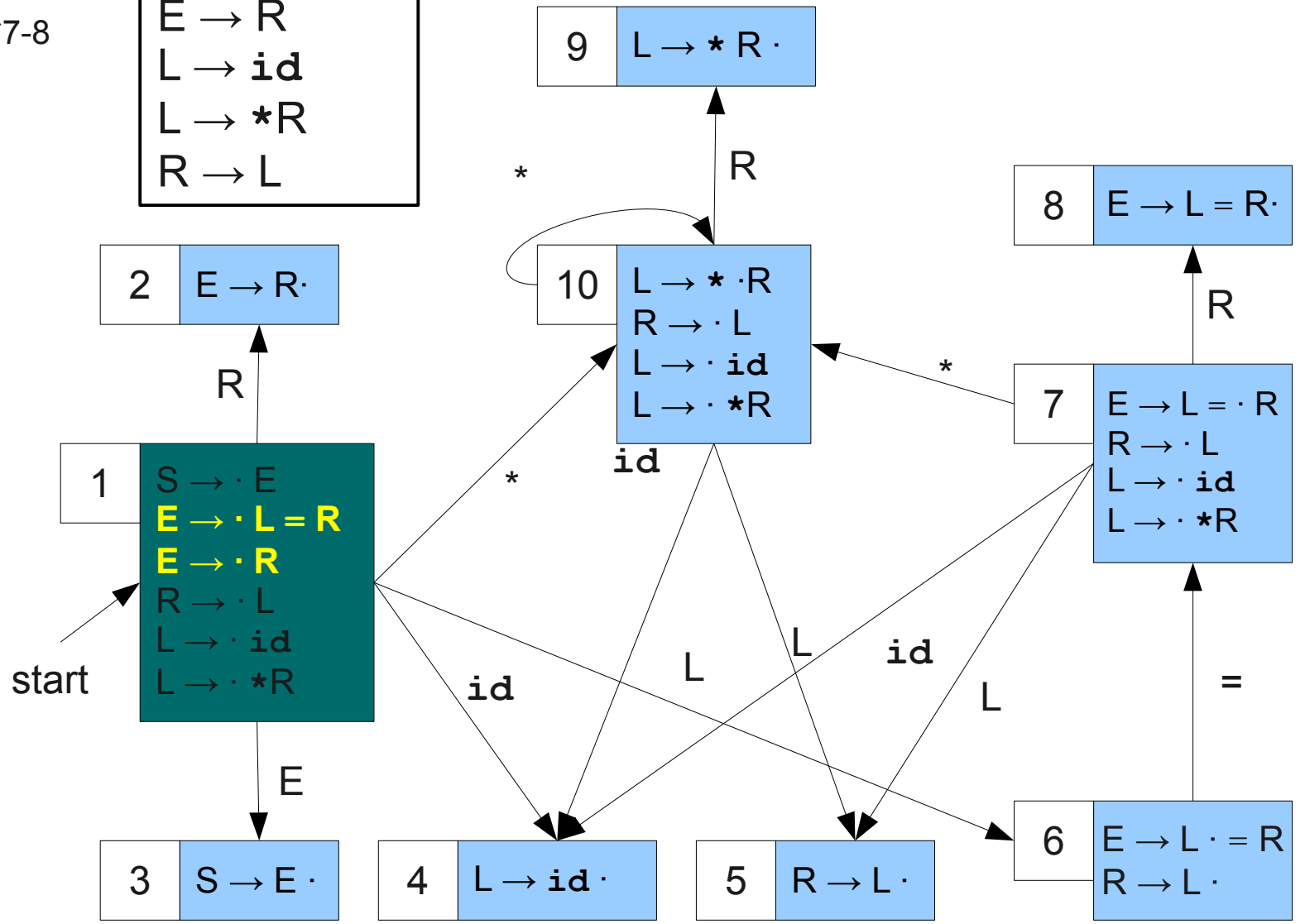

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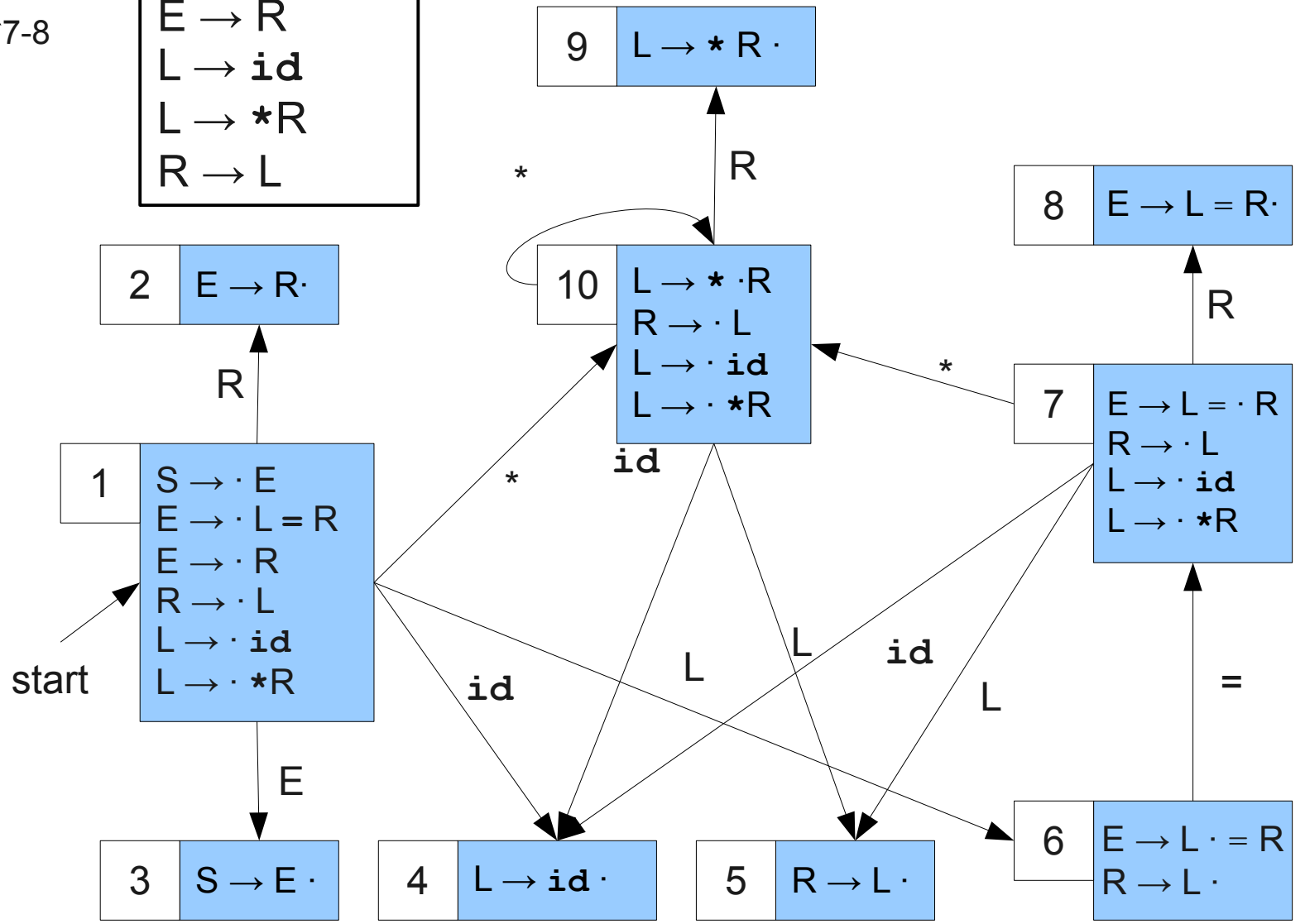

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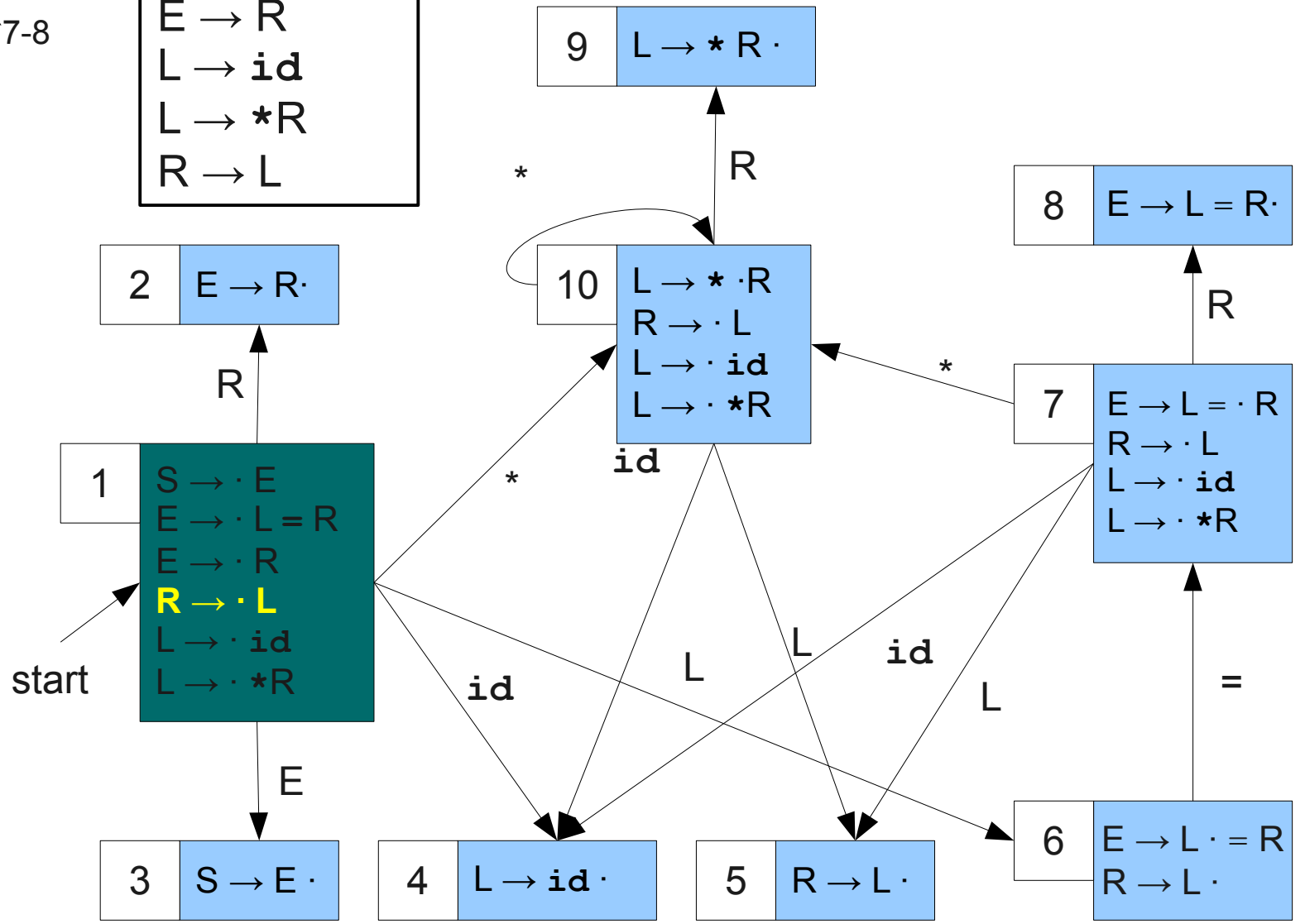

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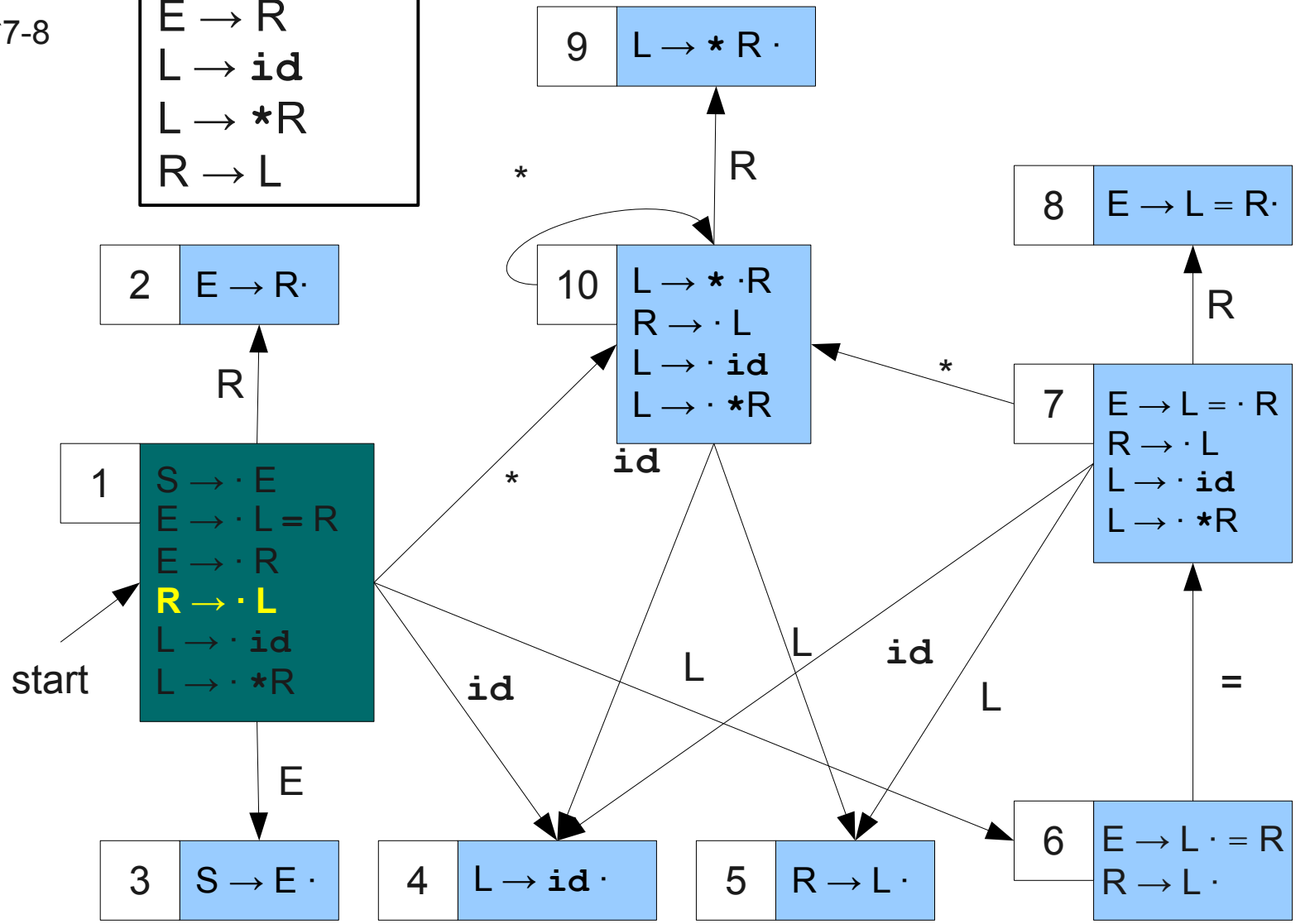

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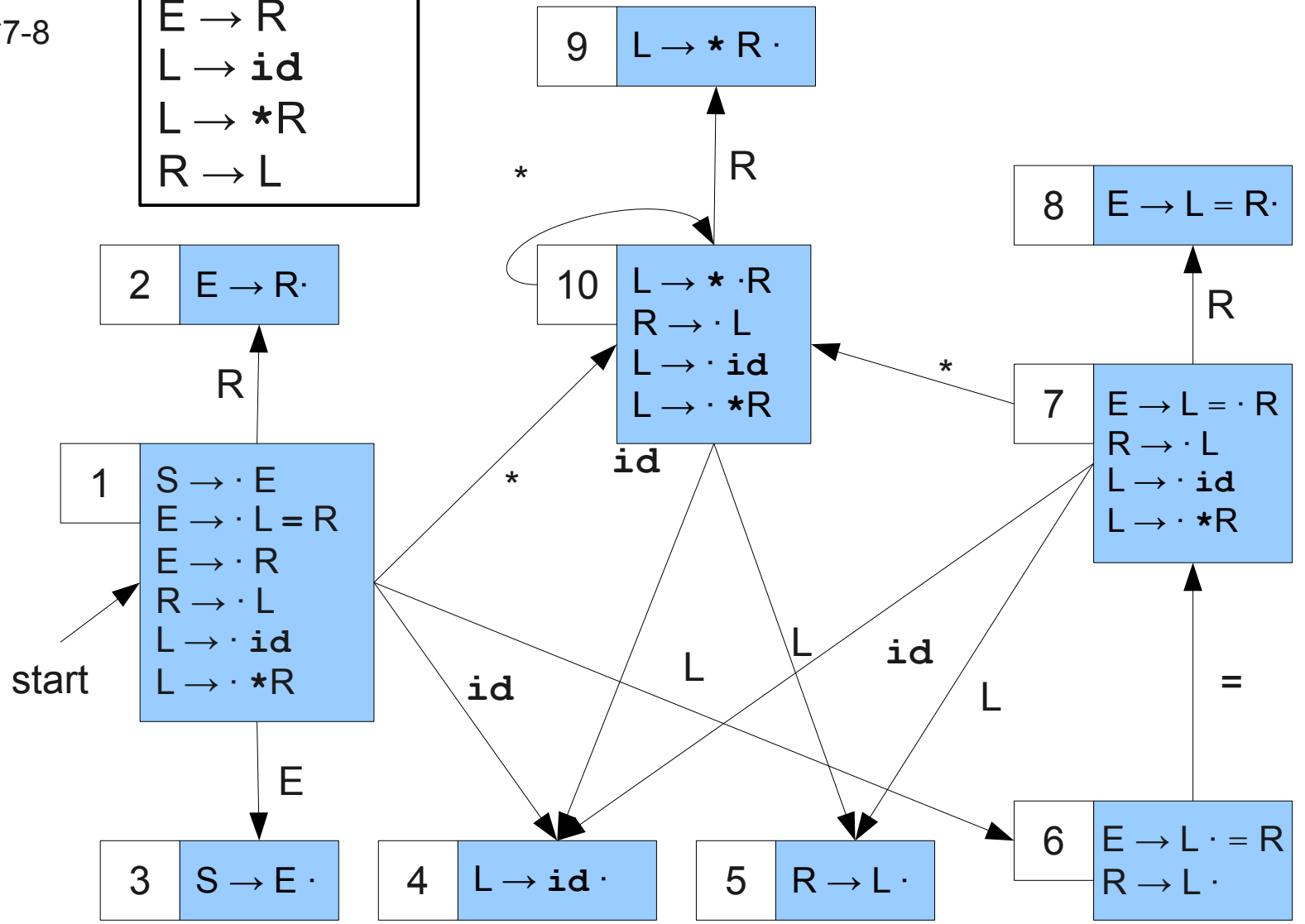

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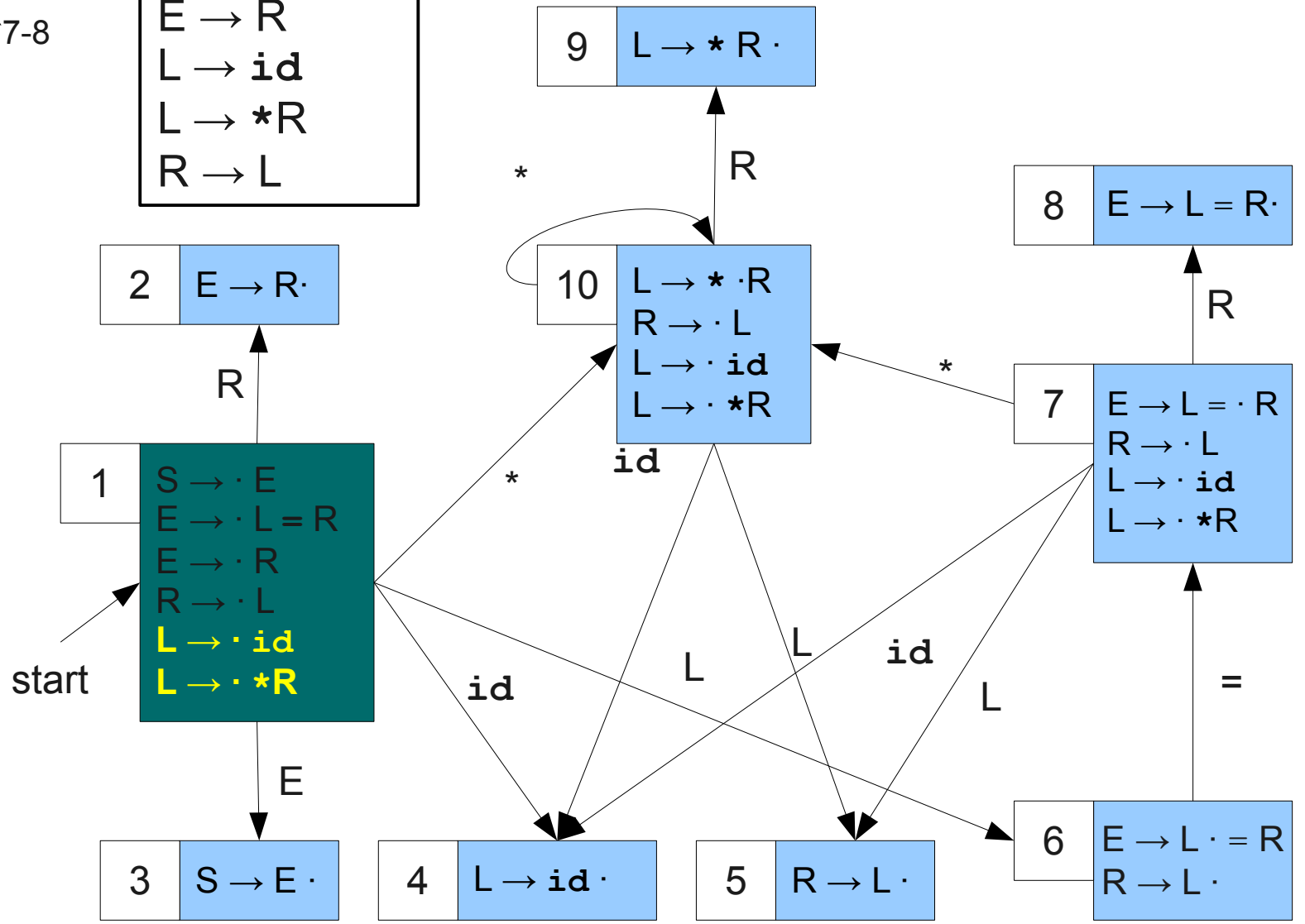

