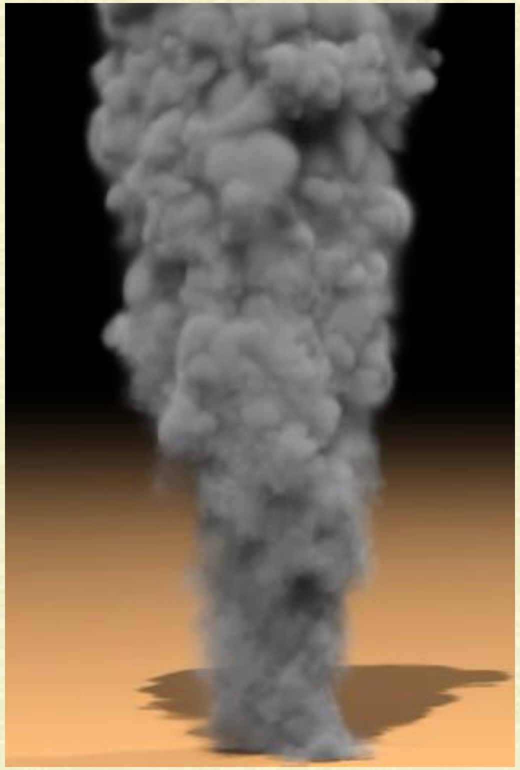
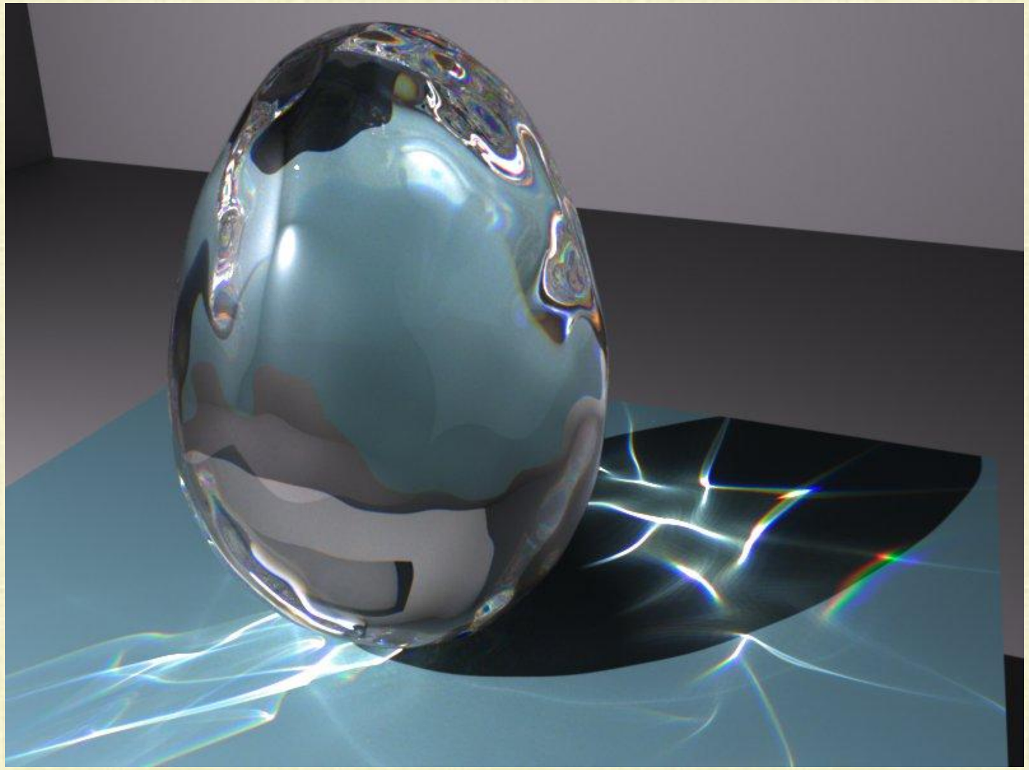
Recursive Ray Tracing

Color Accumulation at an Intersection Point

- A shadow ray is cast to each light source, and the total contribution from all light sources is accumulated

$$(k_R, k_G, k_B)\left(\sum\nolimits_{lights} V_{light} I_{light} \max(0, \cos\theta_{light}) + I_{ambient} \prod\nolimits_{lights} (1 - V_{light})\right)$$

- $V_{light} = 1$ for visible light sources, and $V_{light} = 0$ for occluded light sources
- $I_{ambient}$ is added to fully shadowed regions where $\prod_{lights}(1 - V_{light}) \neq 0$
- To summarize: $(k_R, k_G, k_B)(L_{diffuse} + L_{ambient})$

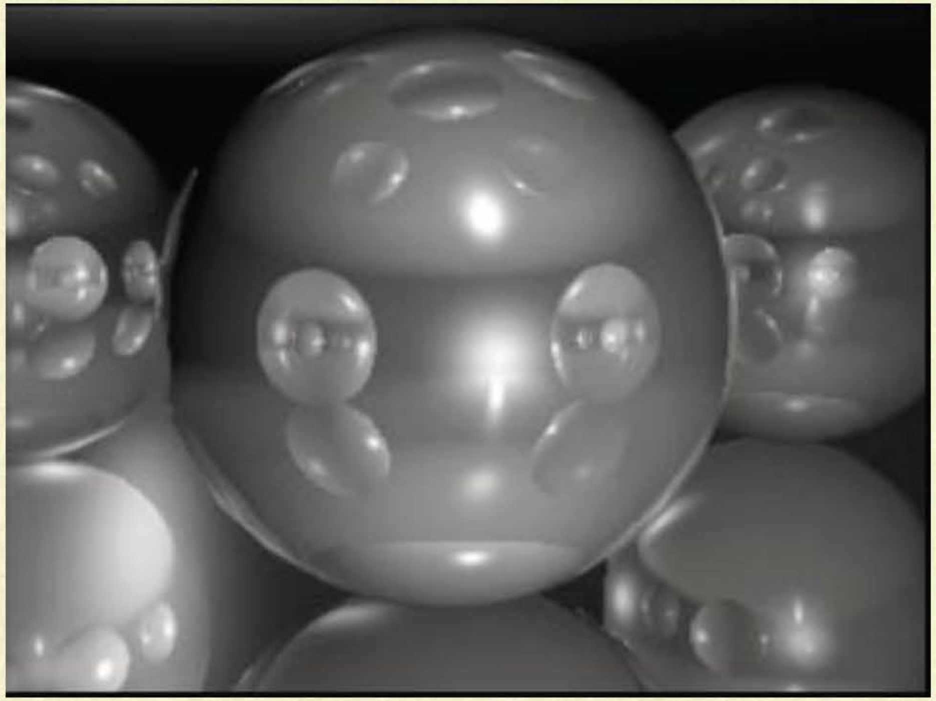
- Mirror-like reflective properties can *also* contribute to the color at an intersection point
- Transparency allows other objects to show through a surface, and thus those objects can *also* contribute color to an intersection point
- In summary: $(k_R, k_G, k_B)(L_{diffuse} + L_{ambient}) + L_{reflect} + L_{transmit}$

Additional Light

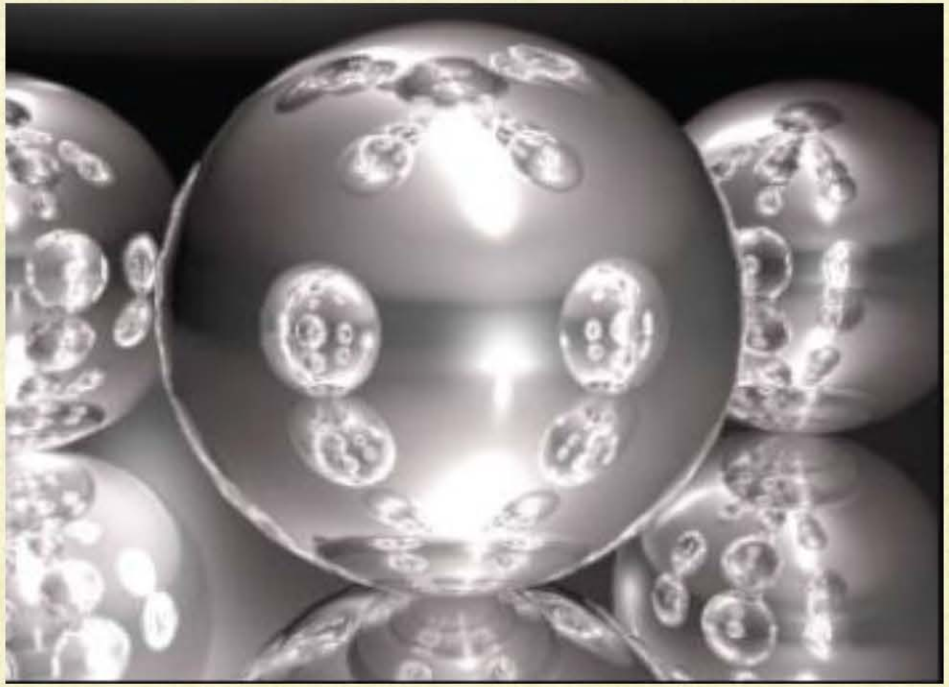
- Reflection and Transmission add light to a pixel making it brighter
- Thus, scaling coefficients are added in front of every lighting contribution

$$(k_R, k_G, k_B)\left(k_d L_{diffuse} + k_a L_{ambient}\right) + k_r L_{reflect} + k_t L_{transmit}$$

- Typically, coefficients are adjusted relative to each other to get the desired “look”; then, all the coefficients are scaled together to get the appropriate overall brightness/darkness



less reflection (darker)



more reflection (brighter)

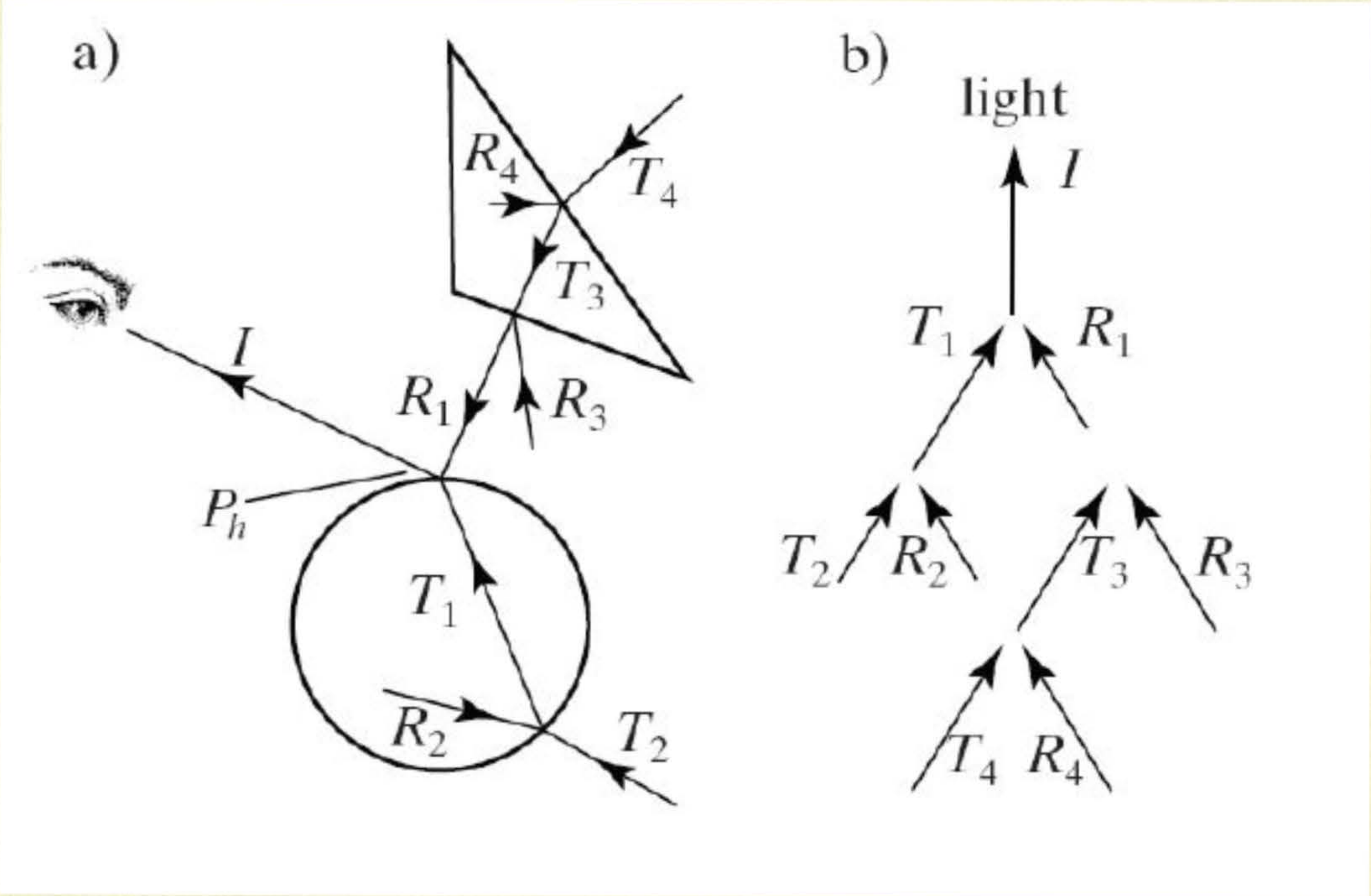
Recursion

- $L_{reflect}$ and $L_{transmit}$ are treated in exactly the same way as pixel colors are treated

