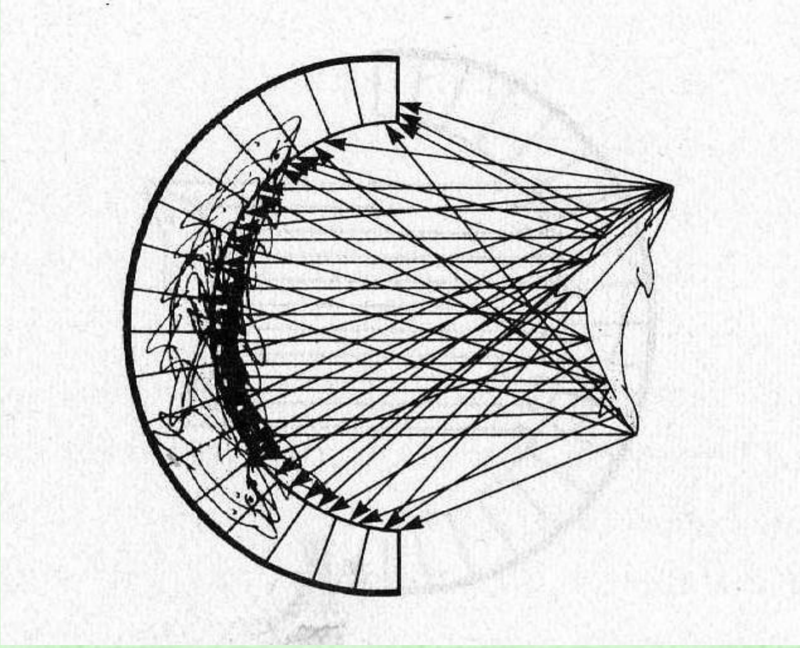
The Virtual World

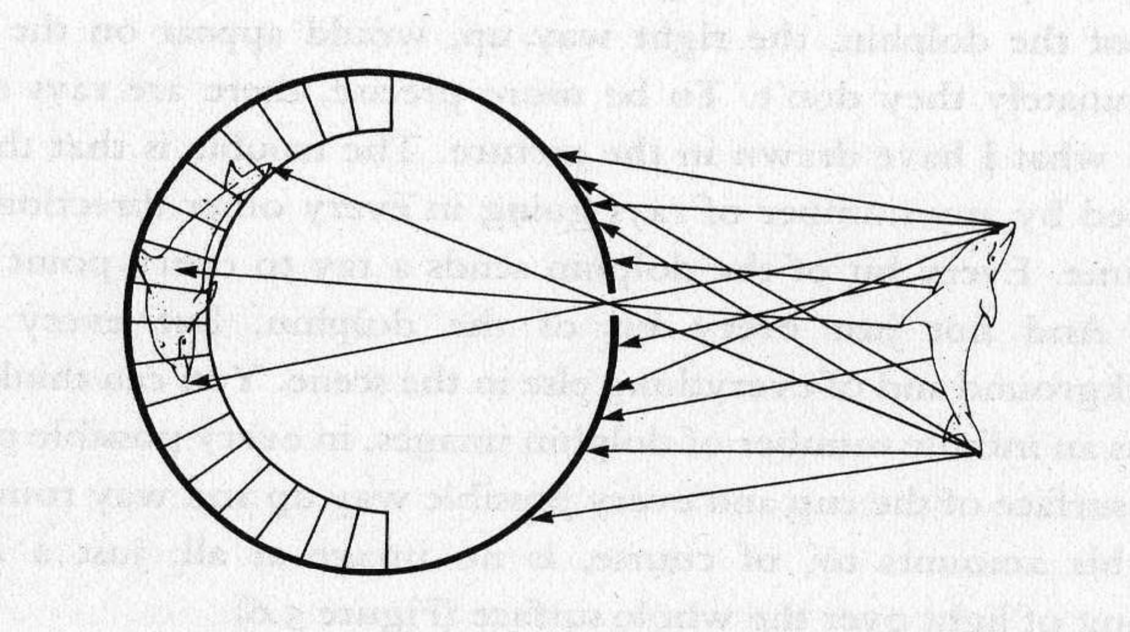
Building a Virtual World

- Goal: mimic human vision in a virtual world (with the computer)
  - Cheating for efficiency, using knowledge about light and our eye (e.g. from the last lecture)
- Create a virtual camera, place it somewhere, and point it at something
- Put film containing pixels with RGB values ranging from 0-255 into the camera
  - Taking a picture creates film data as the final image
- Place objects into the world, including a floor/ground, walls, ceiling/sky, etc.
  - Two step process: (1) make objects (geometric modeling), (2) place objects (transformations)
  - Making objects is itself a two step process: (1) build geometry (geometric modeling), (2) paint geometry (texture mapping)
- Put lights into the scene (so that it's not completely dark)
- Finally, snap the picture:
  - "Code" emits light from virtual light sources, bounces that light off of virtual geometry, and follows that light into the camera and onto the film
  - We will consider both scanline rendering and ray tracing for the taking this picture

Pupil

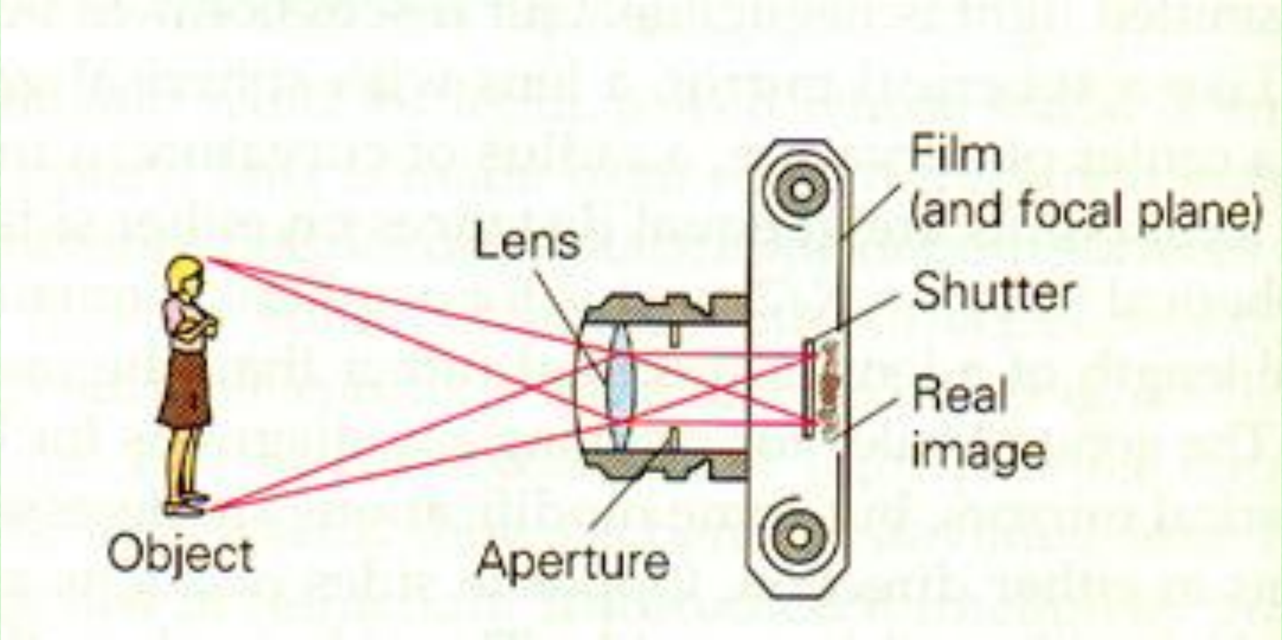
- Light emanates off of every point of an object outwards in every direction
  - That's why we can all look at the same spot on the same object and see it
  - Light is leaving from that point/spot on the object and entering each of our eyes
- Without a pupil, light from every part of an object would hit the same cone on our eye, blurring the image
- The (small) pupil restricts the entry of light so that each cone only receives light emanating from a small region on the object, giving interpretable spatial detail

