Bottom-Up Parsing

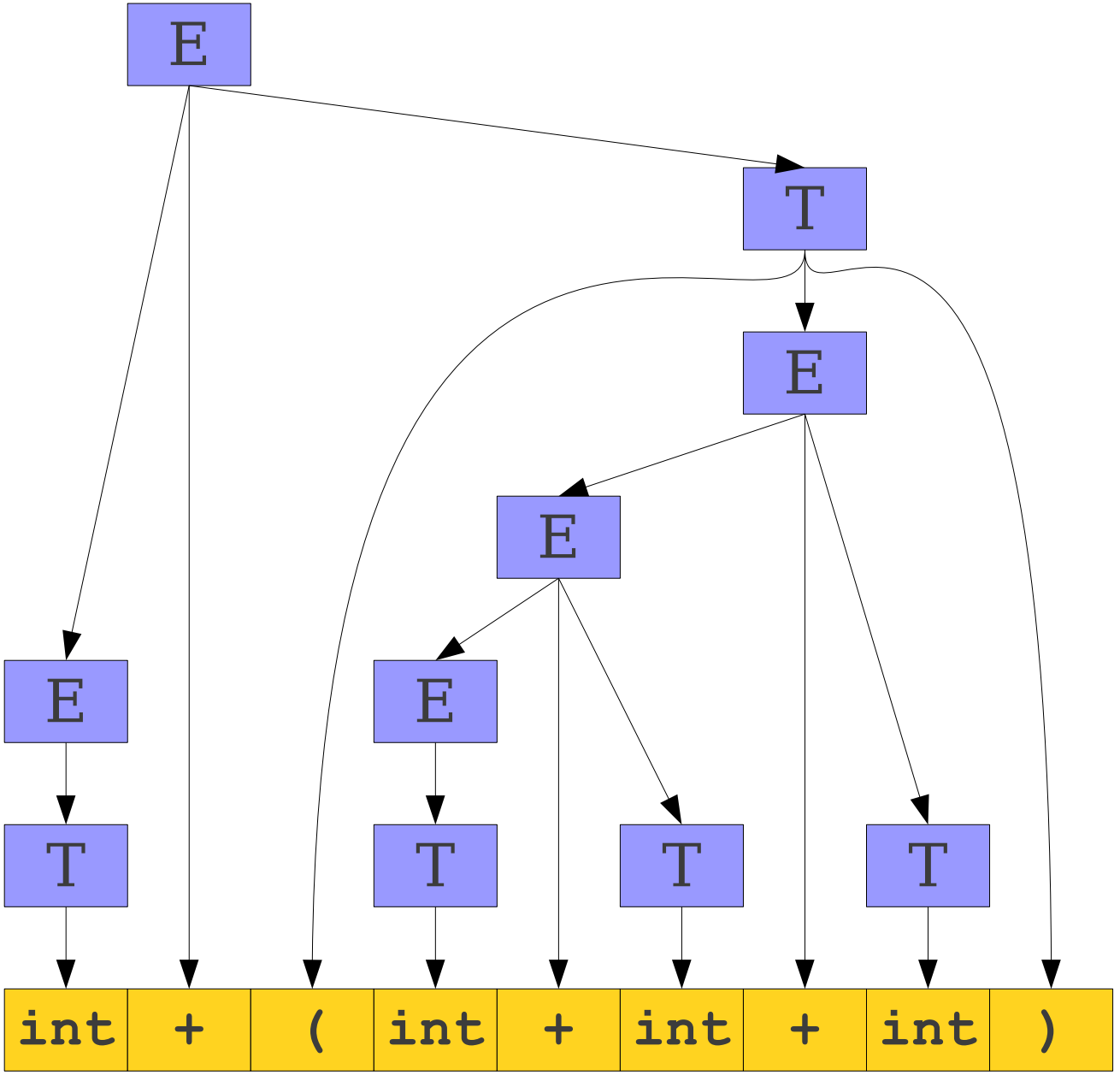
What is Bottom-Up Parsing?

- Idea: Apply productions **in reverse** to convert the user's program to the start symbol.
- As with top-down, could be done with a DFS or BFS, though this is rarely done in practice.
- We'll be exploring four **directional**, **predictive** bottom-up parsing techniques:
  - **Directional**: Scan the input from left-to-right.
  - **Predictive**: Guess which production should be inverted.

Bottoms Up!

One View of a Bottom-Up Parse

E → T
E → E + T
T → int
T → (E)



A Second View of a Bottom-Up Parse

$E \rightarrow T$
$E \rightarrow E + T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$

$$
\begin{aligned}
& \texttt{int} + (\texttt{int} + \texttt{int} + \texttt{int}) \\
\Rightarrow\ & T + (\texttt{int} + \texttt{int} + \texttt{int}) \\
\Rightarrow\ & E + (\texttt{int} + \texttt{int} + \texttt{int}) \\
\Rightarrow\ & E + (T + \texttt{int} + \texttt{int}) \\
\Rightarrow\ & E + (E + \texttt{int} + \texttt{int}) \\
\Rightarrow\ & E + (E + T + \texttt{int}) \\
\Rightarrow\ & E + (E + \texttt{int}) \\
\Rightarrow\ & E + (E + T) \\
\Rightarrow\ & E + (E) \\
\Rightarrow\ & E + T \\
\Rightarrow\ & E
\end{aligned}
$$

A Second View of a Bottom-Up Parse

$E \rightarrow T$
$E \rightarrow E + T$
$T \rightarrow \texttt{int}$
$T \rightarrow (E)$

$$
\begin{aligned}
& \texttt{int} + (\texttt{int} + \texttt{int} + \texttt{int}) \\
\Rightarrow\ & T + (\texttt{int} + \texttt{int} + \texttt{int}) \\
\Rightarrow\ & E + (\texttt{int} + \texttt{int} + \texttt{int}) \\
\Rightarrow\ & E + (T + \texttt{int} + \texttt{int}) \\
\Rightarrow\ & E + (E + \texttt{int} + \texttt{int}) \\
\Rightarrow\ & E + (E + T + \texttt{int}) \\
\Rightarrow\ & E + (E + \texttt{int}) \\
\Rightarrow\ & E + (E + T) \\
\Rightarrow\ & E + (E) \\
\Rightarrow\ & E + T \\
\Rightarrow\ & E
\end{aligned}
$$

A left-to-right, bottom-up parse is a rightmost derivation traced in reverse.

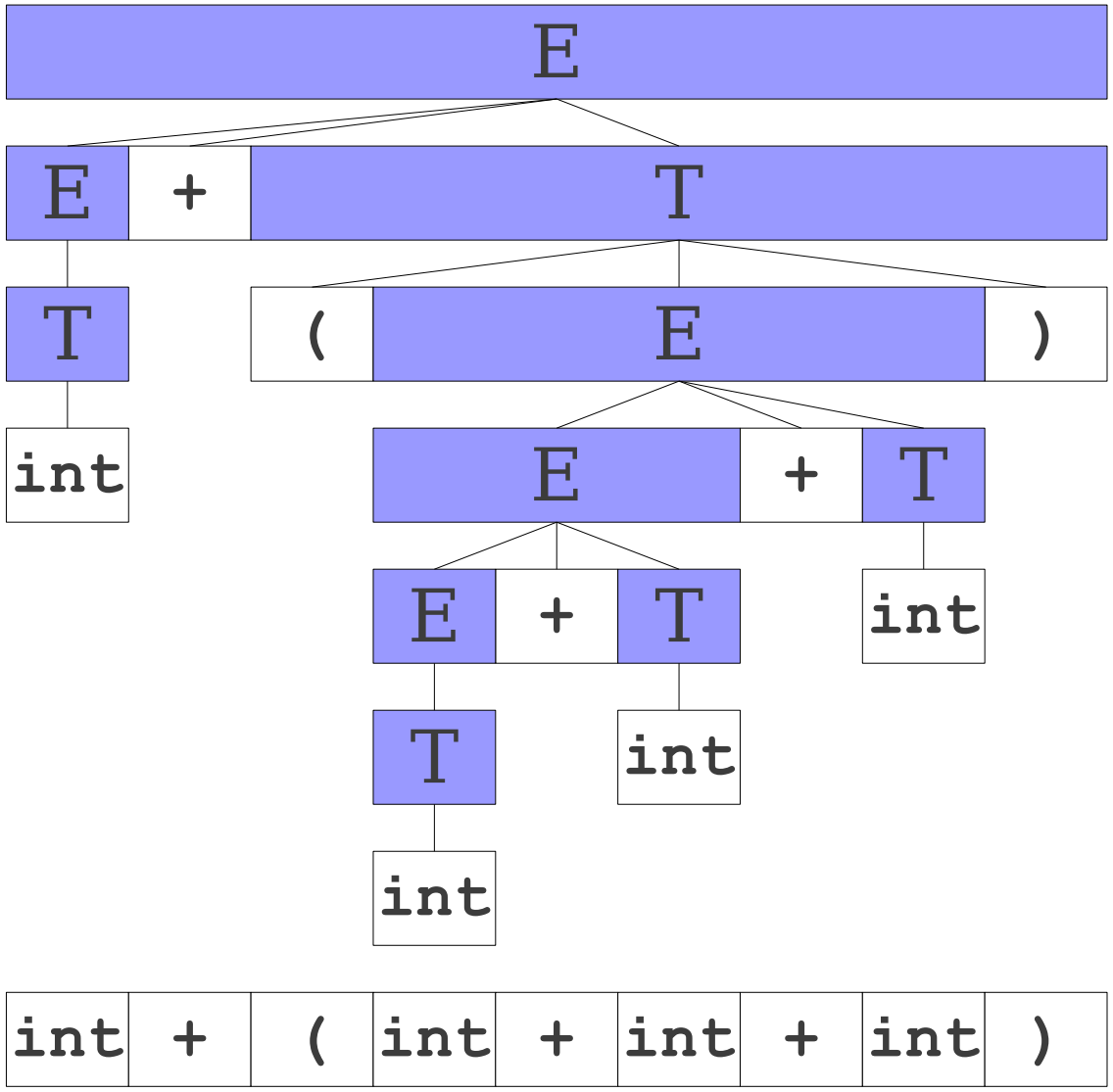
A Third View of a Bottom-Up Parse

```
  int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E
```

Each step in this bottom-up parse is called a **reduction**. We **reduce** a substring of the sentential form back to a nonterminal.

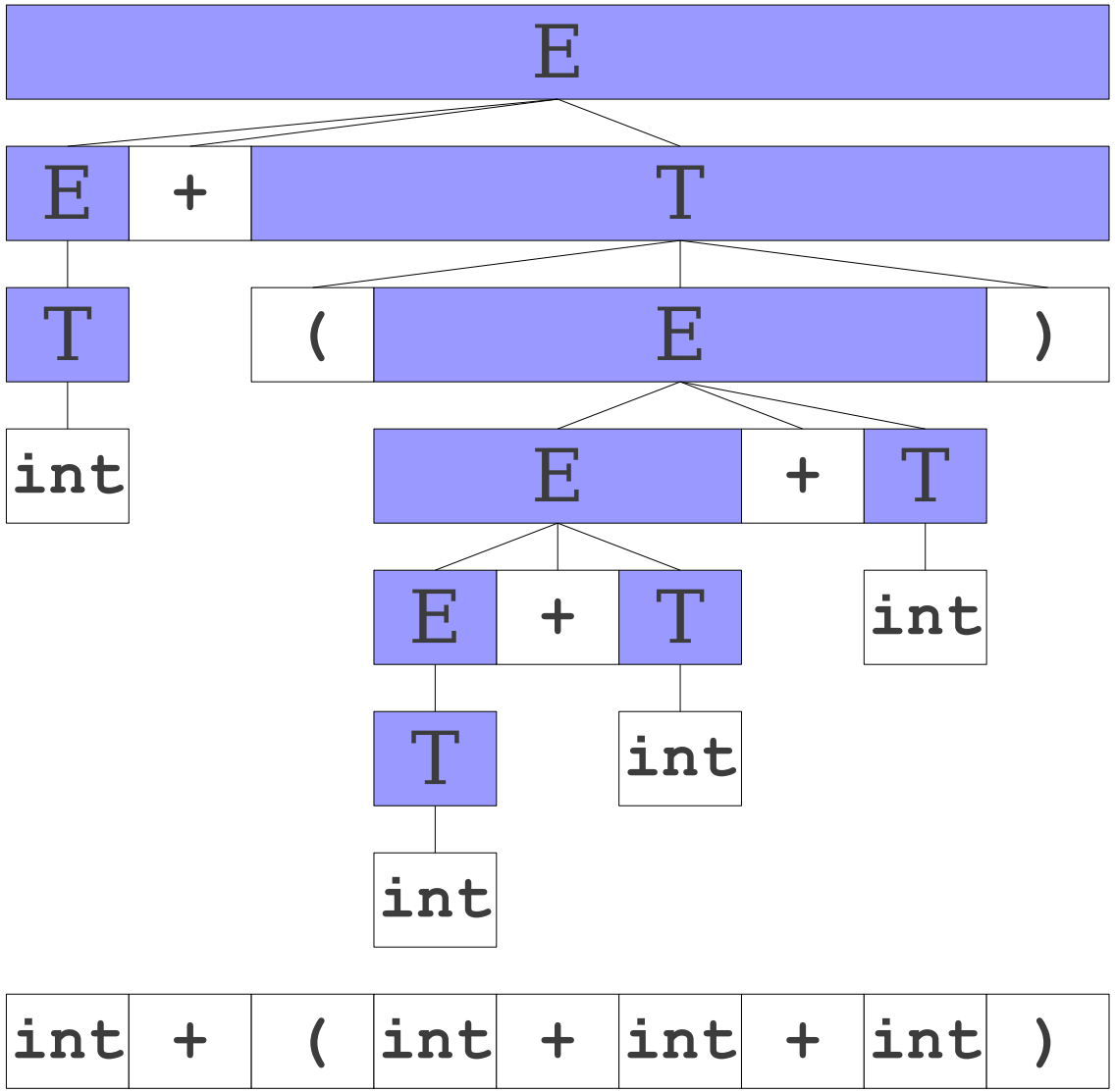
A Third View of a Bottom-Up Parse

```
  int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E
```



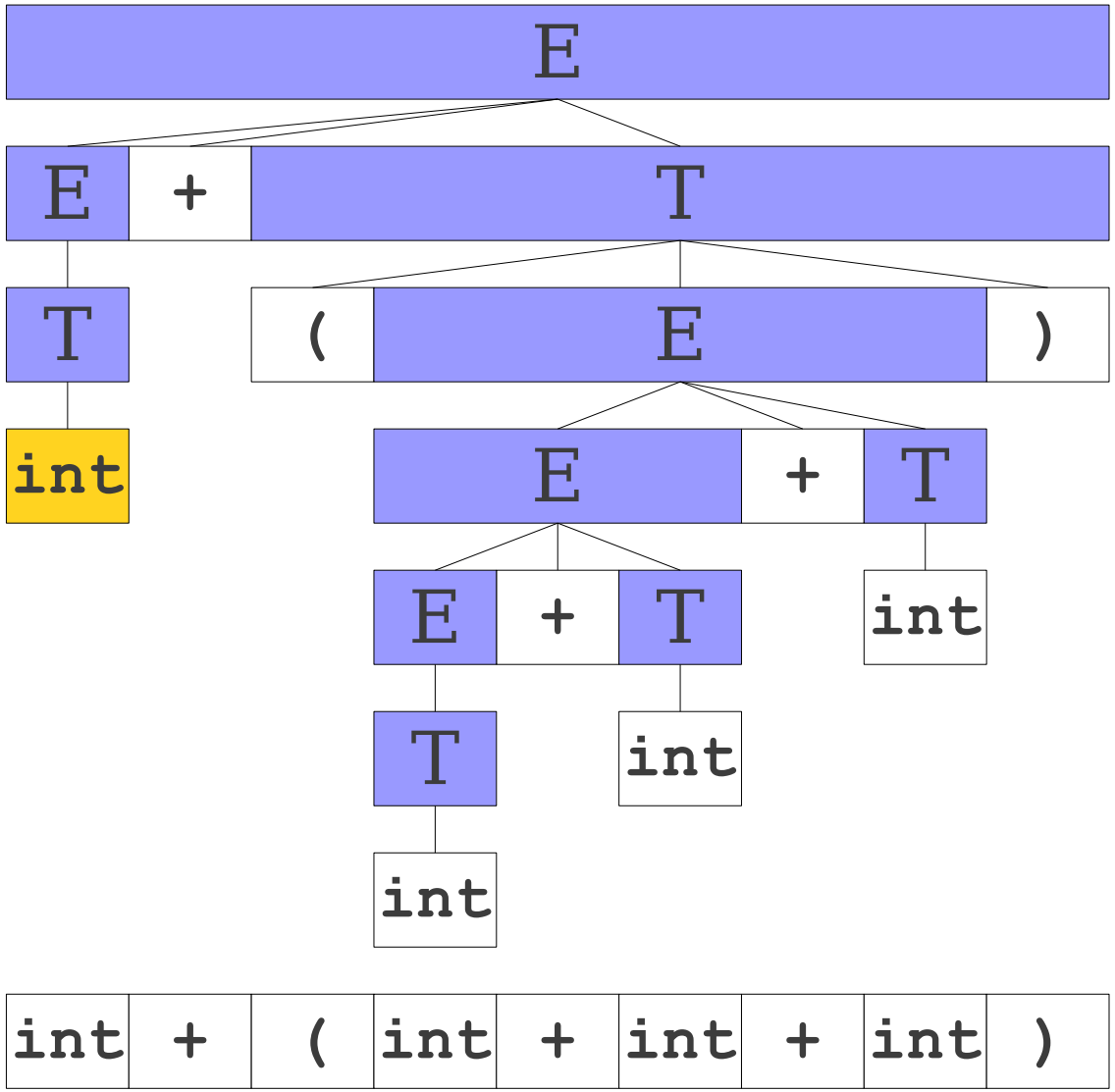
A Third View of a Bottom-Up Parse

```
  int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E
```