- A ray is constructed for the reflection direction and intersected with scene geometry (in exactly the same way as was done for camera rays through pixels)
  - the result is stored in $L_{reflect}$
- A ray is constructed for the transmission direction and intersected with scene geometry (in exactly the same way as was done for camera rays through pixels)
  - the result is stored in $L_{transmit}$

- $L_{reflect}$ and $L_{transmit}$ depend on the color computed from whatever object geometry was intersected by their corresponding rays
- Those intersection points will have colors of their own (computed as usual via shadow rays, diffuse shading, and ambient shading)
- In addition, the color of those intersections can also depend on subsequent reflection and transmission, meaning that even more rays need to be spawned before a color can be computed

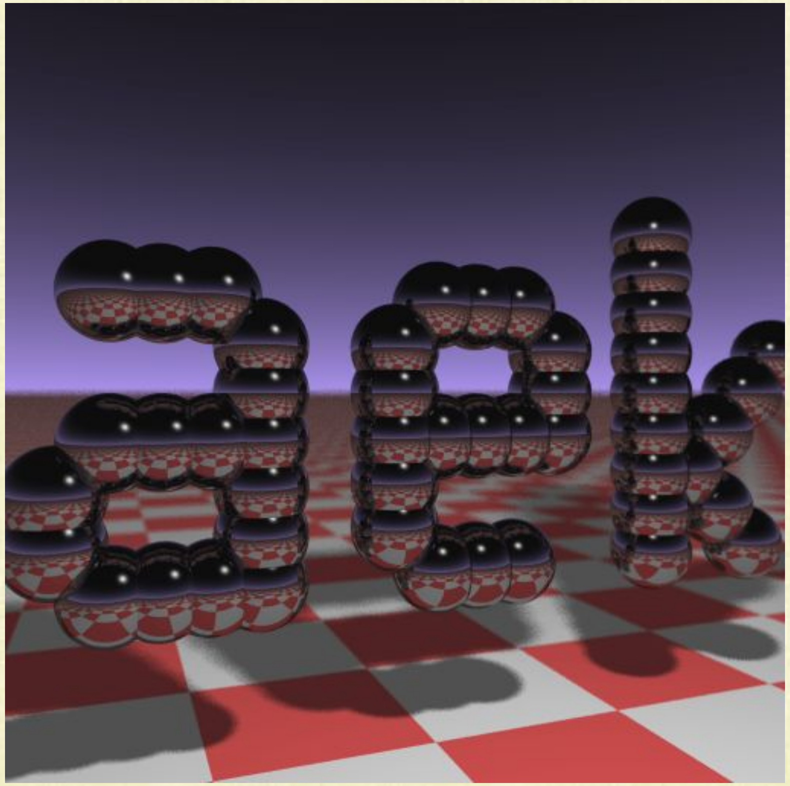
Ray Tree Example



Code Simplicity

- Recursion allows for stunning imagery with minimal code, as demonstrated by these 1337 characters printed on the back of a business card

```
#include <stdlib.h>   // card > aek.ppm
#include <stdio.h>
#include <math.h>
typedef int i;typedef float f;struct v{
f x,y,z;v operator+(v r){return v(x+r.x
,y+r.y,z+r.z);}v operator*(f r){return
v(x*r,y*r,z*r);}f operator%(v r){return
x*r.x+y*r.y+z*r.z;}v(){}v operator^(v r
){return v(y*r.z-z*r.y,z*r.x-x*r.z,x*r.
y-y*r.x);}v(f a,f b,f c){x=a;y=b;z=c;}v
operator!(){return*this*(1/sqrt(*this%*
this));}};i G[]={247570,280596,280600,
249748,18578,18577,231184,16,16};f R(){
return(f)rand()/RAND_MAX;}i T(v o,v d,f
&t,v&n){t=1e9;i m=0;f p=-o.z/d.z;if(.01
<p)t=p,n=v(0,0,1),m=1;for(i k=19;k--;)
for(i j=9;j--;)if(G[j]&1<<k){v p=o+v(-k
,0,-j-4);f b=p%d,c=p%p-1,q=b*b-c;if(q>0
){f s=-b-sqrt(q);if(s<t&&s>.01)t=s,n=!(
p+d*t),m=2;}}return m;}v S(v o,v d){f t
;v n;i m=T(o,d,t,n);if(!m)return v(.7,
.6,1)*pow(1-d.z,4);v h=o+d*t,l=!(v(9+R(
),9+R(),16)+h*-1),r=d+n*(n%d*-2);f b=l%
n;if(b<0||T(h,l,t,n))b=0;f p=pow(l%r*(b
>0),99);if(m&1){h=h*.2;return((i)(ceil(
h.x)+ceil(h.y))&1?v(3,1,1):v(3,3,3))*(b
*.2+.1);}return v(p,p,p)+S(h,r)*.5;}i
main(){printf("P6 512 512 255 ");v g=!v
(-6,-16,0),a=!(v(0,0,1)^g)*.002,b=!(g^a
)*.002,c=(a+b)*-256+g;for(i y=512;y--;)
for(i x=512;x--;){v p(13,13,13);for(i r
=64;r--;){v t=a*(R()-.5)*99+b*(R()-.5)*
99;p=S(v(17,16,8)+t,!(t*-1+(a*(R()+x)+b
*(y+R())+c)*16))*3.5+p;}printf("%c%c%c"
,(i)p.x,(i)p.y,(i)p.z);}}
```



http://fabiensanglard.net/rayTracing_back_of_business_card/

Termination

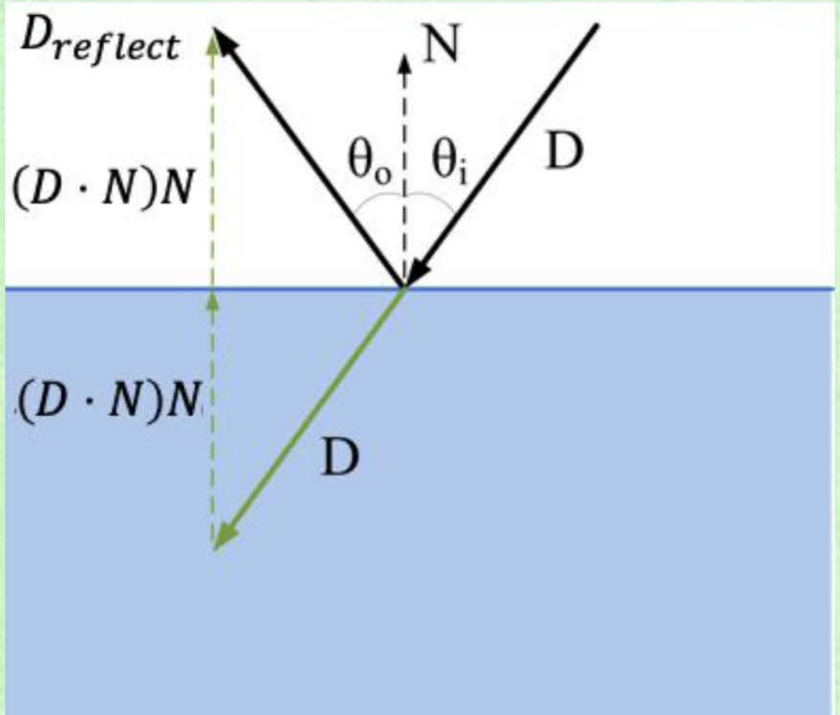
- If every subsequently intersected point continued to depend on reflected/transmitted rays, rays would be spawned indefinitely (never terminating)
- Eventually one hits the recursion limit (depending on hardware) that prevents stack overflow

- If k_d and k_a are nonzero frequently enough, the reflected/transmitted contributions are eventually diminished enough that one can terminate the recursion
- Just need to add a final <u>arbitrary value</u> for $L_{reflect}$ and/or $L_{transmit}$ (without tracing the associated ray)

- In some cases (mirrors, bubbles, etc.), there is little to no diffuse/ambient lighting and nearly 100% of the lighting is recursively sought after via reflected/transmitted rays
- In those cases, any arbitrary values will show up as the final pixel color
- This can look terrible, e.g. a black pixel in a mist of bubbles
- One often struggles to choose realistic termination colors (common choices are the sky color, the background color, etc.)

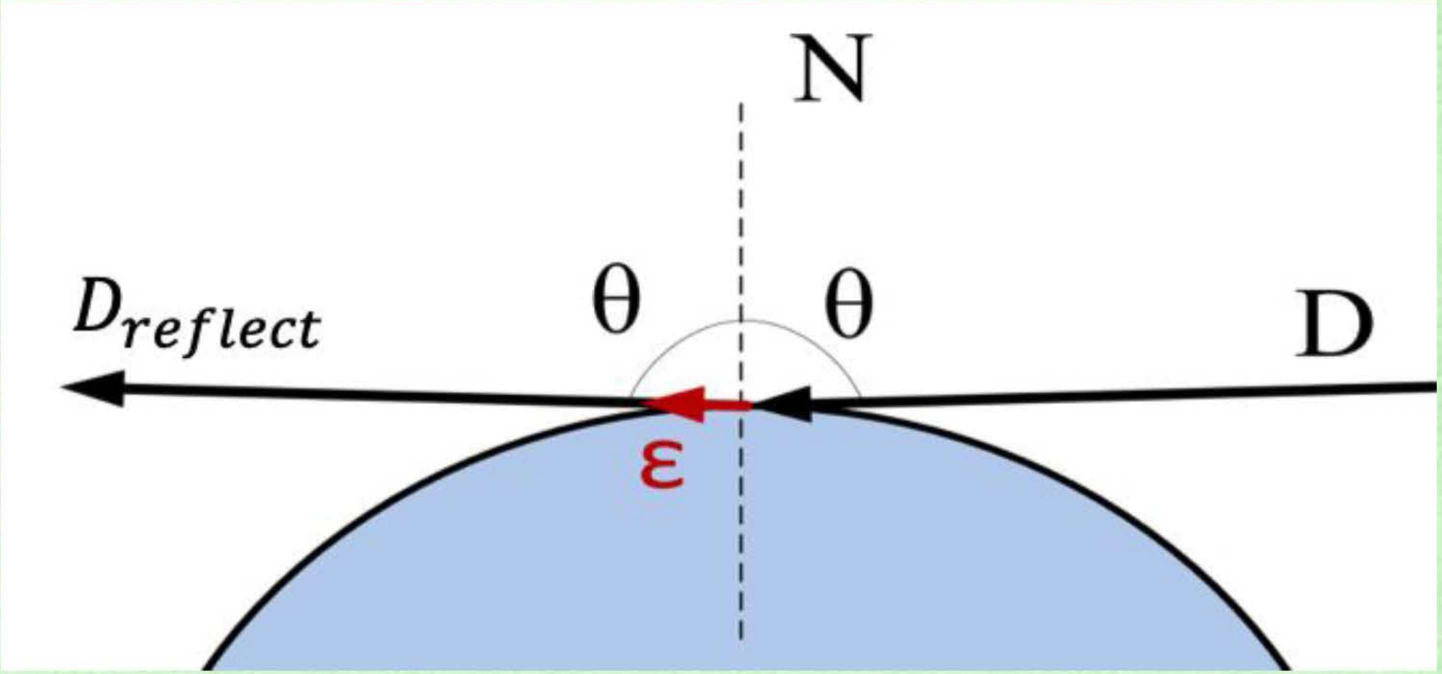
Reflected Ray

- Given an incoming ray $R(t) = A + Dt$ with direction D, and local (outward) unit normal to the geometry N, the angle of incidence is defined via $D \cdot N = -\|D\|_2 \cos\theta_i$
- For mirror reflection, the incoming/outgoing rays make the same angle with N, i.e. $\theta_o = \theta_i$, and those rays and the normal are all coplanar
- Thus, the reflected ray direction is $D_{reflect} = D - 2(D \cdot N)N$
- Then, the reflected ray is $R_{reflect}(t) = R(t_{int}) + D_{reflect}t$