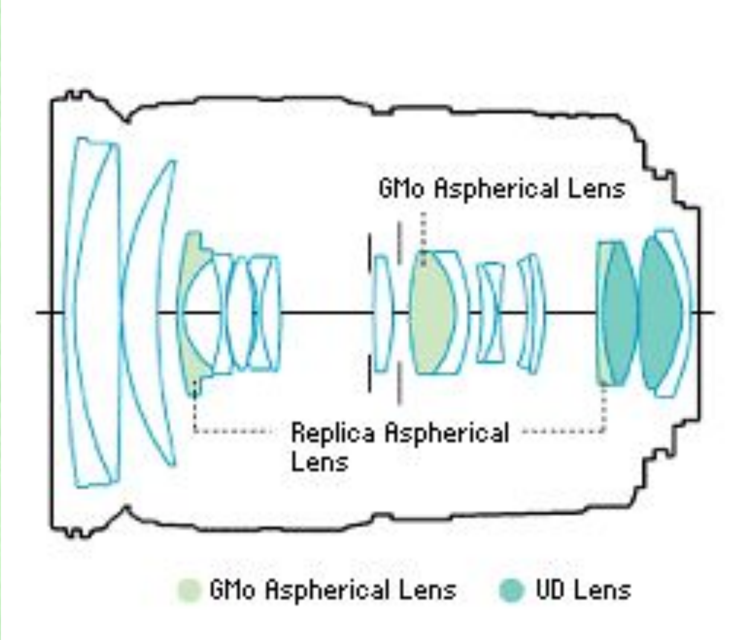




Aperture

- Cameras are similar to the eye, except with mechanical as opposed to biological components
- Instead of cones, the camera has mechanical pixels
- Instead of a pupil, the camera has a small (adjustable) aperture that light passes through
- The camera also typically has a hefty/complex lens system



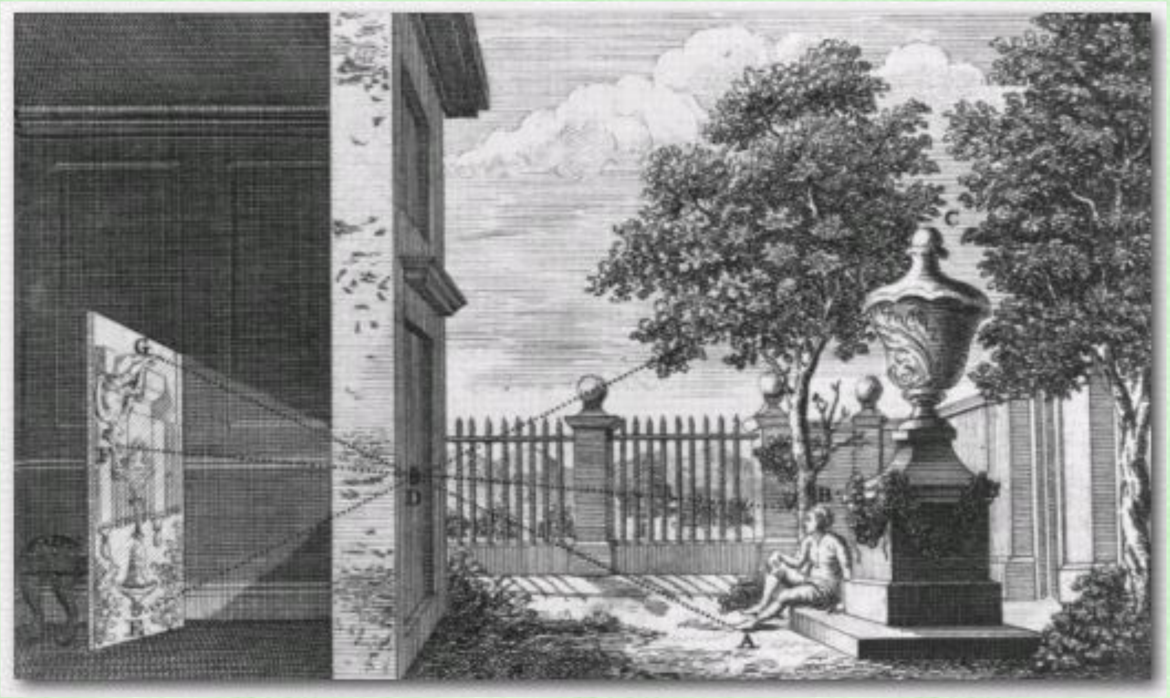


Aside: Lens Flare

- Many camera complexities are not often properly accounted for in virtual worlds
- Thus, some complex effects like depth of field, motion blur, chromatic aberration, lens flare, etc. have to be approximated/modeled in other ways (as we will discuss later)
- Particularly complex is lens flare, which is caused by light being reflected/scattered by lenses in the lens system
- This is caused in part by material inhomogeneities in the lens, and depends on the geometry of lens surfaces and characteristic planes, absorption/dispersion of lens elements, antireflective coatings, diffraction, etc.