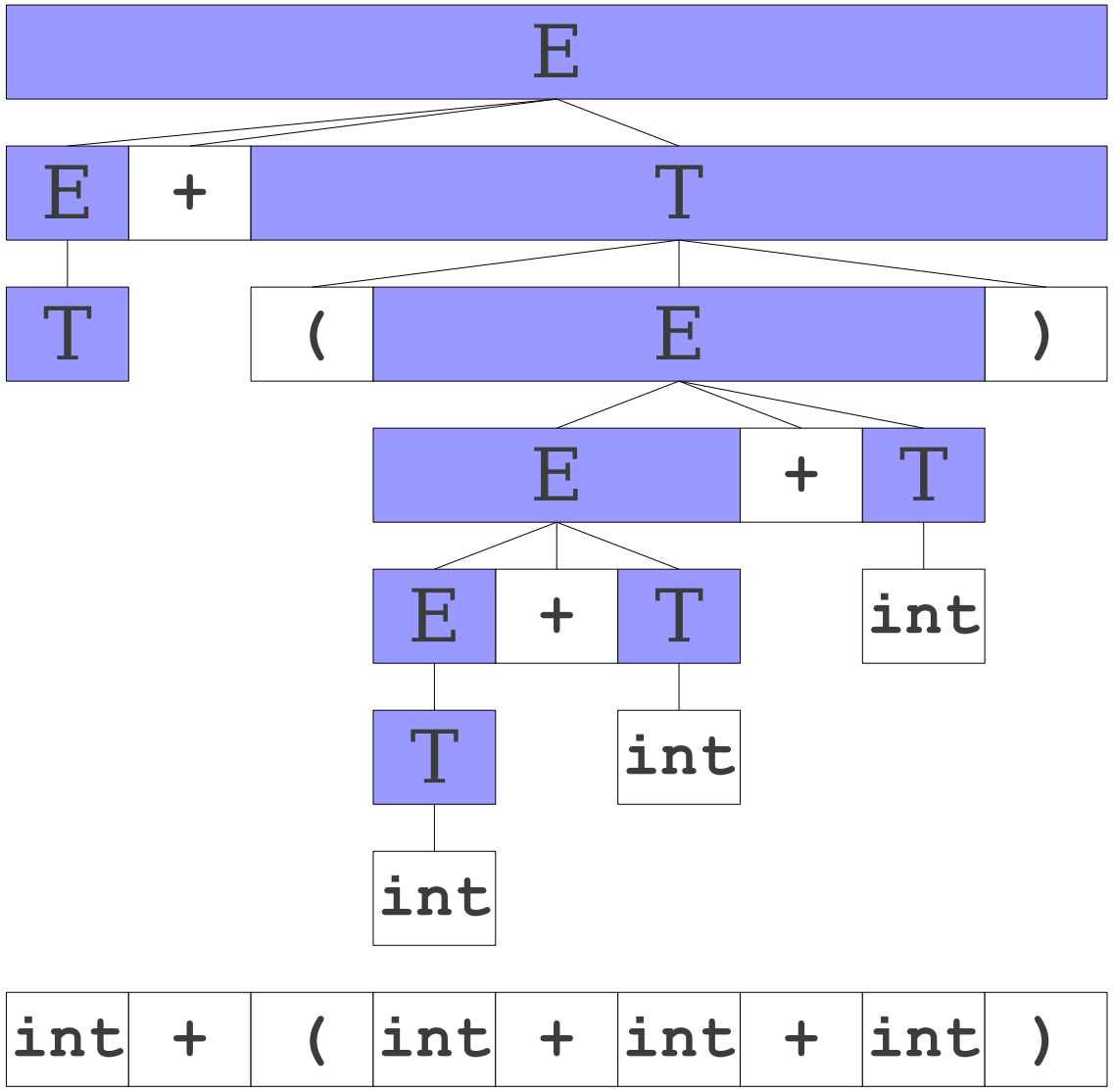
A Third View of a Bottom-Up Parse

```
  int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E
```