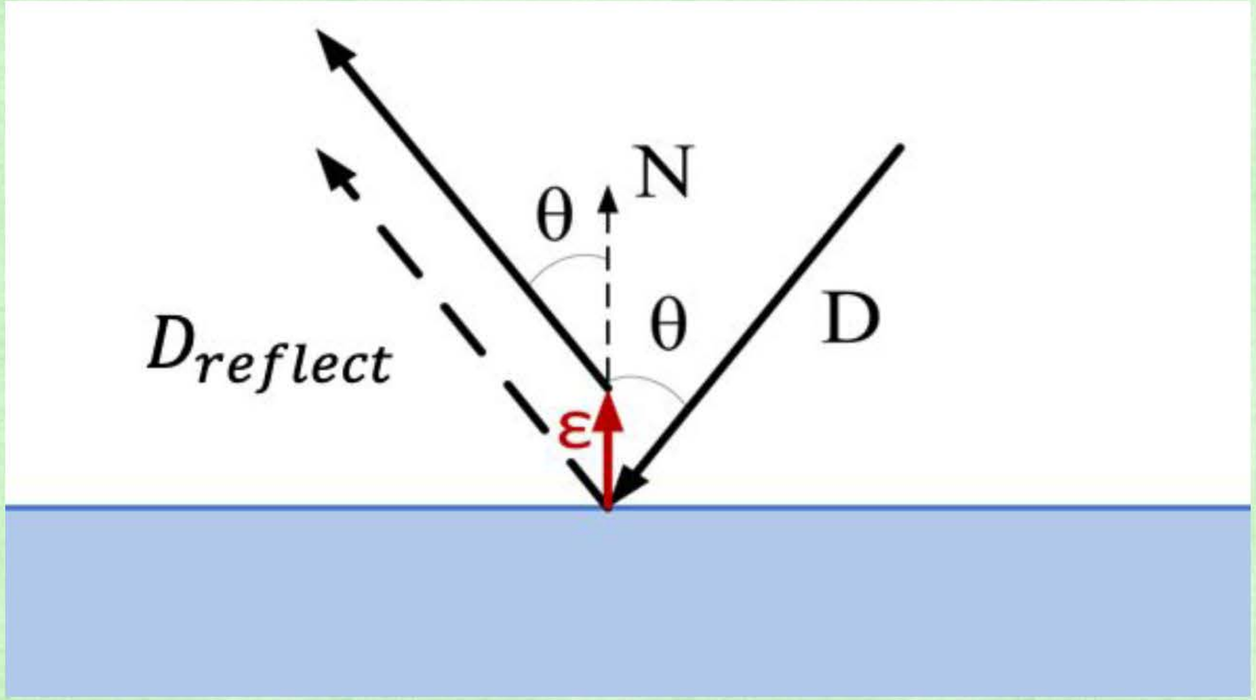
Spurious Self-Occlusion

- Numerical precision issues can cause the reflected ray to incorrectly re-intersect the same object near $R(t_{int})$, similar to the issues for shadow rays
- Once again, one can use $t \in (\epsilon, \infty)$ for some $\epsilon > 0$ large enough to avoid the ray incorrectly re-intersecting the same object
- However, grazing rays near the object's silhouette may still incorrectly re-intersect the object

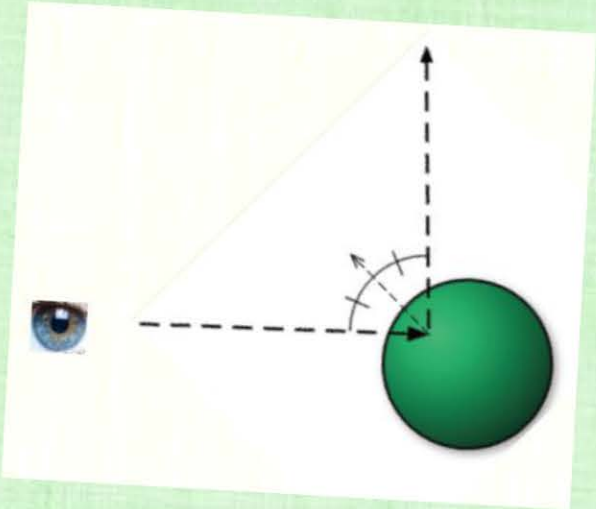


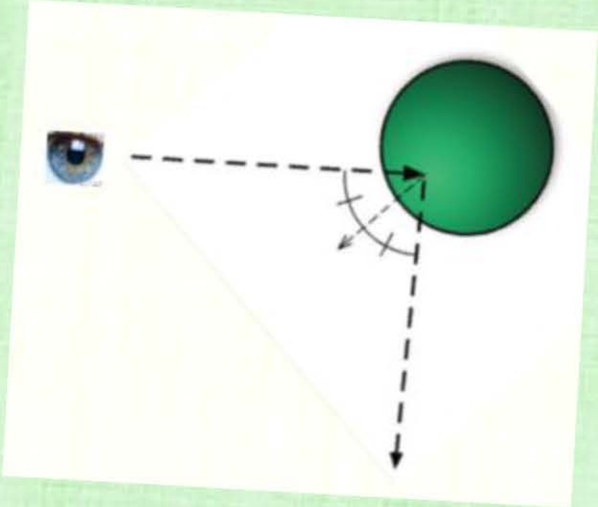
Spurious Self-Occlusion

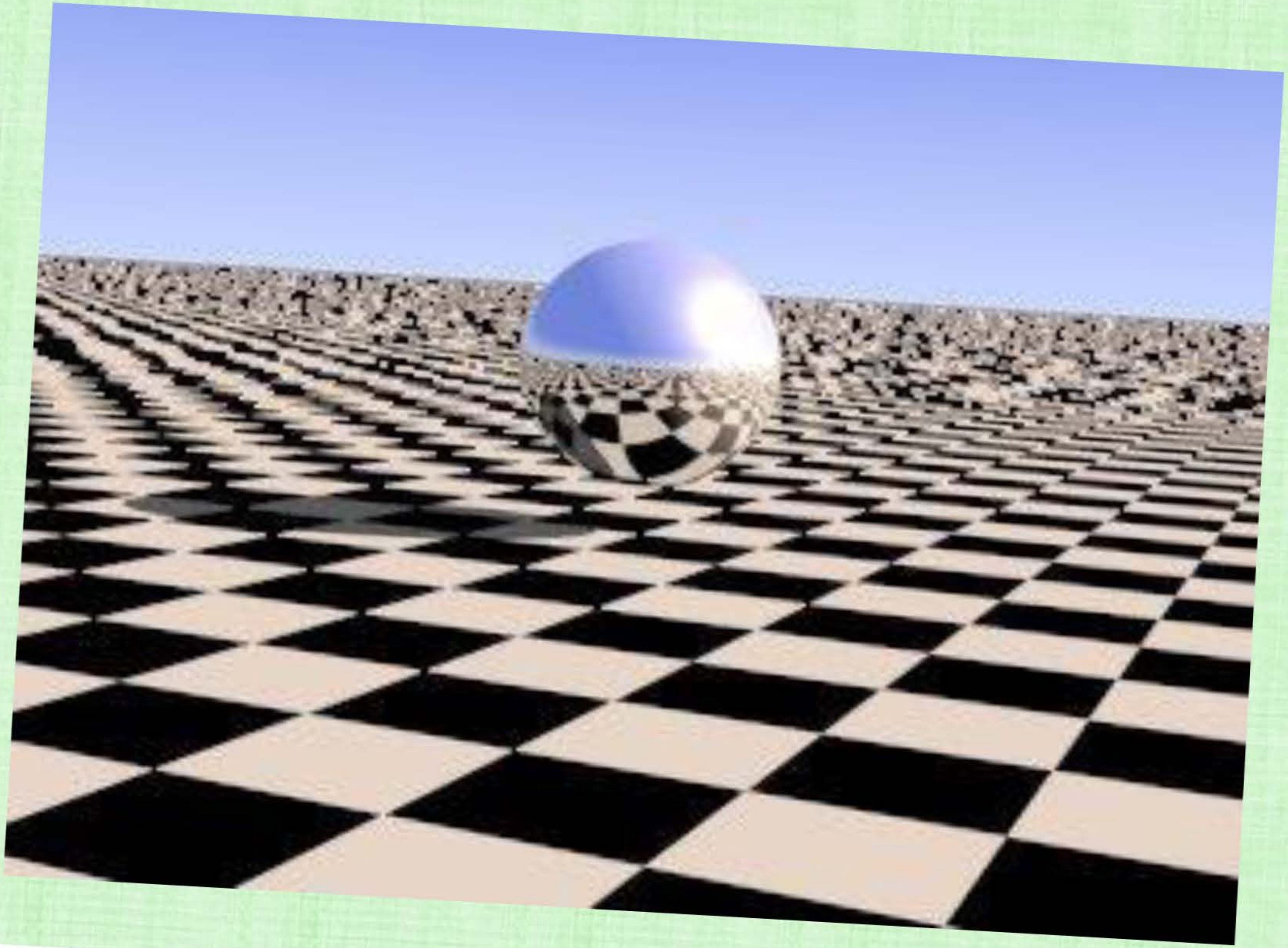
- Alternatively, perturb the starting point of the reflected ray to be slightly away from the object, e.g. from $R(t_{int})$ to $R(t_{int}) + \epsilon N$
- Perturbed reflected rays do not have the ray direction modified (unlike shadow rays)
- The new reflected ray is $R_{reflect}(t) = R(t_{int}) + \epsilon N + D_{reflect}t$ with $t \in (0, \infty)$
- This works well, but one needs to be careful that the new starting point does not fall inside (or too close to) nearby geometry



Reflections

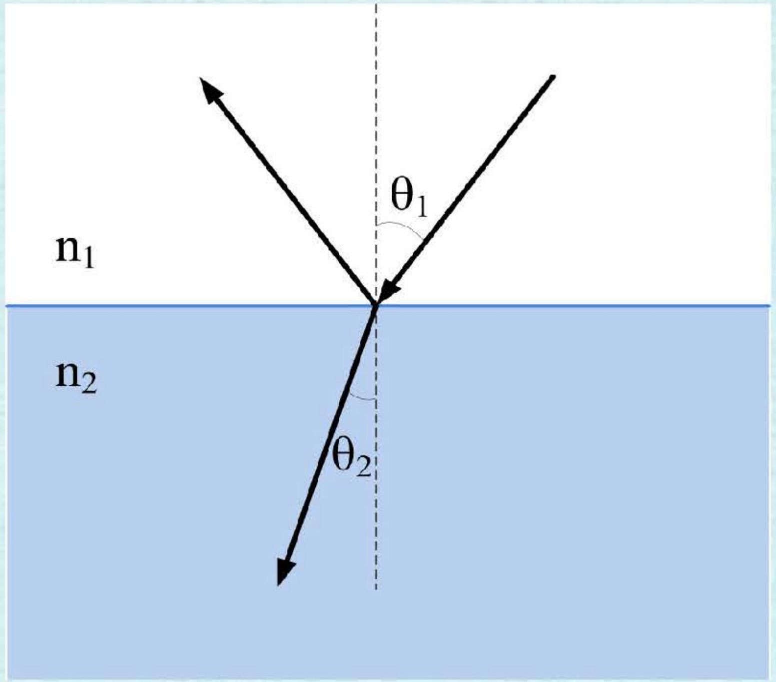


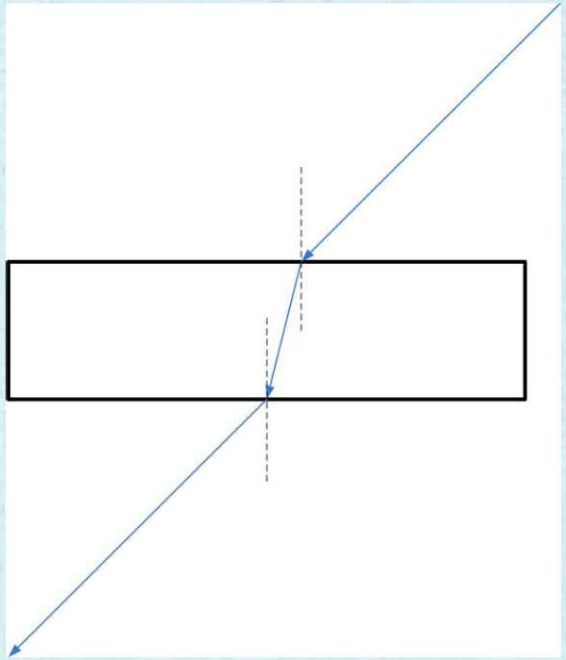




Transmission

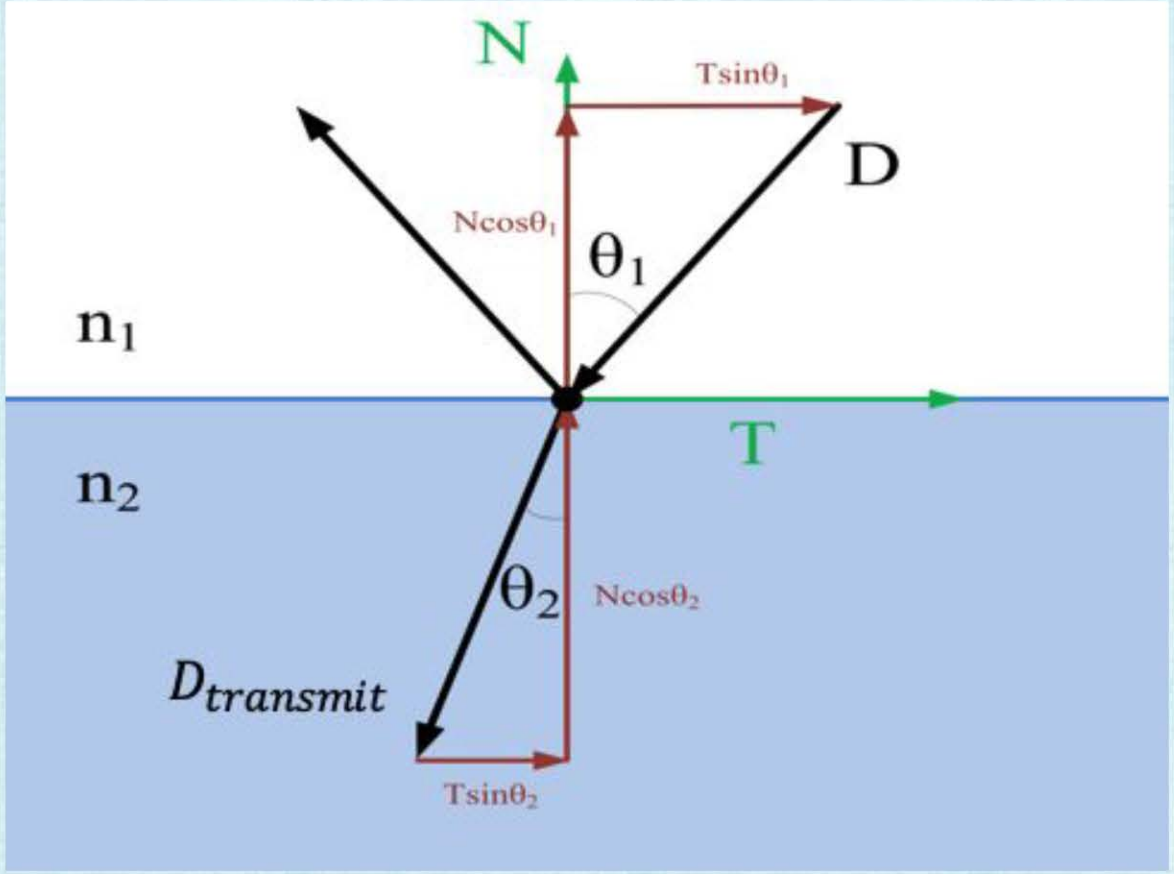
- The relationship between the angle of incidence and angle of transmission (refraction) for light passing through a boundary between two different isotropic media is given by Snell's Law
- $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$ where θ_1 and θ_2 are the incoming/outgoing angles, v_1 and v_2 are the phase velocities and n_1 and n_2 are the indices of refraction