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$L_{1\text{-}6} \rightarrow \mathbf{id}$

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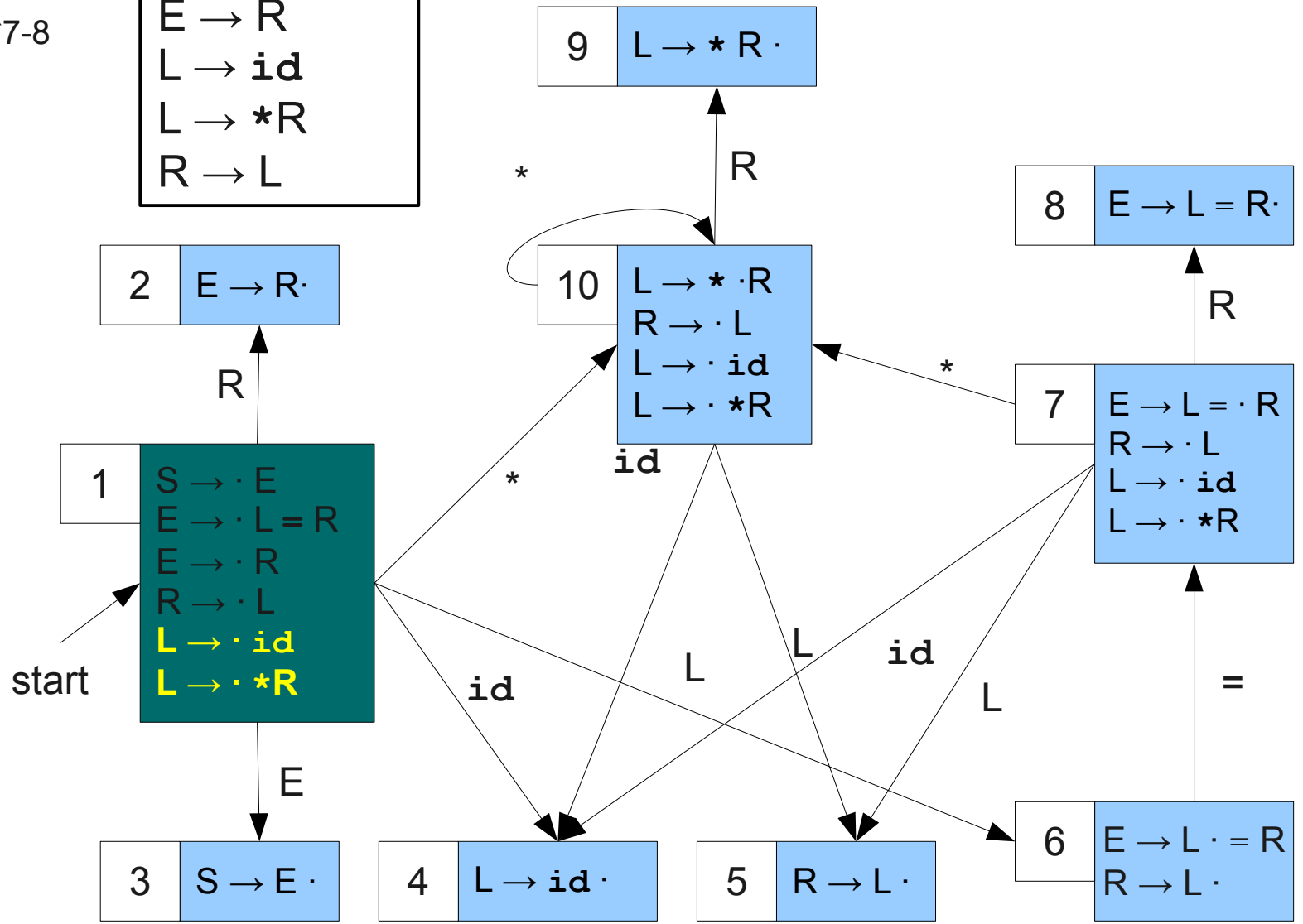

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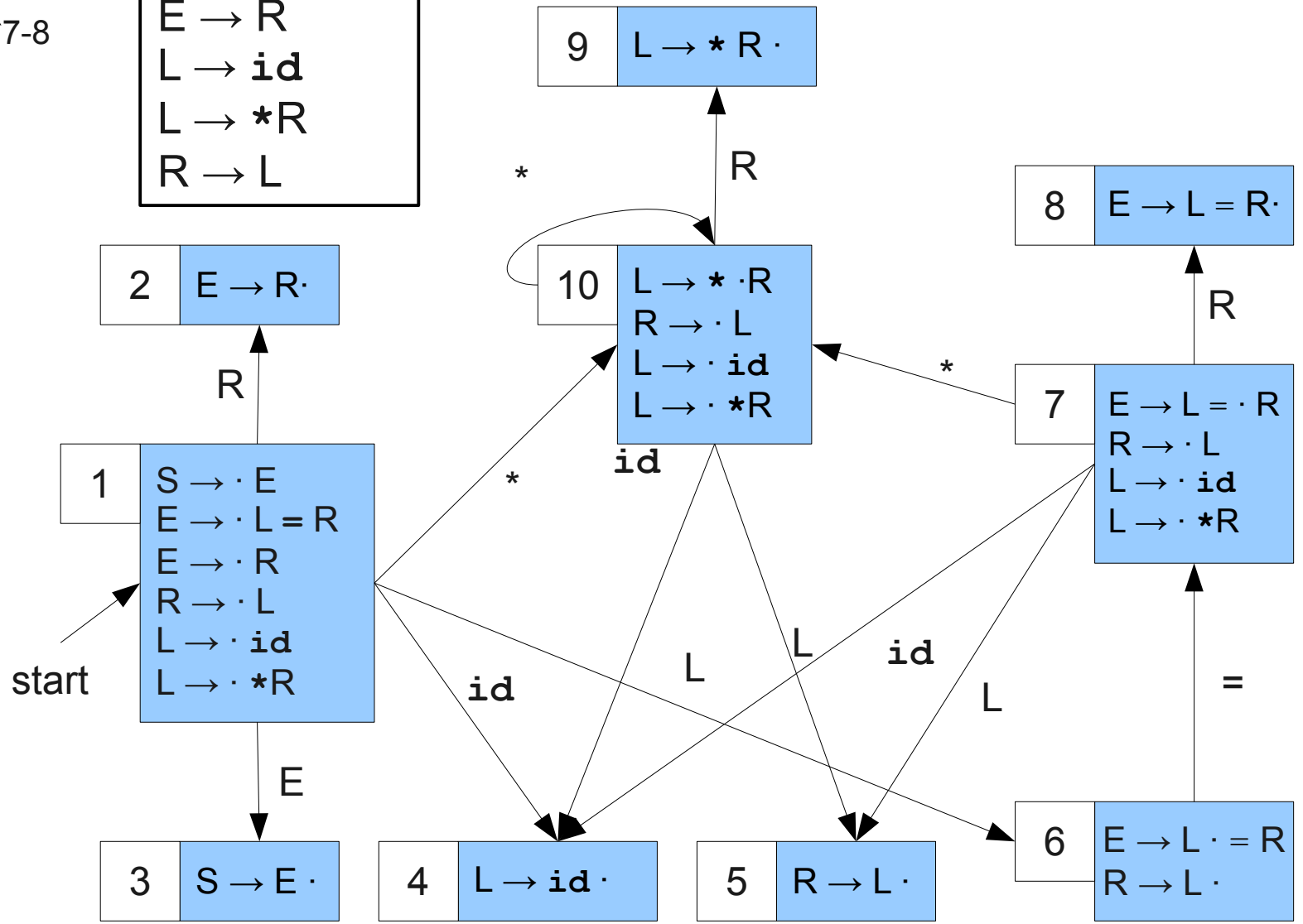

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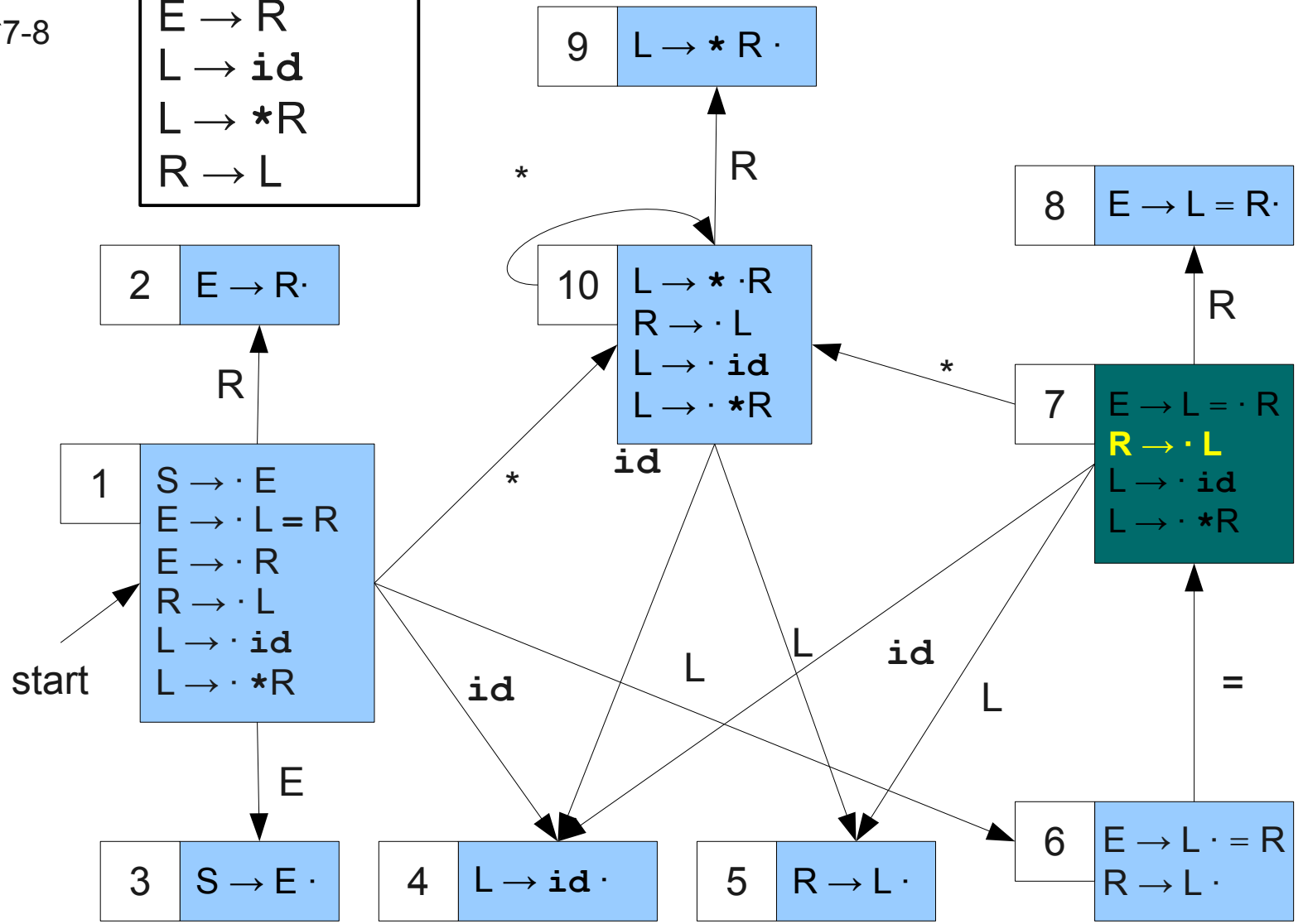


Augmenting the Grammar

$S_1 \rightarrow E_{1\text{-}3}$

$E_{1\text{-}3} \rightarrow L_{1\text{-}6} = R_{7\text{-}8}$

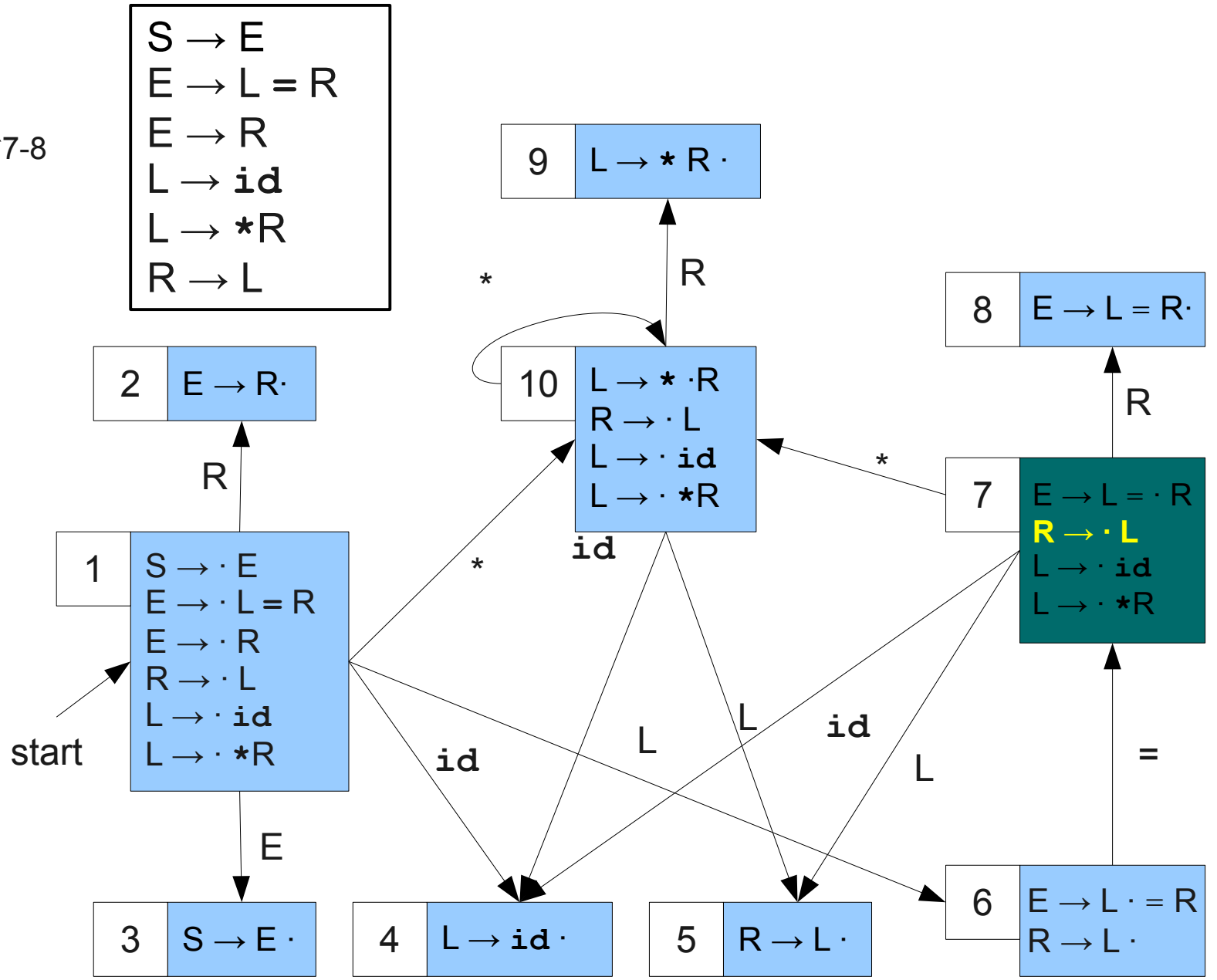
$E_{1\text{-}3} \rightarrow R_{1\text{-}2}$

$R_{1\text{-}2} \rightarrow L_{1\text{-}6}$

$L_{1\text{-}6} \rightarrow \mathbf{id}$

$L_{1\text{-}6} \rightarrow * R_{10\text{-}9}$

$R_{7\text{-}8} \rightarrow L_{7\text{-}5}$



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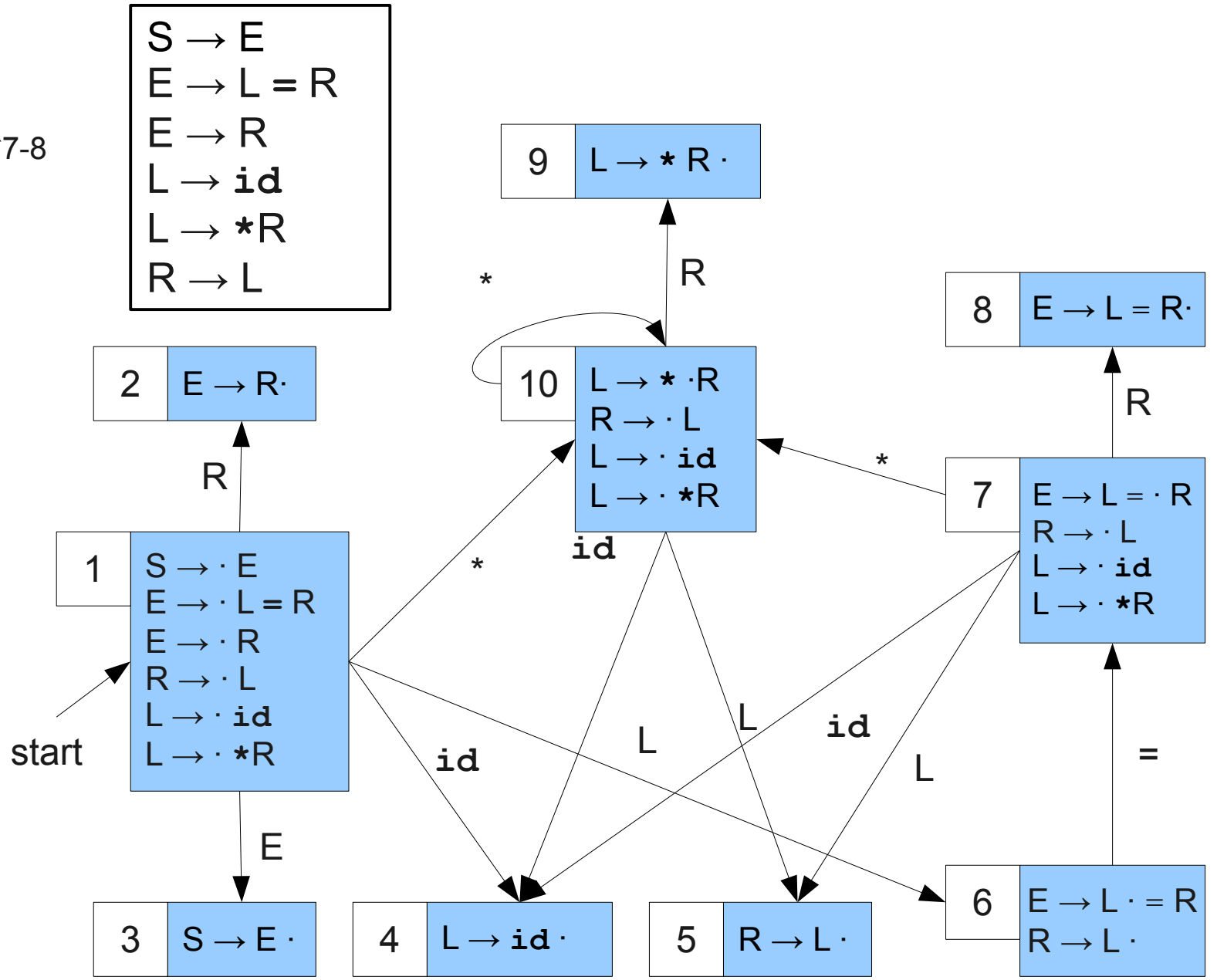

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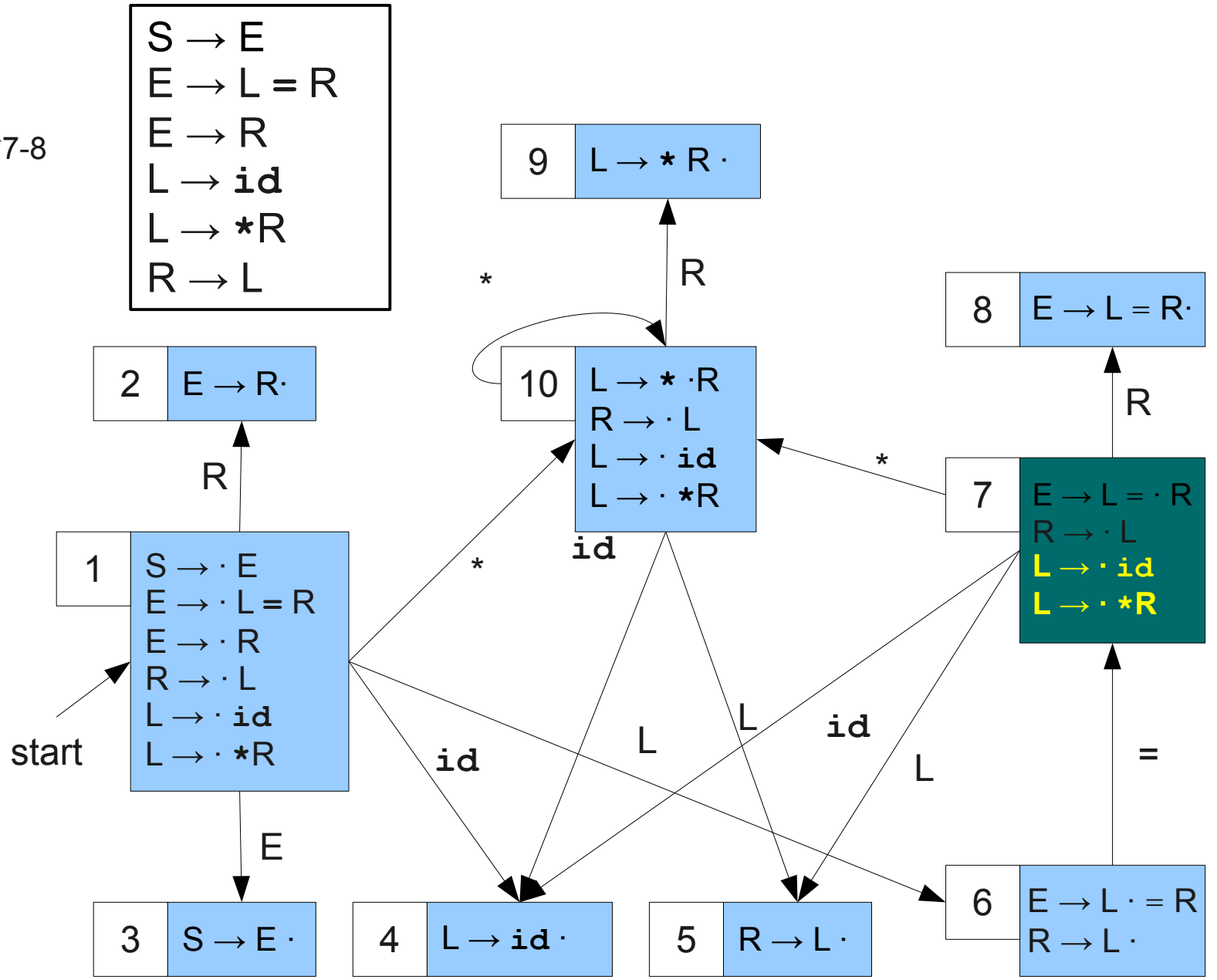
$R_{7\text{-}8} \rightarrow L_{7\text{-}5}$