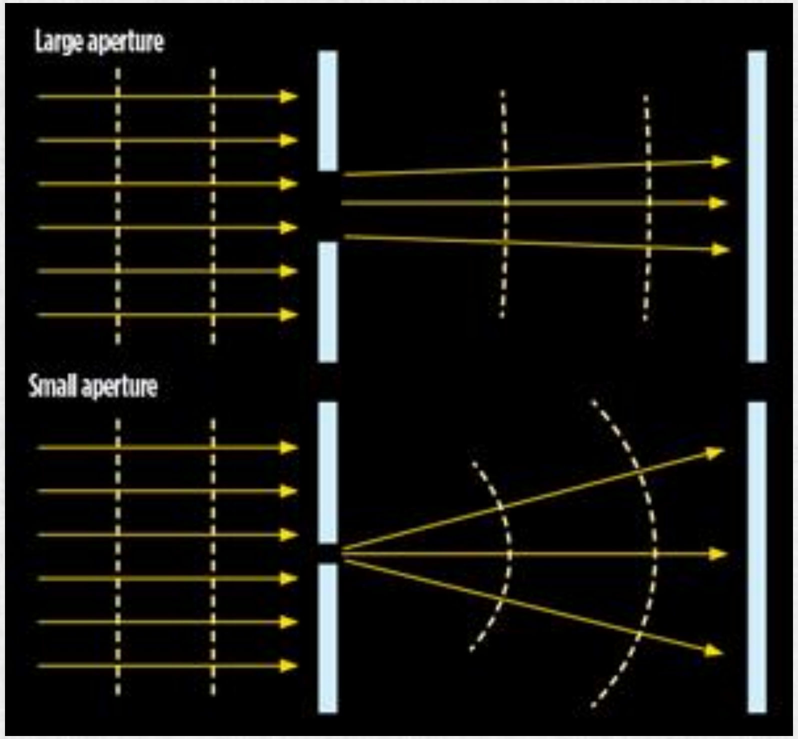
Pinhole Camera

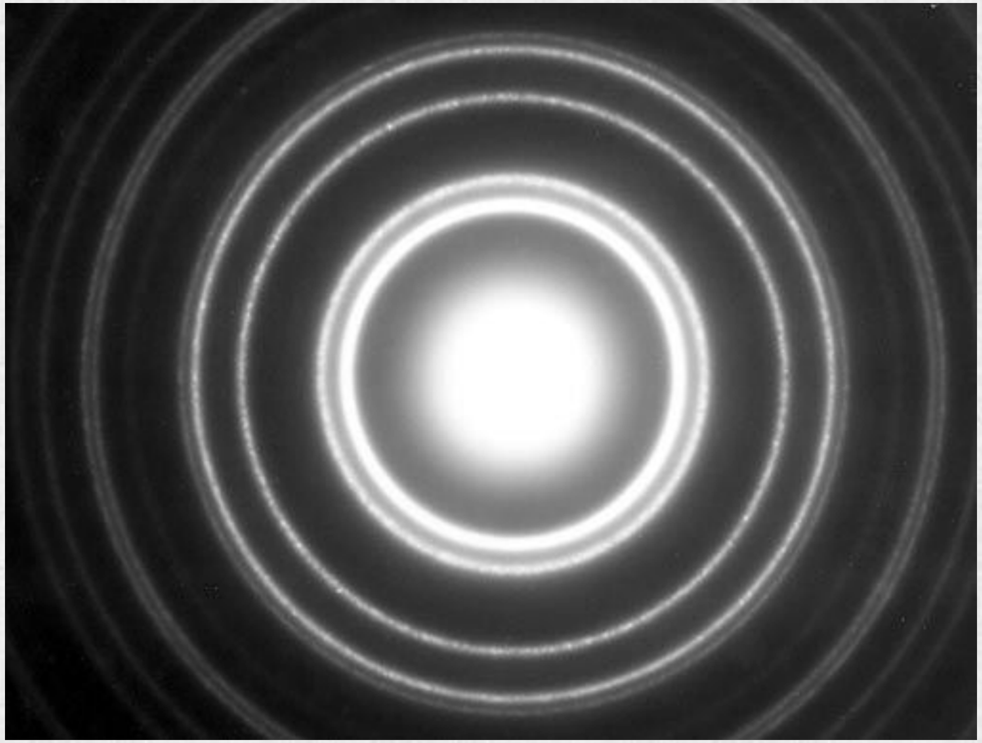
- The pupil/aperture has to have a finite size in order for light to get through
- If too small, not enough light enters and the image is too dark/noisy to interpret
  - In addition, light can diffract (instead of traveling in straight lines) distorting the image
- If too large, light from a large area of an object hits the same cone causing blurring
- Luckily, the virtual world can use a single point for the aperture (without worrying about dark or distorted images)



Aside: Diffraction

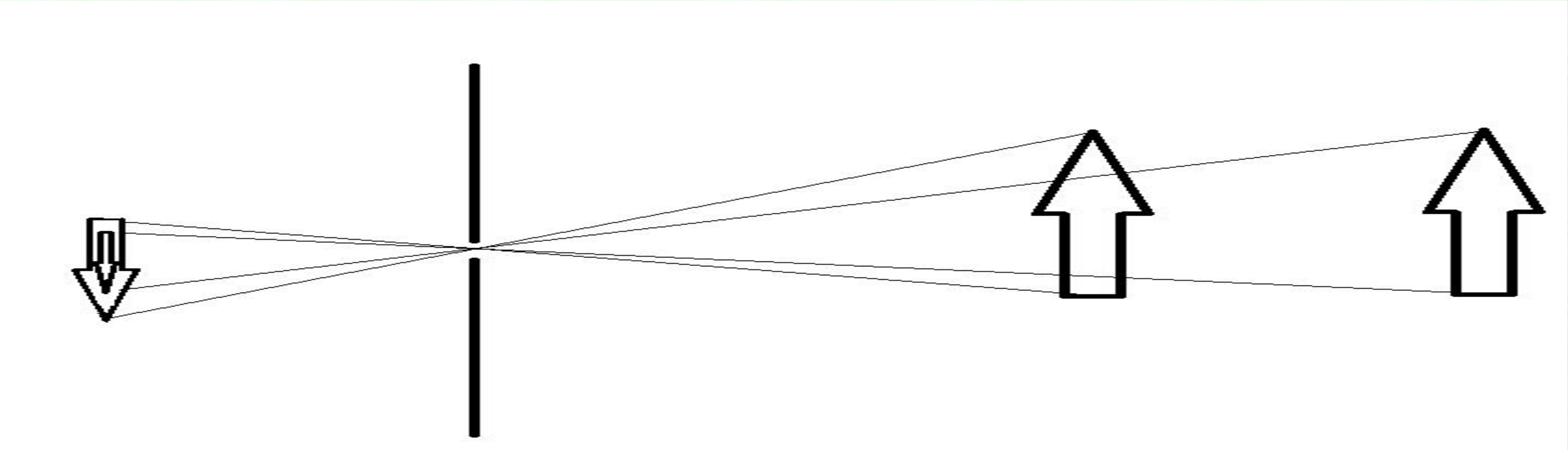
- Light spread out as it goes through small openings
- This happens when the camera aperture is too small (diffraction limited)
- Creates constructive/destructive interference of light waves (the Airy disk effect)





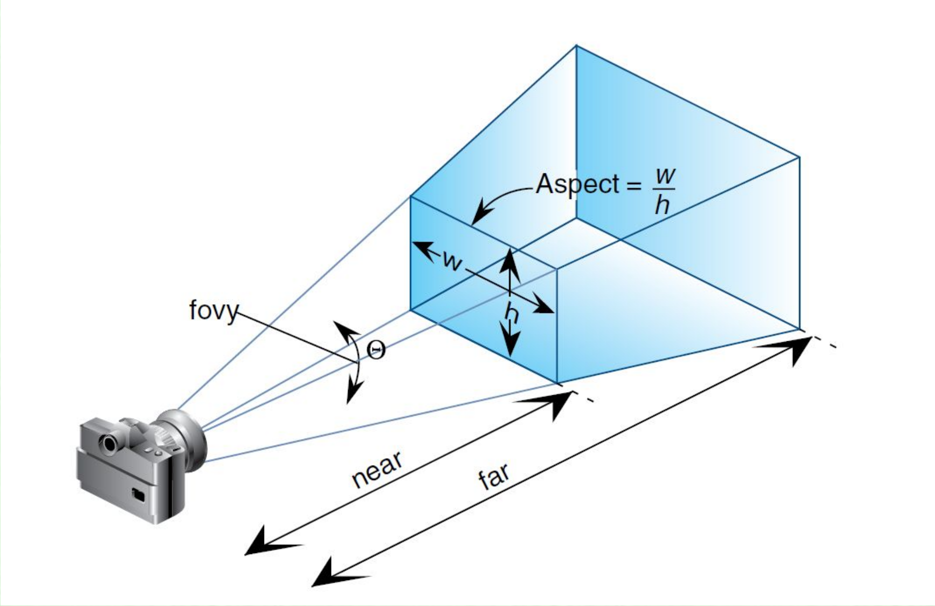
Pinhole Camera

- Light leaving any point travels in straight lines
- We only care about the lines that hit the pinhole (a single point)
- Infinite depth of field; i.e., everything is in focus (no blurring)
- An upside down image is formed by the intersection of these lines with an image plane
- More distant objects subtend smaller visual angles and appear smaller
- Objects occlude objects behind them



Virtual World Camera

- Trick: Move the film out in front of the pinhole, so that the image is not upside down
- Only render (compute an image for) objects further away from the camera than the film
- Add a back clipping plane for efficiency
- Volume between the film (front clipping plane) and the back clipping plane is the viewing frustum (shown in blue)
    - Make sure near/far (i.e. front/back) clipping planes have enough space between them to contain the scene
    - Make sure objects are inside the viewing frustum
    - Do not set the near clipping plane at the camera aperture!