Transmitted Ray

- D is the (unit) incoming ray direction, N is the (outward) unit normal, and T is the unit tangent in the plane of D and N, so that $D + N\cos\theta_1 + T\sin\theta_1 = 0$
- Let $D_{transmit}$ be the (unit) transmitted ray direction, then $D_{transmit} + T\sin\theta_2 + N\cos\theta_2 = 0$



Transmitted Ray

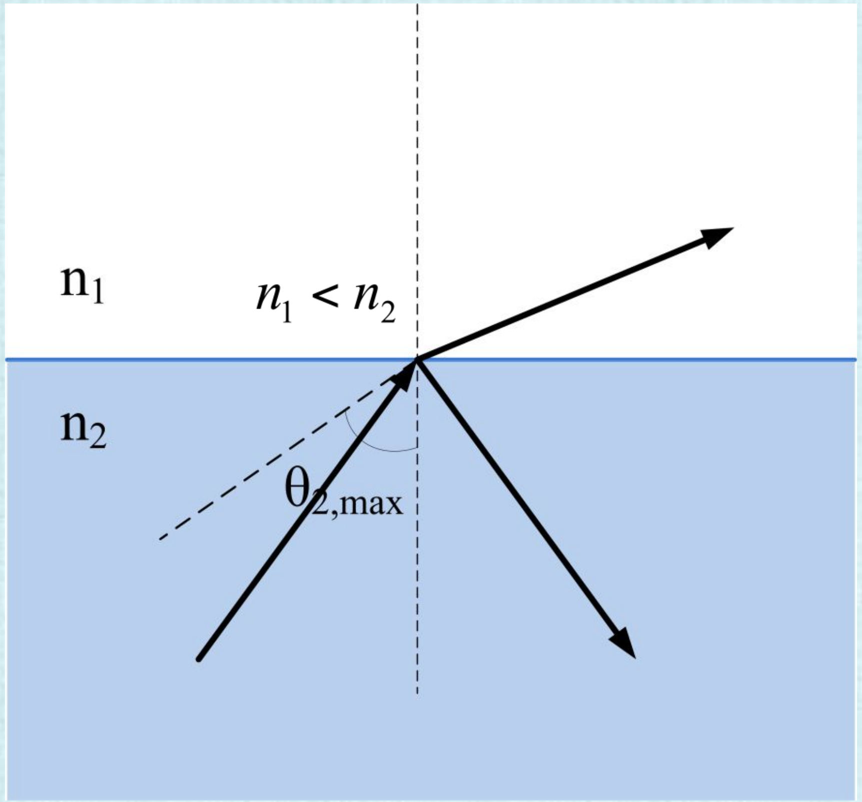
- So $D_{transmit} = -T sin\theta_2 - N cos\theta_2 = (D + N cos\theta_1)\frac{sin\theta_2}{sin\theta_1} - N\sqrt{1 - \sin^2\theta_2}$
- Using Snell's Law, $D_{transmit} = (D + N cos\theta_1)\frac{n_1}{n_2} - N\sqrt{1 - \left(\frac{n_1}{n_2} sin\theta_1\right)^2}$
- $D_{transmit} = D\frac{n_1}{n_2} + N\left(\frac{n_1}{n_2} cos\theta_1 - \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 (1 - \cos^2\theta_1)}\right)$
- Using $cos\theta_1 = -D \cdot N$ gives $D_{transmit} = D\frac{n_1}{n_2} - N\left(\frac{n_1}{n_2} D \cdot N + \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 (1 - (D \cdot N)^2)}\right)$

- If the term under the square root is negative, there is no transmitted ray, and all the light is reflected (total internal reflection)

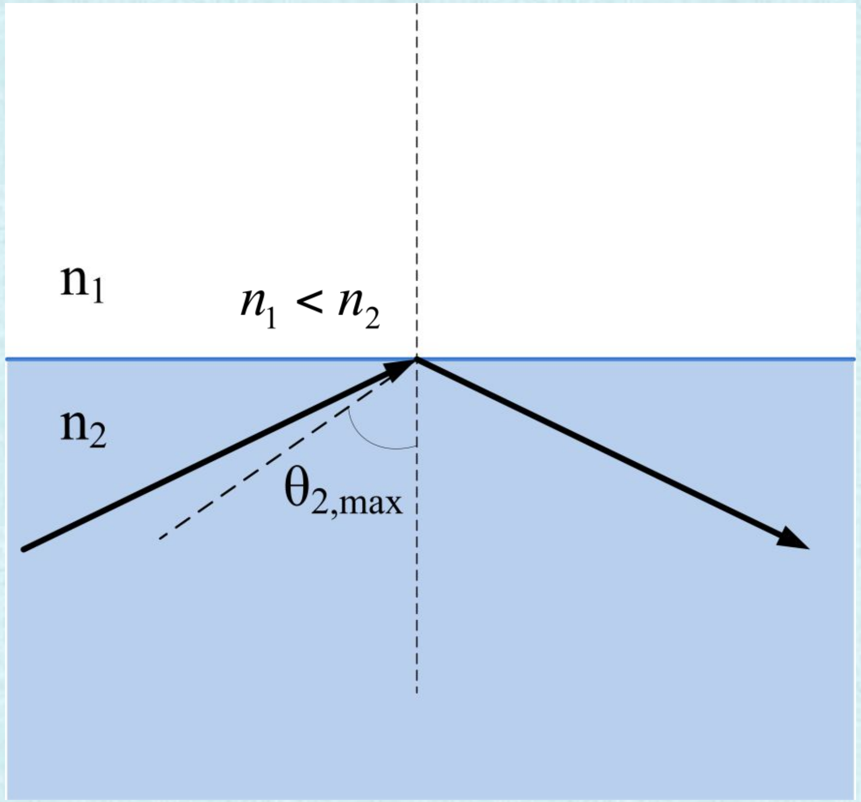
- Note: This equation works regardless of whether n_1 or n_2 is bigger
- Note: Add ε to avoid self intersection, or offset in the negative normal direction (respecting collisions, other geometry, etc. as usual)

Total Internal Reflection

- When light is going from a higher index of refraction to lower index of refraction, no light is transmitted when the incident angle exceeds a critical angle
- In such a case, all the light reflects



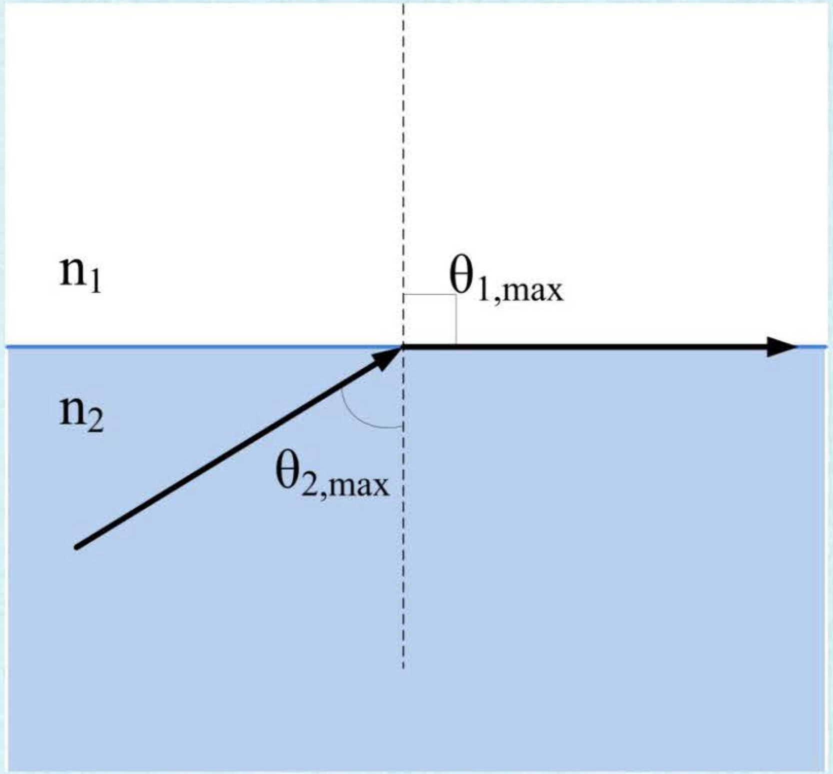
when $\theta_2 < \theta_{2,max}$, both reflection and Transmission occur



when $\theta_2 > \theta_{2,max}$, only reflection occurs

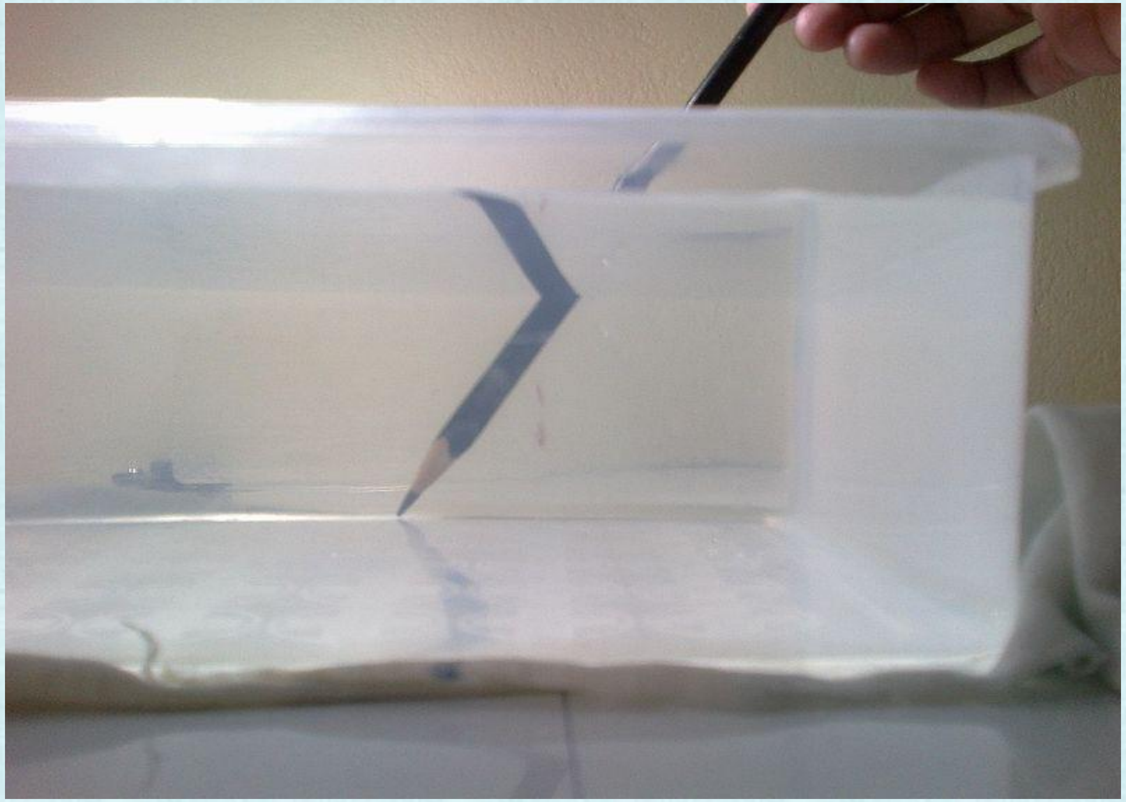
Critical Angle

- When $\theta_1 = \frac{\pi}{2}$, which is the maximum angle for transmission, $\sin\left(\frac{\pi}{2}\right) = 1$ and Snell's Law becomes $\frac{1}{\sin \theta_2} = \frac{n_2}{n_1}$ or $\theta_2 = \arcsin\left(\frac{n_1}{n_2}\right)$
- So for $n_1 < n_2$, the critical angle is $\theta_2 = \arcsin\left(\frac{n_1}{n_2}\right)$