Augmenting the Grammar

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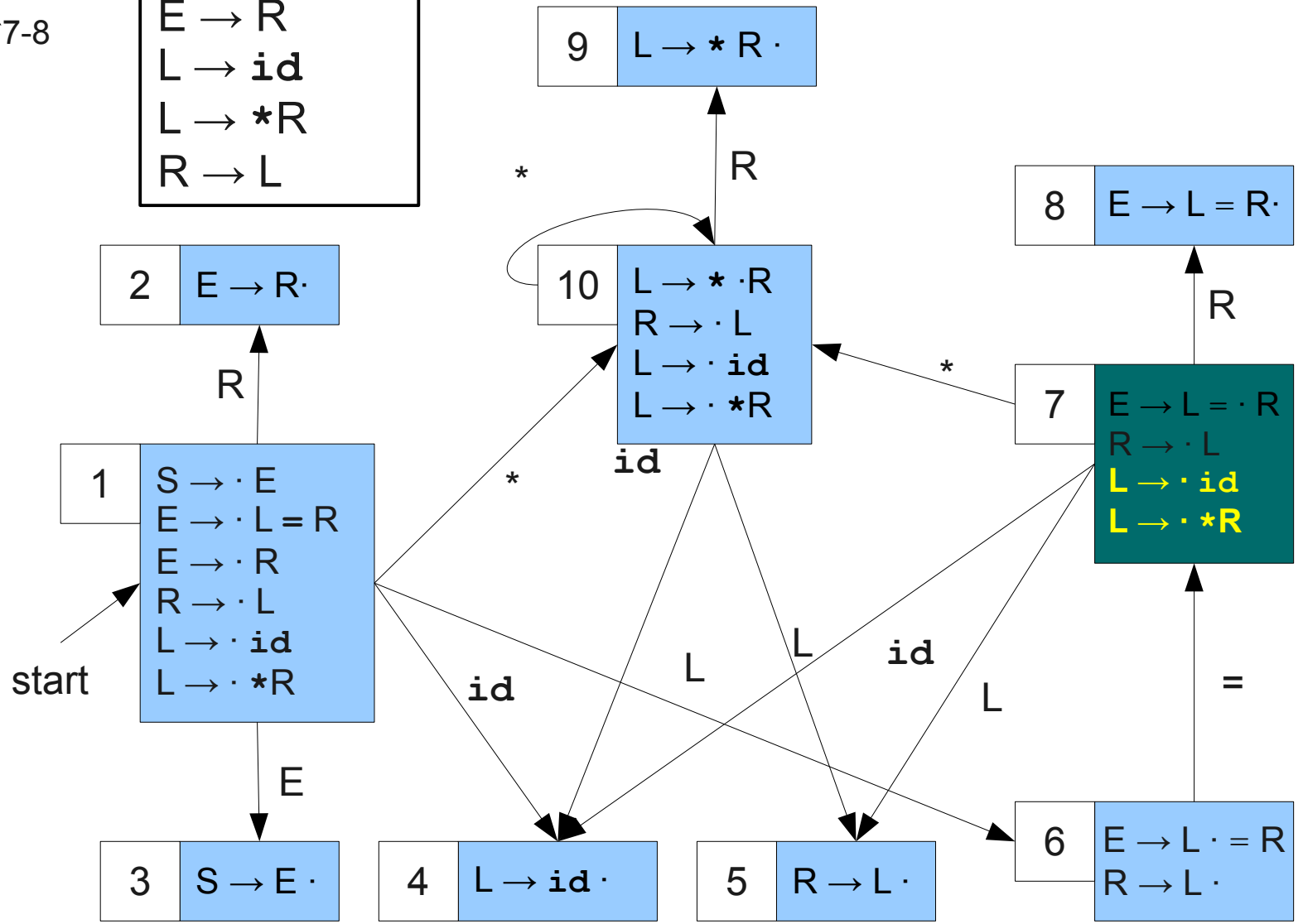
$L_{1\text{-}6} \rightarrow \mathbf{id}$
$L_{1\text{-}6} \rightarrow * R_{10\text{-}9}$
$R_{7\text{-}8} \rightarrow L_{7\text{-}5}$
$L_{7\text{-}5} \rightarrow \mathbf{id}$
$L_{7\text{-}5} \rightarrow *R_{7\text{-}8}$

$S \rightarrow E$
$E \rightarrow L = R$
$E \rightarrow R$
$L \rightarrow \mathbf{id}$
$L \rightarrow *R$
$R \rightarrow L$



Augmenting the Grammar

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$R_{1\text{-}2} \rightarrow L_{1\text{-}6}$
$L_{1\text{-}6} \rightarrow \mathbf{id}$
$L_{1\text{-}6} \rightarrow * R_{10\text{-}9}$
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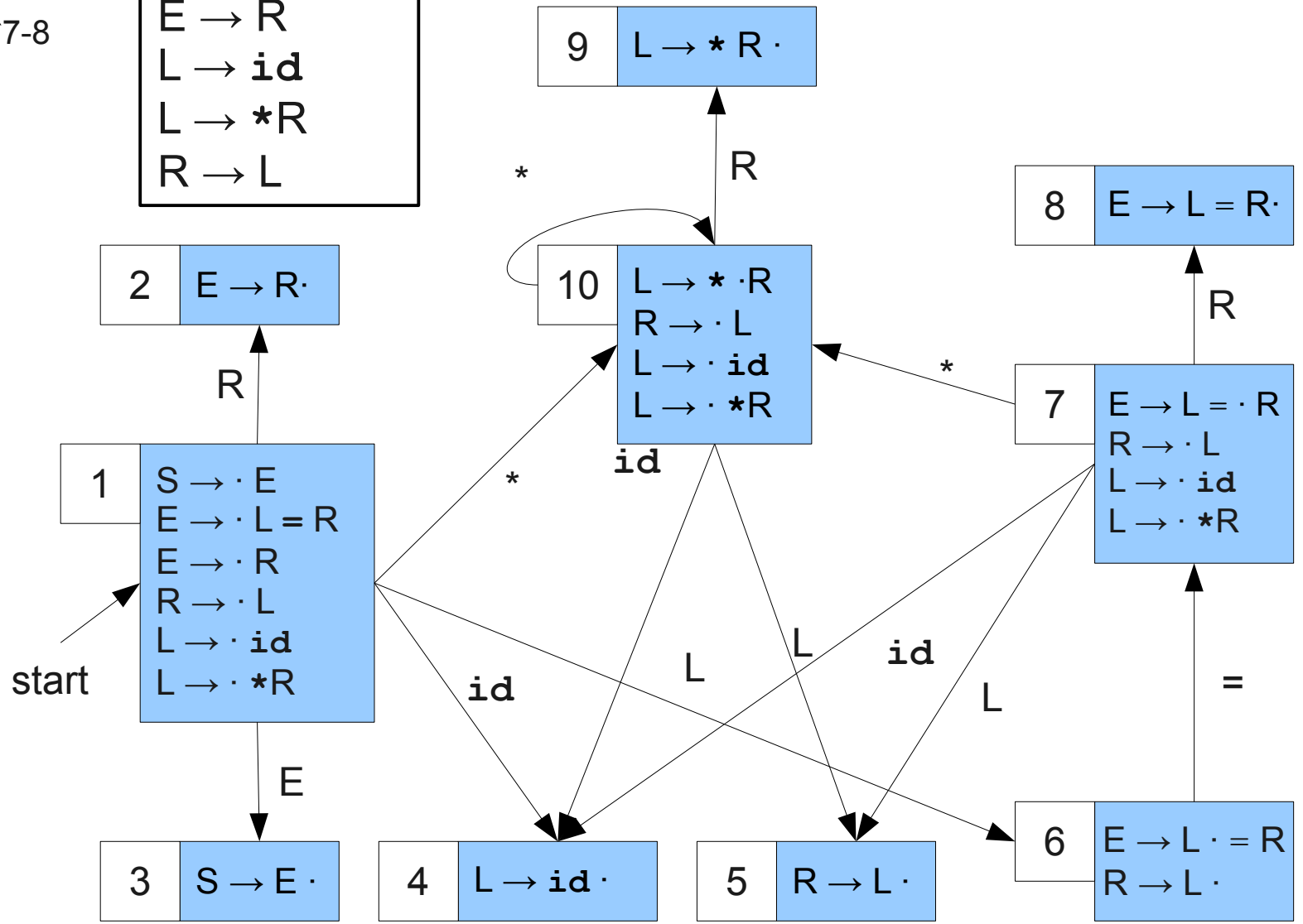
$R \rightarrow L$



Augmenting the Grammar

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$E_{1\text{-}3} \rightarrow R_{1\text{-}2}$
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$L_{1\text{-}6} \rightarrow \mathbf{id}$
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$L_{7\text{-}5} \rightarrow \mathbf{id}$
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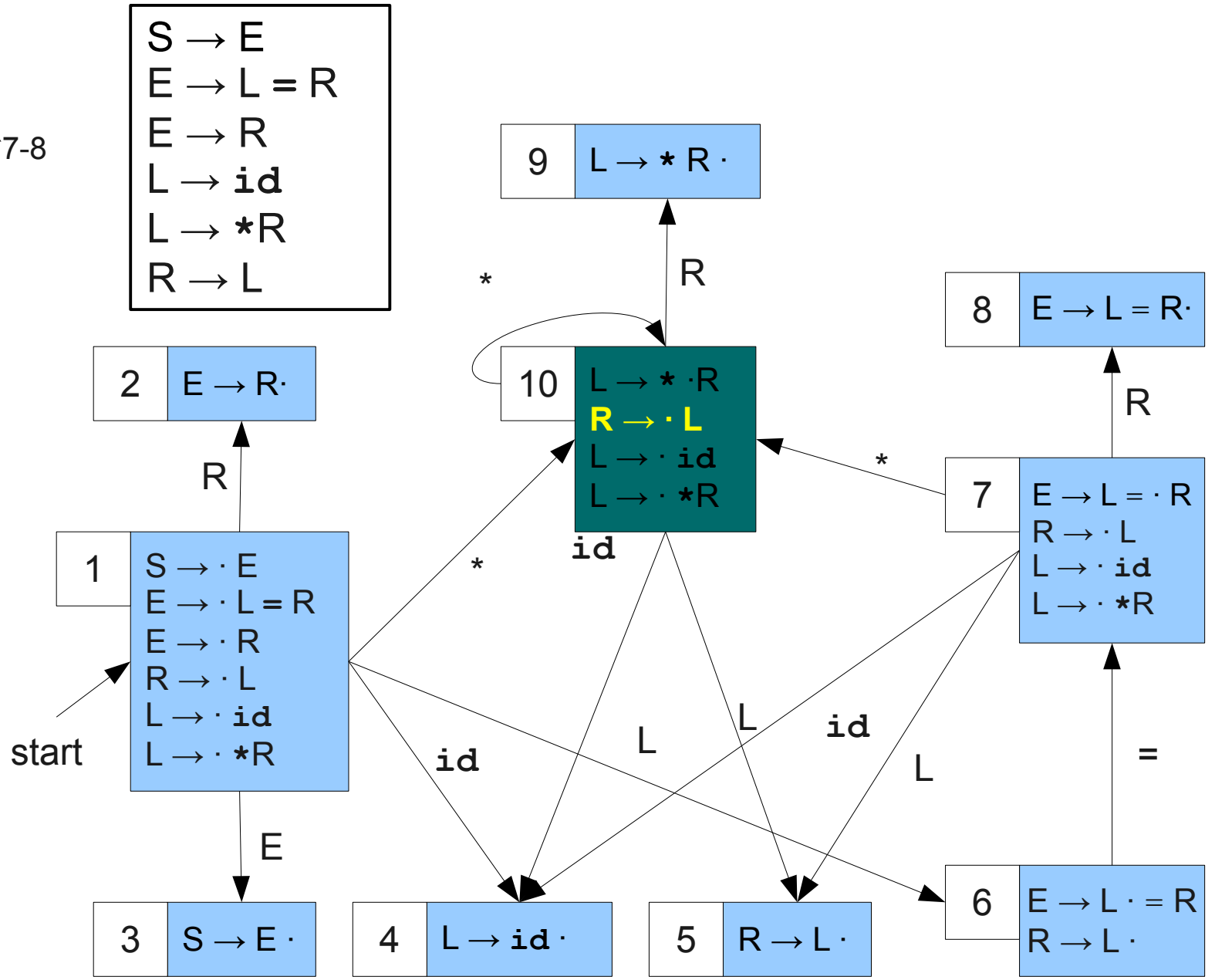

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$R_{1\text{-}2} \rightarrow L_{1\text{-}6}$
$L_{1\text{-}6} \rightarrow \mathbf{id}$
$L_{1\text{-}6} \rightarrow * R_{10\text{-}9}$
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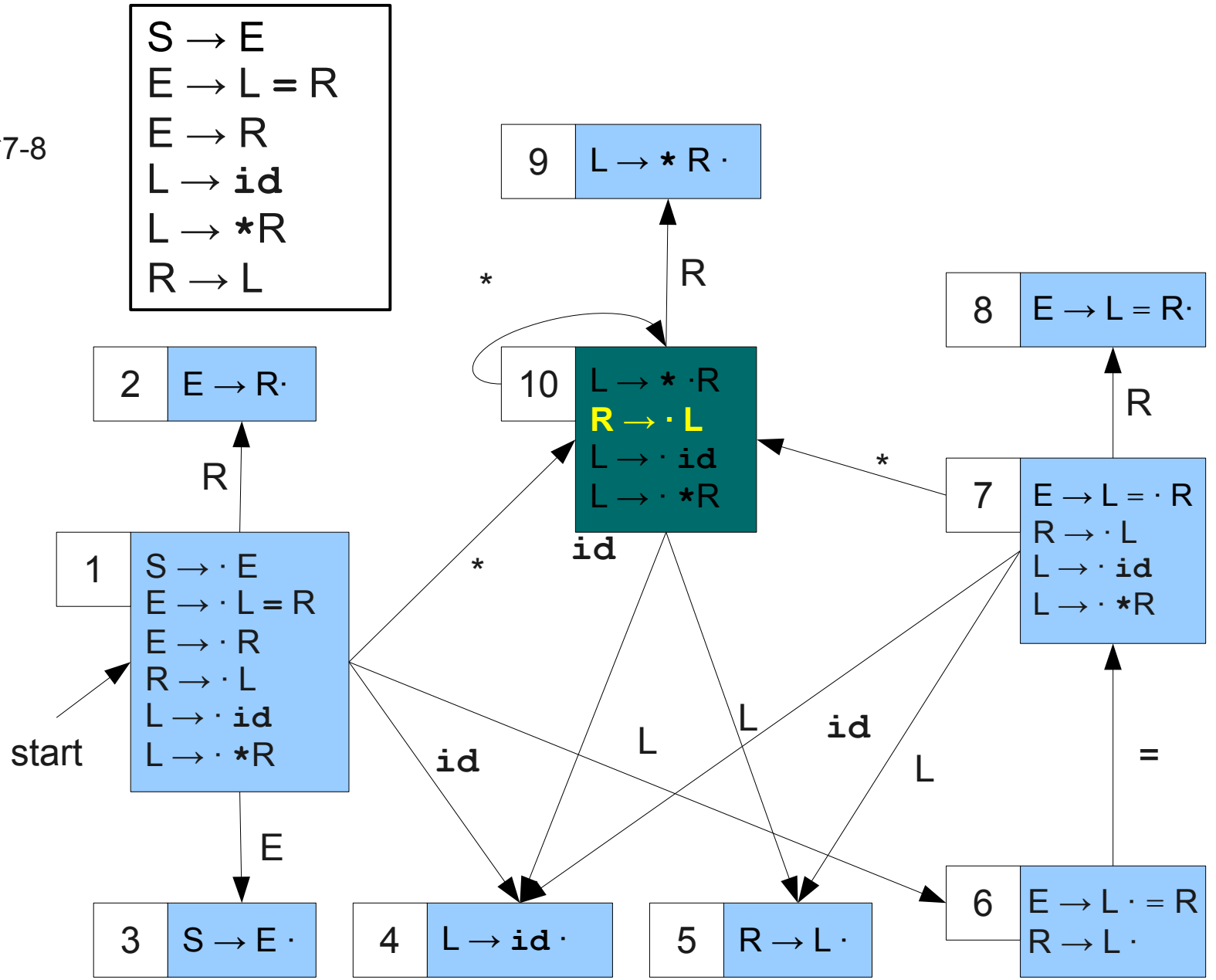
$L_{7\text{-}5} \rightarrow \mathbf{id}$
$L_{7\text{-}5} \rightarrow *R_{10\text{-}9}$
$R_{10\text{-}9} \rightarrow L_{10\text{-}5}$



Augmenting the Grammar

$S_1 \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow L_{1\text{-}6} = R_{7\text{-}8}$
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$L_{1\text{-}6} \rightarrow \mathbf{id}$
$L_{1\text{-}6} \rightarrow * R_{10\text{-}9}$
$R_{7\text{-}8} \rightarrow L_{7\text{-}5}$
$L_{7\text{-}5} \rightarrow \mathbf{id}$
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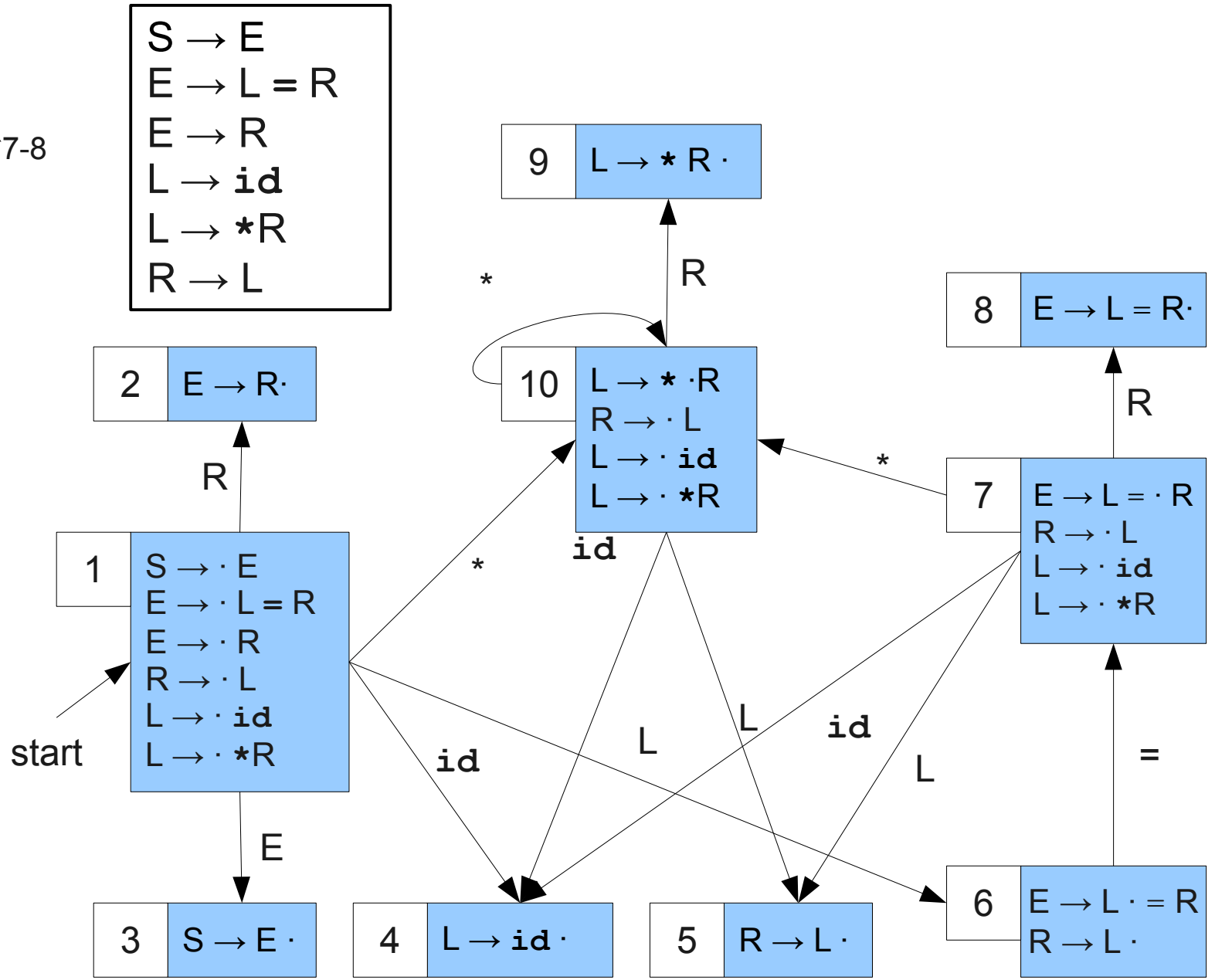
$R_{10\text{-}9} \rightarrow L_{10\text{-}5}$