Cameras Distortion depends on Distance

- Do not put the camera too close to objects of interest!
    - Significant/severe deductions for poor camera placement, fisheye, etc. (because the image will look terrible)
- Set up the scene like a real world scene!
- Get very familiar with the virtual camera!

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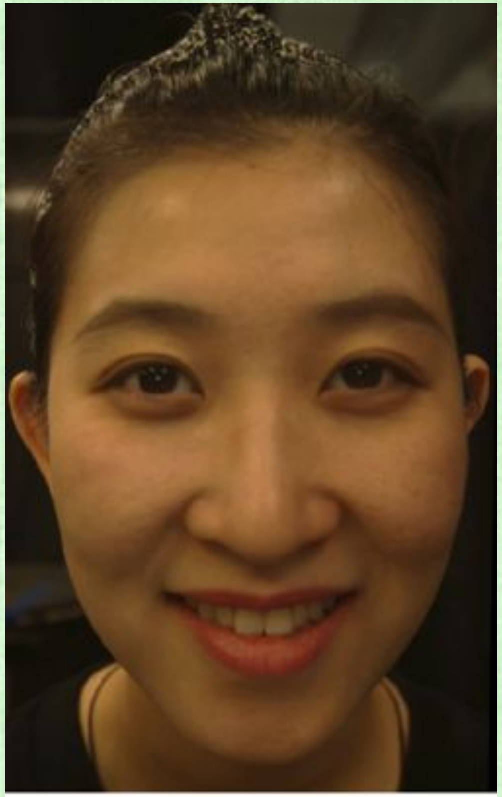

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Eye Distortion?

- Your eye also has distortion

- Unlike a camera, you don’t actually see the signal received on the cones
- Instead, your perceive an image (highly) processed by your brain
- Your eyes constantly move around obtaining multiple images for your brain to work with
- You have two eyes, and see in stereo, so triangulation can be used to estimate depth and undo distortion

- If your skeptical about all this processing, remember that your eye sees this:

Dealing with Objects

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- Let's start with a single 3D point $\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and move it around in the virtual world
- An object is just a collection of points, and as such the methods for handling a single point readily extend to handling entire objects

- Typically, objects are created in a reference space, which we refer to as <u>object space</u>
- After creation, we place objects into the scene, which we refer to as <u>world space</u>
- This may requires rotation, translation, resizing of the object

- When taking a picture, points on the object are projected onto the film, which we refer to as <u>screen space</u>
- Unlike rotation/translation/resizing, the projection onto screen space is highly nonlinear and the source of undesirable distortion

Rotation

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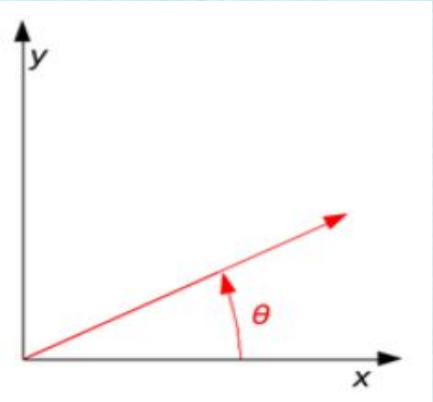
- Consider a single 3D point, $\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$
- In 2D, one can rotate a point clockwise about the origin via:



$$\begin{pmatrix} x^{new} \\ y^{new} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = R(\theta) \begin{pmatrix} x \\ y \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotation

- To rotate a 3D point around the x-axis, y-axis, or z-axis (respectively), one multiplies by:

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Matrix multiplication doesn't commute, i.e. $AB \neq BA$, so the order of rotations matters!
- Rotating about the x-axis and then the y-axis, $R_y(\theta_y)R_x(\theta_x)\vec{x}$, gives a different results than rotating about the y-axis and then the x-axis, $R_x(\theta_x)R_y(\theta_y)\vec{x}$

Line Segments are Preserved

- Consider two points $\vec{p}$ and $\vec{q}$ and the line segment between them:

$$\vec{u}(\alpha) = (1 - \alpha)\vec{p} + \alpha\vec{q}$$

- Here, $\vec{u}(0) = \vec{p}$ and $\vec{u}(1) = \vec{q}$ and $0 \leq \alpha \leq 1$ determine all the points on the line segment

- Multiplying points on the line segment by a rotation matrix R gives:

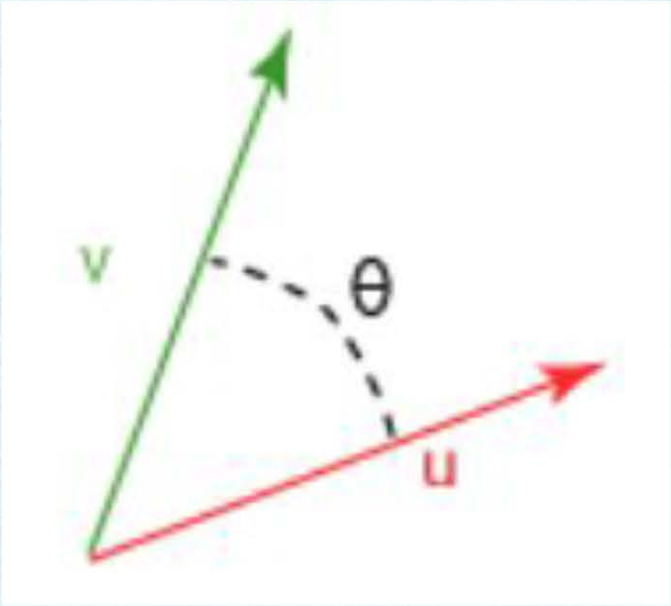
$$R\vec{u}(\alpha) = R\big((1 - \alpha)\vec{p} + \alpha\vec{q}\big) = (1 - \alpha)R\vec{p} + \alpha R\vec{q}$$

- Here, $R\vec{u}(0) = R\vec{p}$ and $R\vec{u}(1) = R\vec{q}$ and $0 \leq \alpha \leq 1$ determine all the points on the new rotated line segment connecting $R\vec{p}$ and $R\vec{q}$
  - i.e., one only need rotate the endpoints to construct the new line segment connecting them

- $\|R\vec{u}(\alpha) - R\vec{p}\|_2^2 = \|R(\vec{u}(\alpha) - \vec{p})\|_2^2 = (\vec{u}(\alpha) - \vec{p})^T R^T R(\vec{u}(\alpha) - \vec{p}) = \|\vec{u}(\alpha) - \vec{p}\|_2^2$ shows that the distance between rotated points is equivalent to the distance between original points

Angles are Preserved

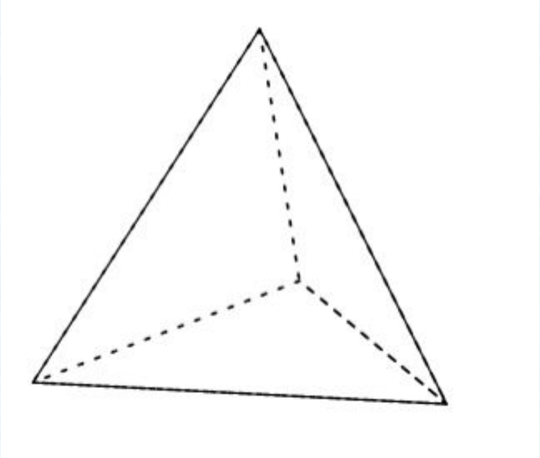
- Consider two line segments $\vec{u}$ and $\vec{v}$ with $\vec{u} \cdot \vec{v} = \|\vec{u}\|_2 \, \|\vec{v}\|_2 \cos(\theta)$ where θ is the angle between them