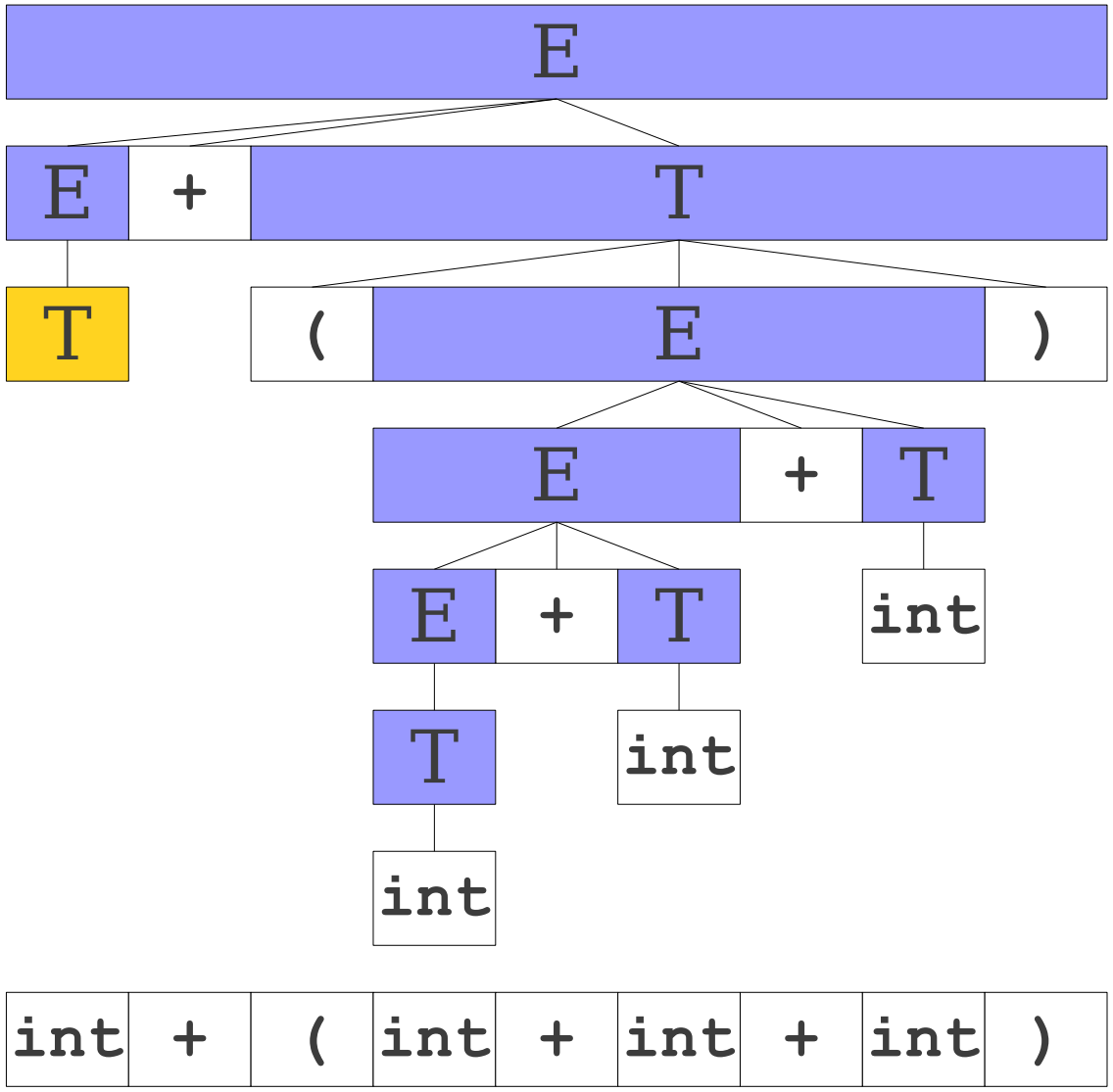
A Third View of a Bottom-Up Parse

⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



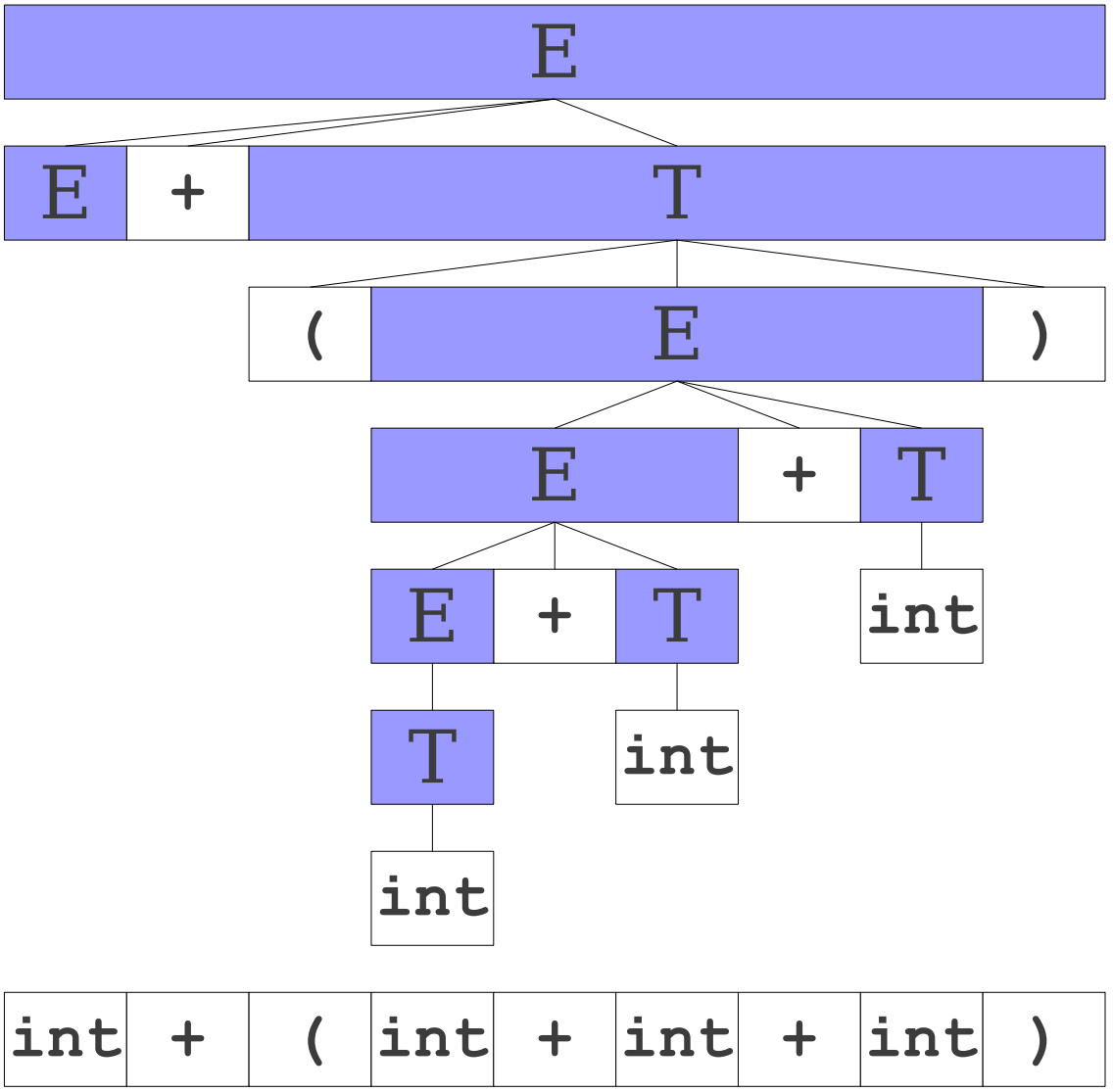
A Third View of a Bottom-Up Parse

⇒ **T** + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



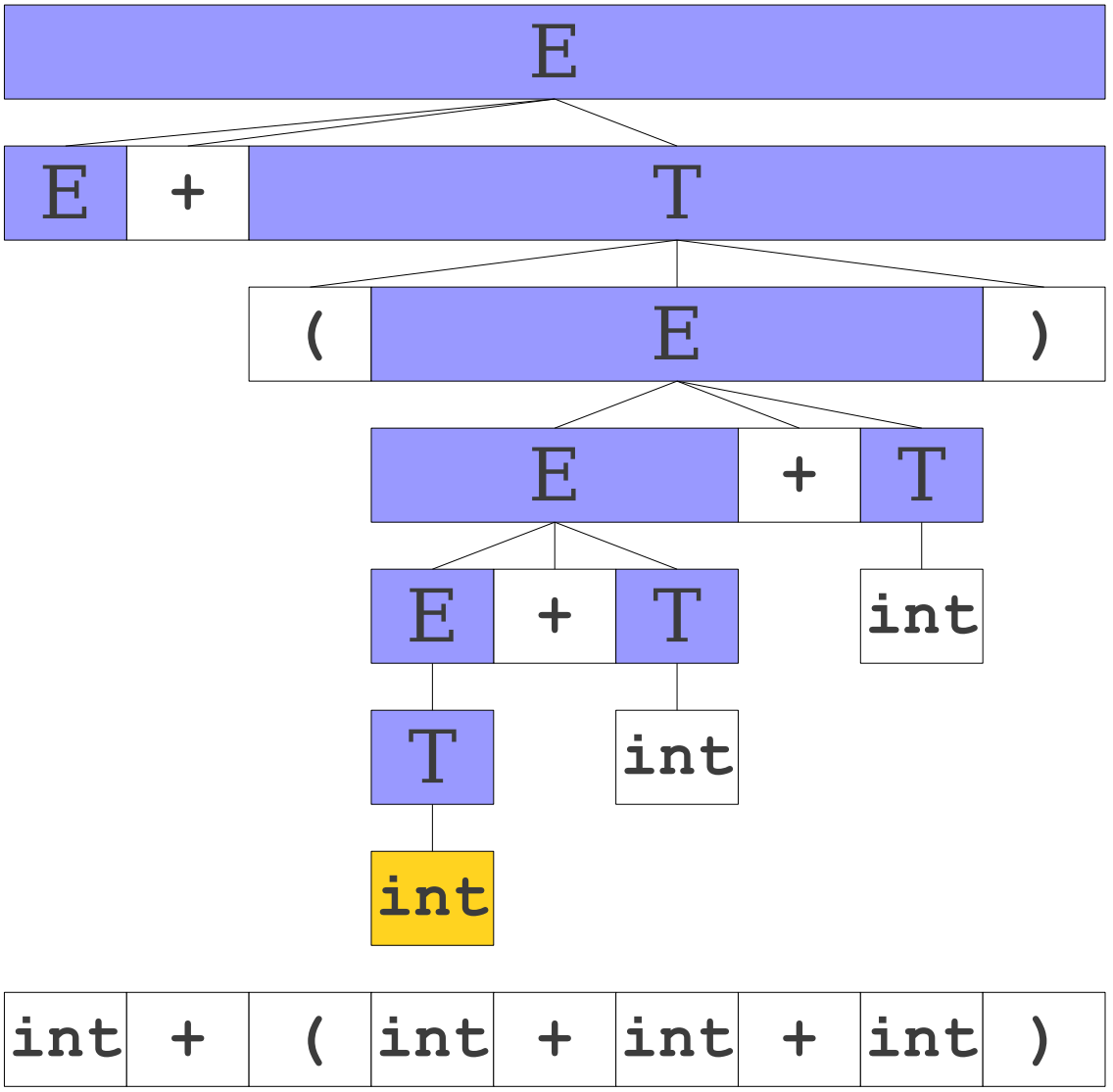
A Third View of a Bottom-Up Parse

⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



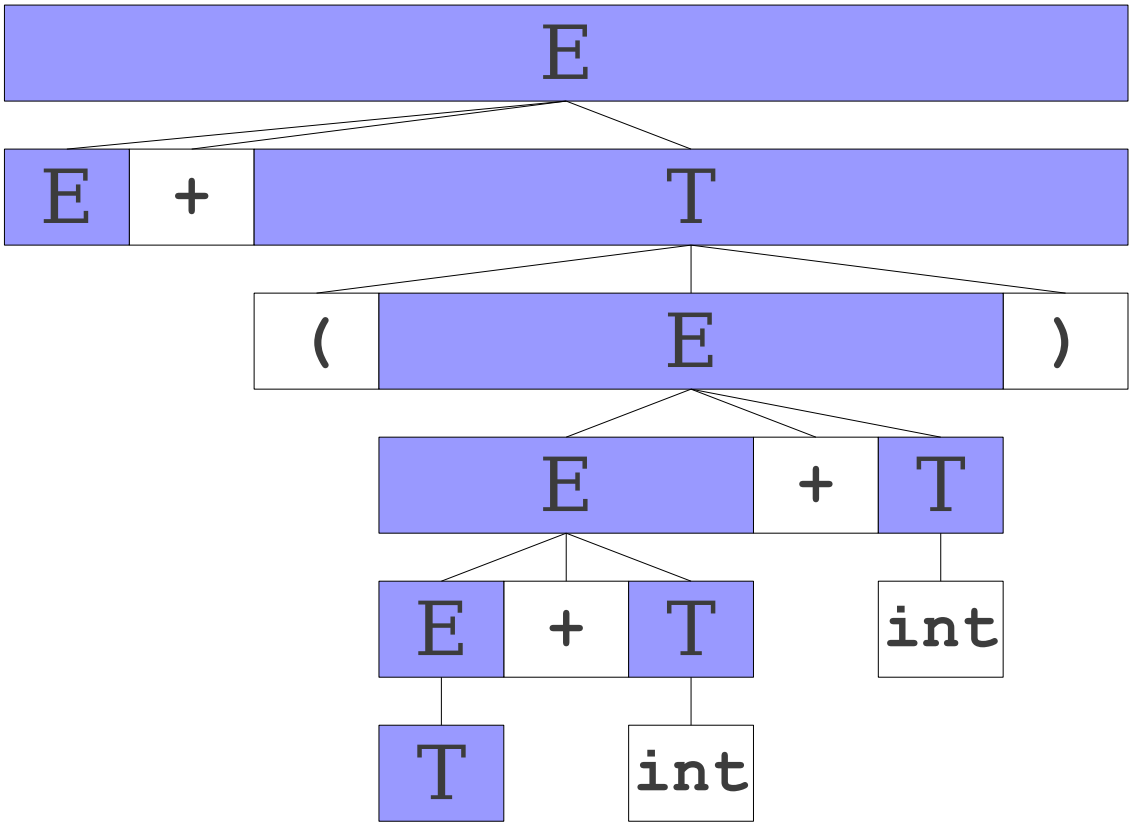
A Third View of a Bottom-Up Parse

⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



A Third View of a Bottom-Up Parse

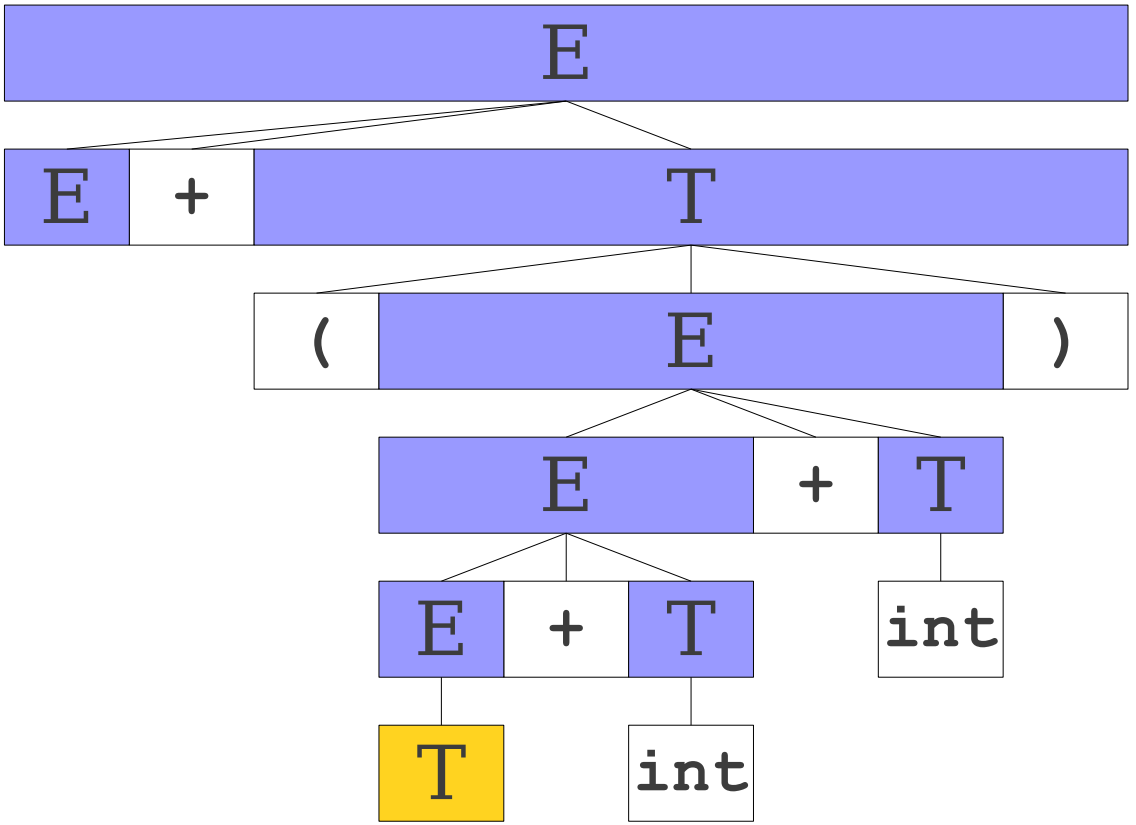
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



int	+	(	int	+	int	+	int	)

A Third View of a Bottom-Up Parse

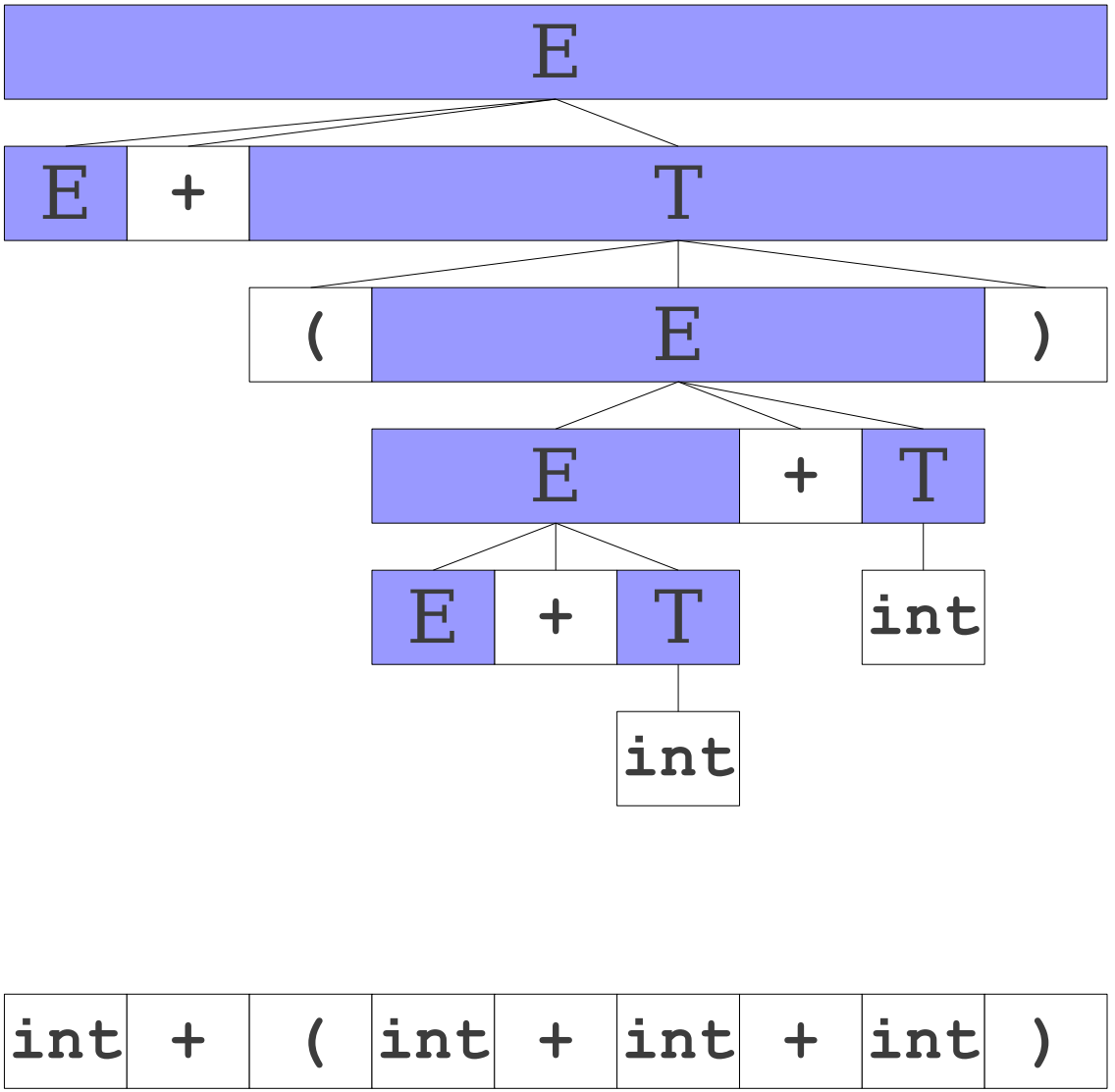
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



int	+	(	int	+	int	+	int	)