Augmenting the Grammar

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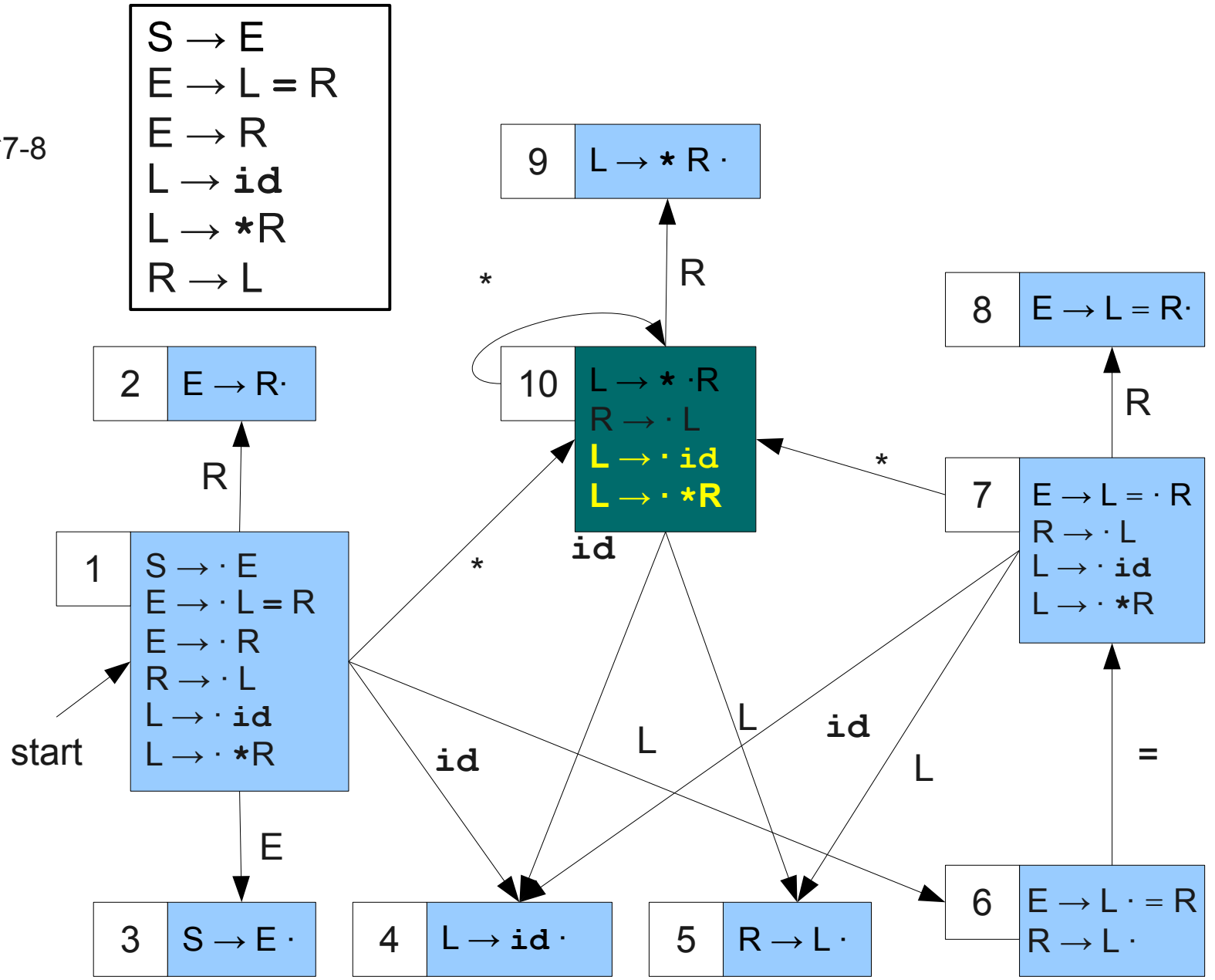
$L_{7\text{-}5} \rightarrow \mathbf{id}$
$L_{7\text{-}5} \rightarrow *R_{10\text{-}9}$
$R_{10\text{-}9} \rightarrow L_{10\text{-}5}$



Augmenting the Grammar

$S_1 \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow L_{1\text{-}6} = R_{7\text{-}8}$
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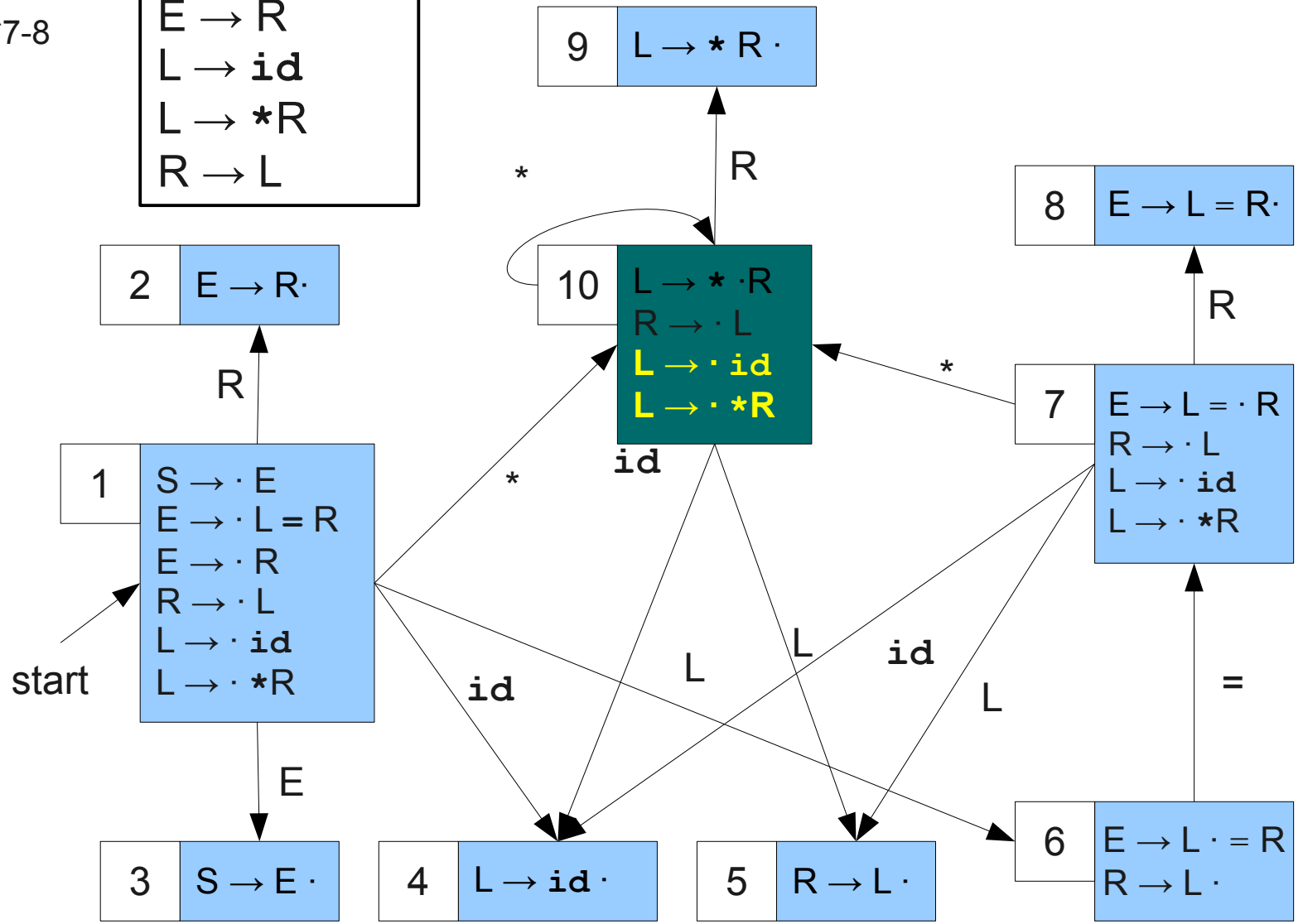
$L_{1\text{-}6} \rightarrow \mathbf{id}$
$L_{1\text{-}6} \rightarrow * R_{10\text{-}9}$
$R_{7\text{-}8} \rightarrow L_{7\text{-}5}$
$L_{7\text{-}5} \rightarrow \mathbf{id}$
$L_{7\text{-}5} \rightarrow *R_{10\text{-}9}$
$R_{10\text{-}9} \rightarrow L_{10\text{-}5}$
$L_{10\text{-}5} \rightarrow \mathbf{id}$
$L_{10\text{-}5} \rightarrow *R_{10\text{-}9}$

S → E
E → L = R
E → R
L → **id**
L → ***R**
R → L



Augmenting the Grammar

$S_1 \rightarrow E_{1\text{-}3}$

$E_{1\text{-}3} \rightarrow L_{1\text{-}6} = R_{7\text{-}8}$

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$R_{1\text{-}2} \rightarrow L_{1\text{-}6}$

$L_{1\text{-}6} \rightarrow \mathbf{id}$

$L_{1\text{-}6} \rightarrow * R_{10\text{-}9}$

$R_{7\text{-}8} \rightarrow L_{7\text{-}5}$

$L_{7\text{-}5} \rightarrow \mathbf{id}$

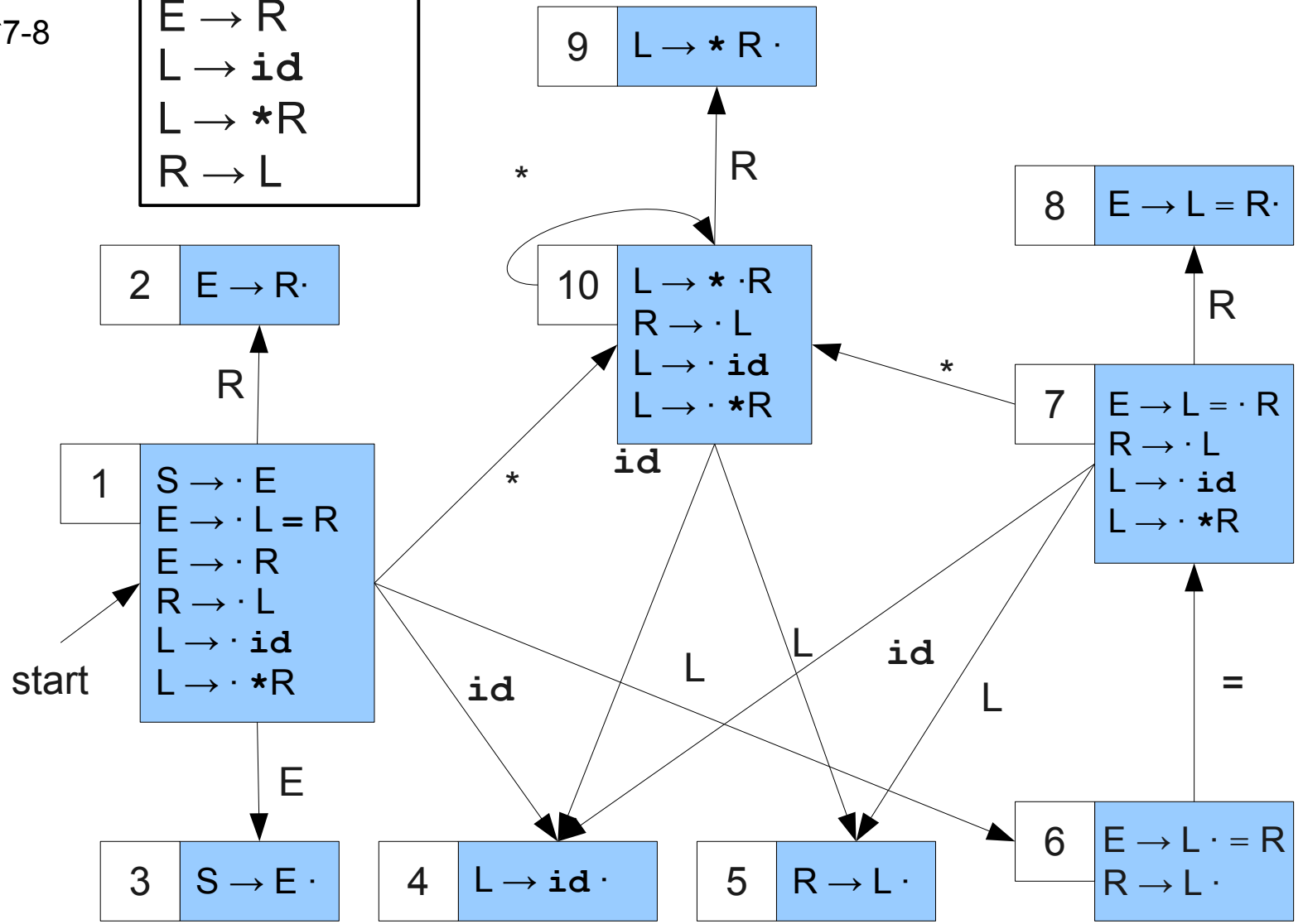
$L_{7\text{-}5} \rightarrow *R_{10\text{-}9}$

$R_{10\text{-}9} \rightarrow L_{10\text{-}5}$

$L_{10\text{-}5} \rightarrow \mathbf{id}$

$L_{10\text{-}5} \rightarrow *R_{10\text{-}9}$

S → E
E → L = R
E → R
L → **id**
L → ***R**
R → L



Constructing Augmented Grammars

- For each item $A \rightarrow \cdot v$ in some state q:
  - Trace out the path *v* takes through the LR(0) automaton starting at q.
  - Replace each nonterminal in *v* with nonterminals annotated with the state transitioned to and from by the edge labeled with that nonterminal.
  - Replace A with a nonterminal annotated with the start and end state of the transition on A out of q.
- Result is a larger grammar with more precise productions.

Why is this Grammar Useful?

- At a high-level, separates out the nonterminals based on their context.
- This makes the FOLLOW sets more precise for their nonterminals.
- In fact, the FOLLOW sets are **surprisingly** precise.

Augmented FOLLOW Sets

$S_1 \rightarrow E_{1\text{-}3}$

$E_{1\text{-}3} \rightarrow L_{1\text{-}6} = R_{7\text{-}8}$

$E_{1\text{-}3} \rightarrow R_{1\text{-}2}$

$R_{1\text{-}2} \rightarrow L_{1\text{-}6}$

$L_{1\text{-}6} \rightarrow$ **id**

$L_{1\text{-}6} \rightarrow$ * $R_{10\text{-}9}$

$R_{7\text{-}8} \rightarrow L_{7\text{-}5}$

$L_{7\text{-}5} \rightarrow$ **id**

$L_{7\text{-}5} \rightarrow$ *$R_{10\text{-}9}$

$R_{10\text{-}9} \rightarrow L_{10\text{-}5}$

$L_{10\text{-}5} \rightarrow$ **id**

$L_{10\text{-}5} \rightarrow$ *$R_{10\text{-}9}$

Augmented FOLLOW Sets

$S_1 \rightarrow E_{1\text{-}3}$

$E_{1\text{-}3} \rightarrow L_{1\text{-}6} = R_{7\text{-}8}$

$E_{1\text{-}3} \rightarrow R_{1\text{-}2}$

$R_{1\text{-}2} \rightarrow L_{1\text{-}6}$

$L_{1\text{-}6} \rightarrow \mathbf{id}$

$L_{1\text{-}6} \rightarrow * R_{10\text{-}9}$

$R_{7\text{-}8} \rightarrow L_{7\text{-}5}$

$L_{7\text{-}5} \rightarrow \mathbf{id}$

$L_{7\text{-}5} \rightarrow *R_{10\text{-}9}$

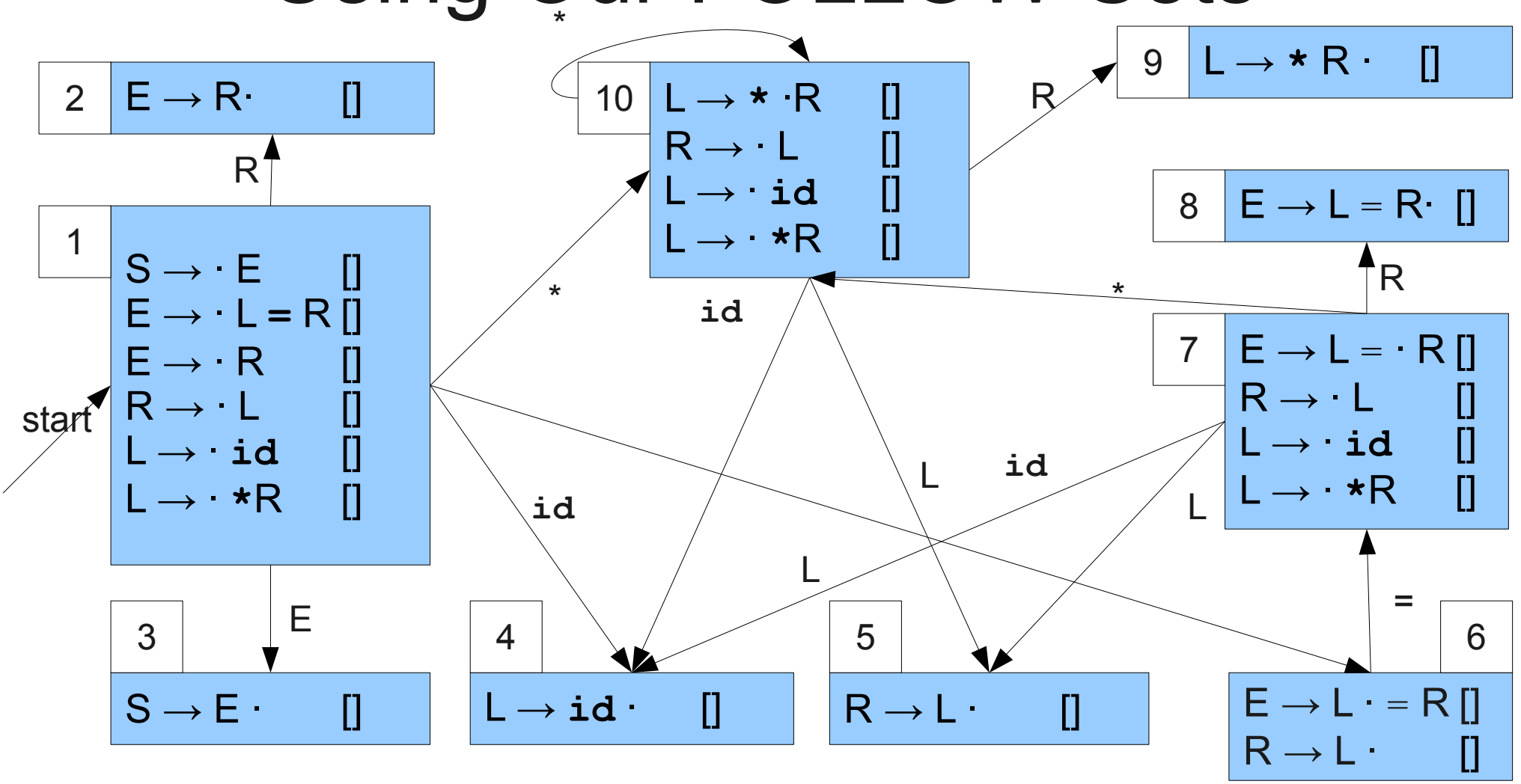
$R_{10\text{-}9} \rightarrow L_{10\text{-}5}$

$L_{10\text{-}5} \rightarrow \mathbf{id}$

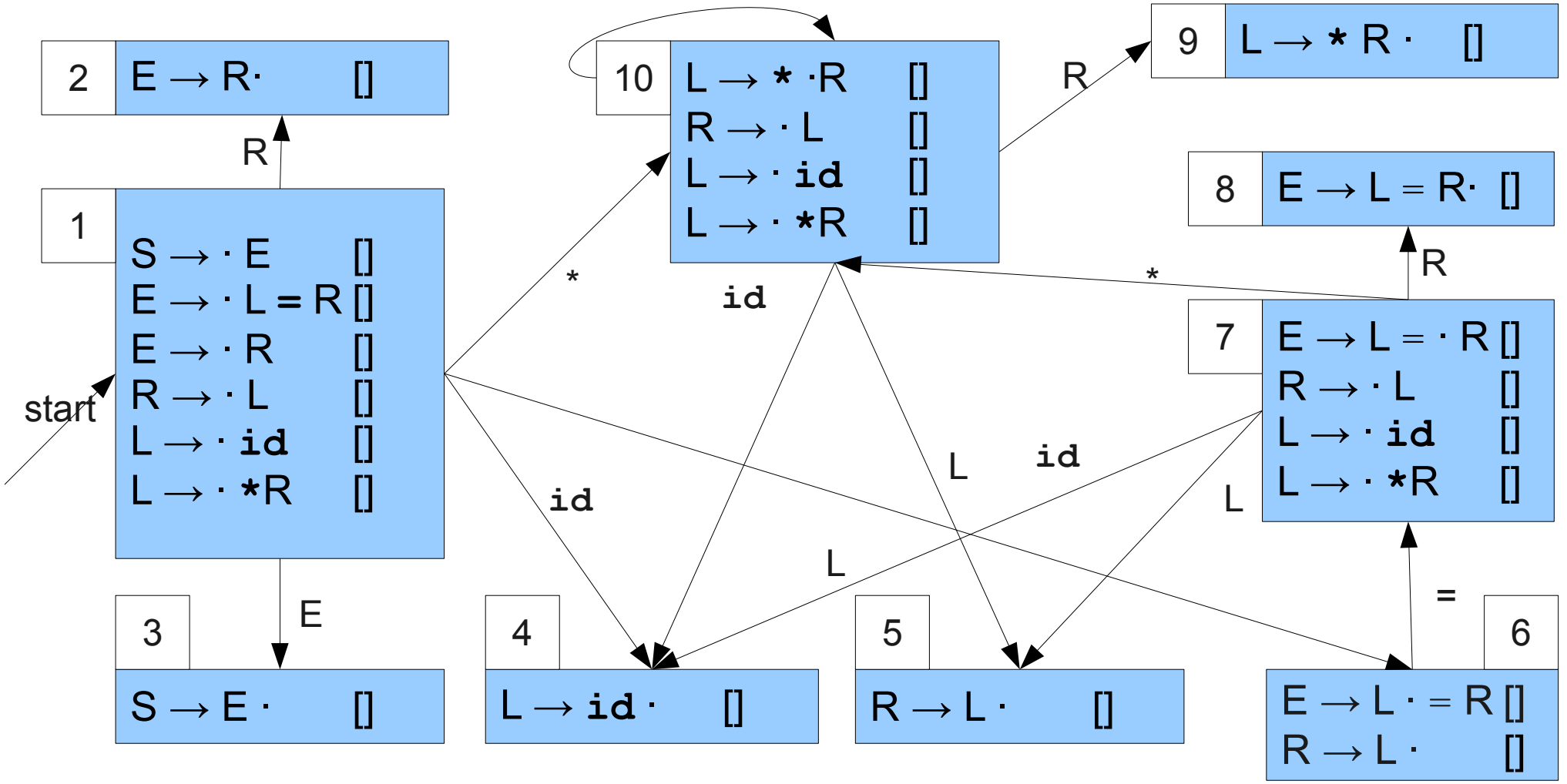
$L_{10\text{-}5} \rightarrow *R_{10\text{-}9}$