- $R\vec{u} \cdot R\vec{v} = \vec{u}^T R^T R \vec{v} = \vec{u}^T \vec{v} = \|\vec{u}\|_2 \, \|\vec{v}\|_2 \cos(\theta) = \|R\vec{u}\|_2 \, \|R\vec{v}\|_2 \cos(\theta)$
- So, the angle θ between $\vec{u}$ and $\vec{v}$ is the same as the the angle θ between $R\vec{u}$ and $R\vec{v}$

Shape is Preserved

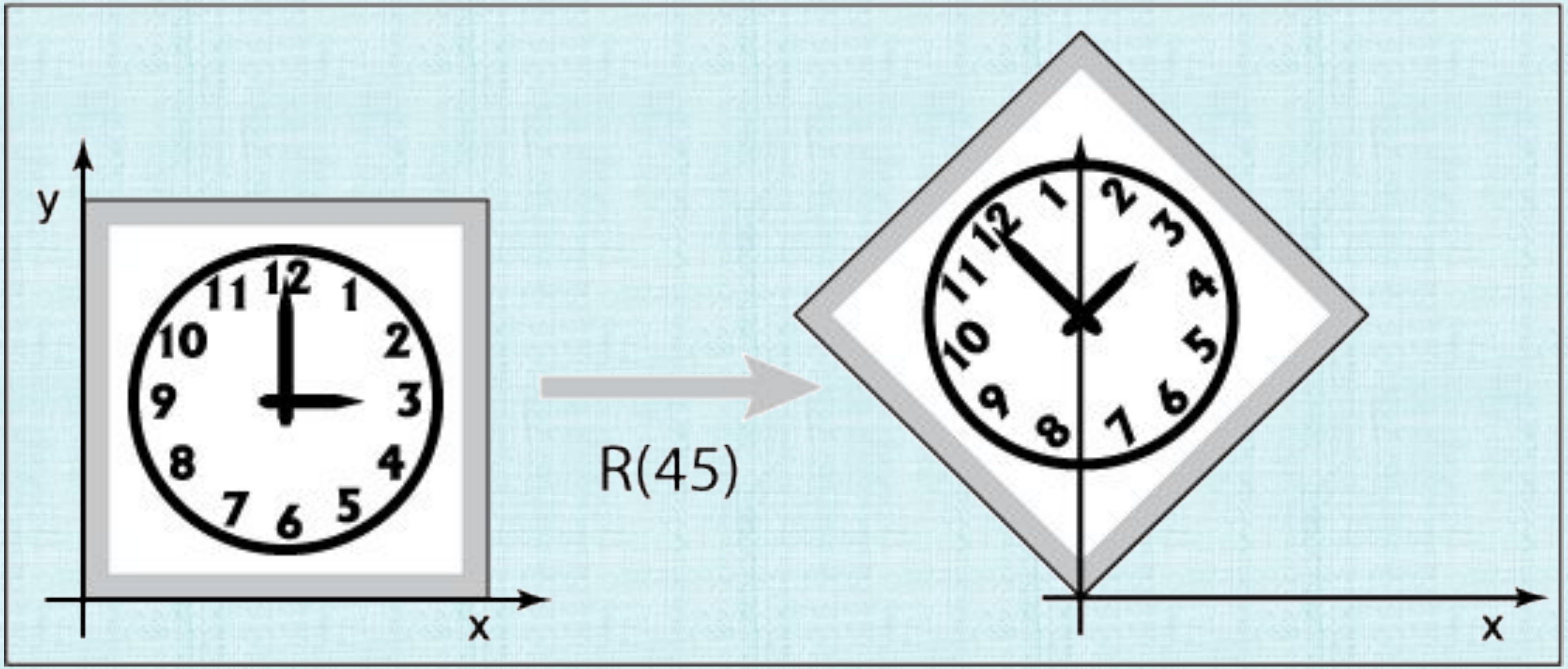
- In continuum mechanics, one measures the deformation of a material by a tensor called the strain
- The six unique entries in the nonlinear Green strain tensor are computed by comparing an undeformed tetrahedron to its deformed counterpart
- Given a tetrahedron in 3D, it is fully determined by one point and three line segments (the dotted lines in the figure)



- The 3 lengths of these three line segments and the 3 angles between any two of them are used to compare the undeformed tetrahedron to its deformed counterpart
- Since we proved these were all identical under rotations, rotations are shape preserving

Shape is Preserved

- Thus we can rotate entire objects without changing them



Scaling (or Resizing)

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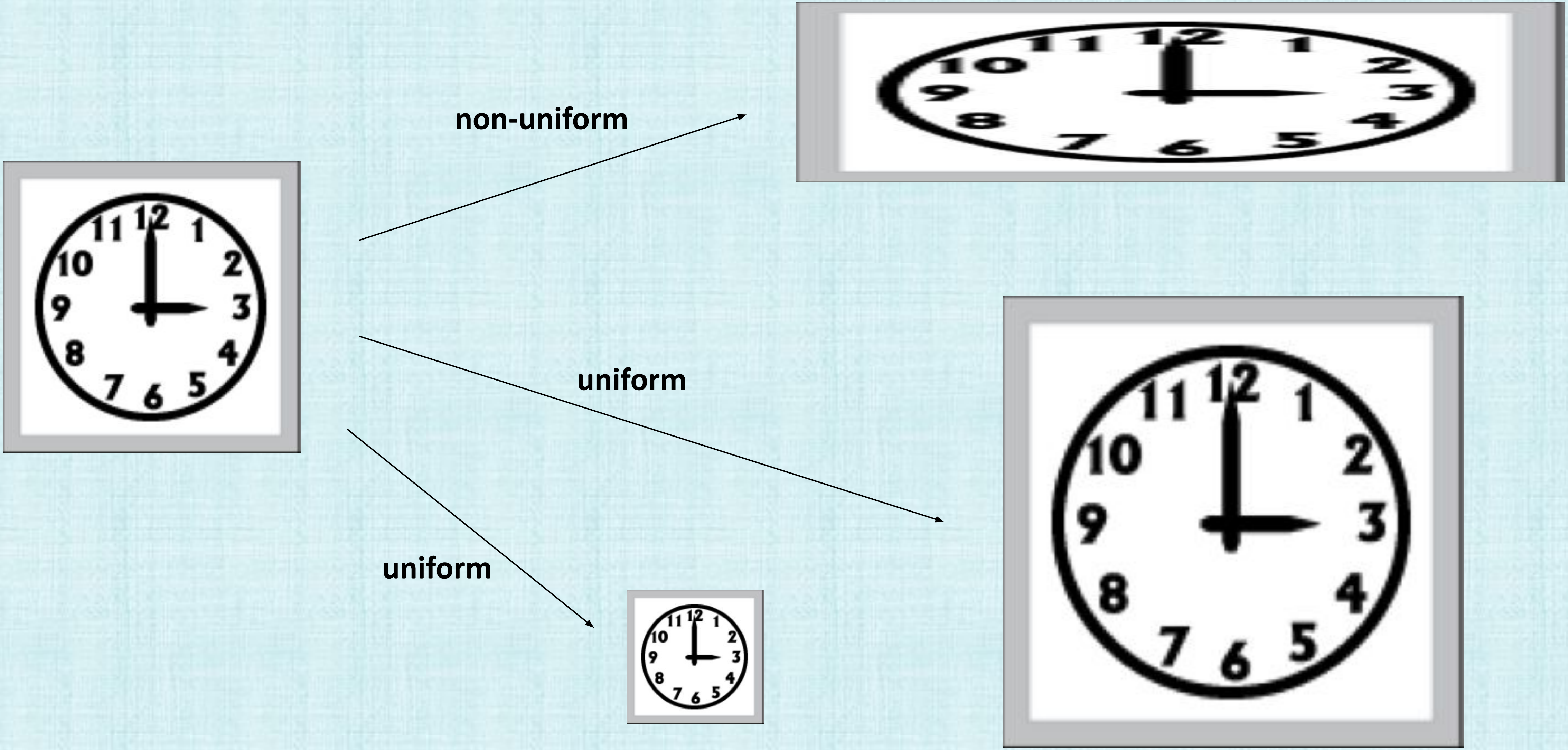
- A scaling matrix has the form $S = \begin{pmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_3 \end{pmatrix}$ and can both scale and shear the object
- Generally speaking, shearing an object changes lengths/angles creating significant distortion
- When $s_1 = s_2 = s_3$, one has pure scaling of the form $S = \begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{pmatrix} = sI$
- The distributive law of matrix multiplication guarantees that line segments map to line segments, and $\|S\vec{u}(\alpha) - S\vec{p}\|_2^2 = s\|\vec{u}(\alpha) - \vec{p}\|_2^2$ implies that the distance between scaled points is increased/decreased by a factor of s
- $S\vec{u} \cdot S\vec{v} = \mathrm{s}^2\vec{u} \cdot \vec{v} = s^2\|\vec{u}\|_2 \, \|\vec{v}\|_2 \cos(\theta) = \|S\vec{u}\|_2 \, \|S\vec{v}\|_2 \cos(\theta)$ shows that angles between line segments are preserved
- Thus, when using uniform scaling, objects grow/shrink but look the same as far as proportions are concerned (they are mathematically <u>similar</u>)