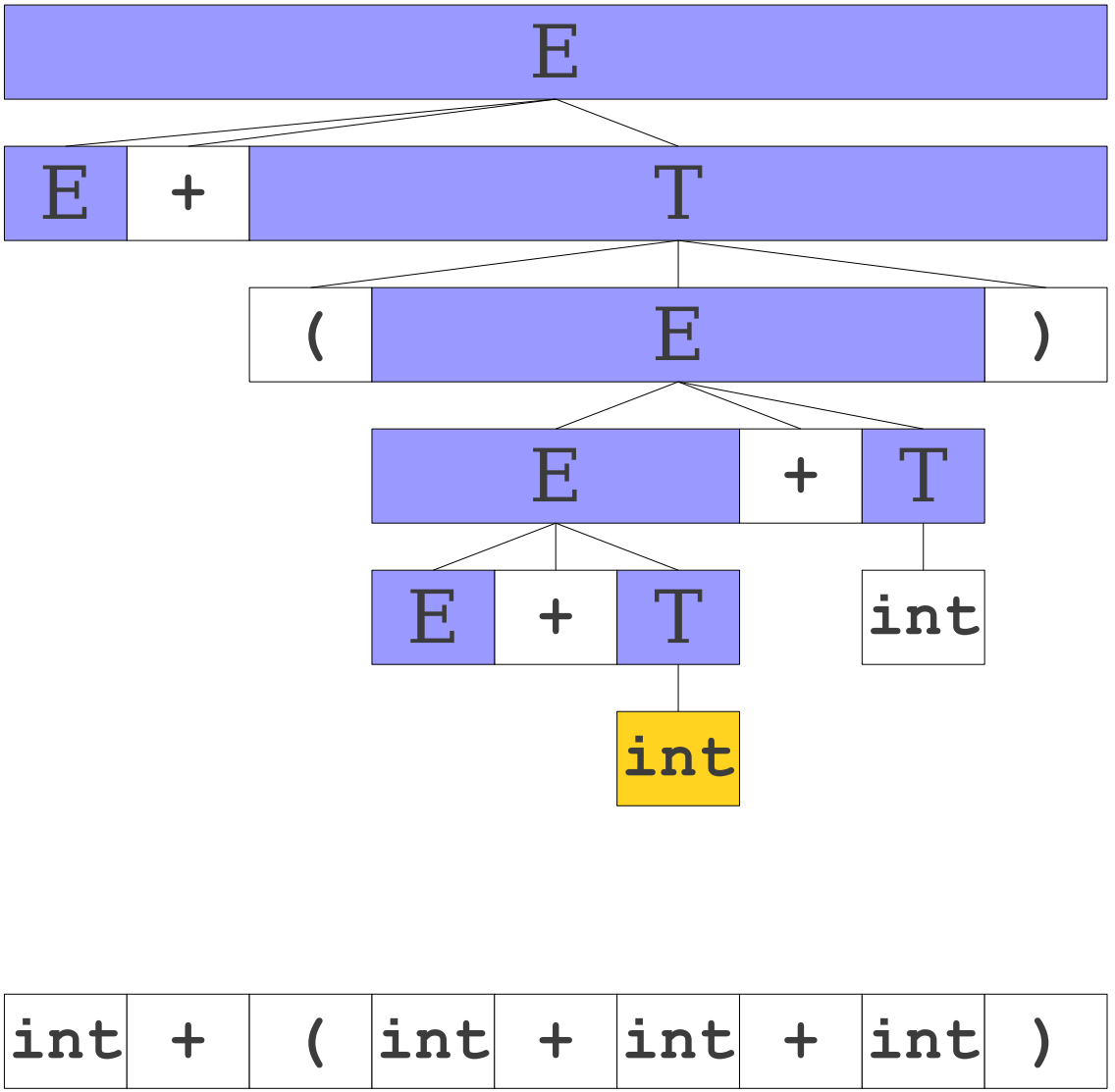
A Third View of a Bottom-Up Parse

⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E

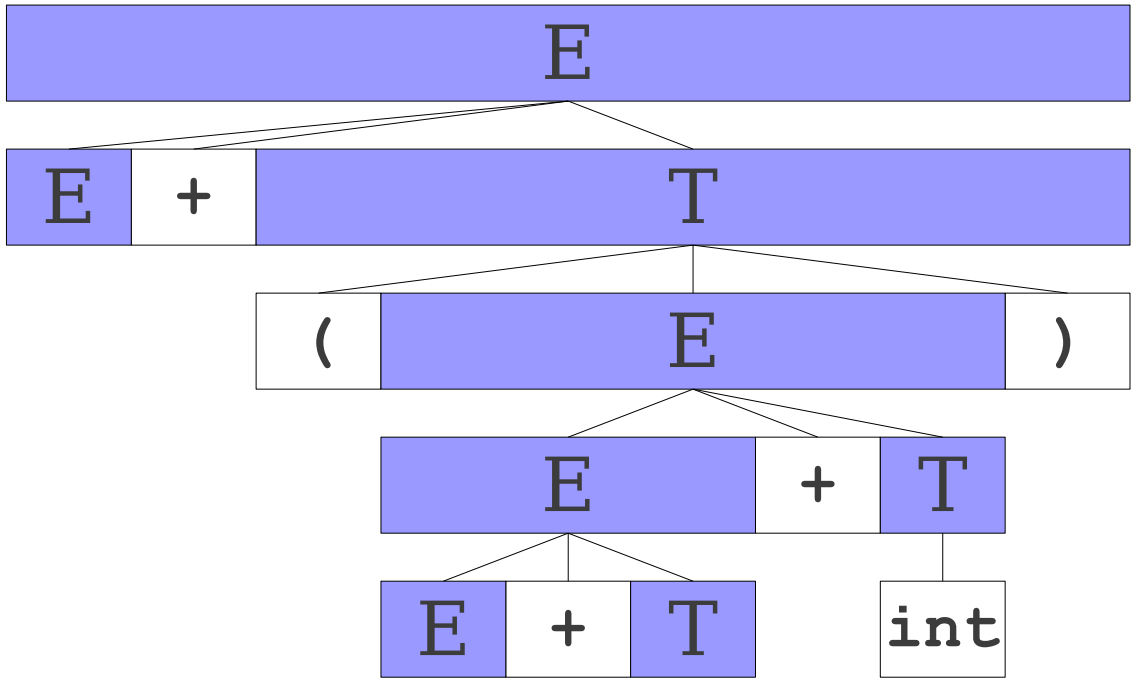


A Third View of a Bottom-Up Parse

⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



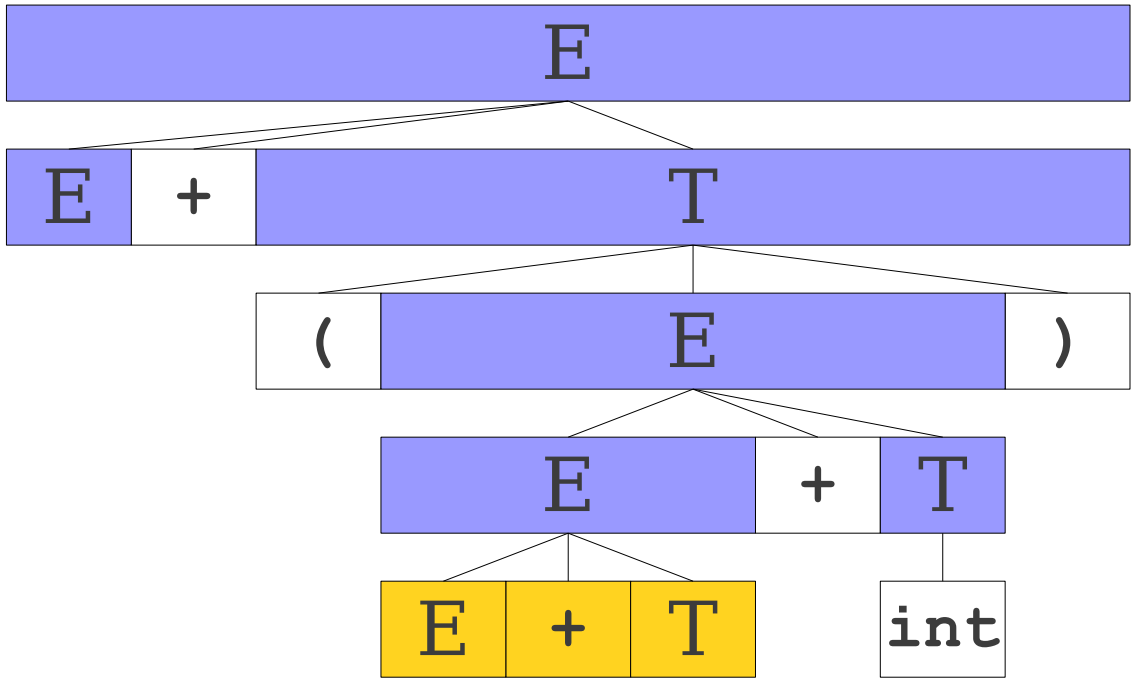
A Third View of a Bottom-Up Parse



⇒ **E + (E + T + int)**
⇒ **E + (E + int)**
⇒ **E + (E + T)**
⇒ **E + (E)**
⇒ **E + T**
⇒ **E**

int	+	(	int	+	int	+	int	)

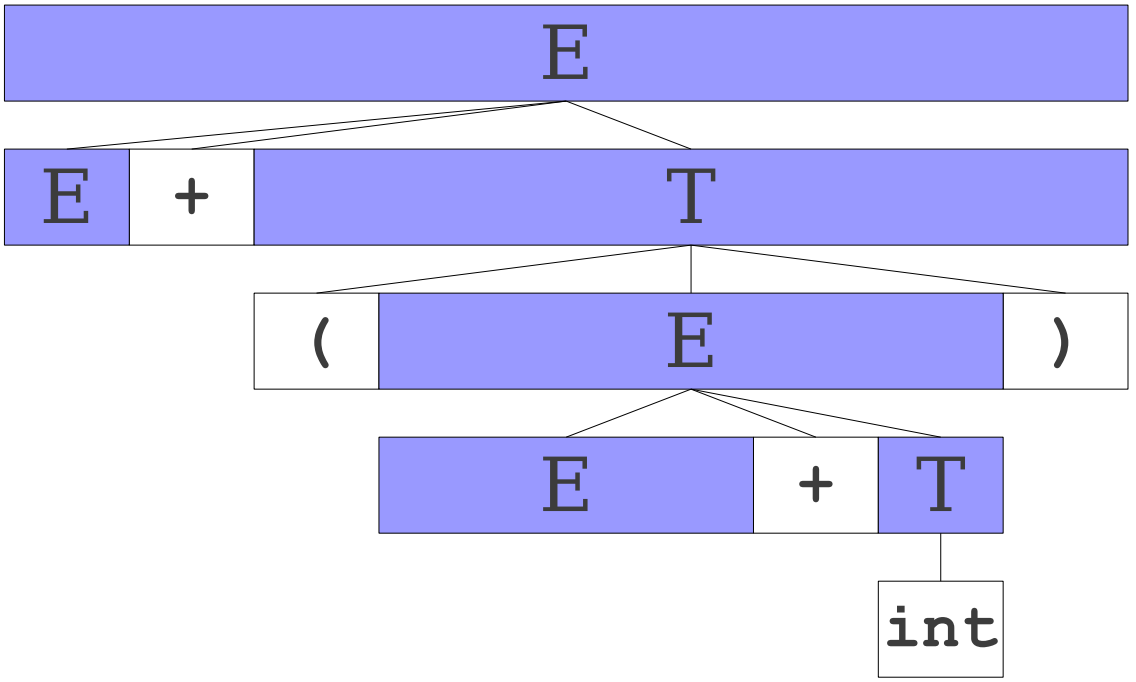
A Third View of a Bottom-Up Parse



⇒ E + (**E + T** + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E

int	+	(	int	+	int	+	int	)

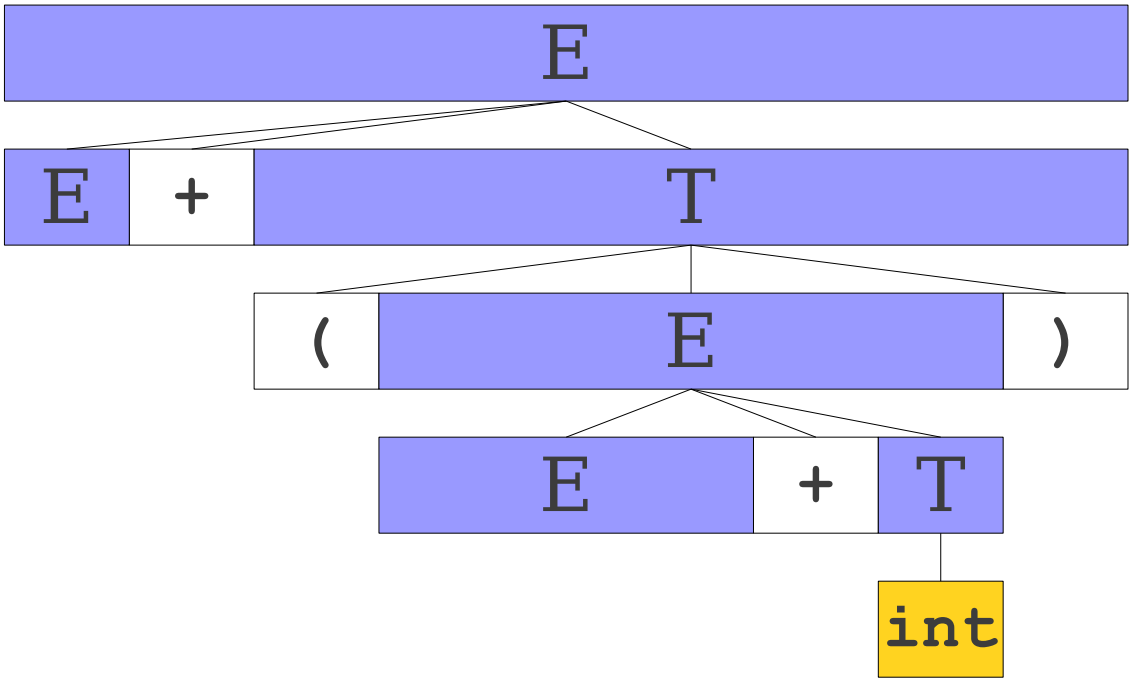
A Third View of a Bottom-Up Parse



⇒ **E + (E + int)**
⇒ **E + (E + T)**
⇒ **E + (E)**
⇒ **E + T**
⇒ **E**

int	+	(	int	+	int	+	int	)

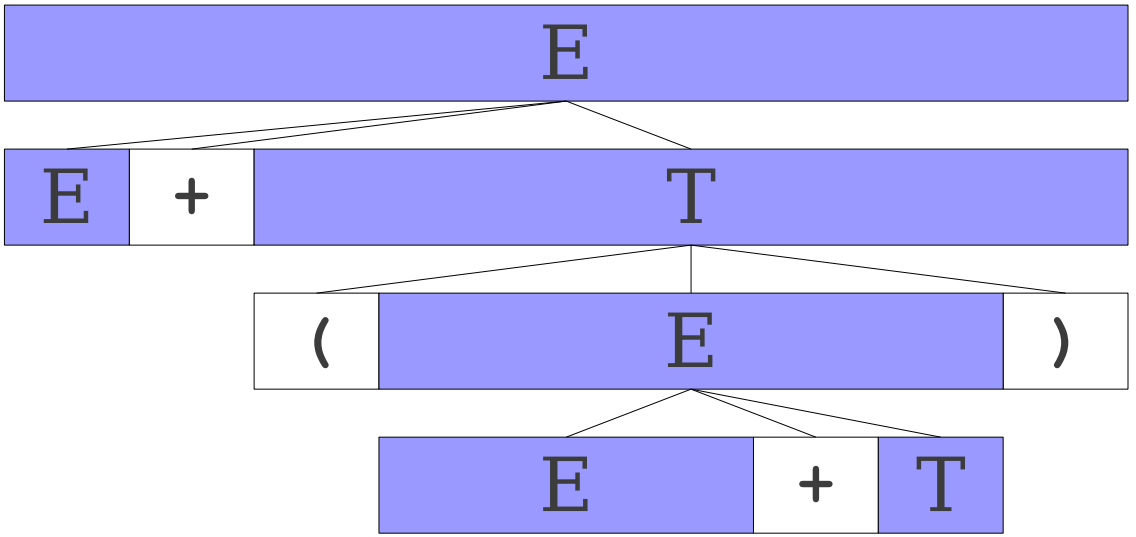
A Third View of a Bottom-Up Parse



⇒ **E + (E + int)**
⇒ **E + (E + T)**
⇒ **E + (E)**
⇒ **E + T**
⇒ **E**

int	+	(	int	+	int	+	int	)

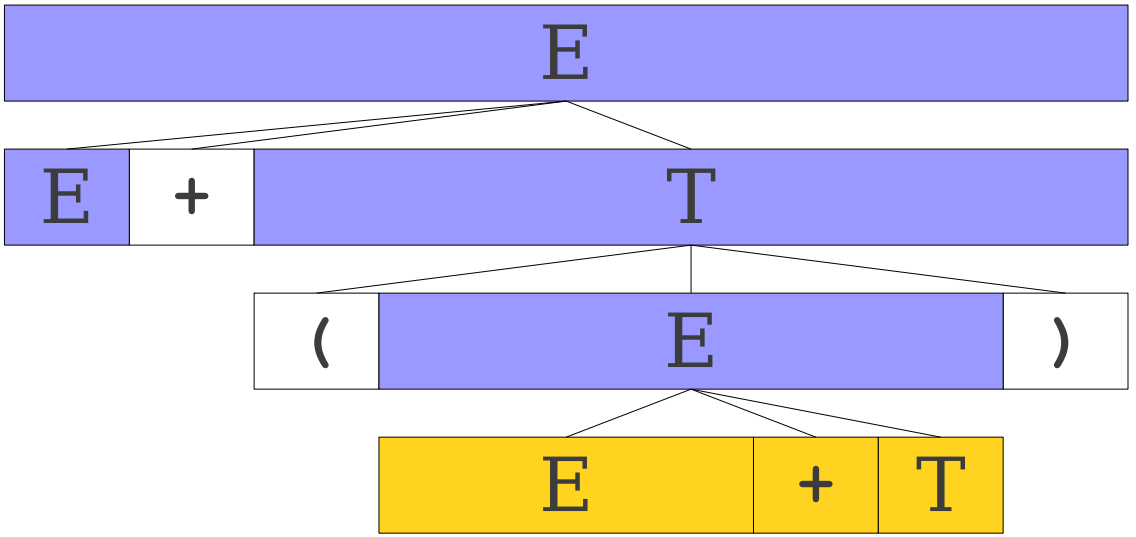
A Third View of a Bottom-Up Parse



⇒ **E + (E + T)**
⇒ **E + (E)**
⇒ **E + T**
⇒ **E**

int	+	(	int	+	int	+	int	)

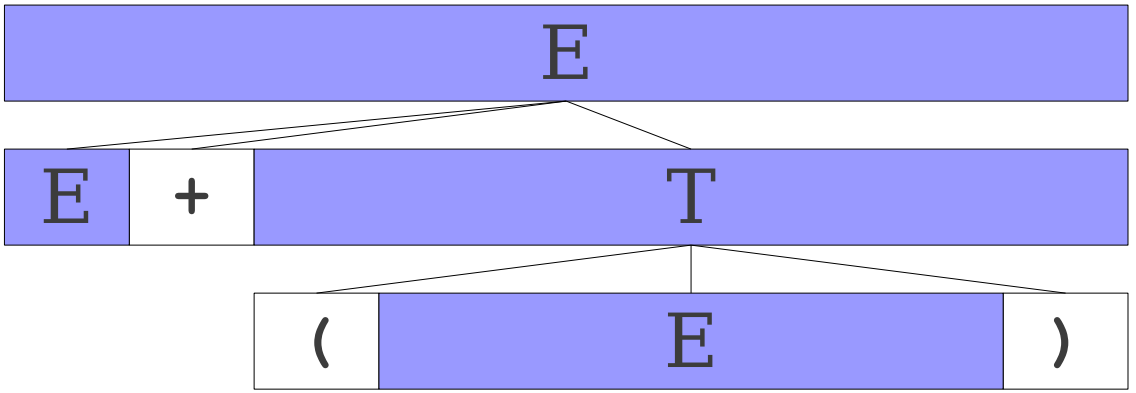
A Third View of a Bottom-Up Parse



⇒ **E + (E + T)**
⇒ **E + (E)**
⇒ **E + T**
⇒ **E**

int	+	(	int	+	int	+	int	)

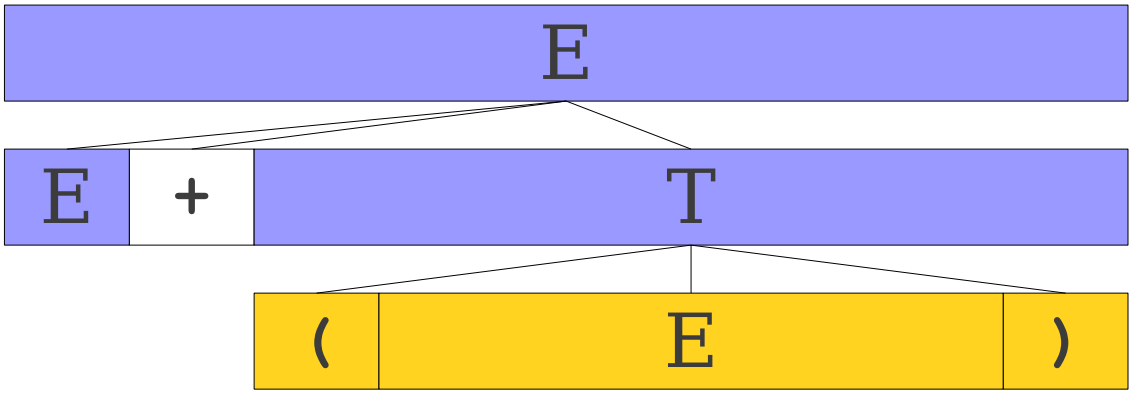
A Third View of a Bottom-Up Parse



⇒ **E + (E)**
⇒ **E + T**
⇒ **E**

int	+	(	int	+	int	+	int	)

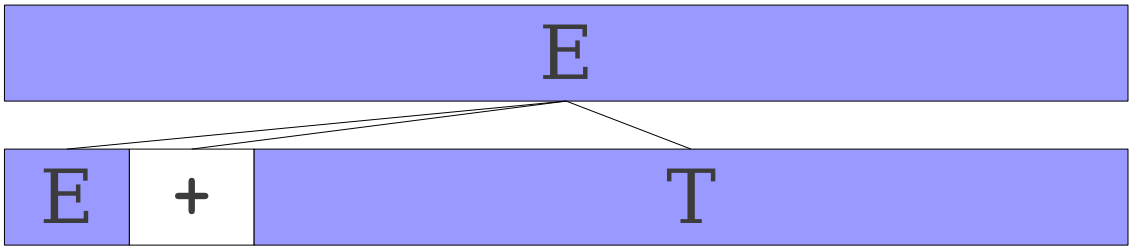
A Third View of a Bottom-Up Parse



⇒ **E + (E)**
⇒ **E + T**
⇒ **E**

int	+	(	int	+	int	+	int	)

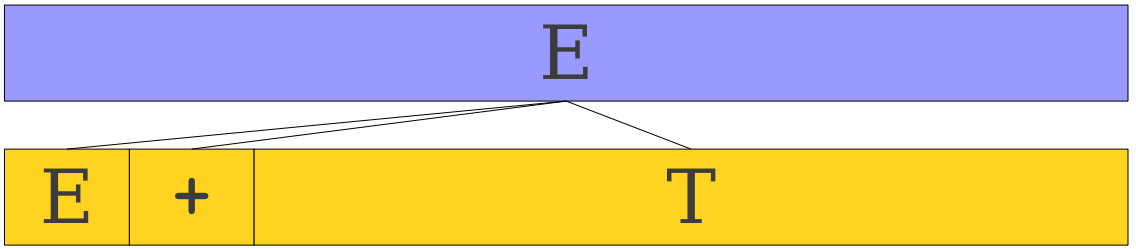
A Third View of a Bottom-Up Parse



⇒ **E + T**
⇒ **E**

int	+	(	int	+	int	+	int	)

A Third View of a Bottom-Up Parse



⇒ **E + T**

⇒ **E**

`int`	`+`	`(`	`int`	`+`	`int`	`+`	`int`	`)`

A Third View of a Bottom-Up Parse

⇒ **E**

`int`	`+`	`(`	`int`	`+`	`int`	`+`	`int`	`)`

Handles

- The **handle** of a parse tree T is the leftmost complete cluster of leaf nodes.
- A left-to-right, bottom-up parse works by iteratively searching for a handle, then reducing the handle.

