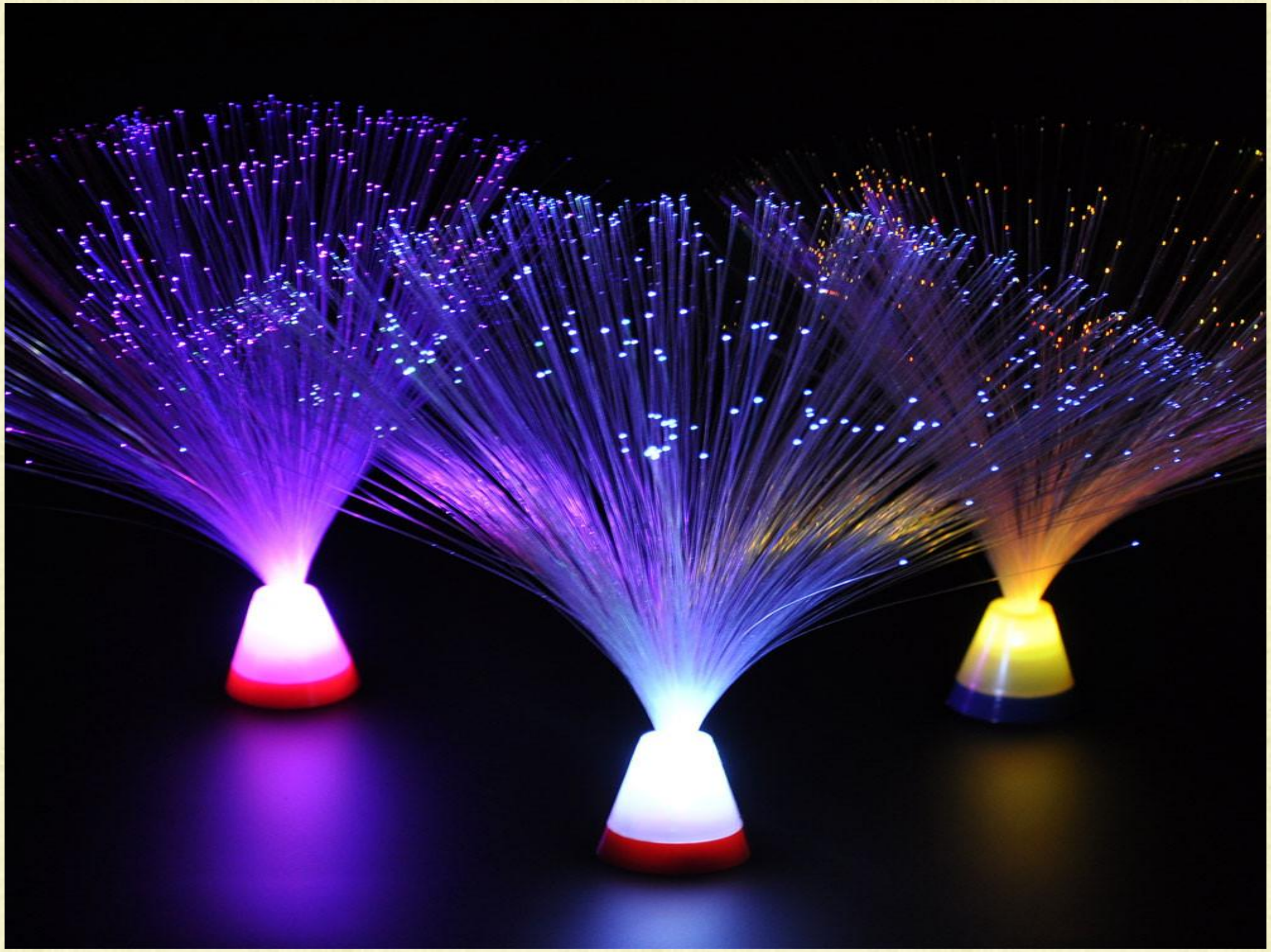
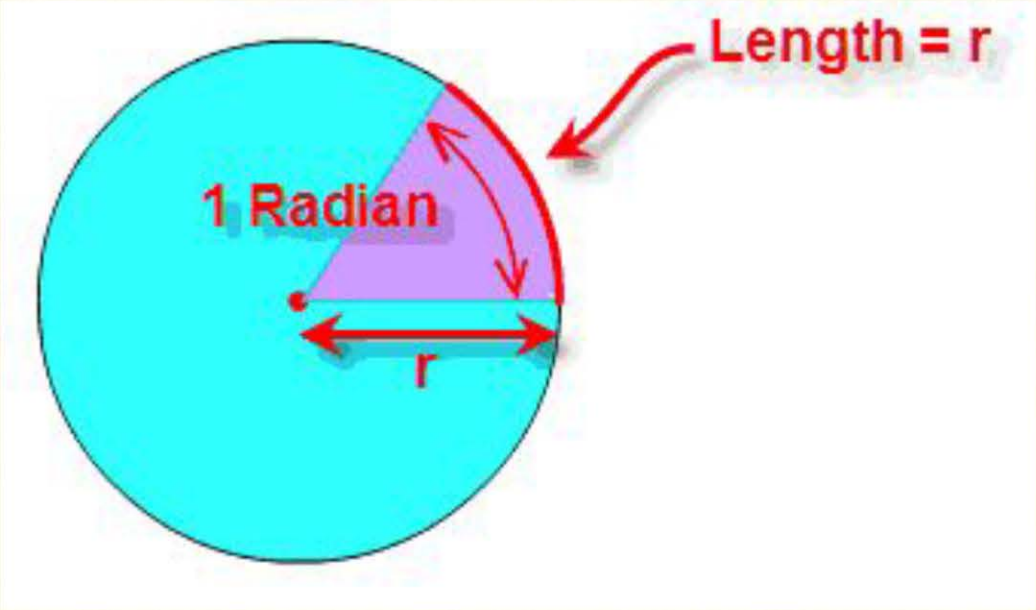