S_1	$E_{1\text{-}3}$	$L_{1\text{-}6}$	$L_{7\text{-}5}$	$L_{10\text{-}5}$	$R_{1\text{-}2}$	$R_{7\text{-}8}$	$R_{10\text{-}9}$
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets

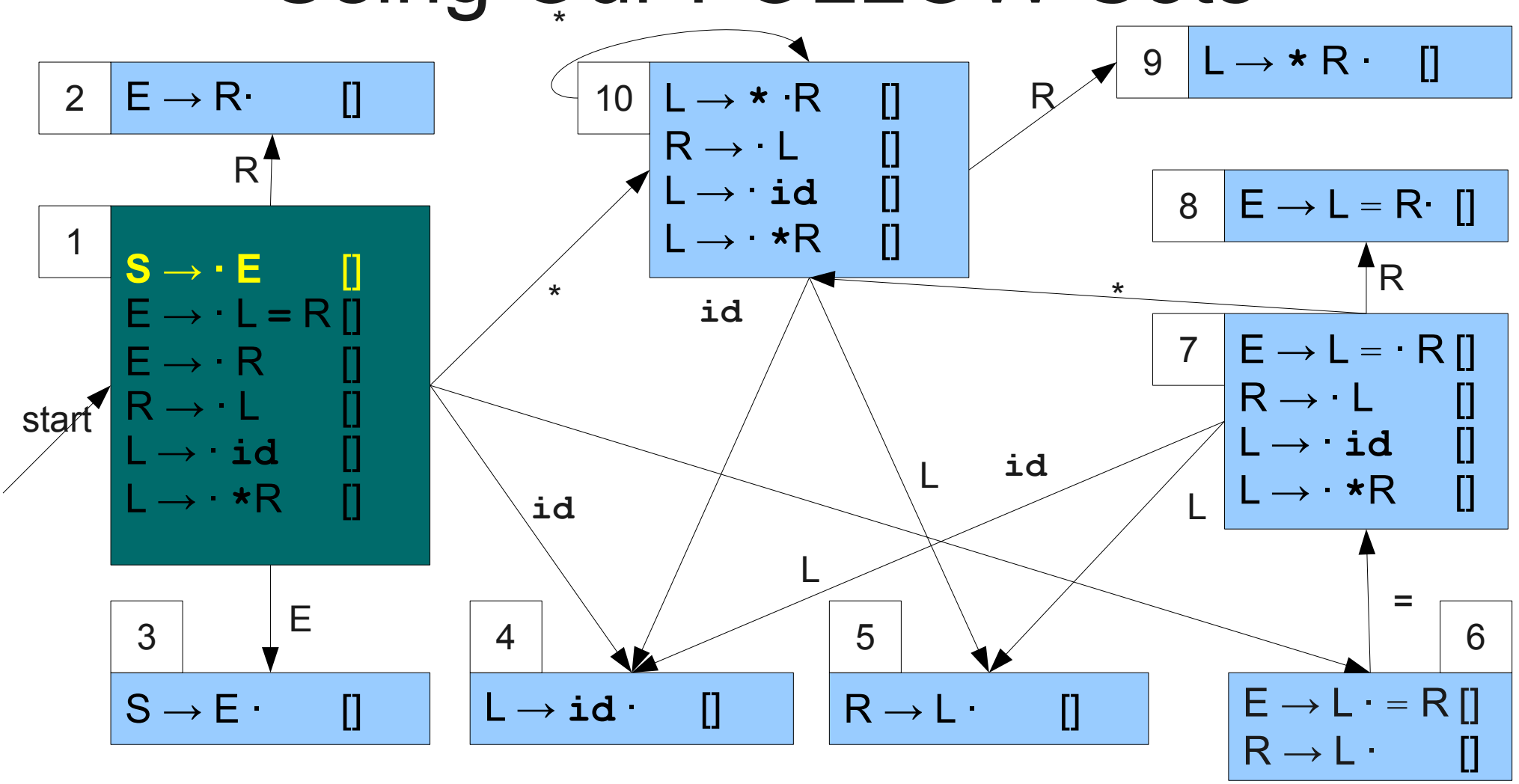


Using Our FOLLOW Sets



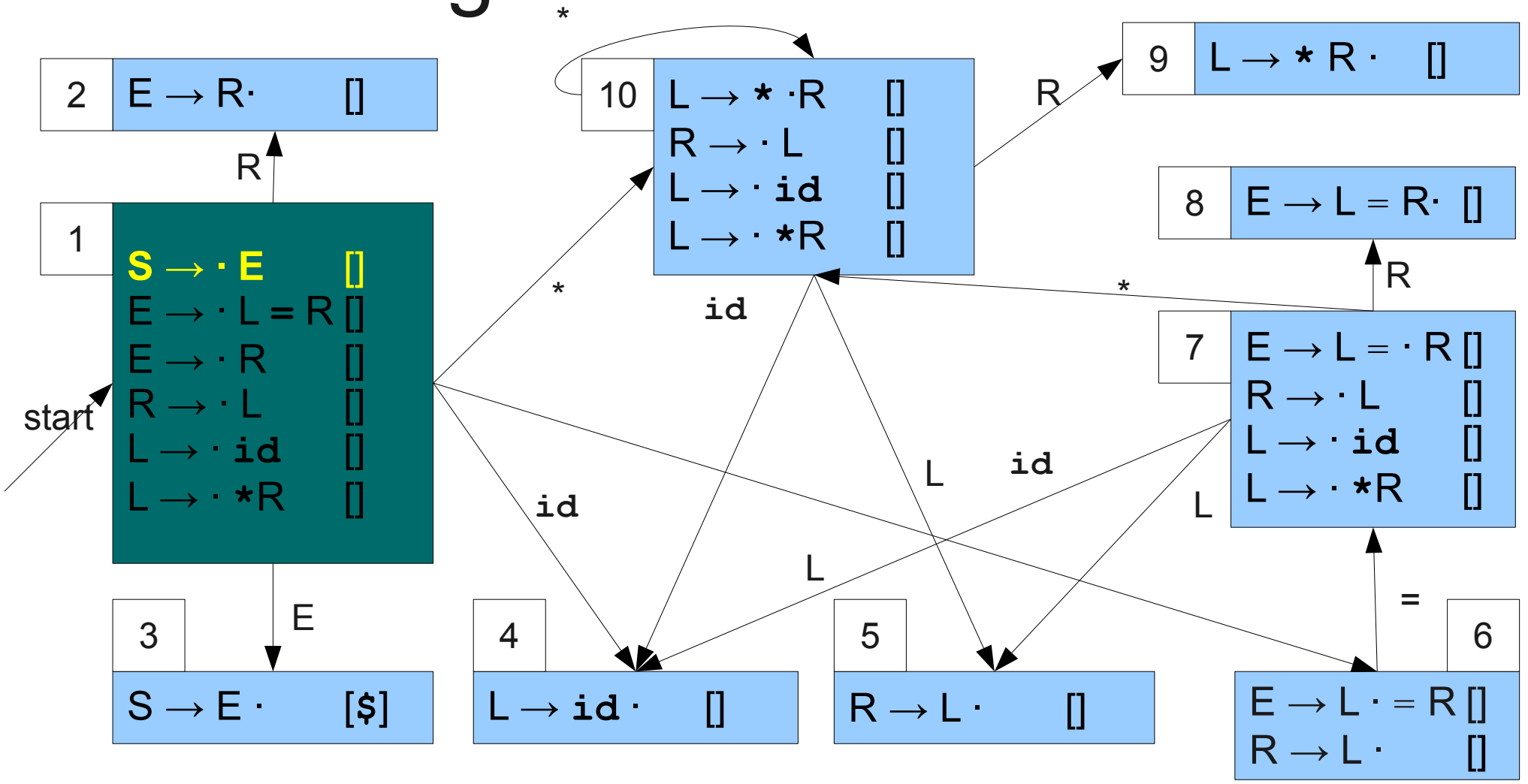
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



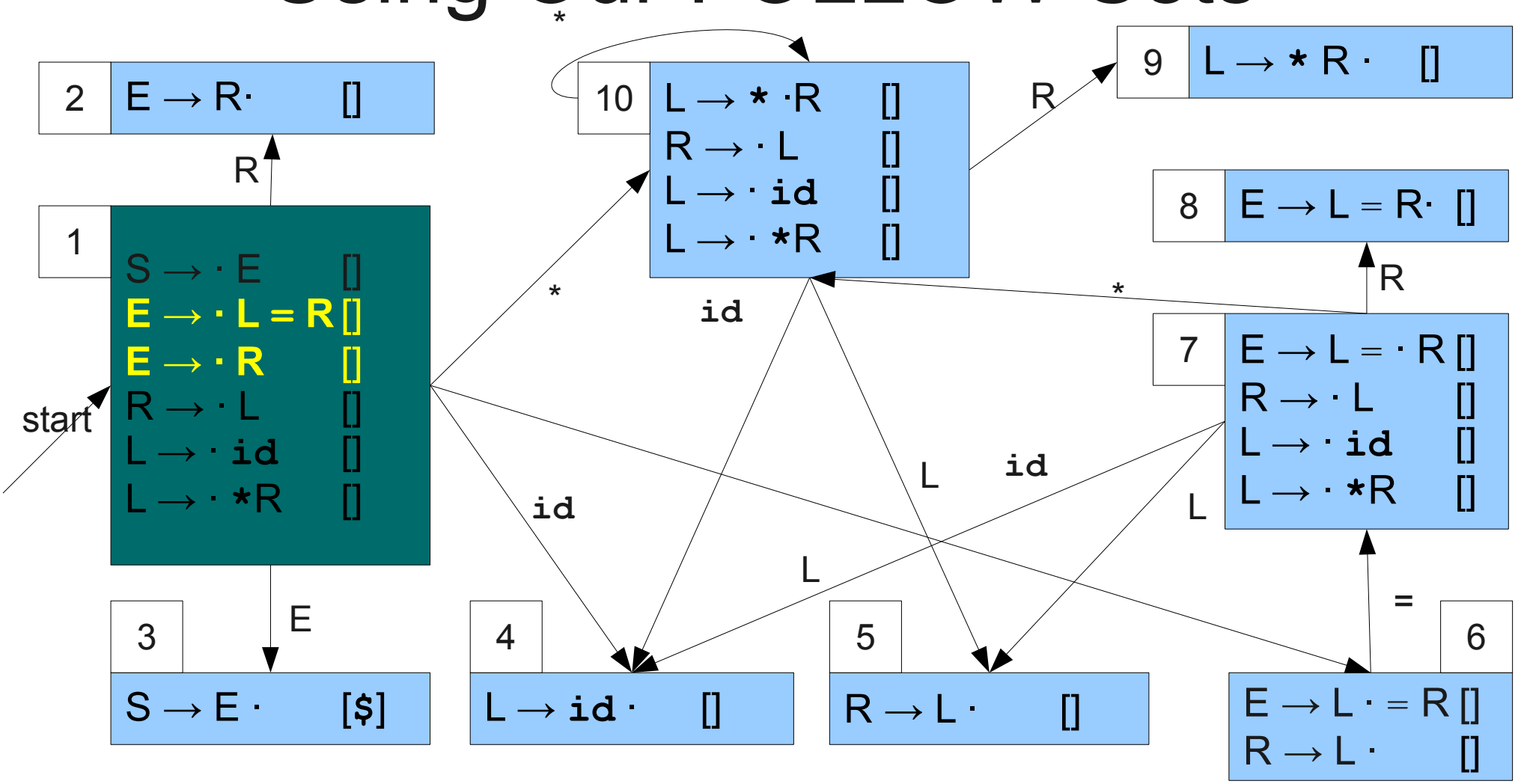
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



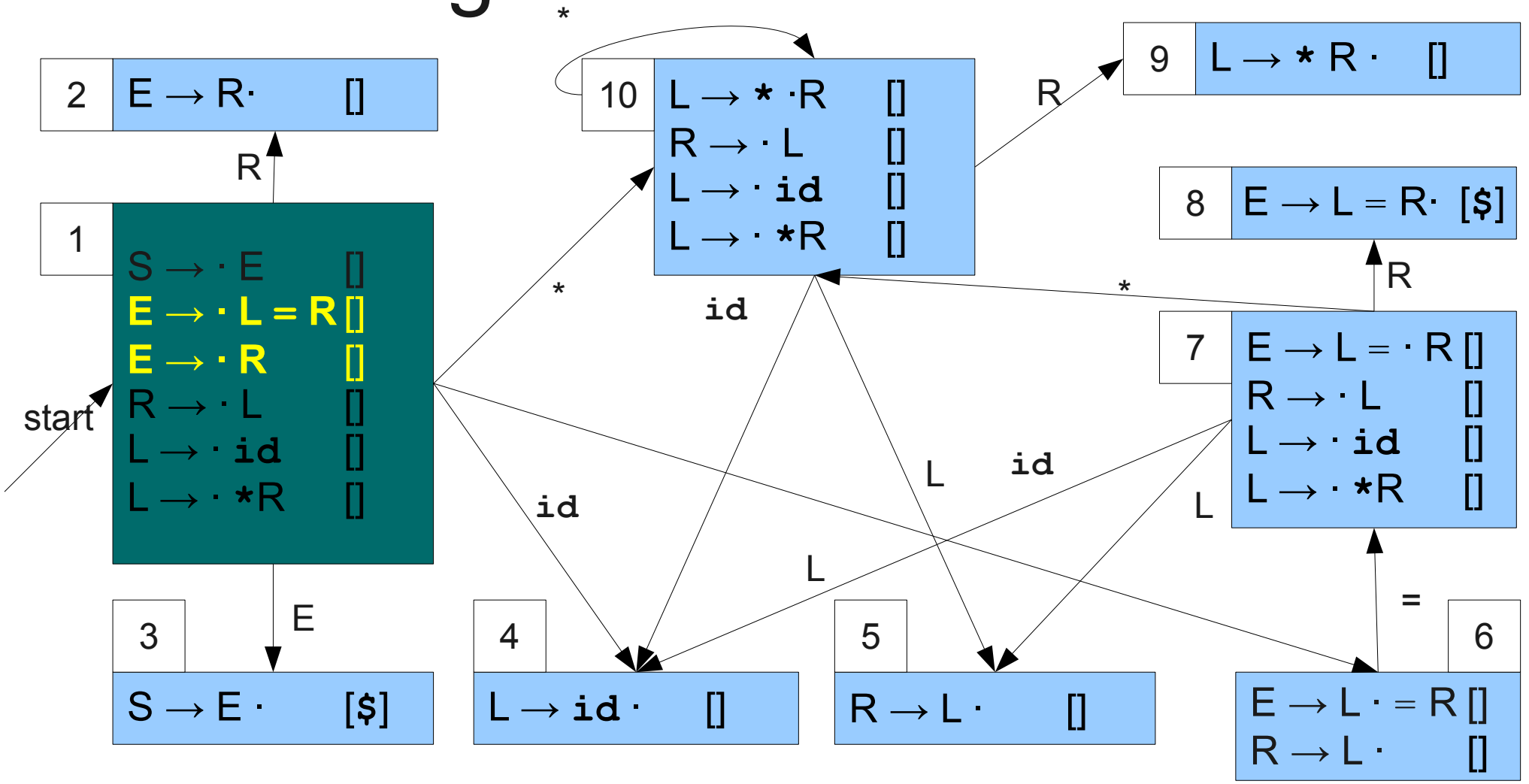
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



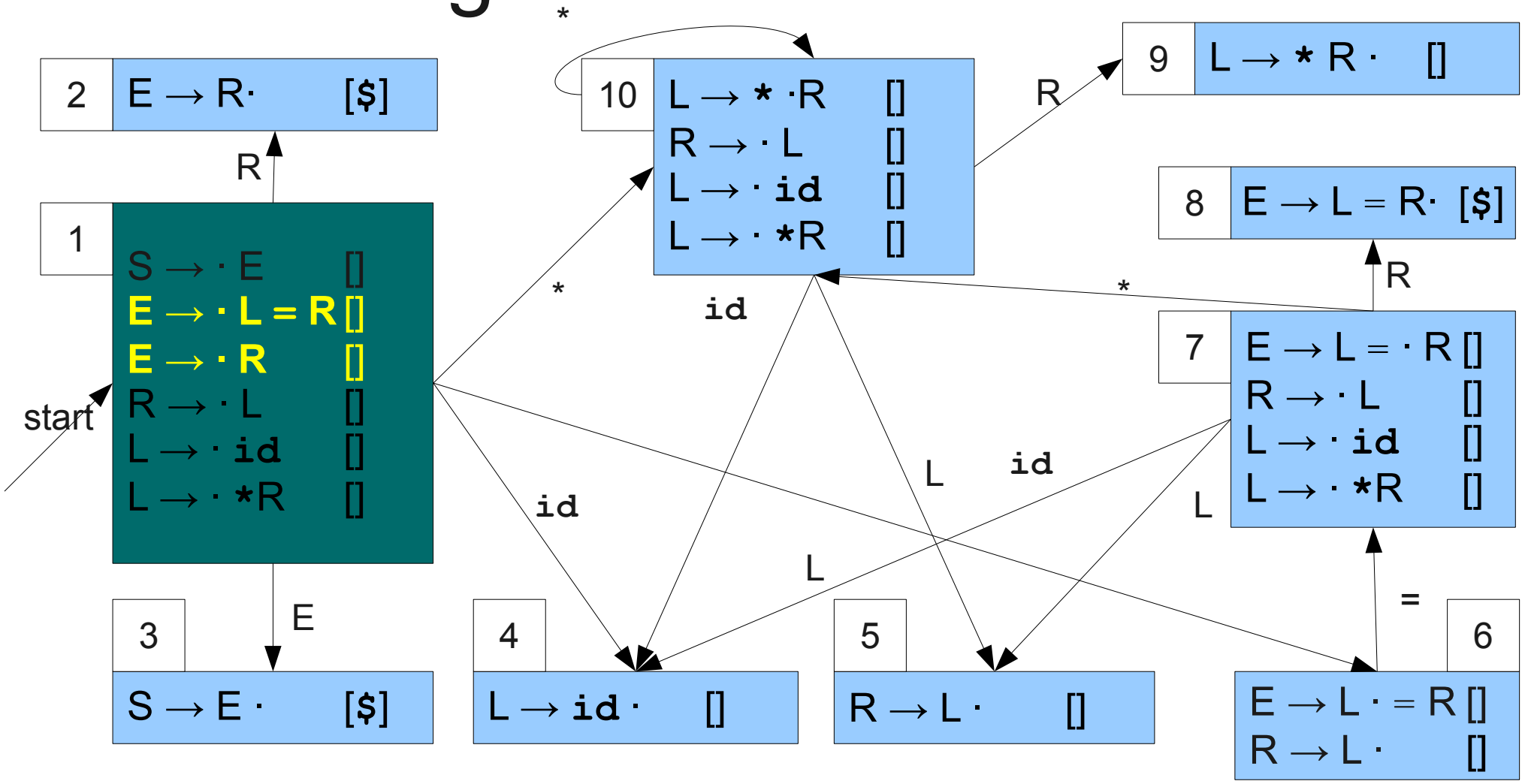
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



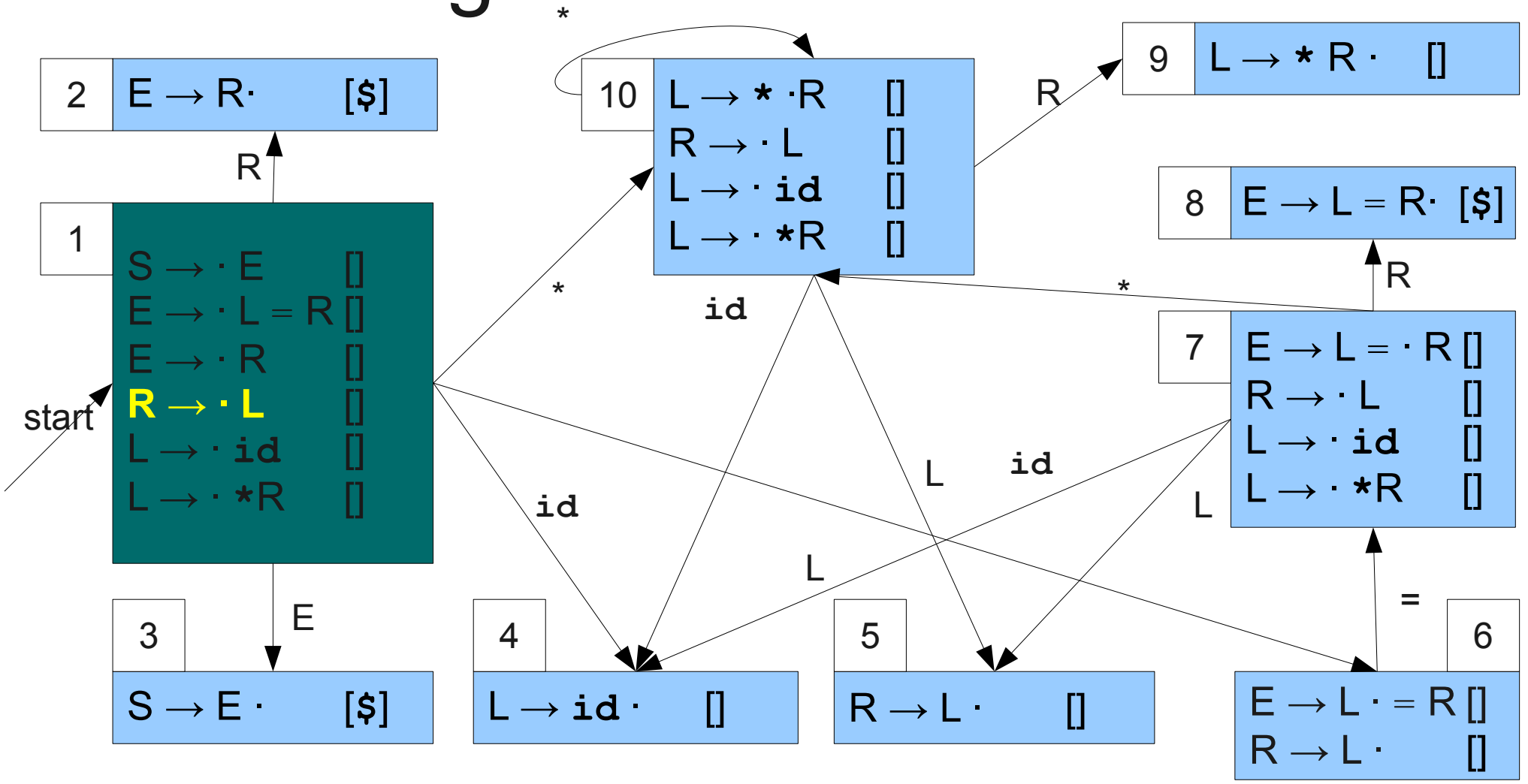
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