Summarizing Our Intuition

- Our first intuition (reconstructing the parse tree bottom-up) motivates how the parsing should work.
- Our second intuition (rightmost derivation in reverse) describes the order in which we should build the parse tree.
- Our third intuition (handle pruning) is the basis for the bottom-up parsing algorithms we will explore.

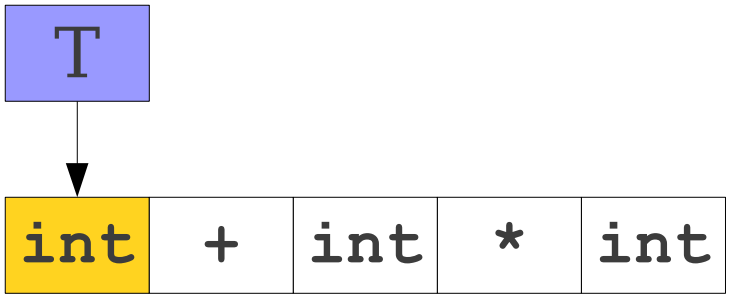
A Detail about Handles

E → **F**
E → **E** `+` **F**
F → **F** `*` **T**
F → **T**
T → `int`
T → `(` **E** `)`

`int`	`+`	`int`	`*`	`int`

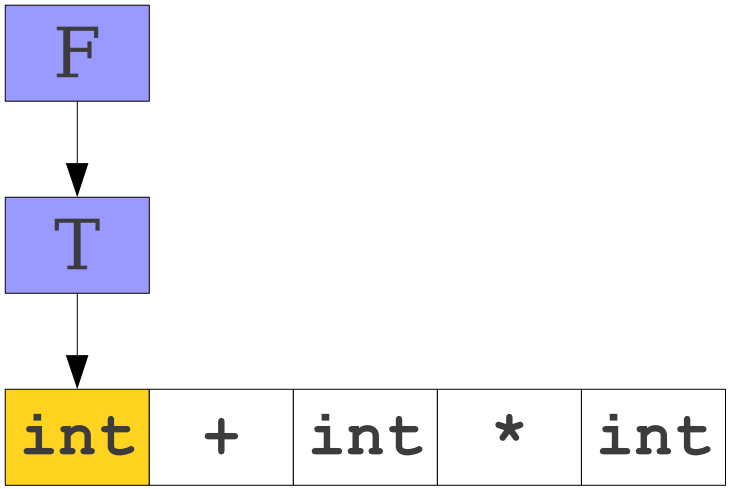
A Detail about Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



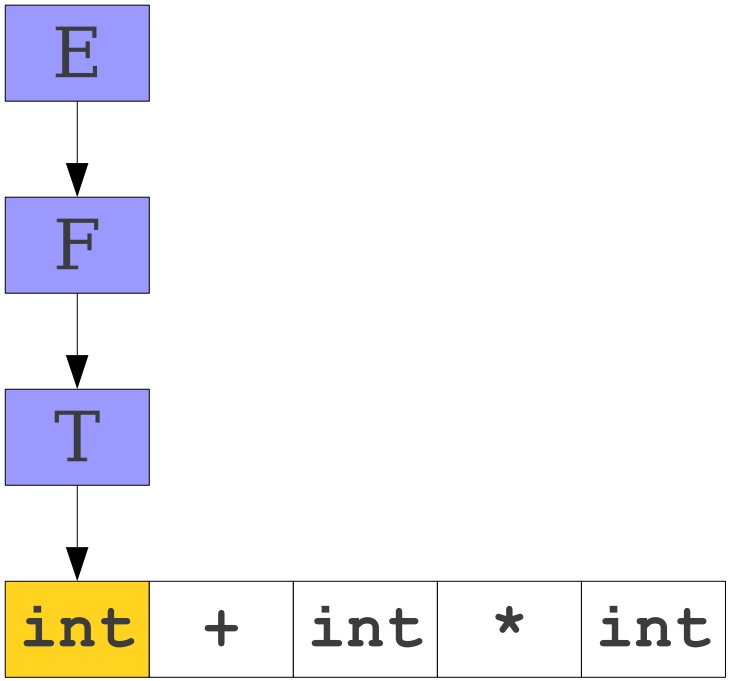
A Detail about Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



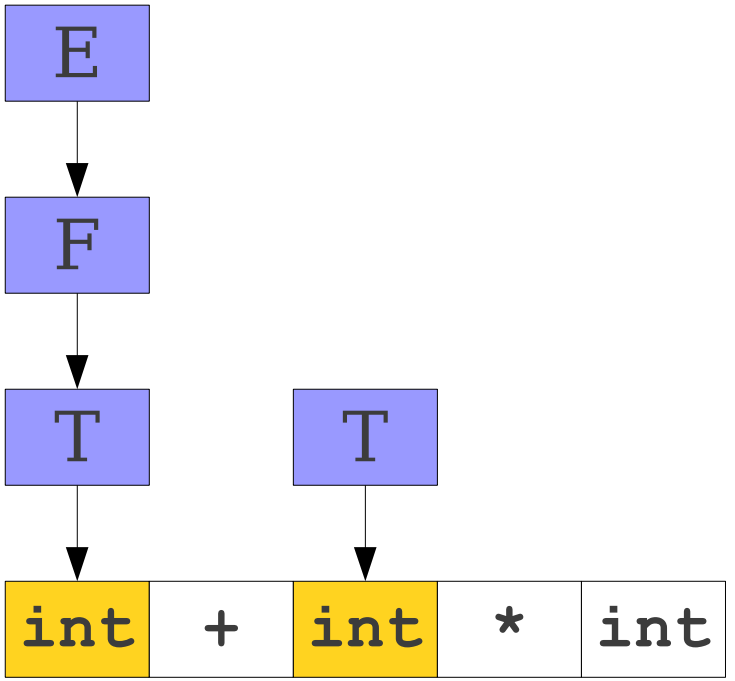
A Detail about Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



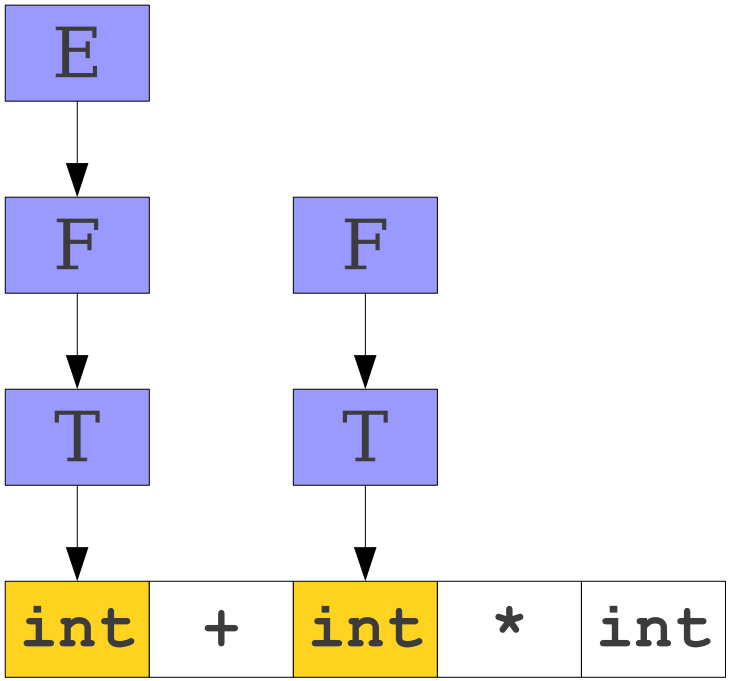
A Detail about Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



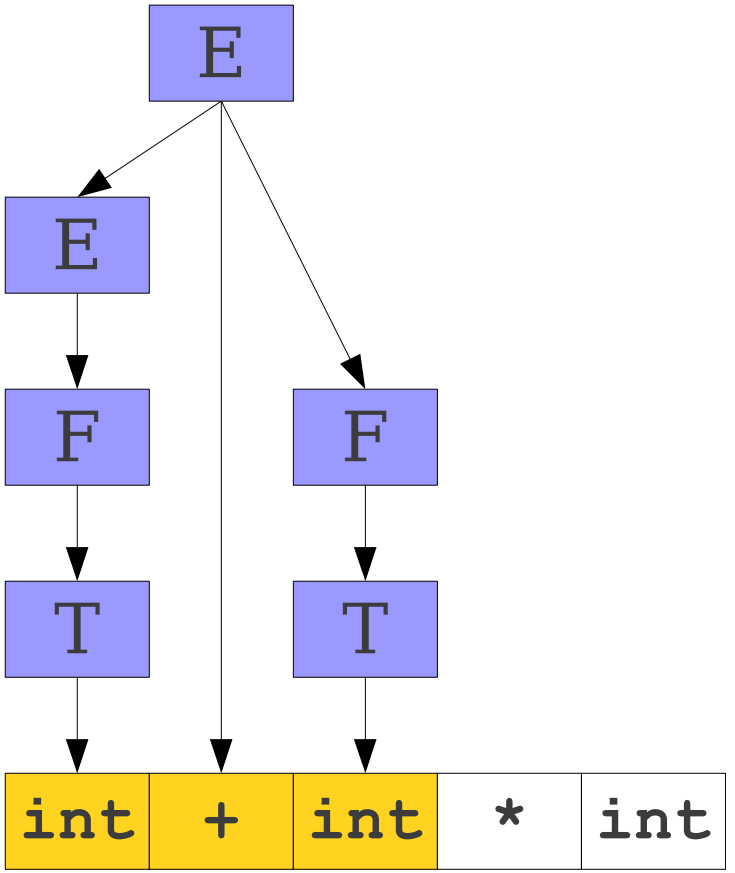
A Detail about Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



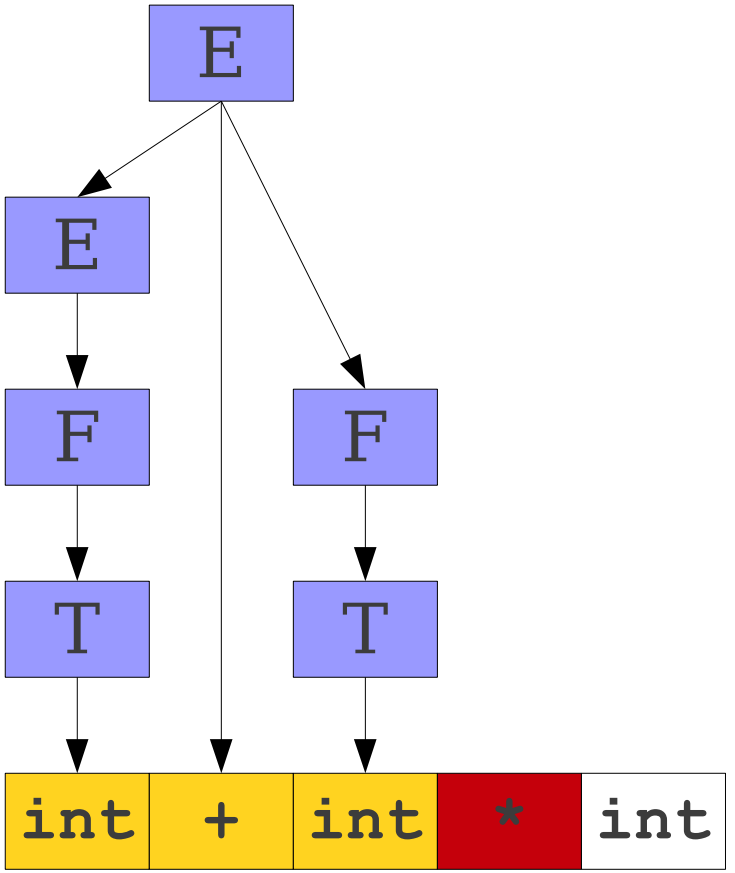
A Detail about Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



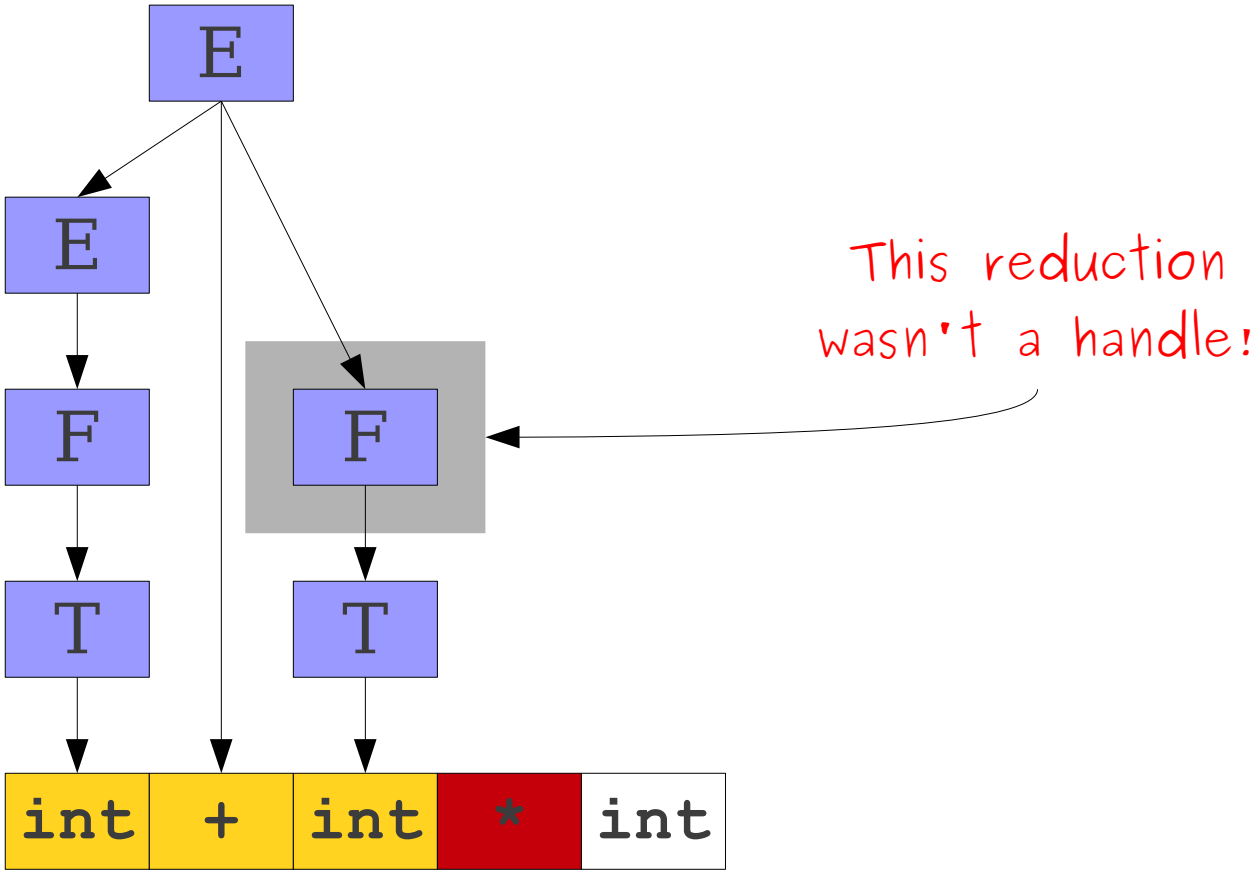
A Detail about Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



A Detail about Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



The leftmost reduction
isn't always the handle.