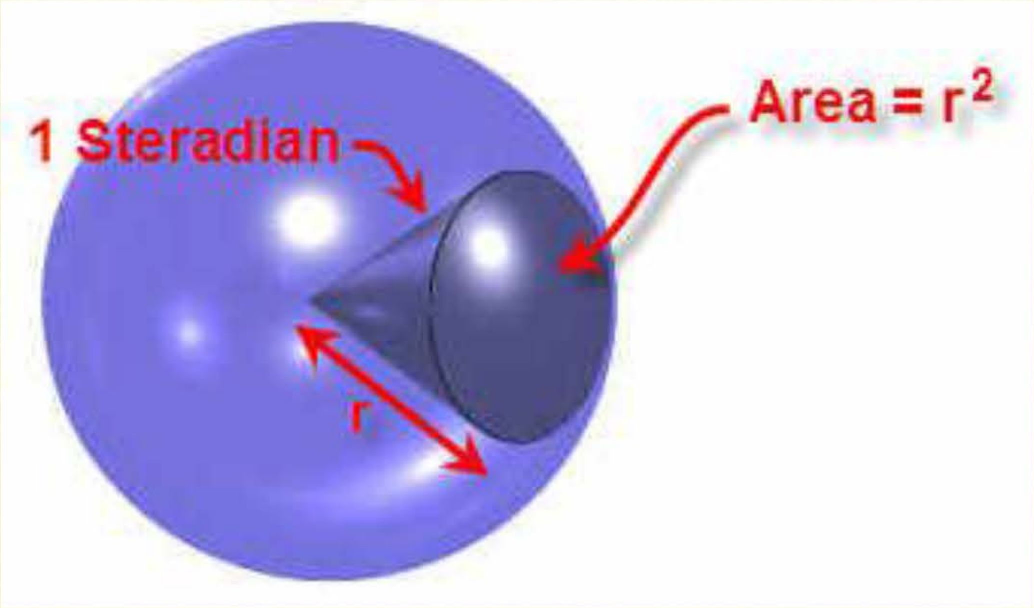
Optics