Scaling (or Resizing)



Homogenous Coordinates

- In order to deal with transformations via matrix multiplication, one uses homogeneous coordinates
- The homogeneous coordinates of a 3D point $\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ are $\vec{x}_H = \begin{pmatrix} xw \\ yw \\ zw \\ w \end{pmatrix}$ for any $w \neq 0$
- Dividing any homogenous coordinates by its fourth component gives $\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$ or $\begin{pmatrix} \vec{x} \\ 1 \end{pmatrix}$
- We convert all our 3D points to the form $\vec{x}_H = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$, i.e. $w = 1$, to deal with translations
- For vectors $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$, the homogenous coordinates are $\vec{u}_H = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} \vec{u} \\ 0 \end{pmatrix}$

Homogenous Coordinates

- Let M_{3x3} be a 3x3 rotation or scaling matrix (as discussed previously)
- Then, the transformation of a point $\vec{x}$ is given by $M_{3x3}\vec{x}$

- To produce the same result for $\begin{pmatrix} \vec{x} \\ 1 \end{pmatrix}$, use the 4x4 matrix $\begin{pmatrix} & M_{3x3} & & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} M_{3x3}\vec{x} \\ 1 \end{pmatrix}$

- Similarly, for a vector $\begin{pmatrix} & M_{3x3} & & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ 0 \end{pmatrix} = \begin{pmatrix} M_{3x3}\vec{u} \\ 0 \end{pmatrix}$

Translation

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- To translate a point $\vec{x}$ by some amount $\vec{t} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix}$ one multiplies by a 4x4 matrix

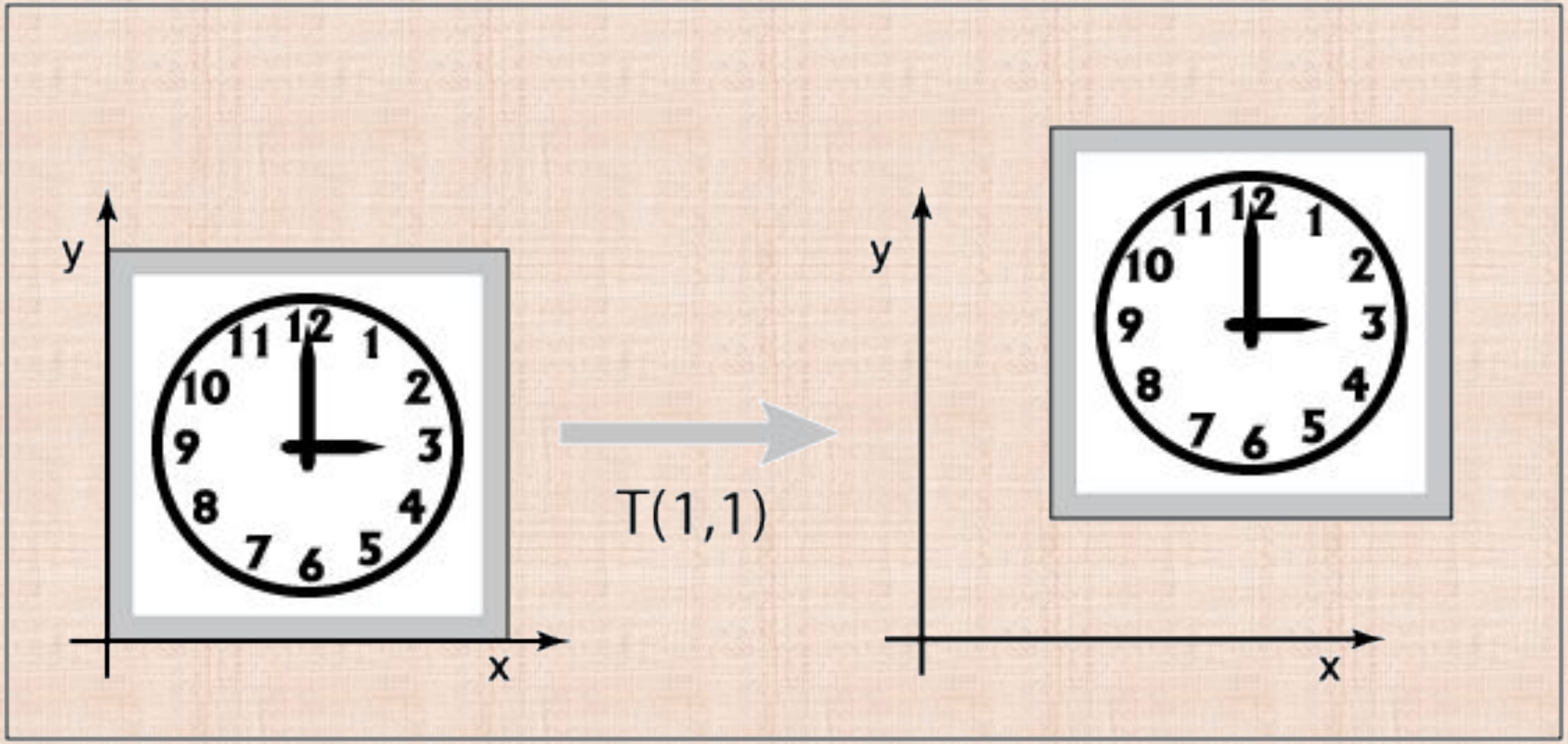
$$\begin{pmatrix} & I_{3x3} & & \begin{matrix} t_1 \\ t_2 \\ t_3 \end{matrix} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \vec{x} + \vec{t} \\ 1 \end{pmatrix}$$ where the 3x3 identity is $I_{3x3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

- For a vector $\begin{pmatrix} & I_{3x3} & & \begin{matrix} t_1 \\ t_2 \\ t_3 \end{matrix} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ 0 \end{pmatrix} = \begin{pmatrix} \vec{u} \\ 0 \end{pmatrix}$, which has no effect (as desired)