Finding Handles

- Where do we look for handles?
  - Where in the string might the handle be?
- How do we search for possible handles?
  - Once we know where to search, how do we identify candidate handles?
- How do we recognize handles?
  - Once we've found a candidate handle, how do we check that it really is the handle?

Question One:

Where are handles?

Where are Handles?

- Recall: A left-to-right, bottom-up parse traces a rightmost derivation in reverse.
- Each time we do a reduction, we are reversing a production applied to the *rightmost* nonterminal symbol.
- Suppose that our current sentential form is $\alpha\gamma\omega$, where γ is the handle and $\mathbf{A} \rightarrow \gamma$ is a production rule.
- After reducing γ back to $\mathbf{A}$, we have the string $\alpha\mathbf{A}\omega$.
- Thus ω must consist purely of terminals, since otherwise the reduction we just did was not for the rightmost terminal.

Why This Matters

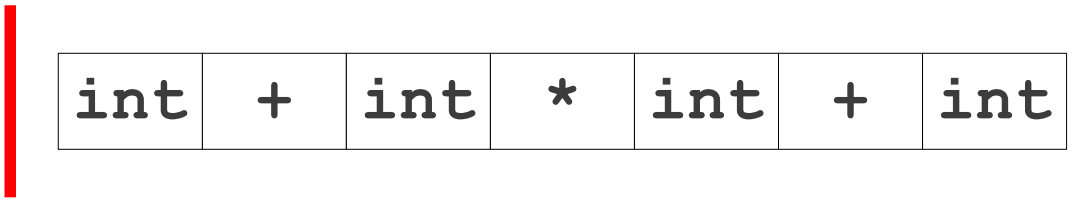
- Suppose we want to parse the string γ.
- We will break γ into two parts, α and ω, where
  - α consists of both terminals and nonterminals, and
  - ω consists purely of terminals.
- Our search for handles will concentrate purely in α.
- As necessary, we will start moving terminals from ω over into α.

Shift/Reduce Parsing

- The bottom-up parsers we will consider are called **shift/reduce** parsers.
  - Contrast with the LL(1) **predict/match** parser.
- Idea: Split the input into two parts:
  - Left substring is our work area; all handles must be here.
  - Right substring is input we have not yet processed; consists purely of terminals.
- At each point, decide whether to:
  - Move a terminal across the split (**shift**)
  - Reduce a handle (**reduce**)

A Sample Shift/Reduce Parse

E → **F**
E → **E + F**
F → **F * T**
F → **T**
T → **int**
T → **(E)**



A Sample Shift/Reduce Parse

E → **F**
E → **E + F**
F → **F * T**
F → **T**
T → **int**
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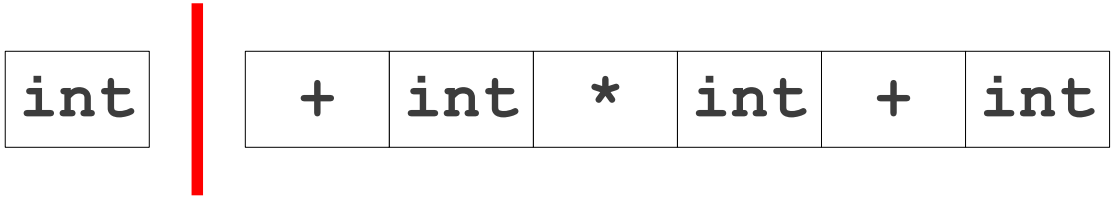
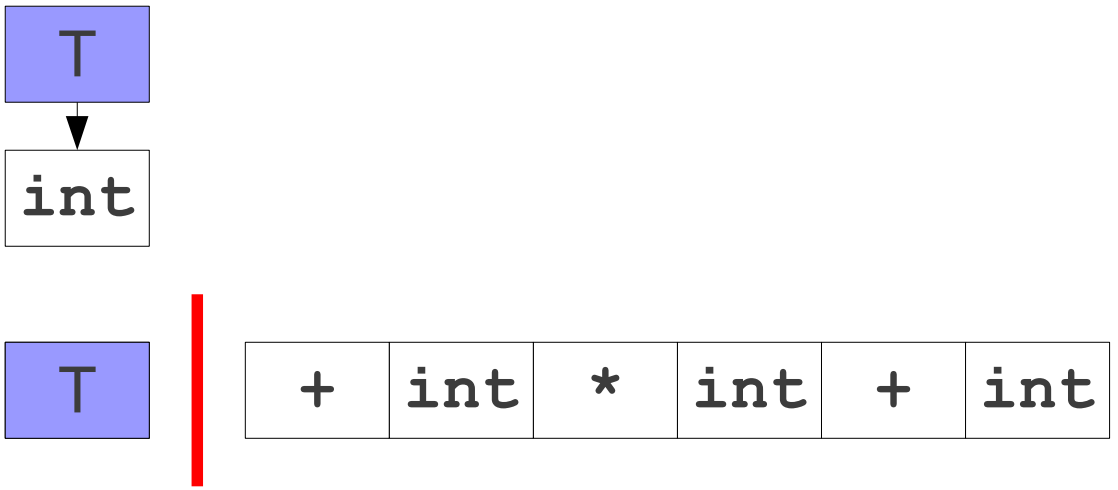


A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



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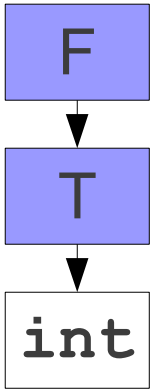

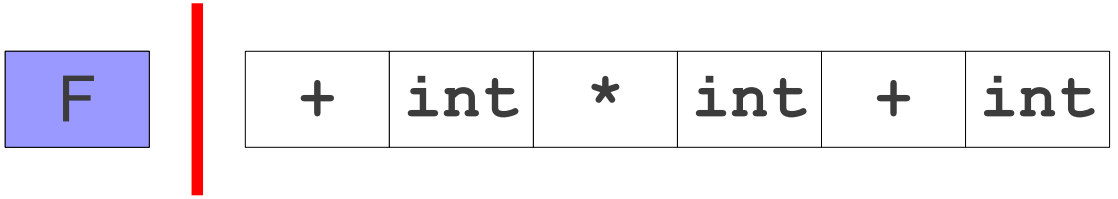

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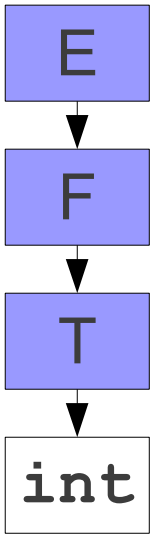

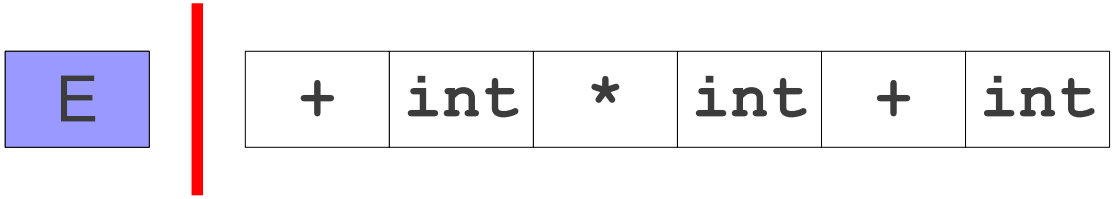

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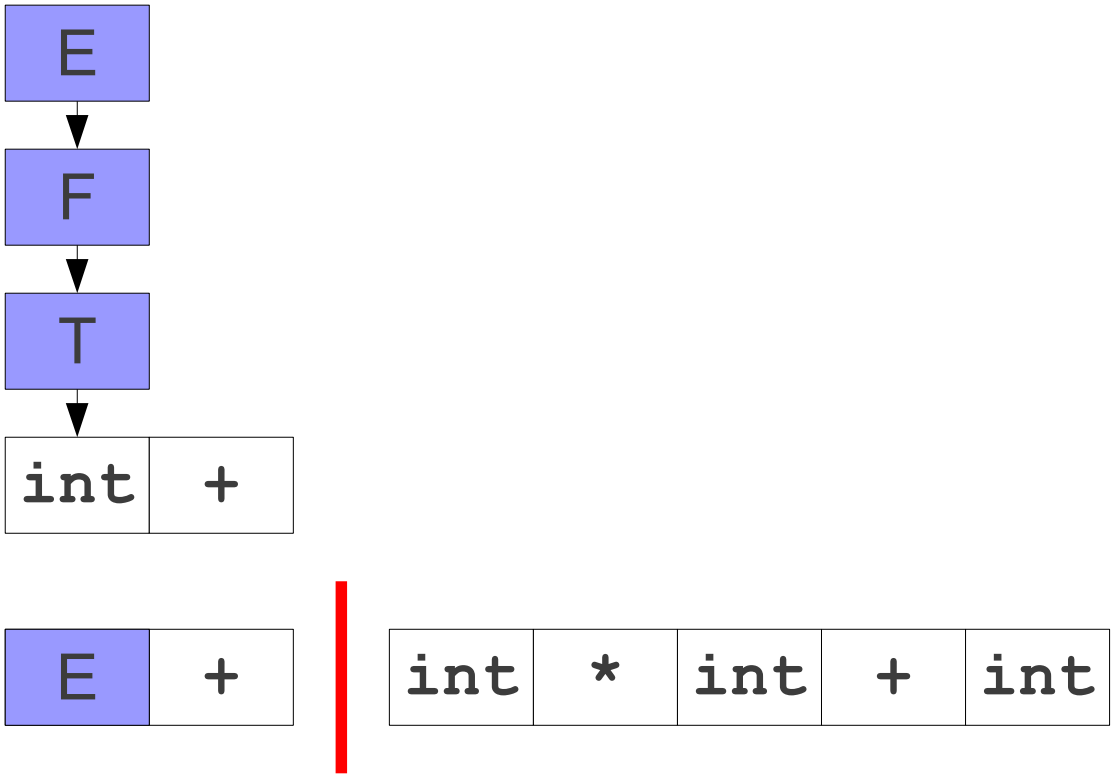

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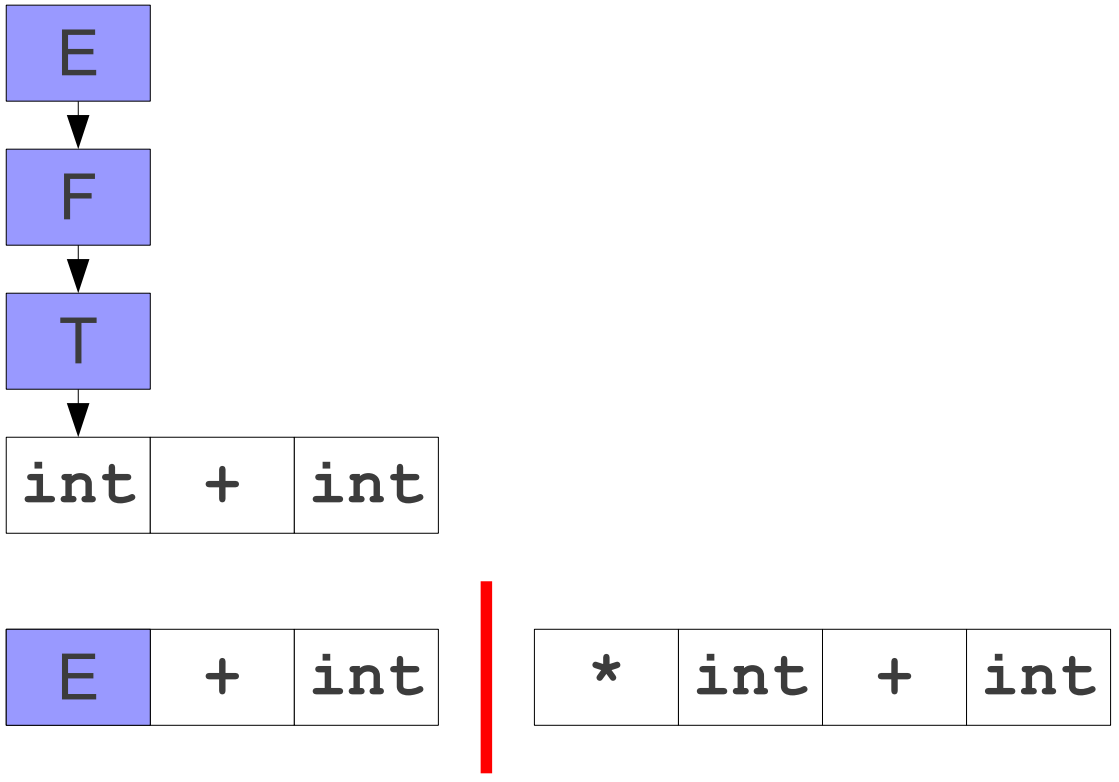

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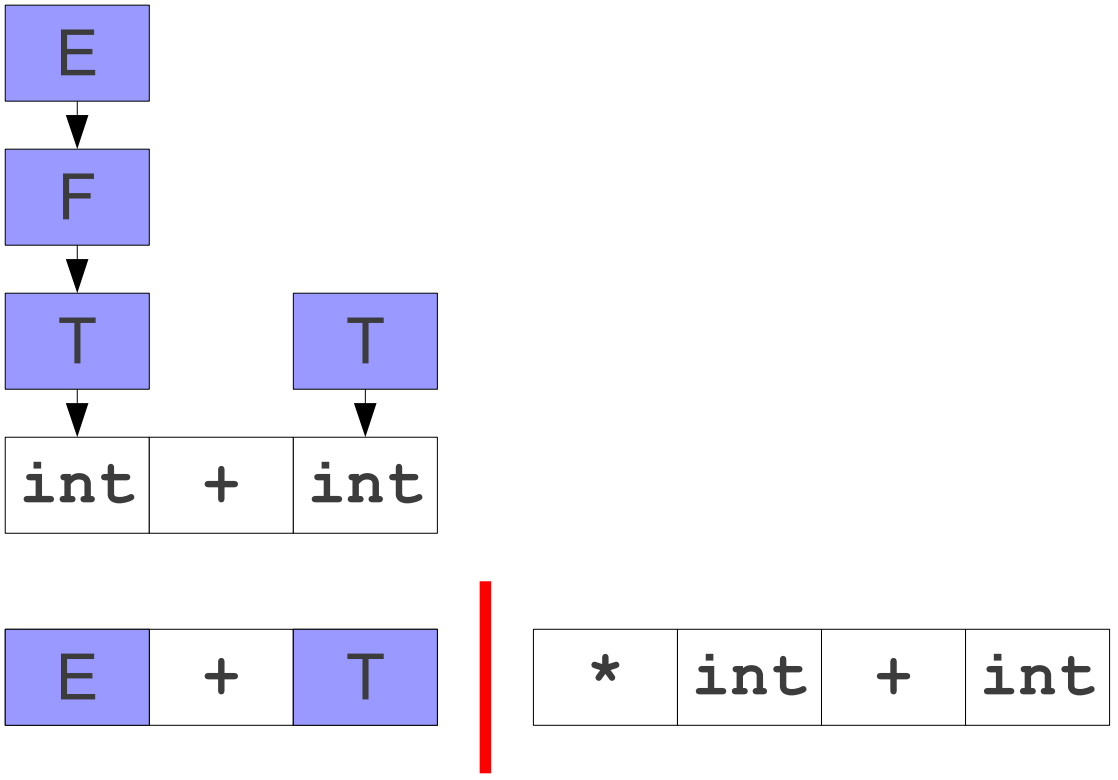