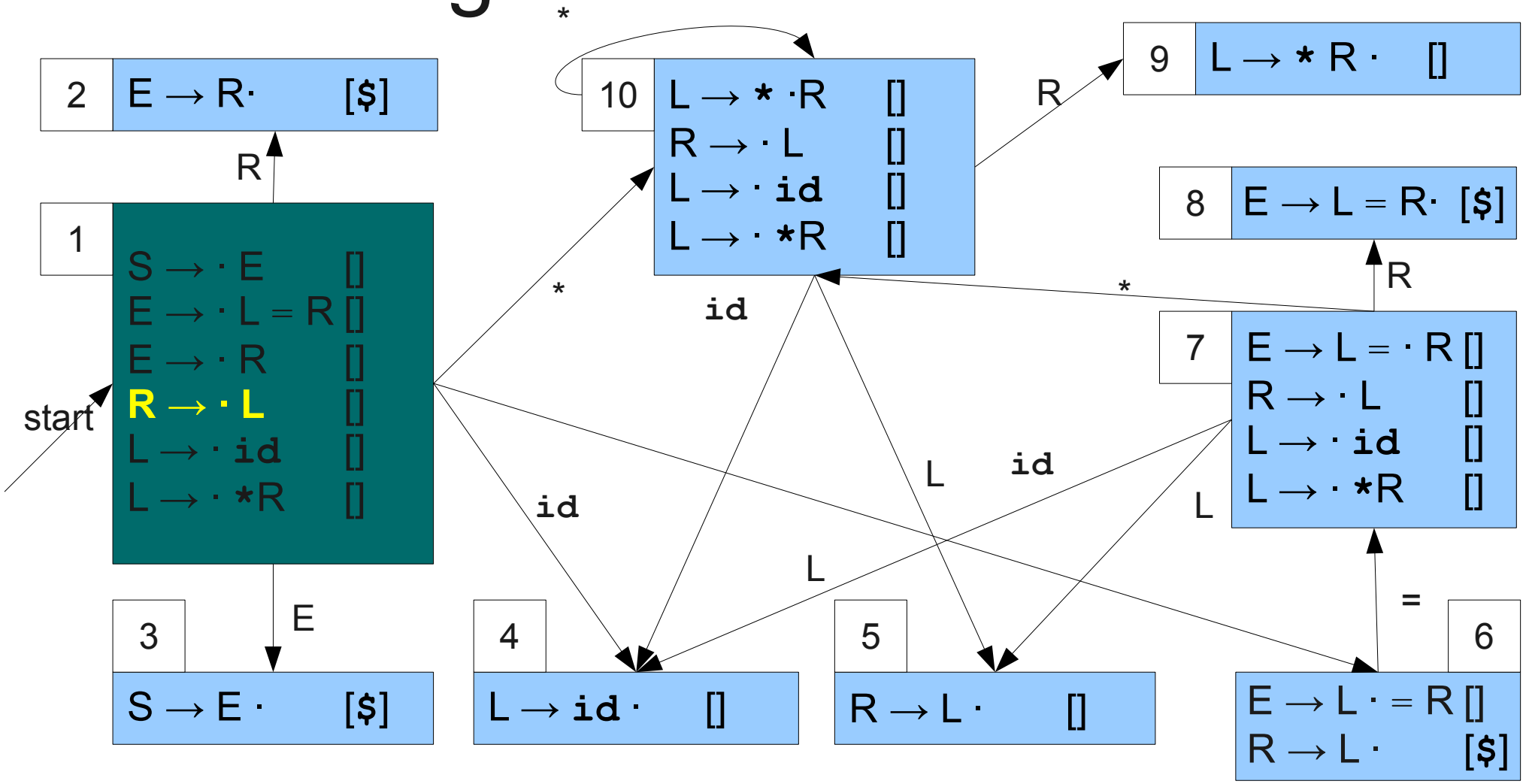
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



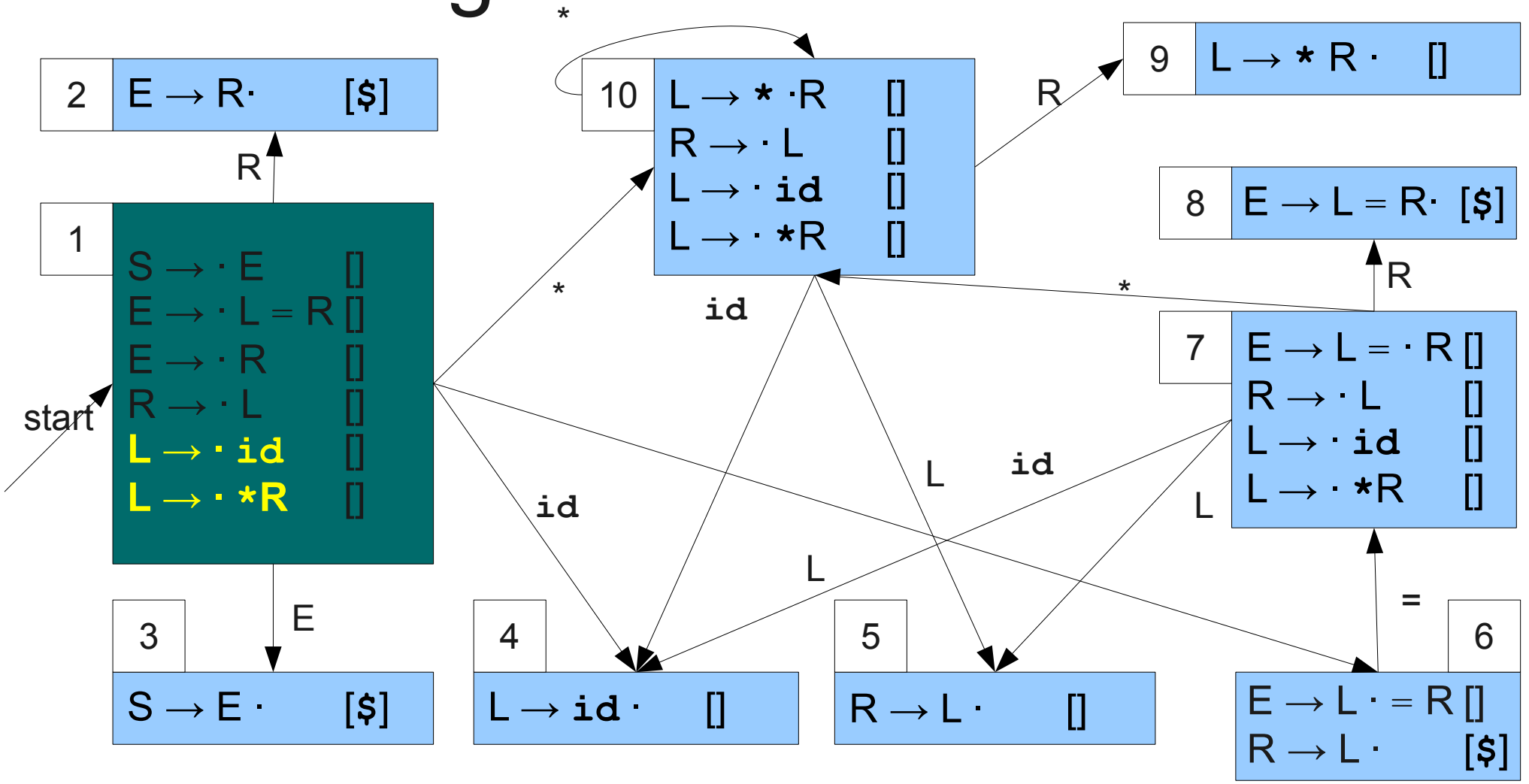
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



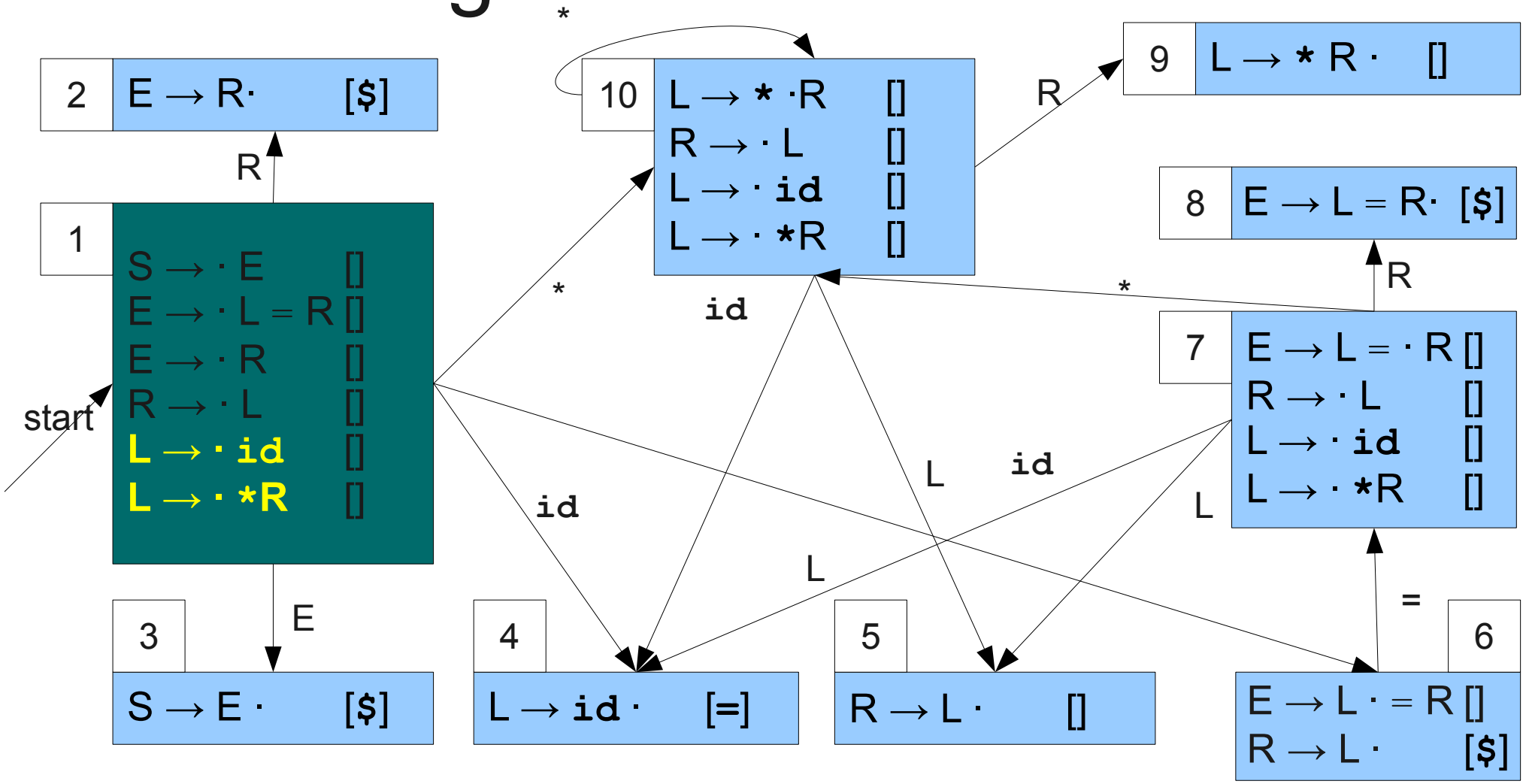
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



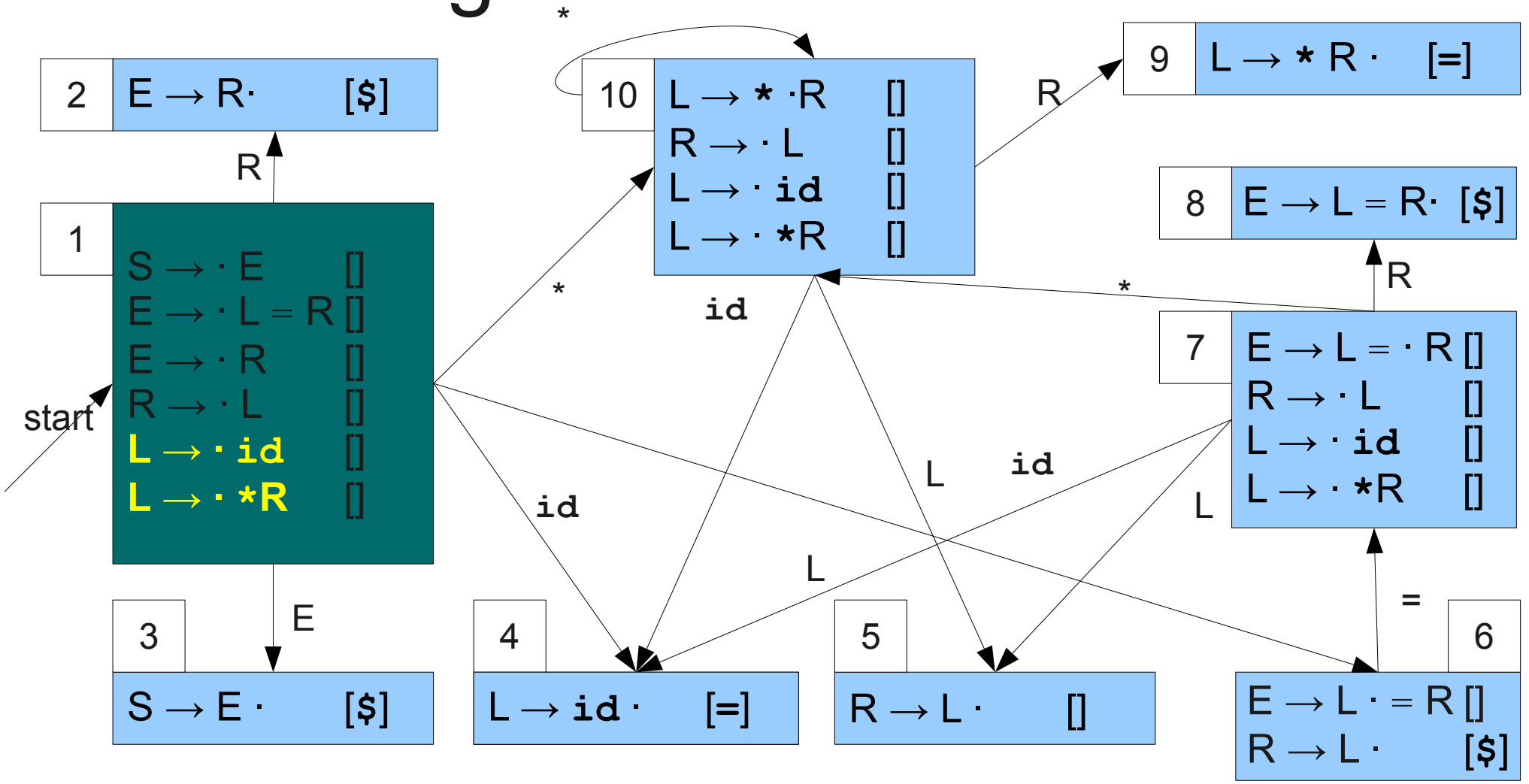
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



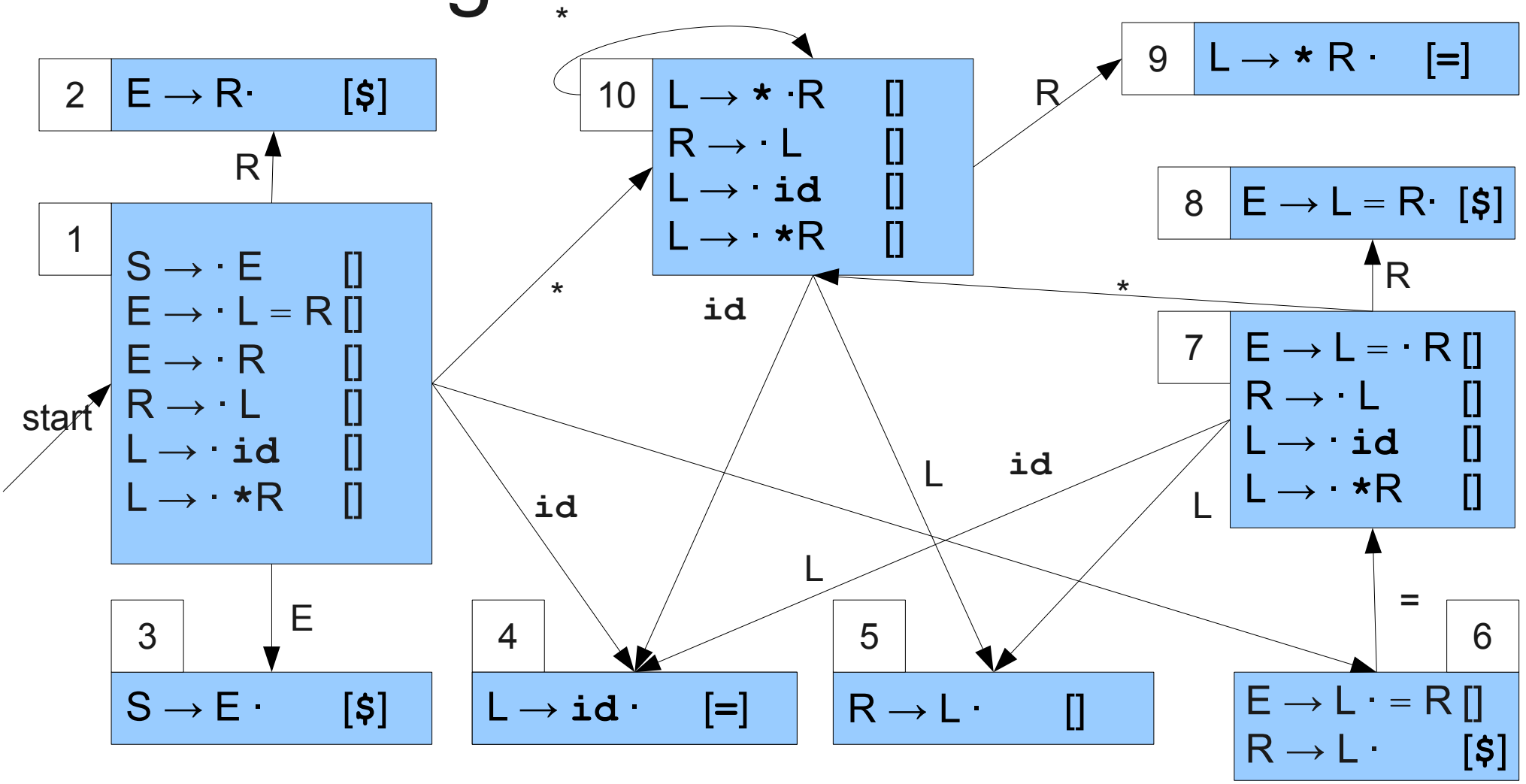
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



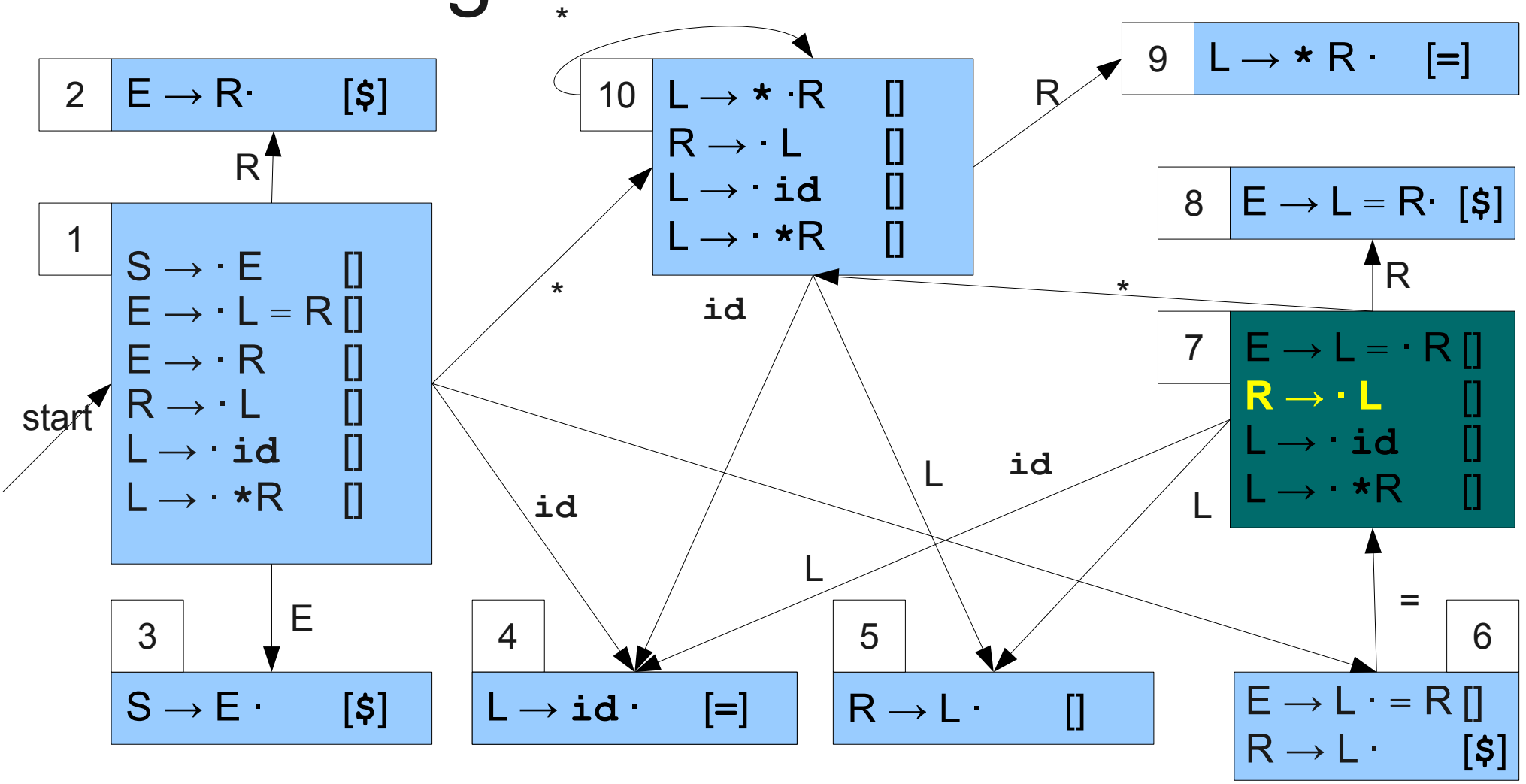
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