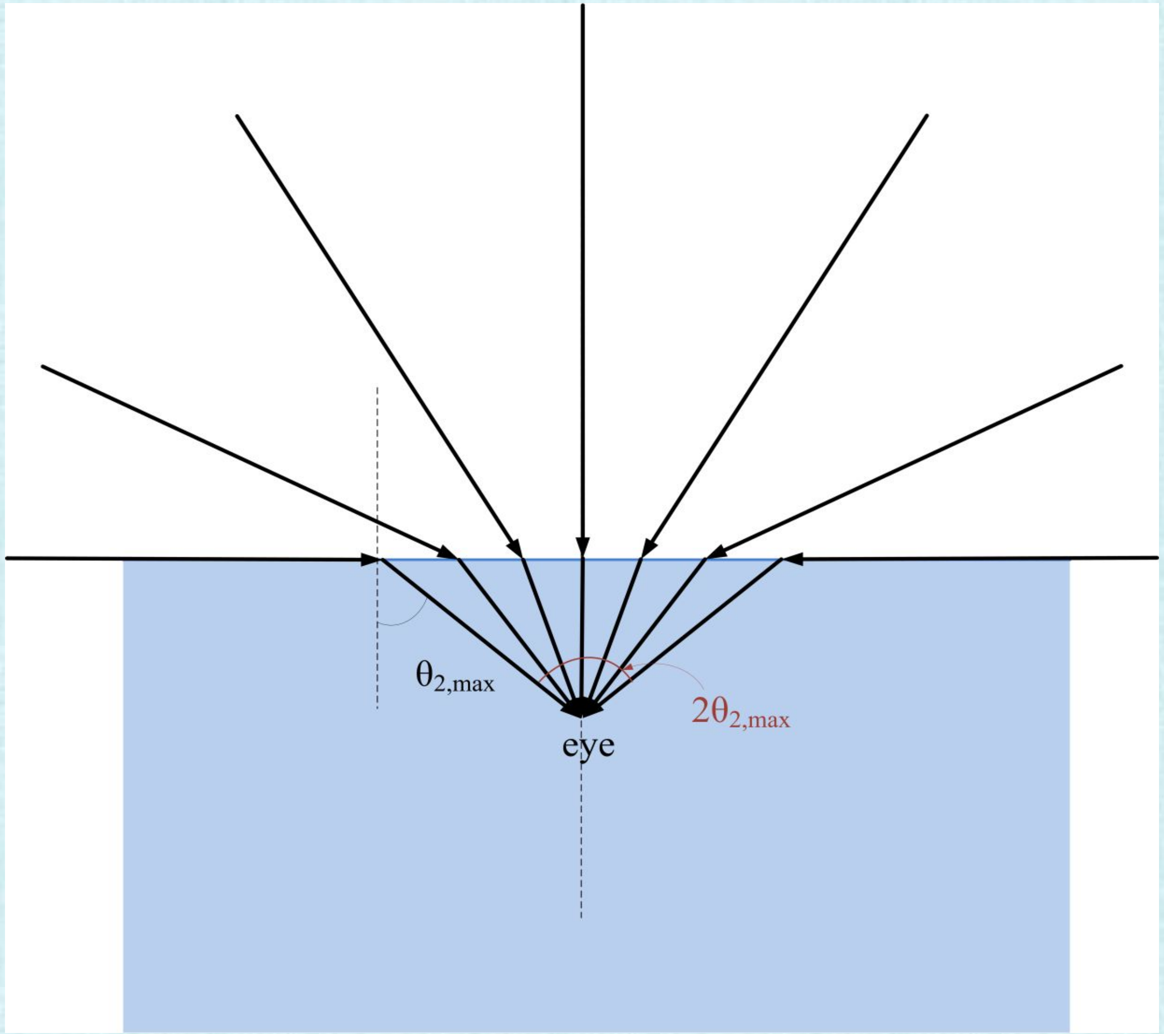
Total Internal Reflection

- Responsible for many interesting and impressive visuals in both glass and water



Snell’s Window

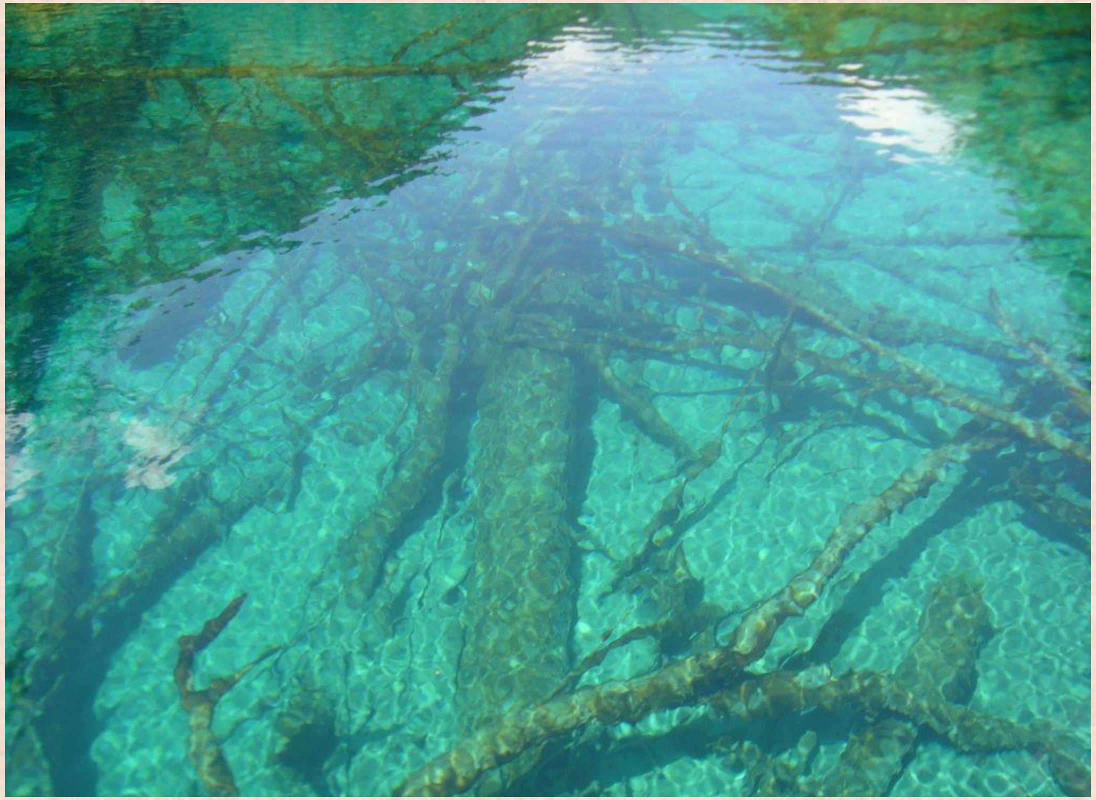
- Yes, fish can see you on the shore!



Snell’s Window

Reflection vs. Transmission

- The amount of transmission vs. reflection decreases as the viewing angle goes from perpendicular (overhead) to parallel (grazing)

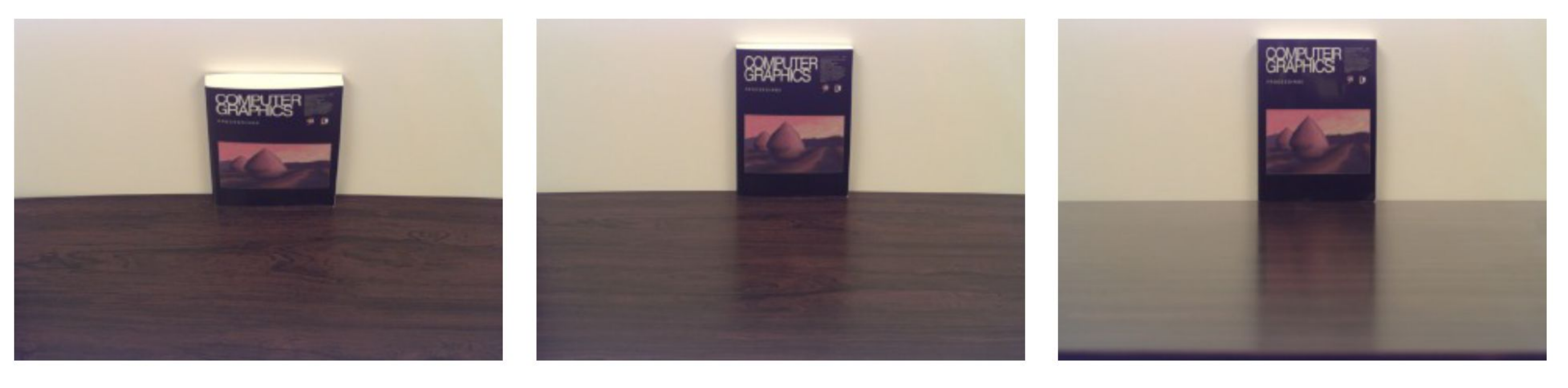


Perpendicular (overhead) view:
high transmission, low reflection

Parallel (grazing) view:
high reflection, low transmission

Reflection vs. Transmission

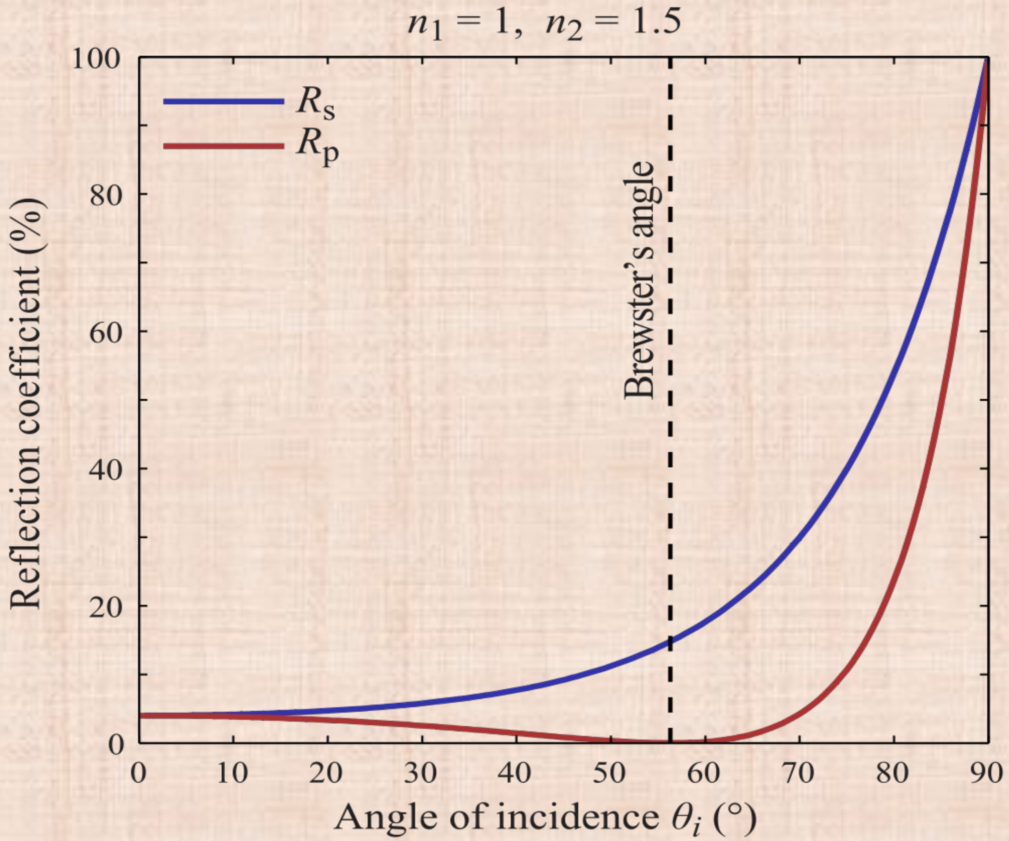
- For opaque objects (without transmission) reflection still behaves similarly



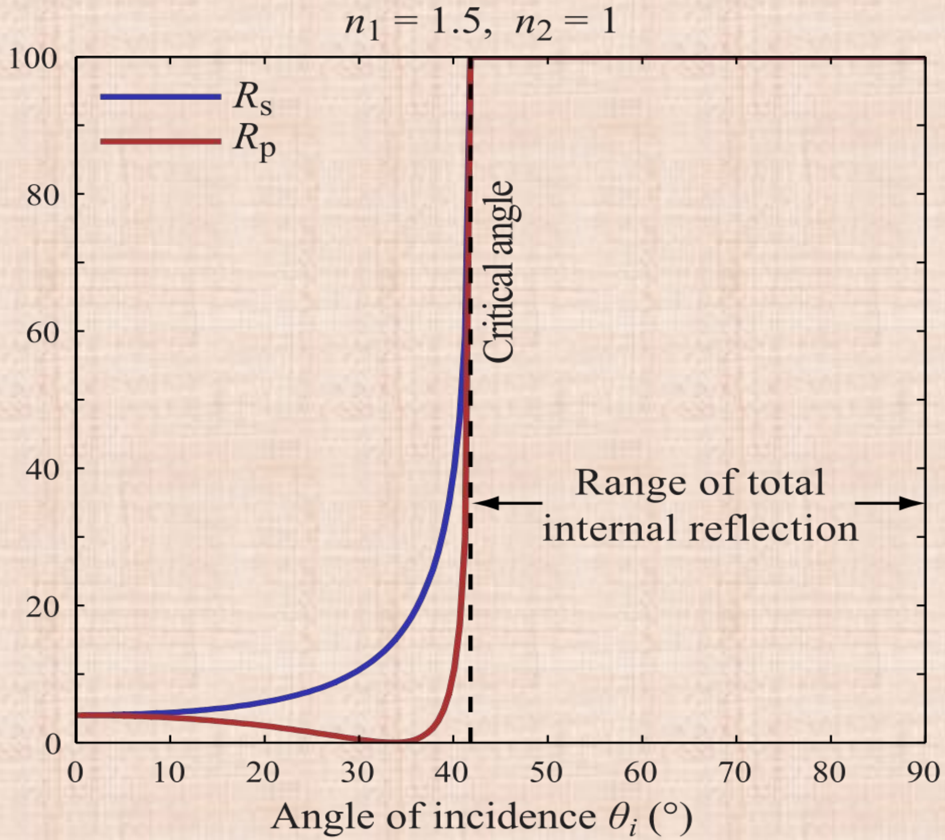
As the viewing angle changes from overhead to a more grazing angle (from left to right), the amount of reflection off of the table increases (and one can better see the book's reflection)