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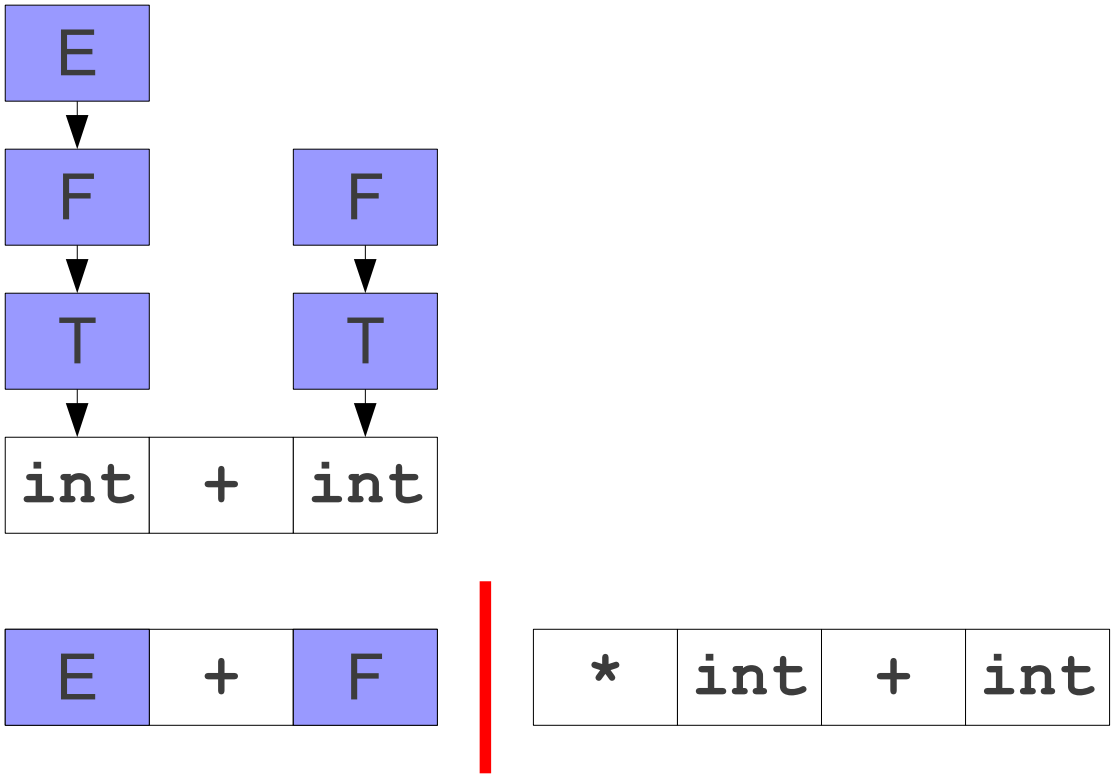

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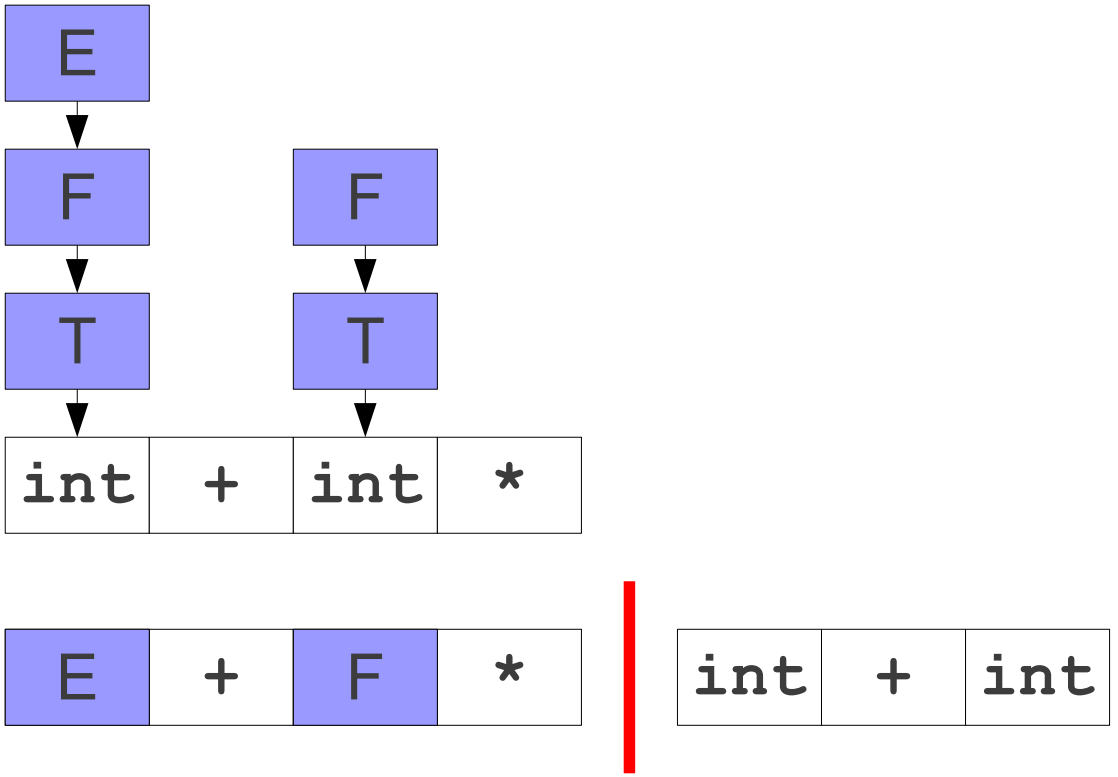

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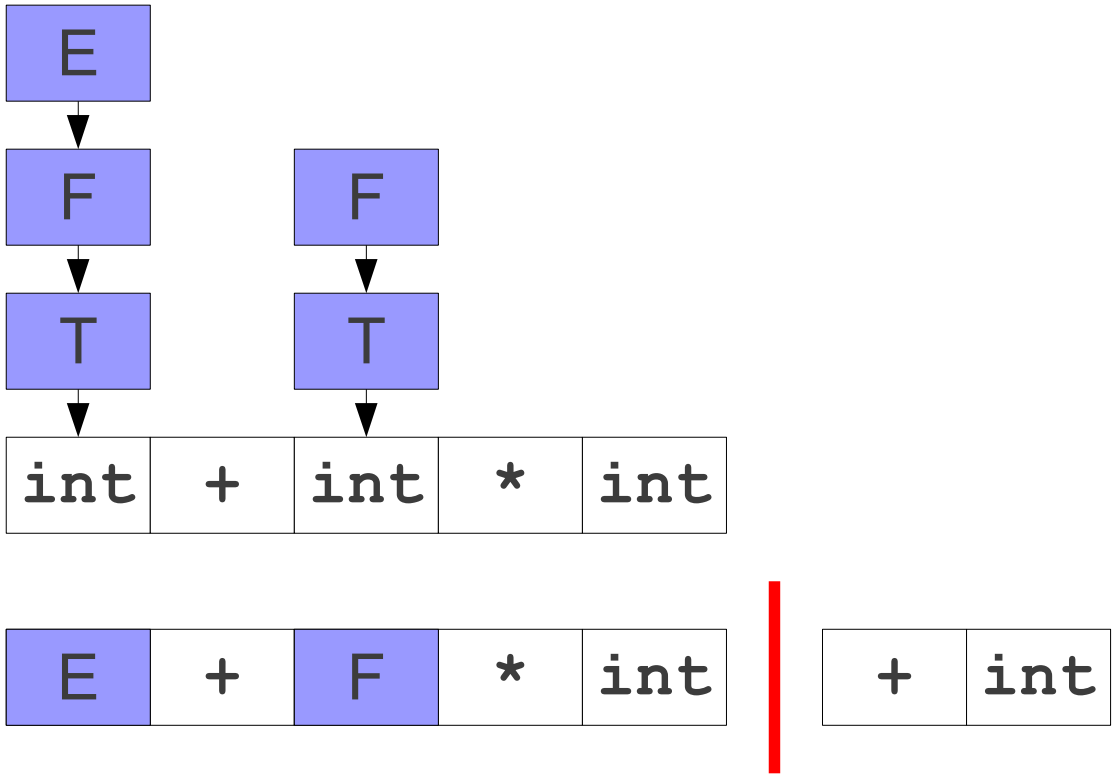

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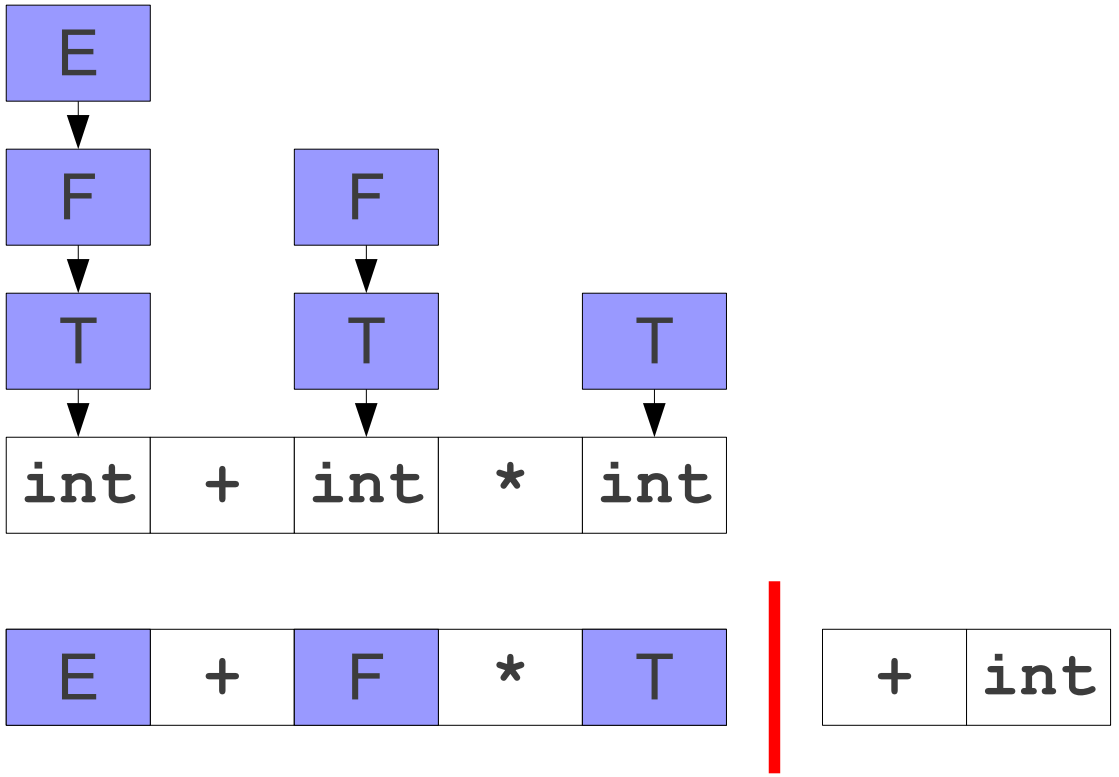

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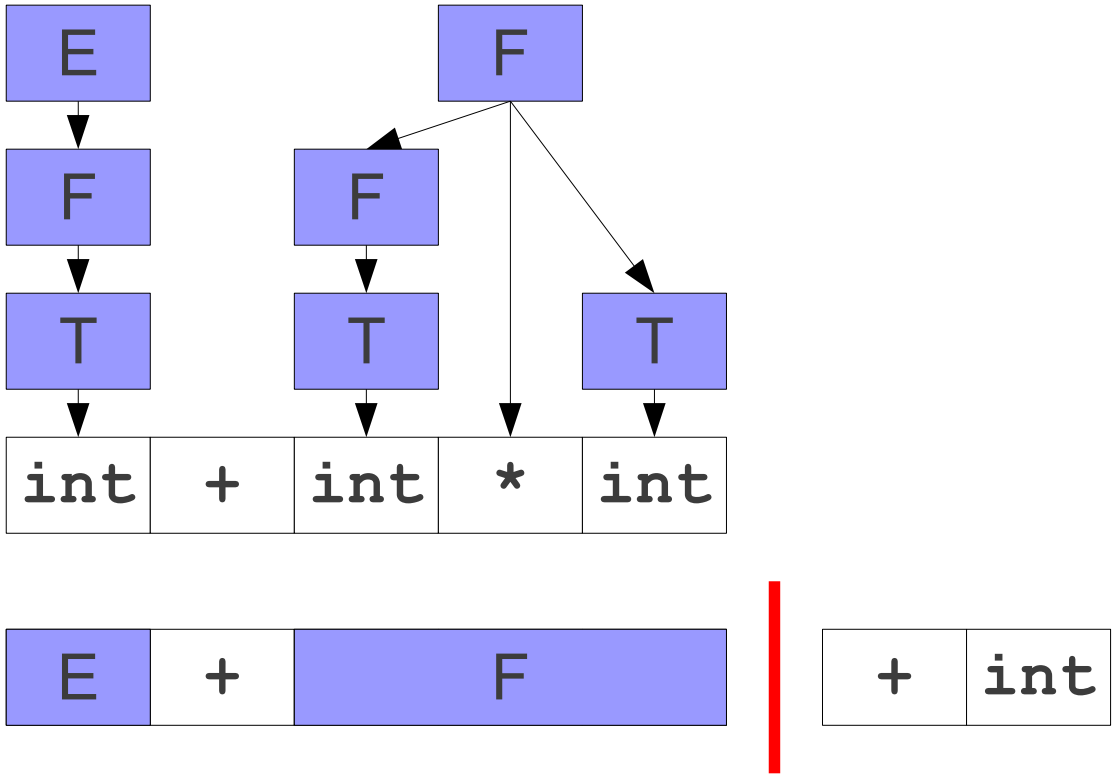

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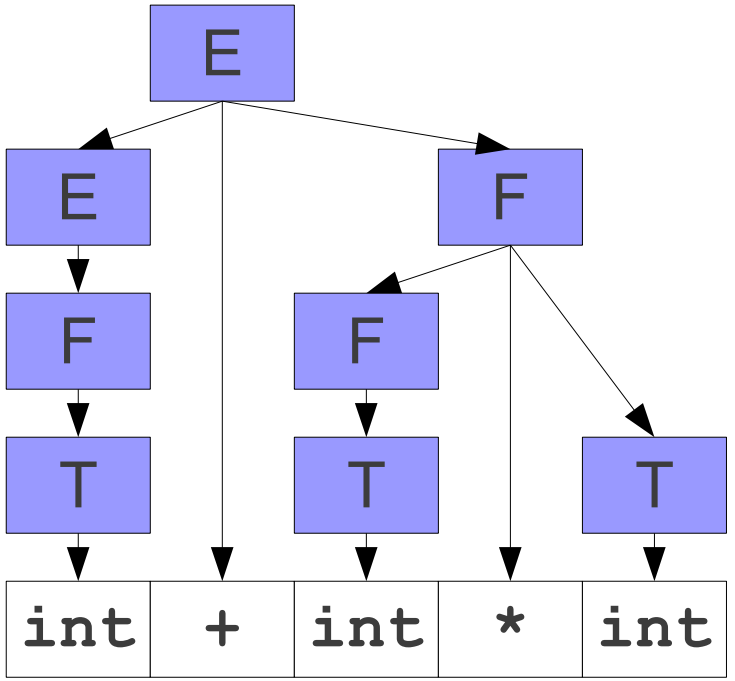