- Translations preserves line segments and angles between them, and thus shapes

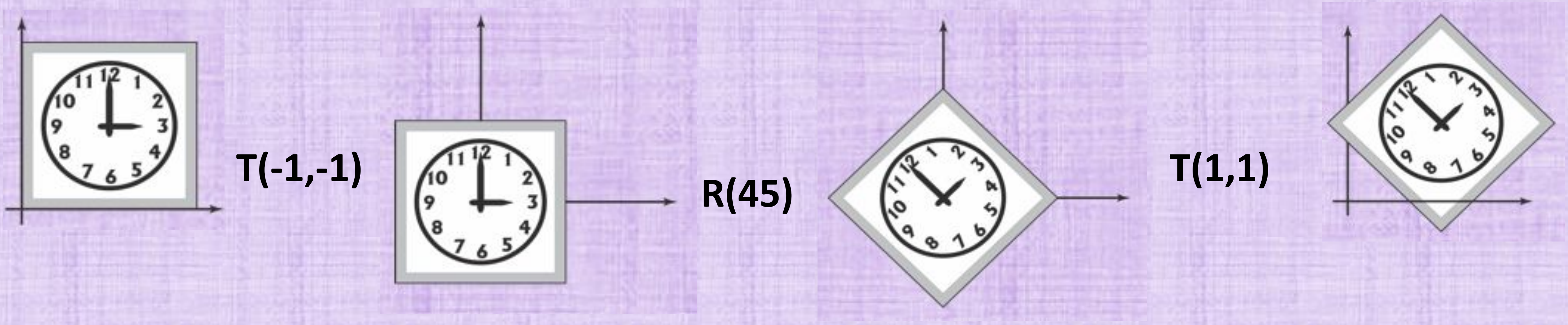
Shape is Preserved

- We can translate entire objects without changing them

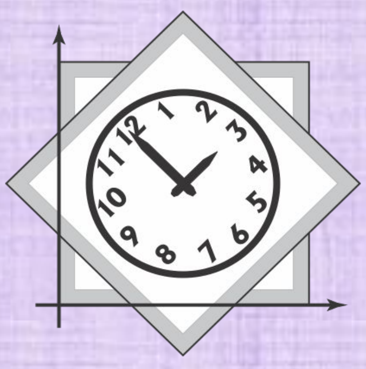


Composite Transforms

- Suppose one wants to rotate 45 degrees about the point (1,1)

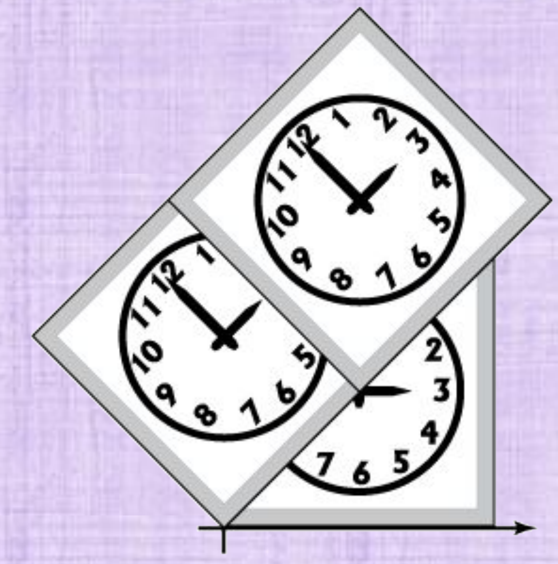


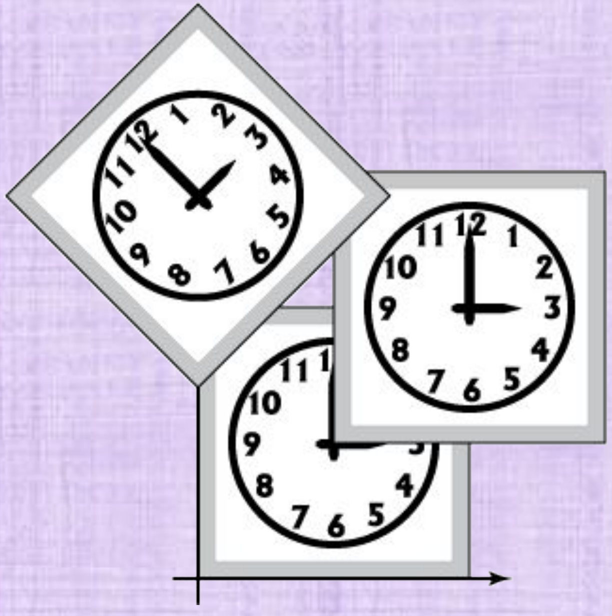
- These transformations can be multiplied together to get a single matrix M=T(1,1)R(45)T(-1,-1) that can be used to multiply every relevant point in the entire object:



Order Matters

- Matrix multiplication does not commute: $AB \neq BA$
- The rightmost transform is applied to the points first

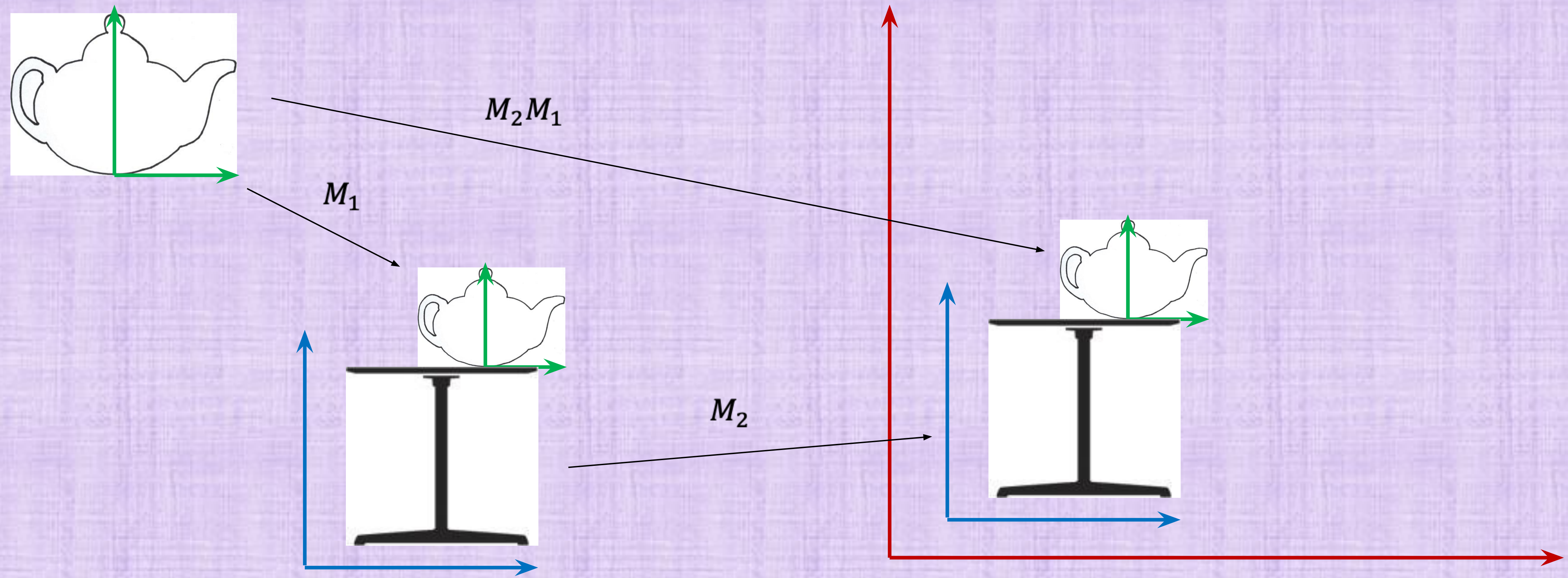




T(1,1)R(45) ≠ R(45)T(1,1)