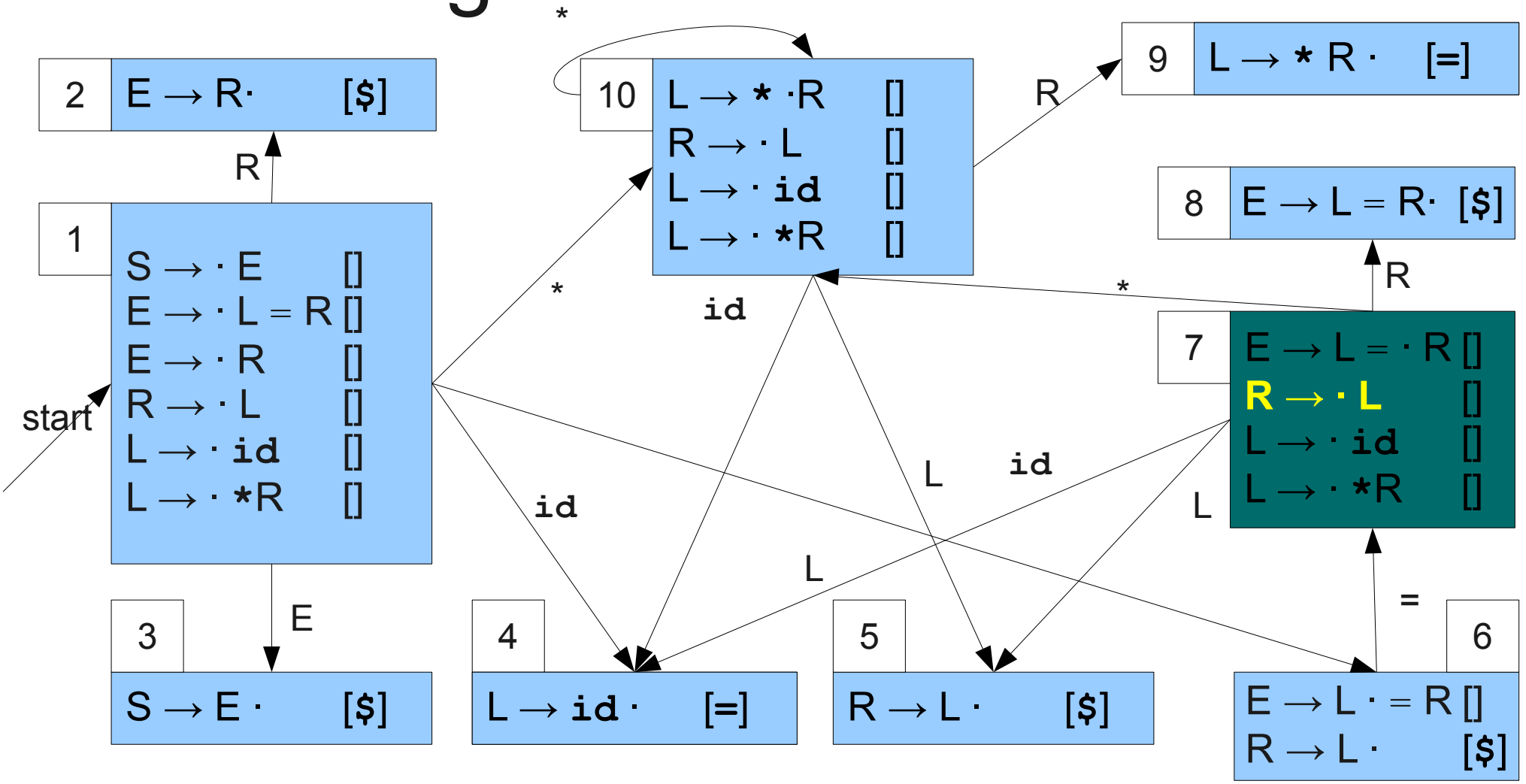
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



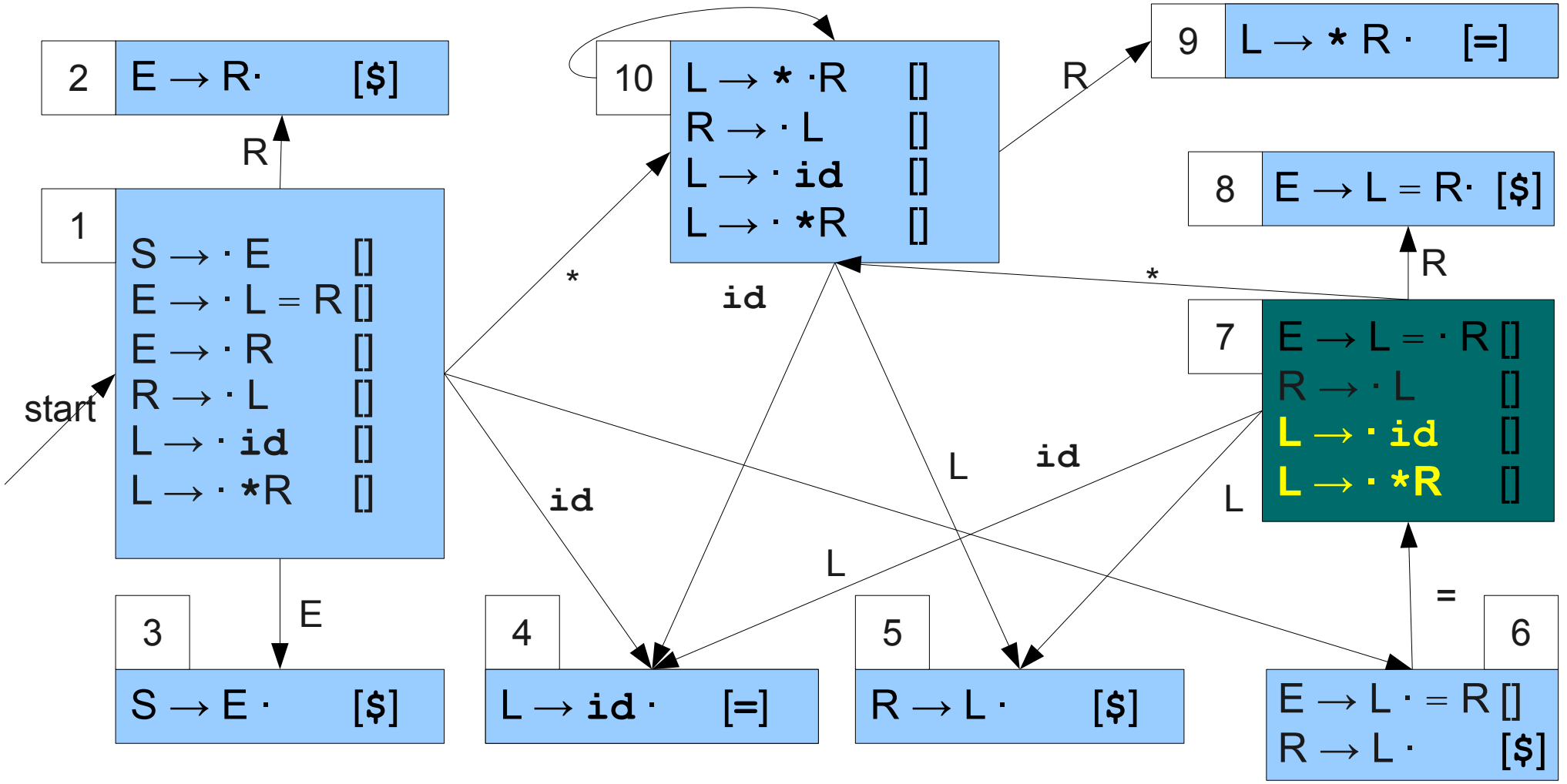
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



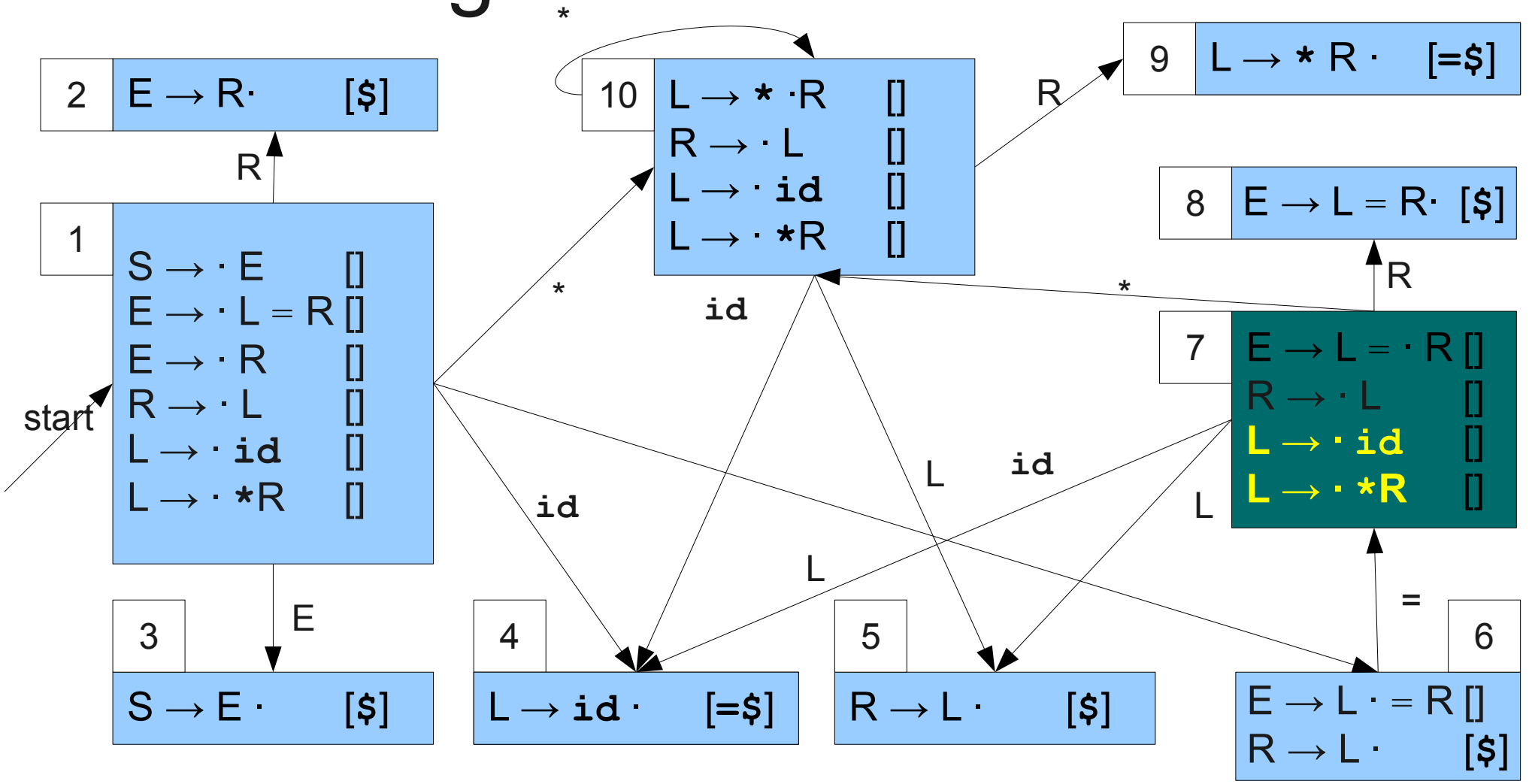
S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



S_1	E_{1-3}	L_{1-6}	L_{7-5}	L_{10-5}	R_{1-2}	R_{7-8}	R_{10-9}
$	$	=	$	=	$	$	=

Using Our FOLLOW Sets



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$	$	=	$	=	$	$	=

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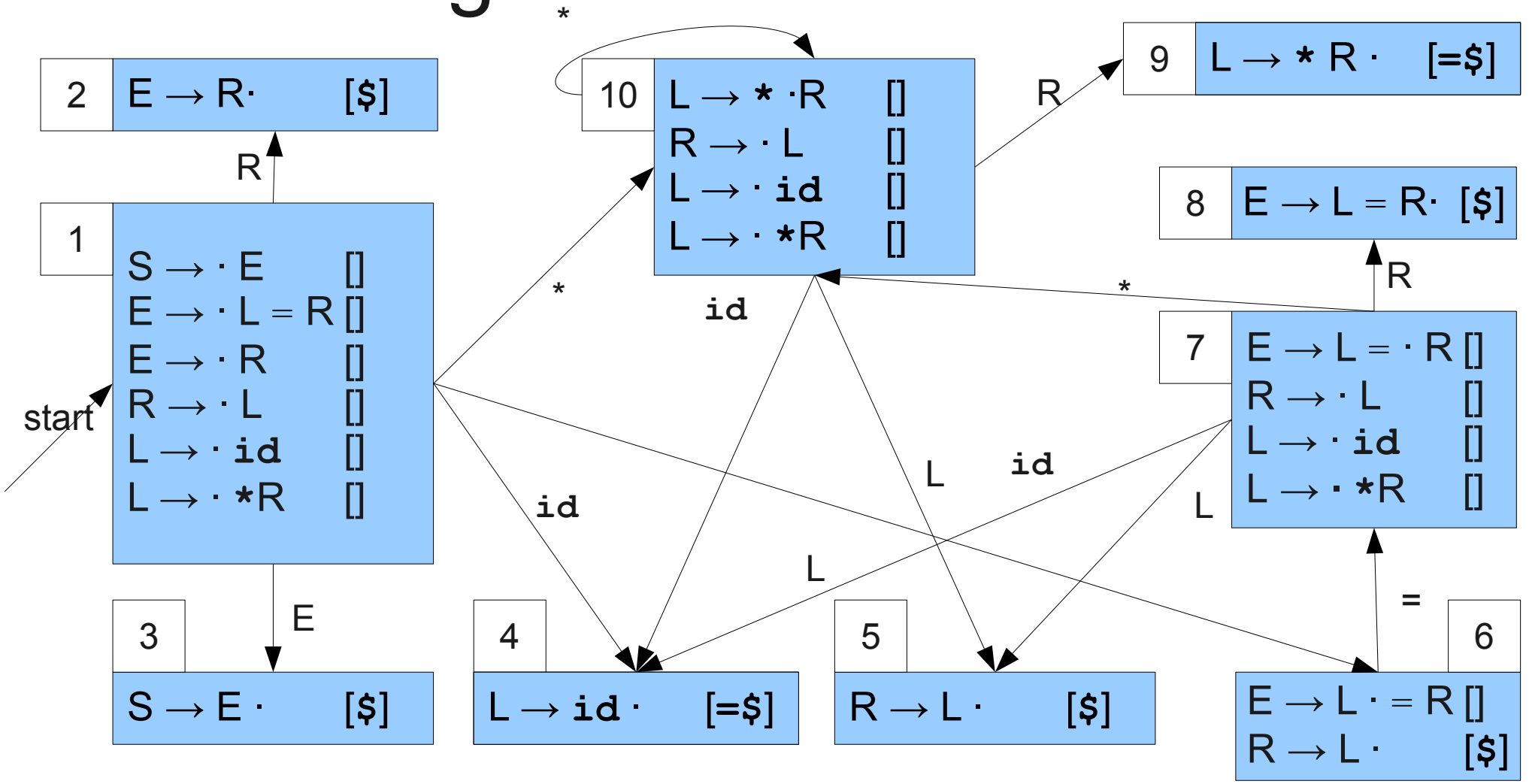

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$	$	=	$	=	$	$	=

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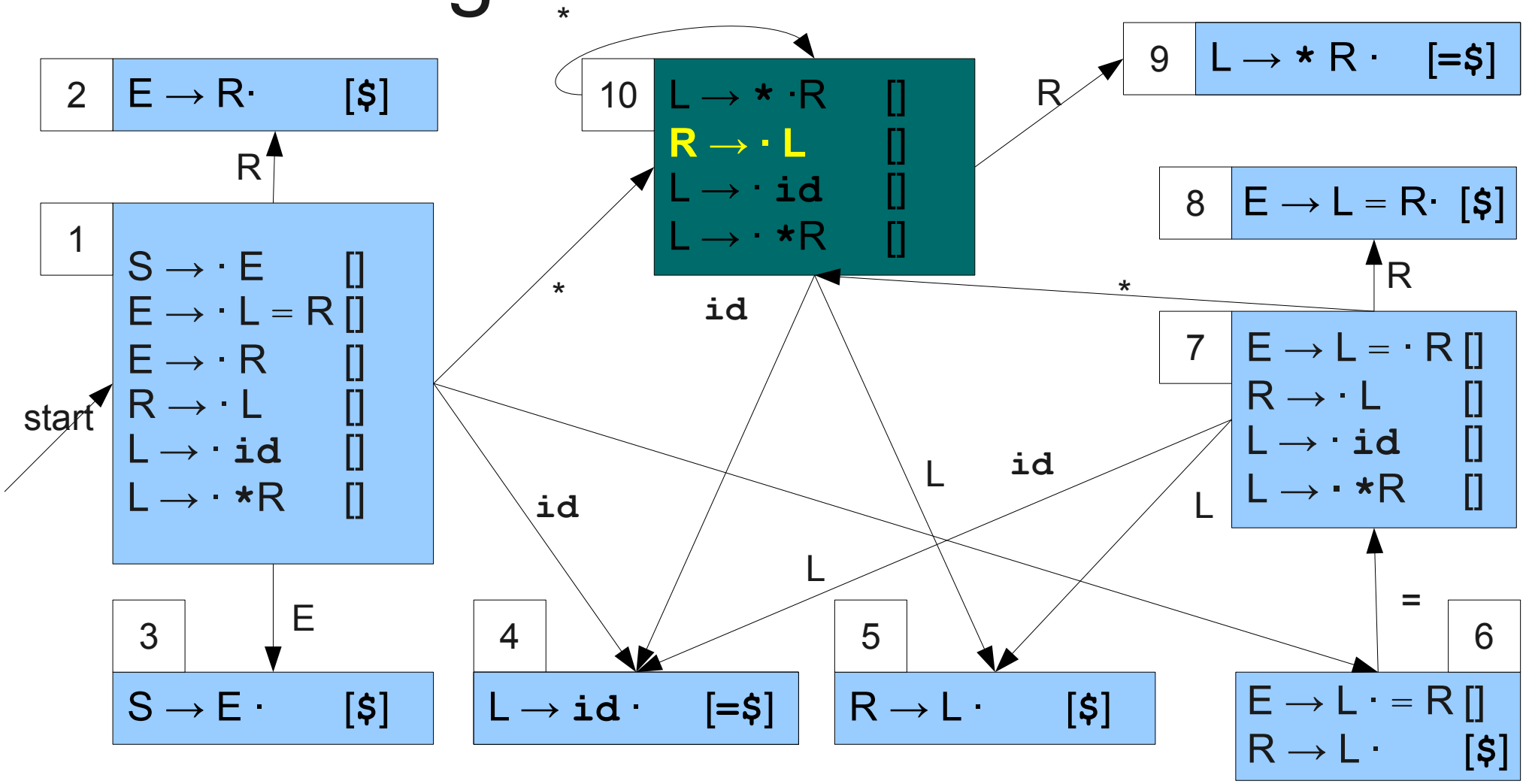

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$	$	=	$	=	$	$	=

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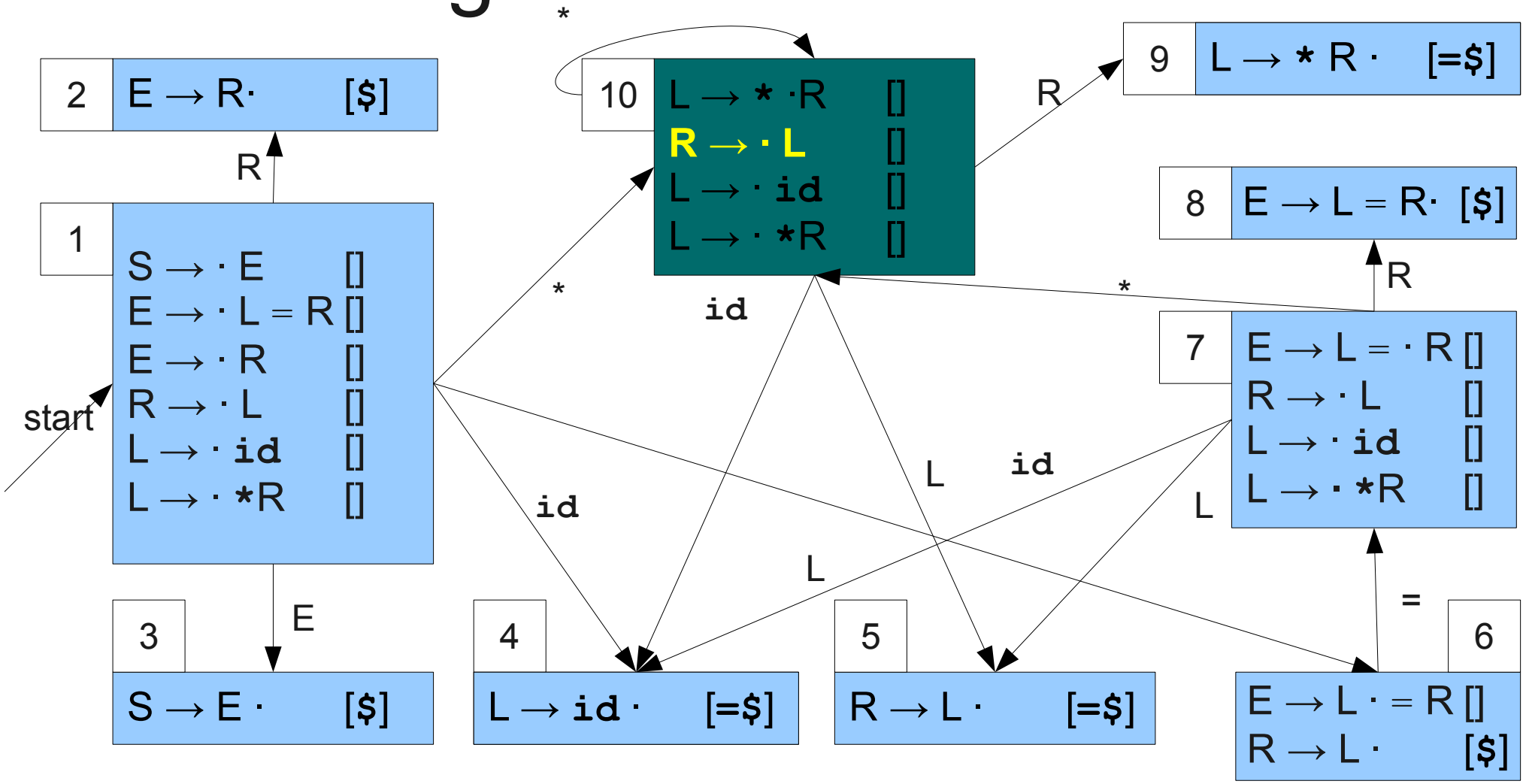