Fresnel Equations

- The proportion of reflection gradually increases as the viewing angle goes from perpendicular (coincident with the normal) to parallel (orthogonal to the normal)



Light entering a denser material
(e.g. from air into water)



Light leaving a denser material
(e.g. exiting water into air)

Fresnel Equations

- Light is polarized into 2 parts based on whether the plane containing the incident, reflected, refracted rays is parallel (p-polarized) or perpendicular (s-polarized) to the electric field
- The Fresnel equations give the fraction of light reflected as:

$$R_p = \left| \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \right|^2 \qquad R_s = \left| \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right|^2$$

- Transmission (if it occurs) is typically calculated as the remaining light:

$$T_p = 1 - R_p \qquad T_s = 1 - R_s$$

- For <u>unpolarized</u> light (a typical assumption in ray tracing), we assume:

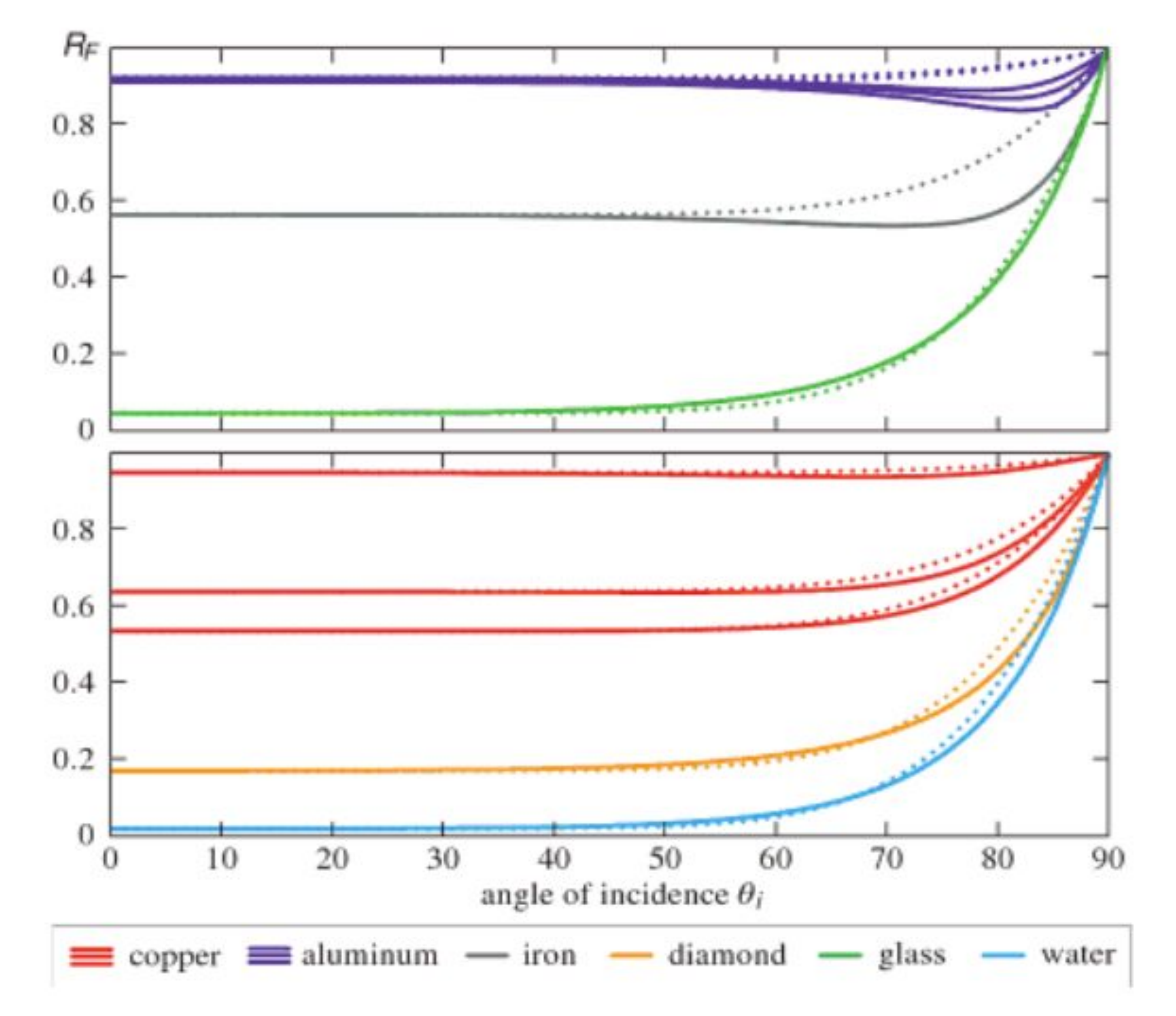
$$R = \frac{R_p + R_s}{2} \qquad T = 1 - R$$

Schlick's Approximation

- Approximate reflection via:

$$R(\theta_i) = R_0 + (1 - R_0)(1 - \cos\theta_i)^5$$

$$R_0 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

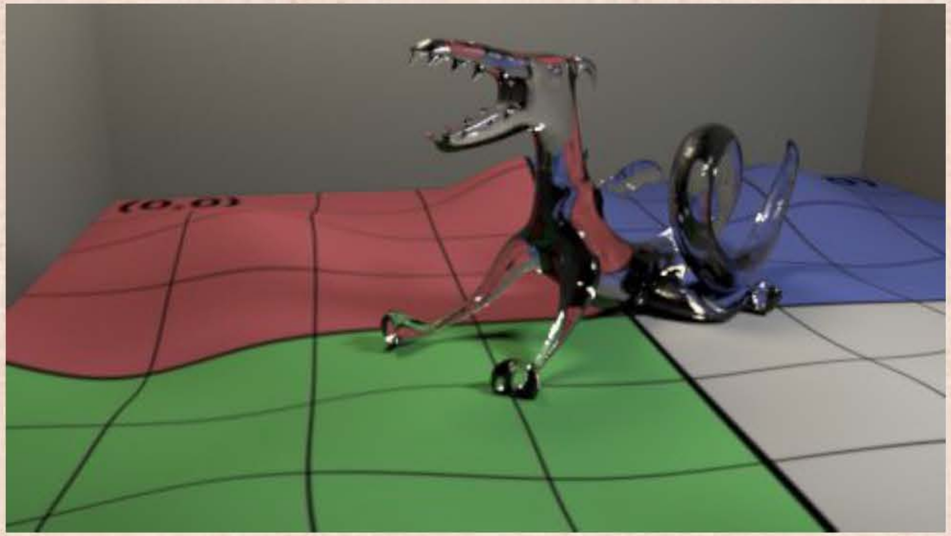


Fresnel (solid lines)
Schlick (dotted lines)

Conductors vs. Dielectrics

- Conductors of electricity (e.g. metals) mostly reflect light (low absorption, zero transmission)
- The amount of light reflected doesn't change much with the viewing angle (e.g. aluminum varies from 90% to 100% as the viewing angle changes from overhead to grazing)
- Thus, can approximate k_r as independent of viewing direction (for conductors)
- In contrast, dielectrics (e.g. glass) have significant variance in reflection vs. transmission with viewing angle

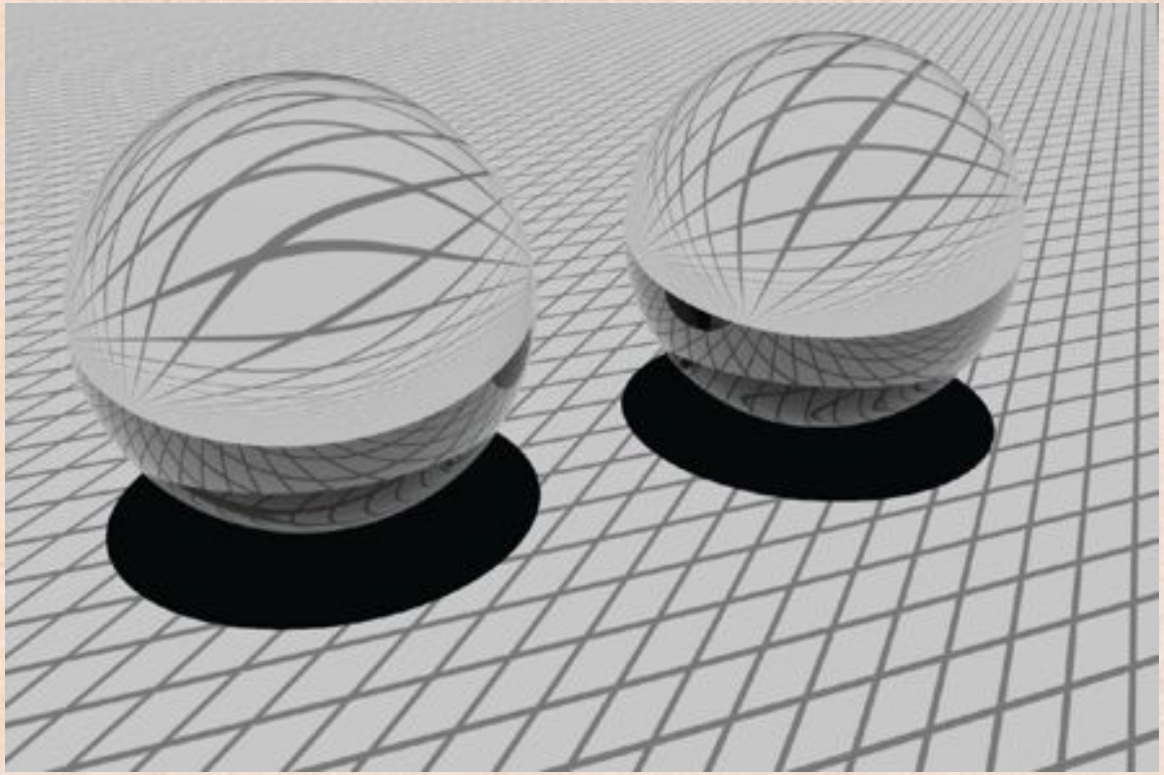
Conductor



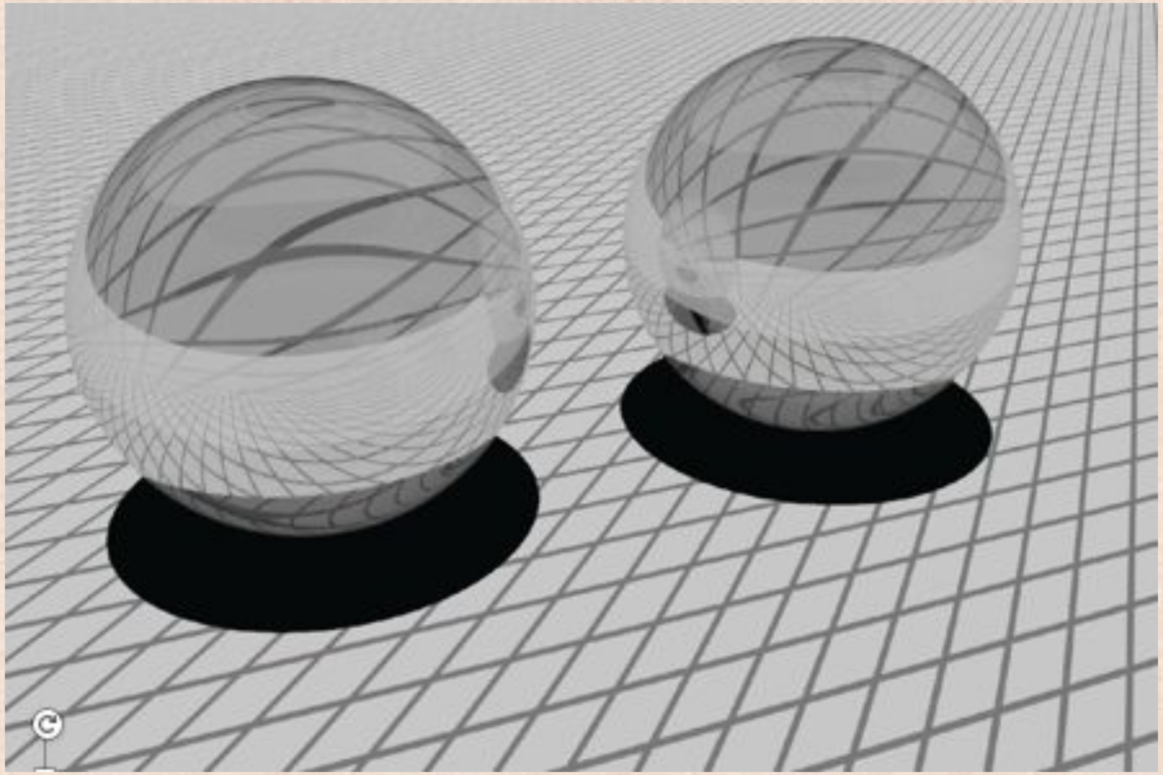
Dielectric

Curved Surfaces

- The viewing angle can vary (from perpendicular to parallel) across the surface of an object
- Thus, the amount of reflection vs. transmission can similarly vary across the object
- Capturing this is especially important for visual realism when rendering dielectrics