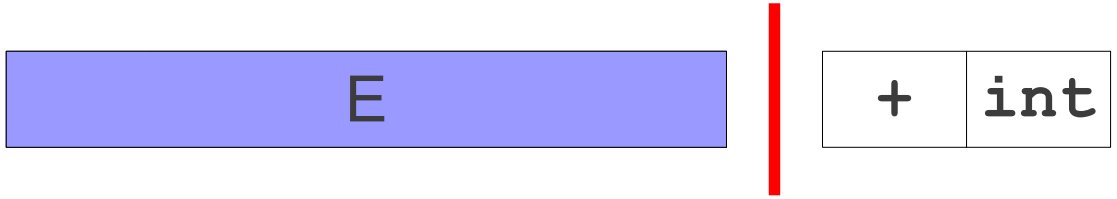

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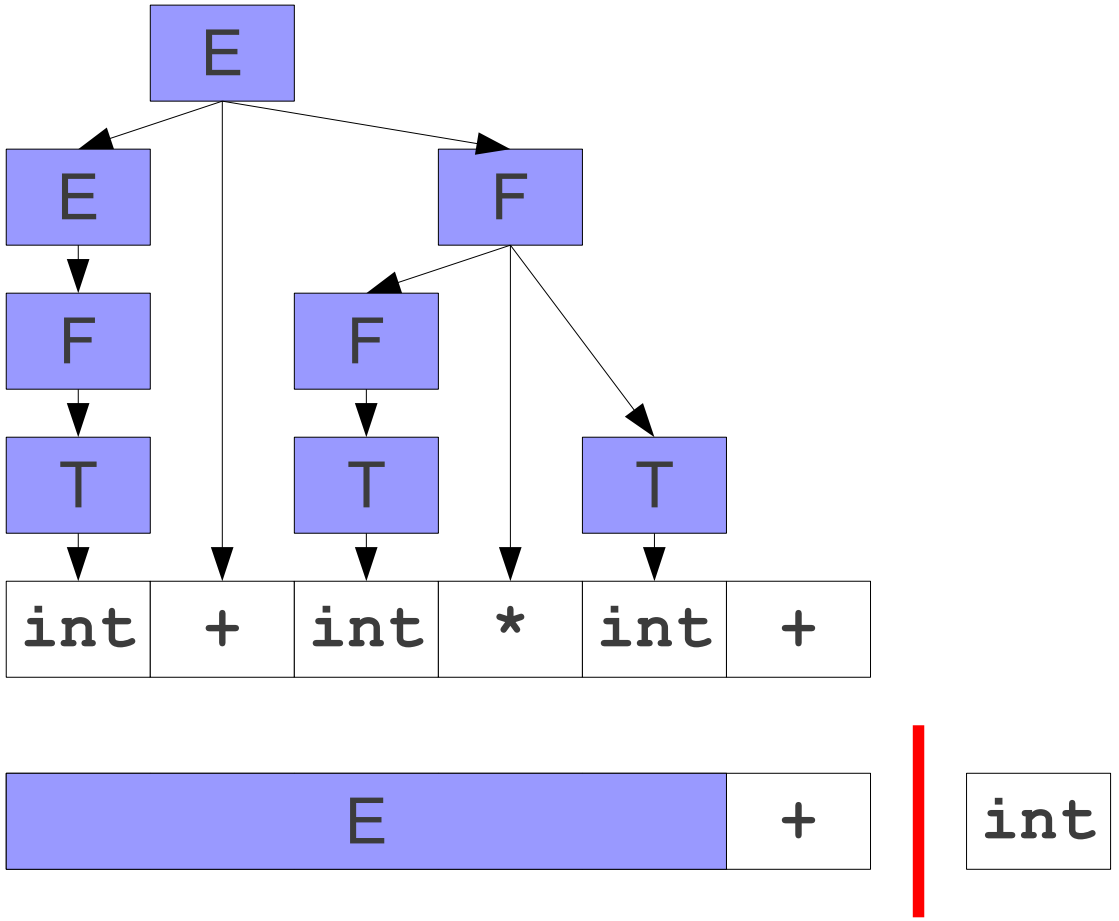

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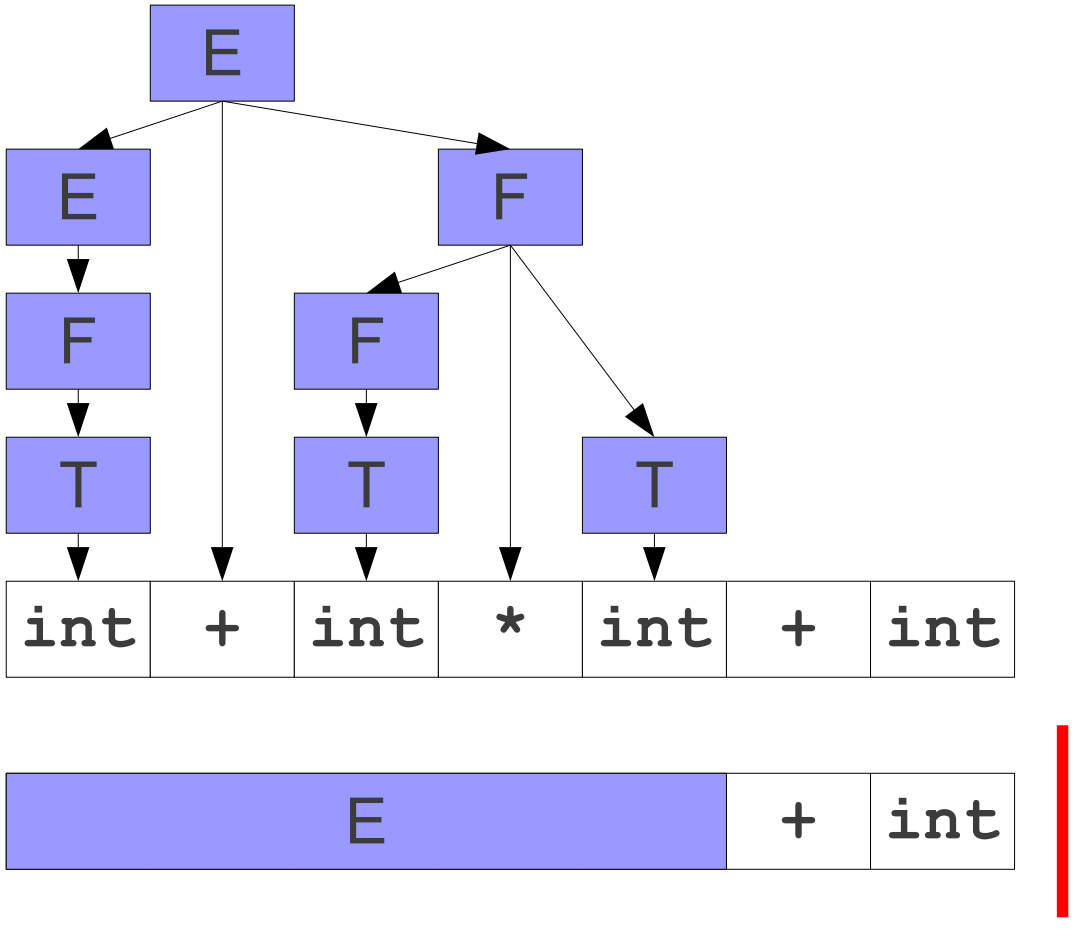

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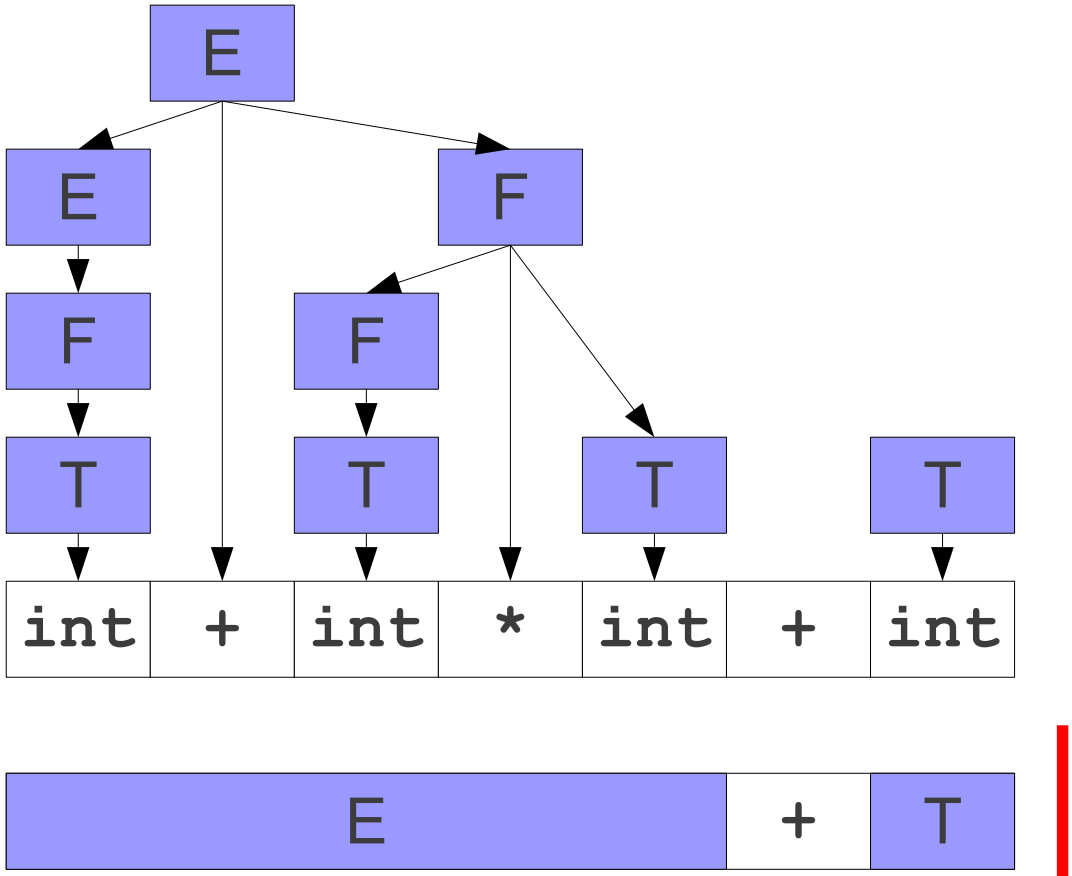

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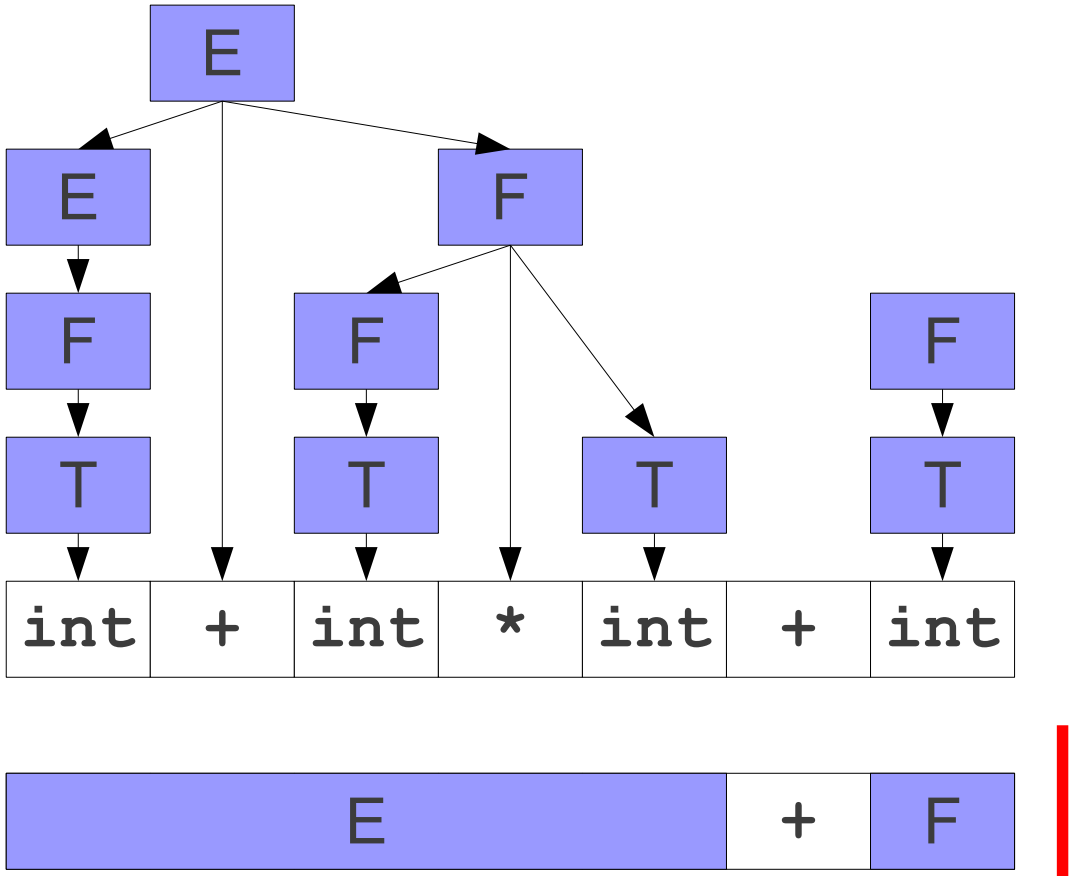

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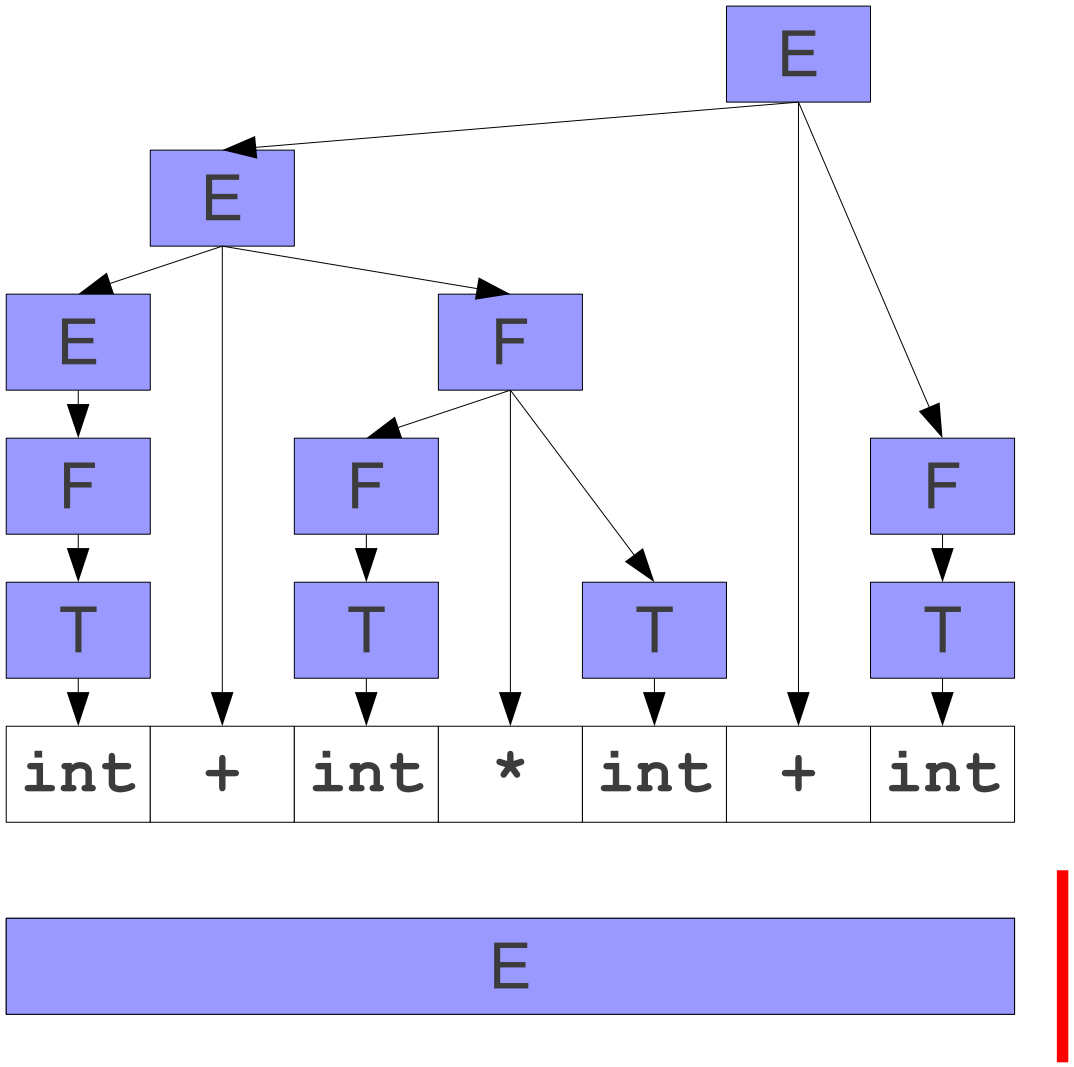

An Important Observation

- All of the reductions we applied were to the far right end of the left area.
- This is not a coincidence; all reductions are always applied all the way to the end of the left area.
- Inductive proof sketch:
  - After no reduces, the first reduction can be done at the right end of the left area.
  - After at least one reduce, the very right of the left area is a nonterminal. This nonterminal must be part of the next reduction, since we're tracing a rightmost derivation backwards.

An Important Corollary

- Since reductions are always at the right side of the left area, we never need to shift from the left to the right.
- No need to “uncover” something to do a reduction.
- Consequently, shift/reduce parsing means
  - **Shift**: Move a terminal from the right to the left area.
  - **Reduce**: Replace some number of symbols at the right side of the left area.

Simplifying our Terminology

- All activity in a shift/reduce parser is at the far right end of the left area.
- Idea: Represent the left area as a stack.
- Shift: Push the next terminal onto the stack.
- Reduce: Pop some number of symbols from the stack, then push the appropriate nonterminal.

Finding Handles

- Where do we look for handles?
  - **At the top of the stack.**
- How do we search for handles?
  - What algorithm do we use to try to discover a handle?
- How do we recognize handles?
  - Once we've found a possible handle, how do we confirm that it's correct?