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$	$	=	$	=	$	$	=

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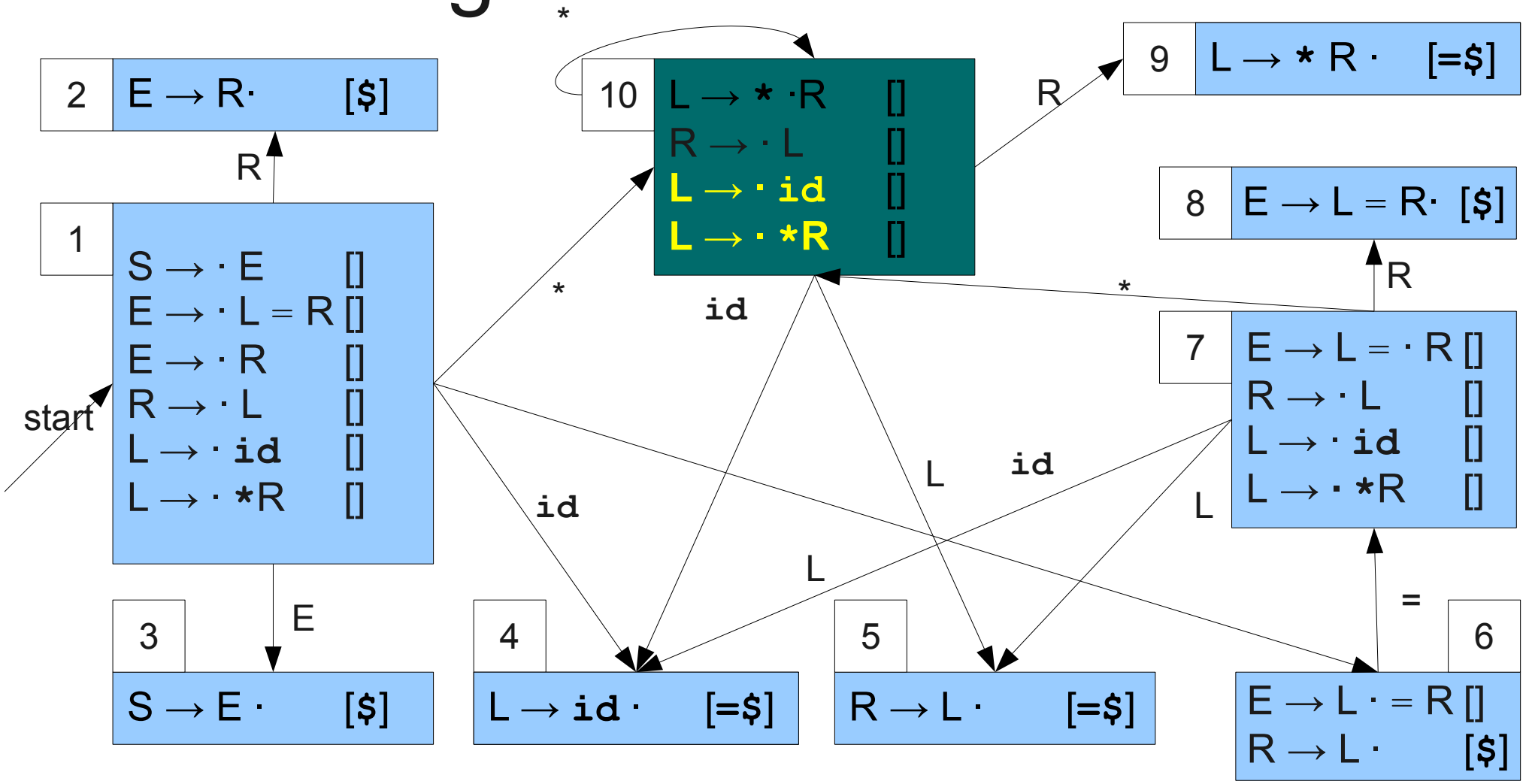

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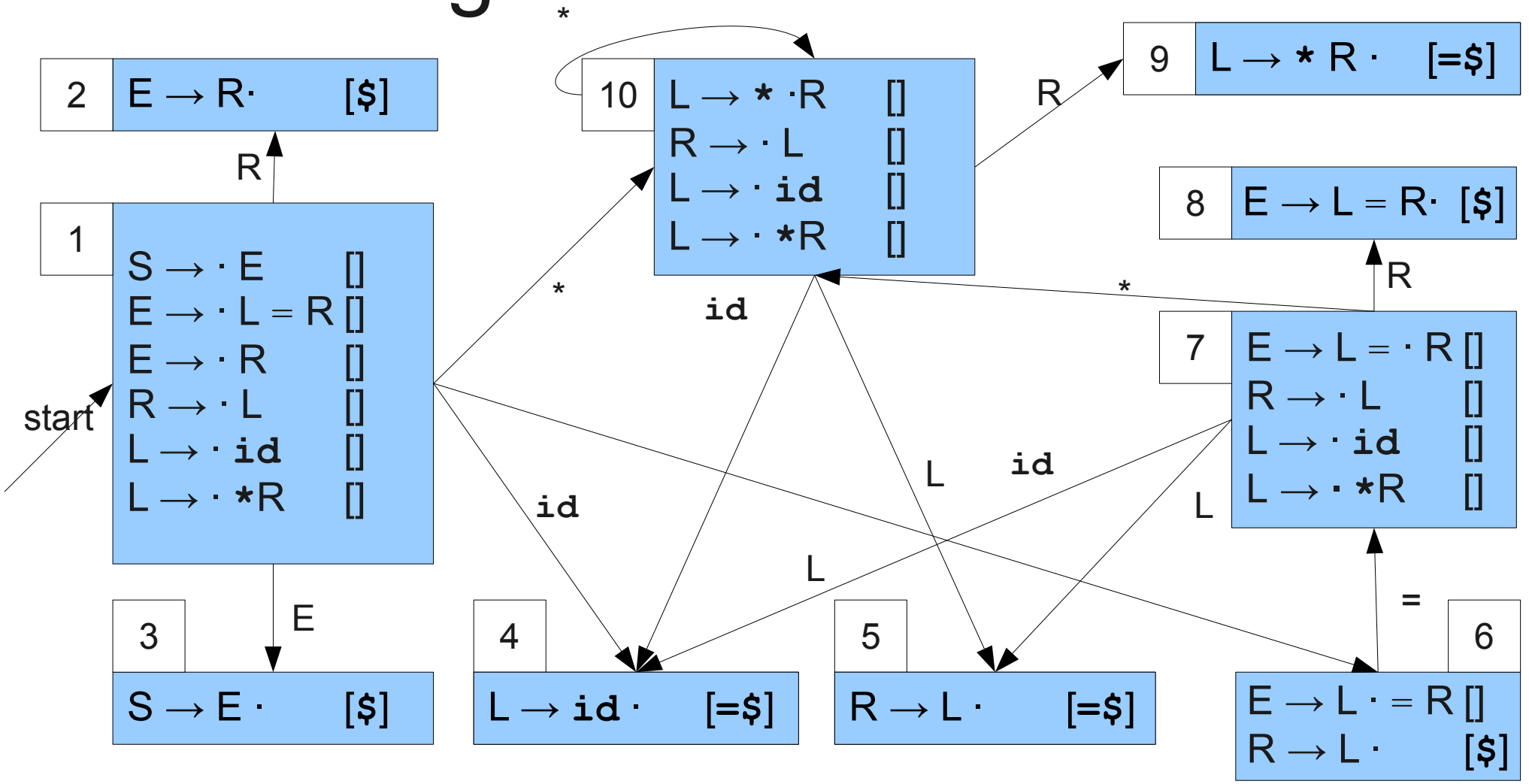

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$	$	=	$	=	$	$	=

Propagating Changes

- For each item $A \rightarrow \cdot v$ in a state q:
  - Let $A_{q\text{-}r}$ be the nonterminal corresponding to A following the transition out of q into some state r.
  - Trace through the automaton along the path labeled by *v*. This will lead to a state containing an item $A \rightarrow v \cdot$
  - Add to the lookahead of $A \rightarrow v \cdot$ the contents of FOLLOW($A_{q\text{-}r}$)

LALR(1)-by-SLR(1)

- Fast and simple construction of LALR(1) lookaheads.
- Four steps:
  - Construct the LR(0) automaton for the grammar.
  - Construct the augmented grammar by replacing nonterminals with new nonterminals based on the LR(0) transitions.
  - Compute the FOLLOW sets for these nonterminals.
  - Propagate changes through the LR(0) automaton.
- **Theorem** (Bermudez and Logothetis): This correctly computes LALR(1) lookaheads.

Summary of LALR(1)

- Along with LL(*k*), one of the most popular parsing algorithms in use today.
- Produced by the **`bison`** parser generator; rarely generated by hand.
- Can handle most, but not all, LR(1) languages.

Practical Bottom-Up Parsing

Where Theory Meets Practice

- We've just covered six powerful parsing algorithms:
  - Leftmost DFS
  - LL(1)
  - LR(0)
  - SLR(1)
  - LALR(1)
  - LR(1)
- How do we make them work in practice?

Two Practical Concerns

- **Ambiguity**
  - Real grammars are often ambiguous.
  - Programmers are **terrible** at eliminating it.
  - How do you build a parser to try to combat it?
- **Error-handling**
  - How do you report errors intelligently?
  - How do you continue parsing after an error?

Parsing an Ambiguous Grammar

- Consider this simple grammar for arithmetic expressions:

  $S \rightarrow E$

  $E \rightarrow E + E$

  $E \rightarrow E * E$

  $E \rightarrow \text{int}$

  $E \rightarrow (E)$

- This grammar is ambiguous.
  - Try parsing int + int * int
- What happens if we try parsing it?

SLR(1) Parsing with Ambiguity

1. $S \rightarrow E$
2. $E \rightarrow E + E$
3. $E \rightarrow E * E$
4. $E \rightarrow (E)$
5. $E \rightarrow int$

FOLLOW(S) = { **$** }

FOLLOW(E) = { **+**, *****, **)** , **$** }

1

$S \rightarrow \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

2

$S \rightarrow E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

3

$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

4

$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

5

$E \rightarrow E + E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

6

$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

7

$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

8

$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

9

$E \rightarrow (E) \cdot$

10

$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

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	int	+	*	(	)	$	E
1							
2							
3							
4							
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9

$E \rightarrow (E) \cdot$

10

$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10						
2							
3							
4							
5							
6							
7							
8							
9							
10							

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$E \rightarrow (E) \cdot$

10

$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			
2							
3							
4							
5							
6							
7							
8							
9							
10							

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$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			s2
2							
3							
4							
5							
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7							
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	int	+	*	(	)	$	E
1	s10			s7			s2
2							
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8							
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$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3					
3							
4							
5							
6							
7							
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9

$E \rightarrow (E) \cdot$

10

$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4				
3							
4							
5							
6							
7							
8							
9							
10							

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$E \rightarrow E \cdot * E$

9

$E \rightarrow (E) \cdot$

10

$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3							
4							
5							
6							
7							
8							
9							
10							

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10

$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3							
4							
5							
6							
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2

$S \rightarrow E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

3

$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

4

$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

5

$E \rightarrow E + E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

6

$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

7

$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

8

$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

9

$E \rightarrow (E) \cdot$

10

$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10						
4							
5							
6							
7							
8							
9							
10							

SLR(1) Parsing with Ambiguity

1. $S \rightarrow E$
2. $E \rightarrow E + E$
3. $E \rightarrow E * E$
4. $E \rightarrow (E)$
5. $E \rightarrow int$

FOLLOW(S) = { **$** }

FOLLOW(E) = { **+**, *****, **)** , **$** }

1

$S \rightarrow \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

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$S \rightarrow E \cdot$
$E \rightarrow E \cdot + E$
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	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			
4							
5							
6							
7							
8							
9							
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4							
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5							
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1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5							
6							
7							
8							
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$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3					
6							
7							
8							
9							
10							

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$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3	s4				
6							
7							
8							
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1

S → · E
E → · E + E
E → · E * E
E → · int
E → · (E)

2

S → E ·
E → E ·+ E
E → E ·* E

3

E → E + ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

4

E → E * ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

5

E → E + E ·
E → E · + E
E → E · * E

6

E → E * E ·
E → E · + E
E → E · * E

7

E → (·E)
E → · E + E
E → · E * E
E → · int
E → · (E)

8

E → (E ·)
E → E ·+ E
E → E ·* E

9

E → (E) ·

10

E → int ·

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4				
6							
7							
8							
9							
10							

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	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2				
6							
7							
8							
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E → (E) ·

10

E → int ·

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2		
6							
7							
8							
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	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6							
7							
8							
9							
10							

SLR(1) Parsing with Ambiguity

1. $S \rightarrow E$
2. $E \rightarrow E + E$
3. $E \rightarrow E * E$
4. $E \rightarrow (E)$
5. $E \rightarrow int$

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S → · E
E → · E + E
E → · E * E
E → · int
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2

S → E ·
E → E ·+ E
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E → E + ·E
E → · E + E
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E → · int
E → · (E)

4

E → E * ·E
E → · E + E
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E → · int
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5

E → E + E ·
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6

E → E * E ·
E → E · + E
E → E · * E

7

E → (·E)
E → · E + E
E → · E * E
E → · int
E → · (E)

8

E → (E ·)
E → E ·+ E
E → E ·* E

9

E → (E) ·

10

E → int ·

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6							
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E → int ·

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2		s3	s4			acc	
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4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3					
7							
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	int	+	*	(	)	$	E
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6		s3	s4				
7							
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5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4				
7							
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5		s3 r2	s4 r2		r2	r2	
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7							
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	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
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4	s10			s7			s6
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$E \rightarrow (E) \cdot$

10

$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4				
9							
10							

SLR(1) Parsing with Ambiguity

1
S → · E
E → · E + E
E → · E * E
E → · int
E → · (E)

1. S → E
2. E → E + E
3. E → E * E
4. E → (E)
5. E → int

FOLLOW(S) = { **$** }

FOLLOW(E) = { **+**, *****, **)** , **$** }

2
S → E ·
E → E ·+ E
E → E ·* E

3
E → E + ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

4
E → E * ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

5
E → E + E ·
E → E · + E
E → E · * E

6
E → E * E ·
E → E · + E
E → E · * E

7
E → (·E)
E → · E + E
E → · E * E
E → · int
E → · (E)

8
E → (E ·)
E → E ·+ E
E → E ·* E

9
E → (E) ·

10
E → int ·

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9							
10							

SLR(1) Parsing with Ambiguity

1. $S \rightarrow E$
2. $E \rightarrow E + E$
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S → · E
E → · E + E
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S → E ·
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E → E + ·E
E → · E + E
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4

E → E * ·E
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E → E + E ·
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	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9							
10							

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E → (E ·)
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E → int ·

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4					
10							

SLR(1) Parsing with Ambiguity

1. $S \rightarrow E$
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$S \rightarrow \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

2

$S \rightarrow E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

3

$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E + E$
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$E \rightarrow \cdot int$
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4

$E \rightarrow E * \cdot E$
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$E \rightarrow \cdot int$
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$E \rightarrow E + E \cdot$
$E \rightarrow E \cdot + E$
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$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

7

$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

8

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$E \rightarrow E \cdot + E$
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9

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	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4				
10							

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$E \rightarrow E + E \cdot$
$E \rightarrow E \cdot + E$
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$E \rightarrow \cdot E + E$
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$E \rightarrow \cdot int$
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$E \rightarrow E \cdot + E$
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9

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10

$E \rightarrow int \cdot$

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4		
10							

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	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10							

SLR(1) Parsing with Ambiguity

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$S \rightarrow \cdot E$
$E \rightarrow \cdot E + E$
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$S \rightarrow E \cdot$
$E \rightarrow E \cdot + E$
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$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

4

$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
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5

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$E \rightarrow E \cdot + E$
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$E \rightarrow E \cdot + E$
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9

$E \rightarrow (E) \cdot$

10

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	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10							

SLR(1) Parsing with Ambiguity

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S → · E
E → · E + E
E → · E * E
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E → · (E)

1. S → E
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5. E → int

FOLLOW(S) = { **$** }

FOLLOW(E) = { **+**, *****, **)** , **$** }

2
S → E ·
E → E ·+ E
E → E ·* E

3
E → E + ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

4
E → E * ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

5
E → E + E ·
E → E · + E
E → E · * E

6
E → E * E ·
E → E · + E
E → E · * E

7
E → (·E)
E → · E + E
E → · E * E
E → · int
E → · (E)

8
E → (E ·)
E → E ·+ E
E → E ·* E

9
E → (E) ·

10
E → int ·

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5					

SLR(1) Parsing with Ambiguity

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$S \rightarrow \cdot E$
$E \rightarrow \cdot E + E$
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$E \rightarrow \cdot E + E$
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$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

4

$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

5

$E \rightarrow E + E \cdot$
$E \rightarrow E \cdot + E$
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	int	+	*	(	)	$	E
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S → · E
E → · E + E
E → · E * E
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S → E ·
E → E ·+ E
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E → E + ·E
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	int	+	*	(	)	$	E
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4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

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	int	+	*	(	)	$	E
1	s10			s7			s2
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3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

Resolving Ambiguity

- Our CFG is ambiguous because our concept of **precedence** is missing from the grammar.
- How can we use this precedence information to avoid LR conflicts?

Precedence Declarations

- Tell the parser generator about the **associativity** and **precedence** of certain rules.
- Productions can be left-associative, right-associative, or nonassociative.
- Productions can have their priorities ranked against one another.

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E → · E * E
E → · int
E → · (E)

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	int	+	*	(	)	$	E
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2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
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4. E → (E)
5. E → int

FOLLOW(S) = { **$** }
FOLLOW(E) = { **+**, *****, **)** , **$** }

2

S → E ·
E → E ·+ E
E → E ·* E

3

E → E + ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

4

E → E * ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

5

E → E + E ·
E → E · + E
E → E · * E

6

E → E * E ·
E → E · + E
E → E · * E

7

E → (·E)
E → · E + E
E → · E * E
E → · int
E → · (E)

8

E → (E ·)
E → E ·+ E
E → E ·* E

9

E → (E) ·

10

E → int ·

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3	s4		r2	r2	
6		r3	s4		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

Resolving Conflicts with Precedence

- When choosing whether to reduce A or shift B:
  - If A has higher priority, **reduce**.
  - If B has higher priority, **shift**.
  - If A and B have the same priority:
    - If A is left-associative, **reduce**.
    - If A is right-associative, **shift**.
    - If A is non-associative, **error**.

SLR(1) Parsing with Ambiguity

1

S → · E
E → · E + E
E → · E * E
E → · int
E → · (E)

1. S → E
2. E → E + E Left-assoc., priority 0
3. E → E * E Left-assoc., priority 1
4. E → (E)
5. E → int

FOLLOW(S) = { **$** }
FOLLOW(E) = { **+**, *****, **)** , **$** }

2

S → E ·
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3

E → E + ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

4

E → E * ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

5

E → E + E ·
E → E · + E
E → E · * E

6

E → E * E ·
E → E · + E
E → E · * E

7

E → (·E)
E → · E + E
E → · E * E
E → · int
E → · (E)

8

E → (E ·)
E → E ·+ E
E → E ·* E

9

E → (E) ·

10

E → int ·

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		r2	s4		r2	r2	
6		r3	r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

SLR(1) Parsing with Ambiguity

1. $S \rightarrow E$
2. $E \rightarrow E + E$ Left-assoc., priority 0
3. $E \rightarrow E * E$ Left-assoc., priority 1
4. $E \rightarrow (E)$
5. $E \rightarrow int$

FOLLOW(S) = { **$** }

FOLLOW(E) = { **+**, *****, **)** , **$** }

1

S → · E
E → · E + E
E → · E * E
E → · int
E → · (E)

2

S → E ·
E → E ·+ E
E → E ·* E

3

E → E + ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

4

E → E * ·E
E → · E + E
E → · E * E
E → · int
E → · (E)

5

E → E + E ·
E → E · + E
E → E · * E

6

E → E * E ·
E → E · + E
E → E · * E

7

E → (·E)
E → · E + E
E → · E * E
E → · int
E → · (E)

8

E → (E ·)
E → E ·+ E
E → E ·* E

9

E → (E) ·

10

E → int ·

	int	+	*	(	)	$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		r2	s4		r2	r2	
6		r3	r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

Error Handling

- What should the parser do when it encounters an error?
- Could just say “syntax error,” but we'd like more detailed messages.
- How do we resume parsing after an error?

Error Productions

- One idea: add productions to the grammar that identify common mistakes.
- For example:

  $E \rightarrow E + E$

  $E \rightarrow E * E$

  $E \rightarrow \text{int}$

  $E \rightarrow (E)$

  $E \rightarrow E\ E$ *(error production)*

  $E \rightarrow E +$ *(error production)*

  $E \rightarrow E *$ *(error production)*

Analysis of Error Productions

- Useful for diagnosing common programmer mistakes.
  - For example, using **implements** instead of **extends** in Java.
- Increases risk of parsing problems.
  - More likelihood for ambiguity.
  - More likelihood grammar won't be accepted by parser generator (i.e. not LALR(1))
- Forces parser generator to anticipate errors.

Panic Mode

- Idea: Augment grammar by adding rules for resuming parsing when Bad Things happen.
- Example:

  E → E + E

  E → E * E

  E → (E)

  E → int

  E → error int

  E → (error)

- Tokens after errors are called **synchronizing tokens**.
- Technique employed by **`bison`** and many other parser generators.

Using Panic Mode

- When parser encounters an error in a configurating set, search for a production containing an error term.
  - Repeatedly pop tokens from the stack until one can be found.
- Shift the error token.
- Resume parsing as normal.

Next Time

- The Limits of Parsing:
  - Parsing ambiguous grammars, take II.
  - Parsing arbitrary CFGs: The Earley Parser
  - Parsing with missing tokens: Intersection Parsing
  - A Most Excellent Parser: The Earley-on-Intersection Algorithm