Correct reflection vs. transmission
(based on viewing angle)



Incorrect reflection vs. transmission
(with no dependence on viewing angle)

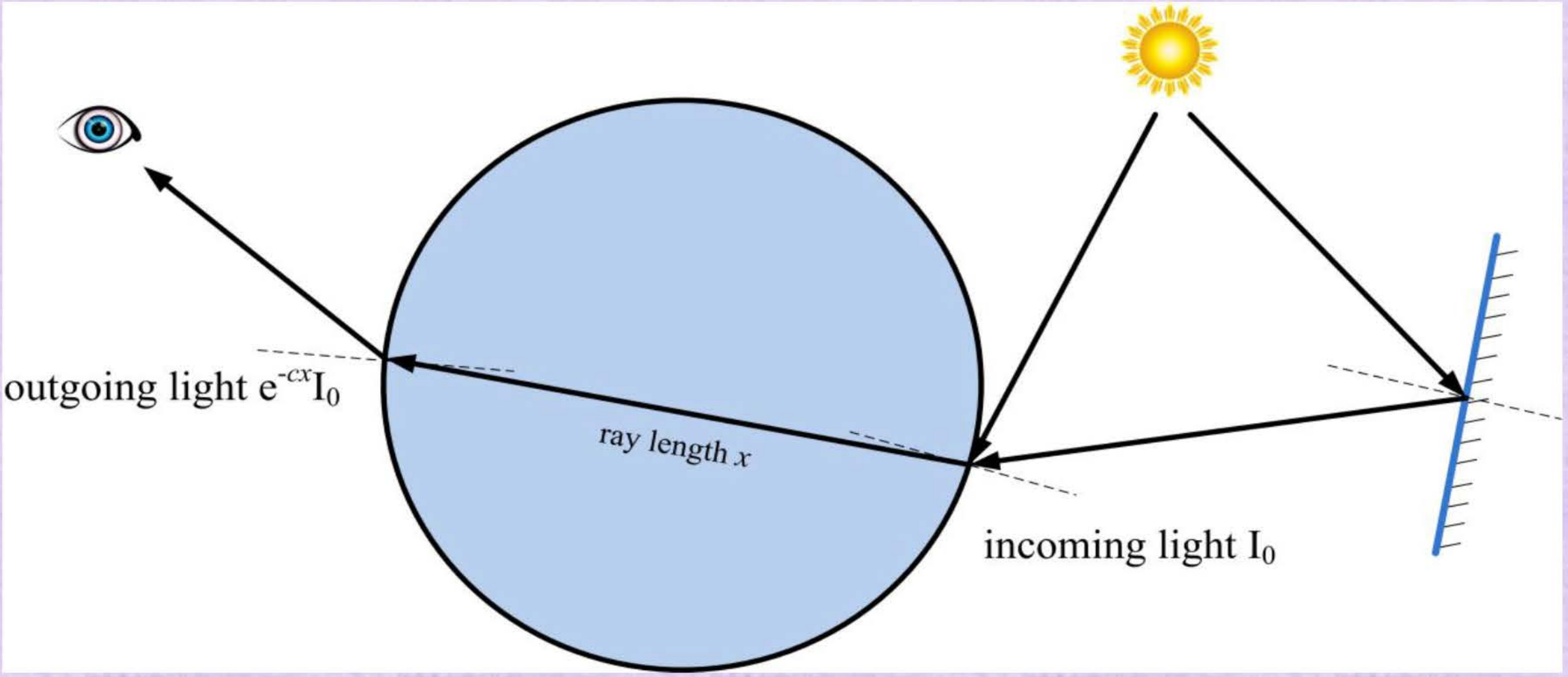
Attenuation

- Light is attenuated as it travels through media (as the light is both absorbed and scattered)
- This attenuation effect is stronger over longer distances
- Different colors are attenuated at different rates

- For example:
  - Shallow water is clear (almost no attenuation)
  - Deeper water attenuates all the red light and looks bluish-green
  - Even deeper water attenuates all the green light too, and looks blue
  - Eventually all the light attenuates, and the color ranges from blackish-blue to black

Beer's Law

- If the media is homogeneous, attenuation along a ray can be described by Beer's Law
- Light with intensity I is attenuated over a distance x via the Ordinary Differential Equation (ODE): $\frac{dI}{dx} = -cI$ where c is the coefficient of attenuation
- The exact solution to this ODE is $I(x) = I_o e^{-cx}$ where I_o is the original unattenuated amount of light

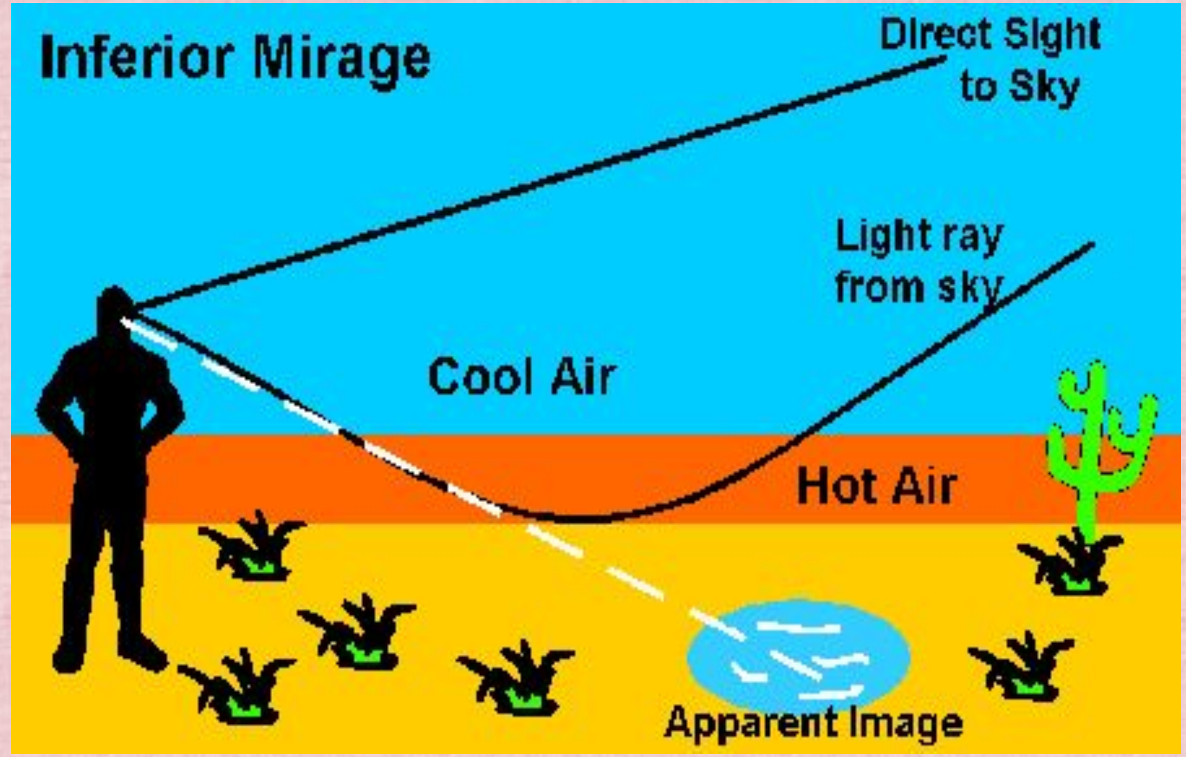


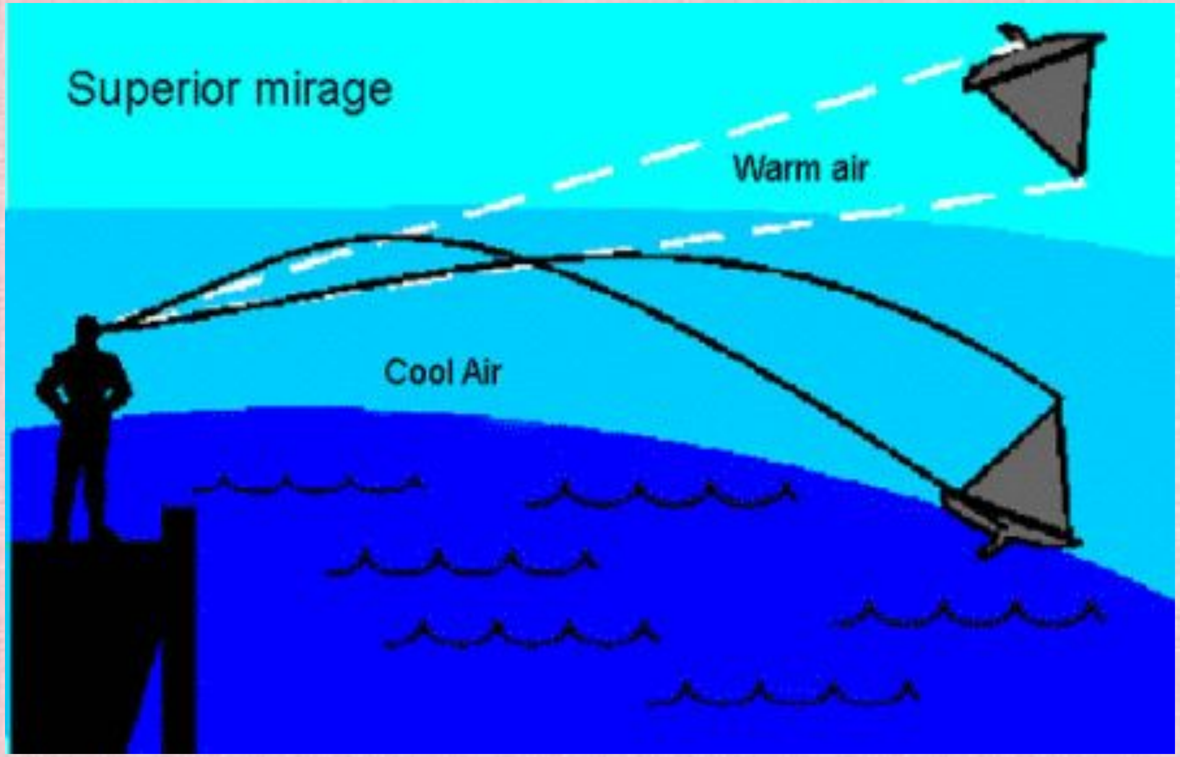
Beer’s Law

- The color of a transparent object is described by three independent attenuation coefficients, one for each color channel (i.e. c_R, c_G, c_B)
- Shadow rays are also attenuated

Atmospheric Refraction

- Light <u>continuously</u> bends (following a curved path) as it passes through varying temperature gases, such as the atmosphere
- This is due to continuous variations in density, and thus similar variations in the refractive index