Hierarchical Transforms

- M_1 transforms the teapot from its object space to the table's object space
- M_2 transforms the table from its object space to world space
- M_2M_1 transforms the teapot from its object space to world space



Using Transformations

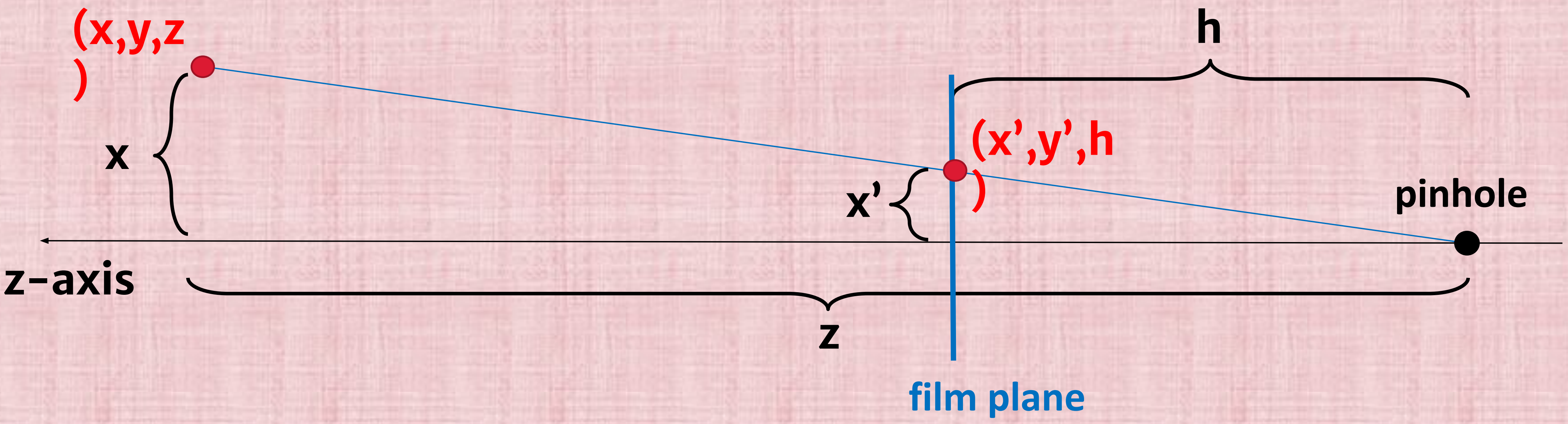
- Create objects (or parts of objects) in convenient coordinate systems
- Assemble objects from their parts (using transformations)
- Then, transform the assembled object into the scene (via hierarchical transformations)
- Can make multiple copies (even of different sizes) of the same object (simply) by adding another transform stack (efficiently, i.e. without creating a new copy of the object)

- <u>Helpful Hint</u>: Always compute composite transforms for objects or sub-objects, and apply the single composite transform to all relevant points (it’s a lot faster)

- <u>Helpful Hint</u>: Orientation is best done first: place the object at the center of the target coordinate system, and rotate it to the desired orientation. Afterwards, translate the object to the correct location.

Screen Space Projection

- Moving geometry from world space to screen space can create significant distortion
- This is because $\frac{1}{z}$ is highly nonlinear



$$\frac{x}{z} = \frac{x'}{h} \Rightarrow x' = h\frac{x}{z}$$ and $$\frac{y}{z} = \frac{y'}{h} \Rightarrow y' = h\frac{y}{z}$$

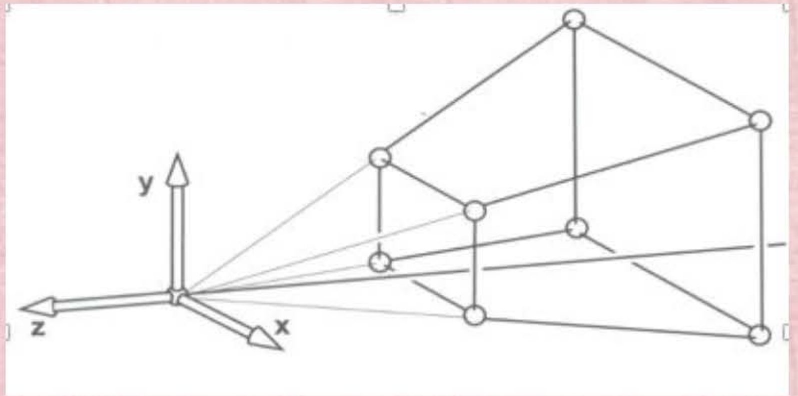
Matrix Form

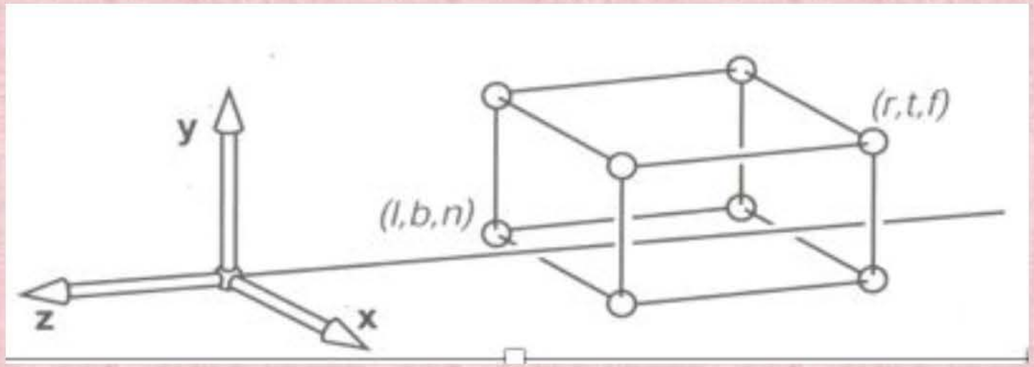
-
- Express the screen space result in homogeneous coordinates as $\begin{pmatrix} x'w' \\ y'w' \\ z'w' \\ w' \end{pmatrix}$
- Setting $w' = z$ gives the desired $\frac{1}{z}$ when dividing by w'
- Consider the following transformation: $\begin{pmatrix} x'w' \\ y'w' \\ z'w' \\ w' \end{pmatrix} = \begin{pmatrix} h & 0 & 0 & 0 \\ 0 & h & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$
- This has $w' = z$, $x'w' = hx$ or $x' = \frac{hx}{z}$, and $y'w' = hy$ or $y' = \frac{hy}{z}$ (as desired)

- Homogenous coordinates allows the nonlinear $\frac{1}{z}$ to be expressed with linear matrix multiplication (so it can be added to the matrix multiplication stack)!

Perspective Projection

- The third equation in the linear system is $z'w' = az + b$ or $z'z = az + b$, but z values are not needed since the projected points all lie on $z = h$ image plane
- However, computing z' as a monotonically increasing function of z allows it to be used to determine occlusions (for alpha channel transparency)
- If $z = n$ is the near clipping plane and $z = f$ is the far clipping plane, these clipping planes can be preserved in z' by setting $z' = n$ and $z' = f$ (respectively)
- This gives 2 equations in 2 unknowns: $n^2 = an + b$ and $f^2 = af + b$ leads to $a = n + f$ and $b = -fn$





- This transforms the viewing frustum into an orthographic volume in screen space