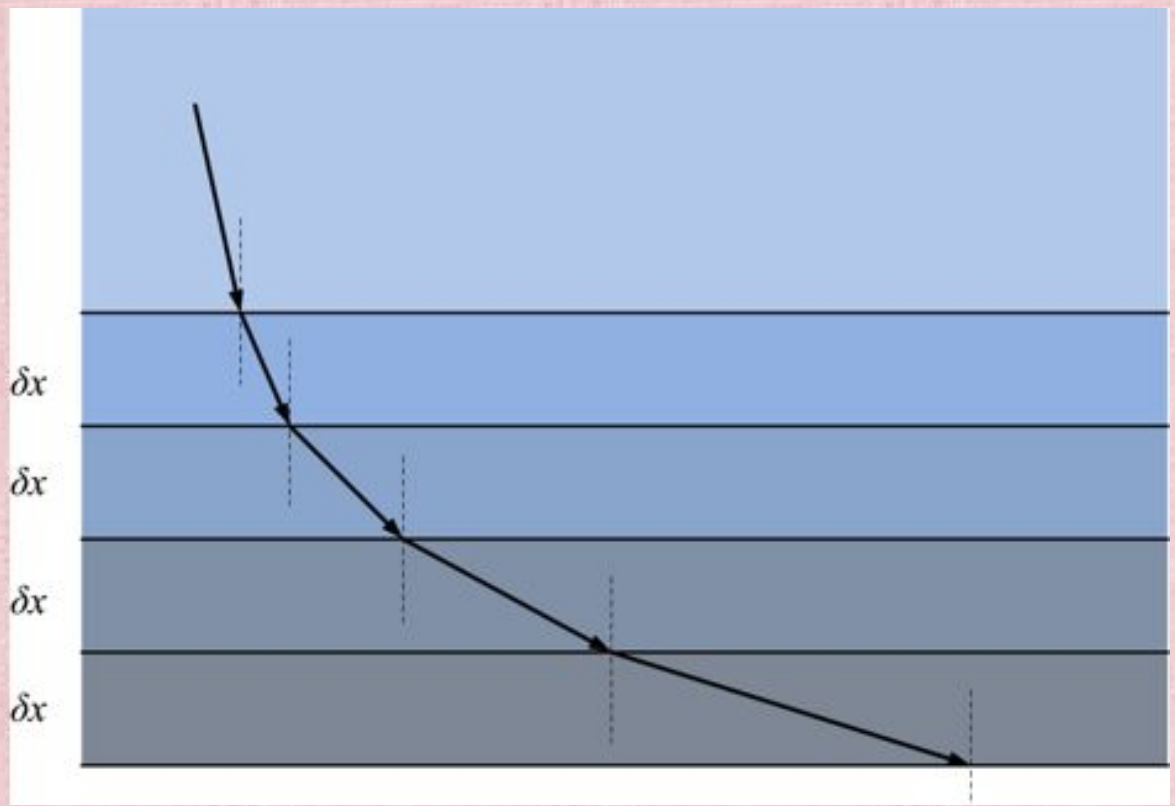
Inferior Mirage

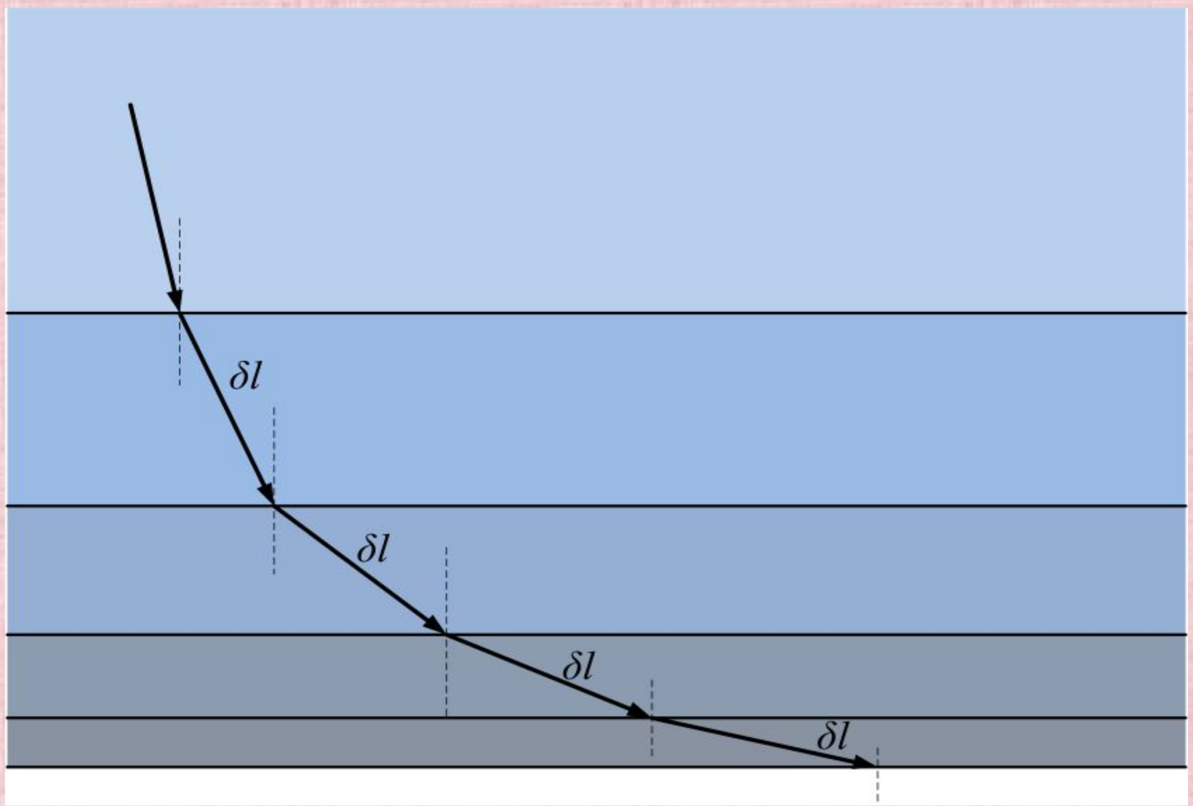
Superior Mirage

Superior Mirage (March 2021, England)

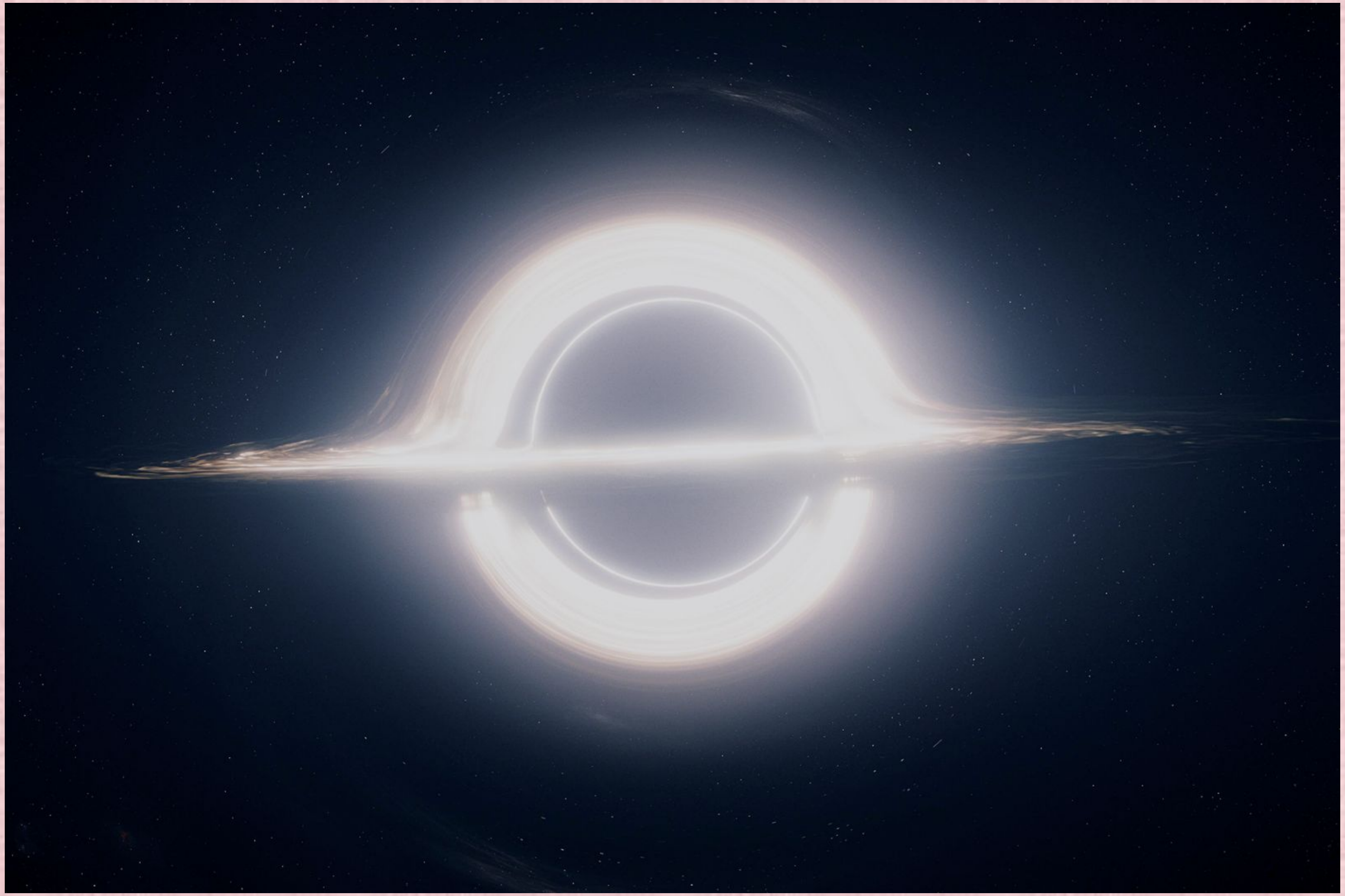
Atmospheric Refraction

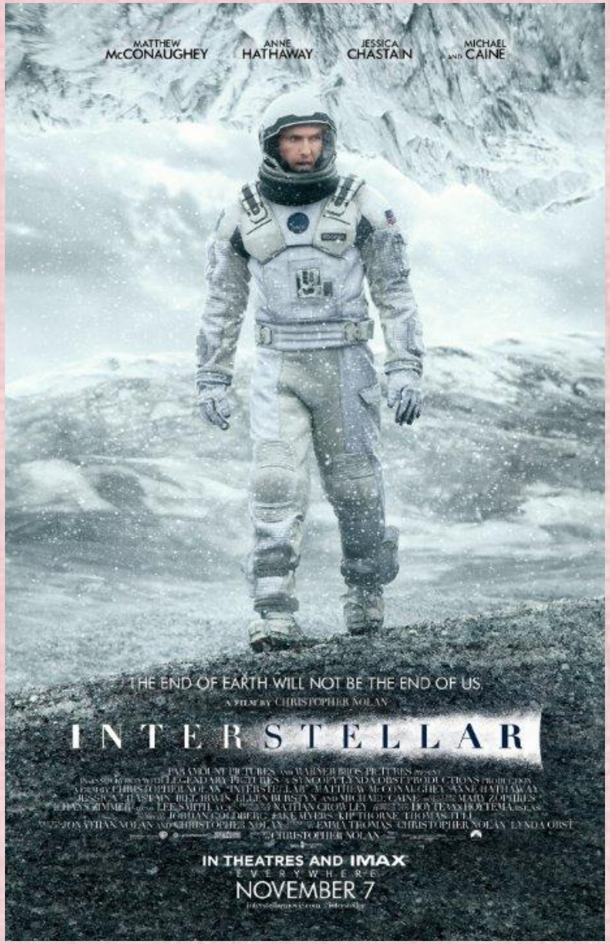
- Bend the rays as they go through varying air densities
- Change the light direction between every interval in the vertical direction (left) or along the ray direction (right)





Gravity can bend light too!





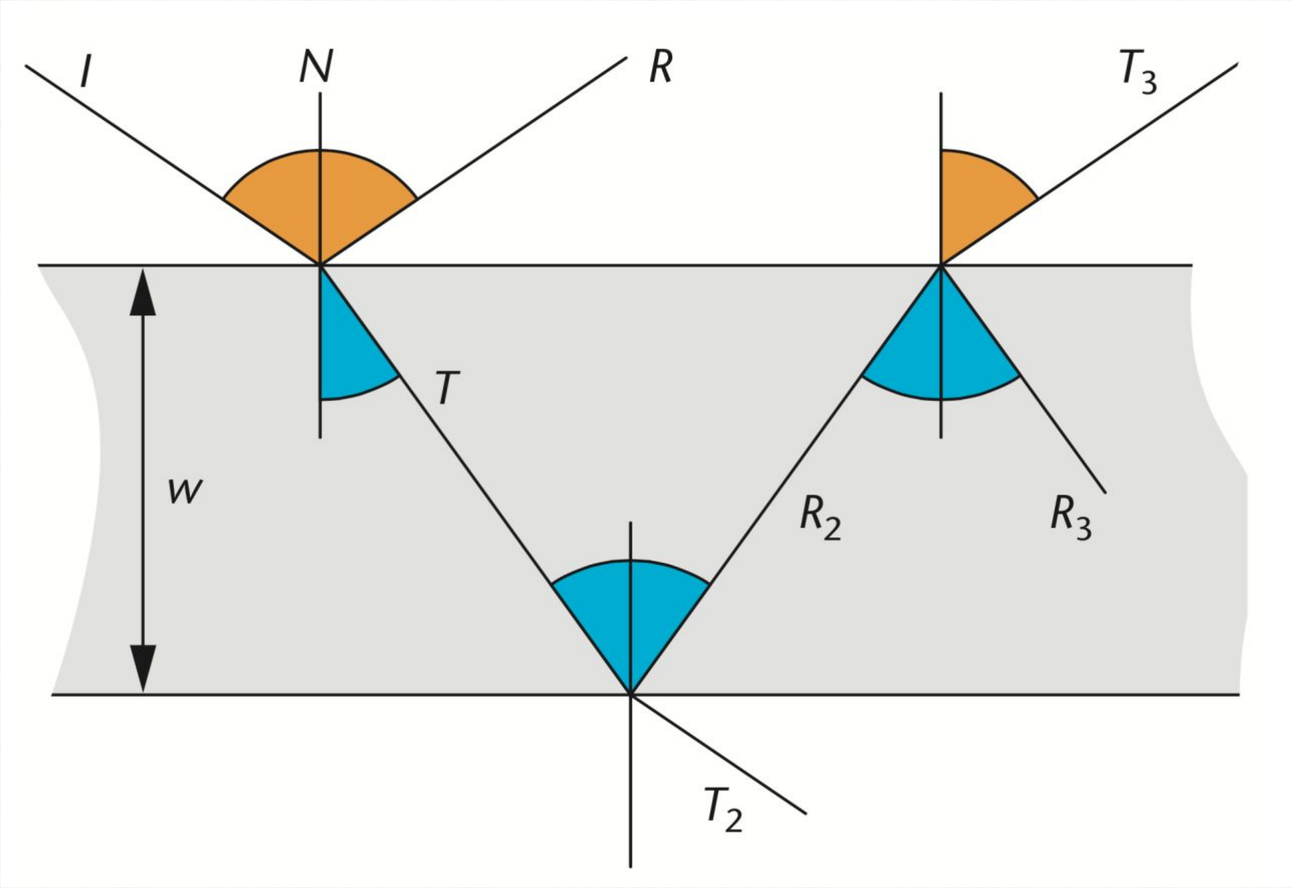
http://www.wired.com/2014/10/astrophysics-interstellar-black-hole/
https://www.businessinsider.com/interstellar-anniversary-learned-about-black-holes-2019-11

Iridescence

- Surface can gradually change color as the viewing angle or the lighting change

Iridescence

- Various light waves are emitted in the same direction giving constructive and destructive interference



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