Solid Angle

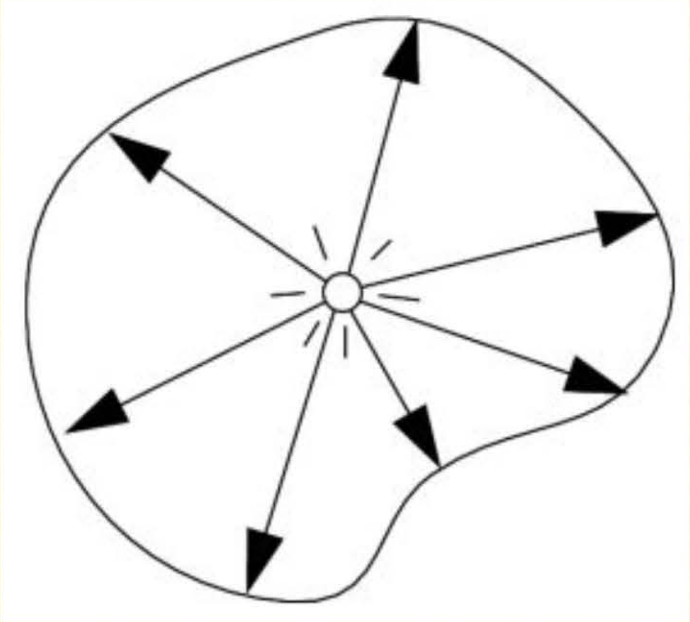
- A two-dimensional angle in 3D defined by a point and a surface patch, measured by a dimensionless unit called a steradian (sr)
- An angle has $\theta = \frac{l_{arc}}{r}$, while a solid angle has $\omega = \frac{A_{on\,sphere}}{r^2}$
- The circumference of a circle is $C = 2\pi r$, while the surface area of a sphere is $4\pi r^2$
- A circle has a total of 2π radians, while a sphere has a total of 4π steradians



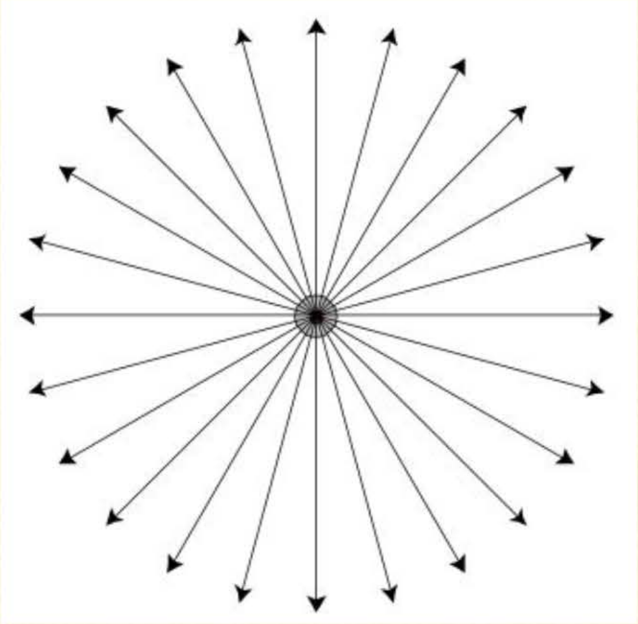


Radiant Intensity of a Light Source

- Power per unit solid angle $I(\omega) = \frac{d\Phi}{d\omega}$
  - where Φ is the light source power (in watts = joules per second)
- Anisotropic light sources: radiant intensity I varies across the light
  - and is thus a function of steradian ω
- Isotropic point light: integrate $d\Phi = I d\omega$ to obtain $\Phi = \int_{sphere} I d\omega = 4\pi I$



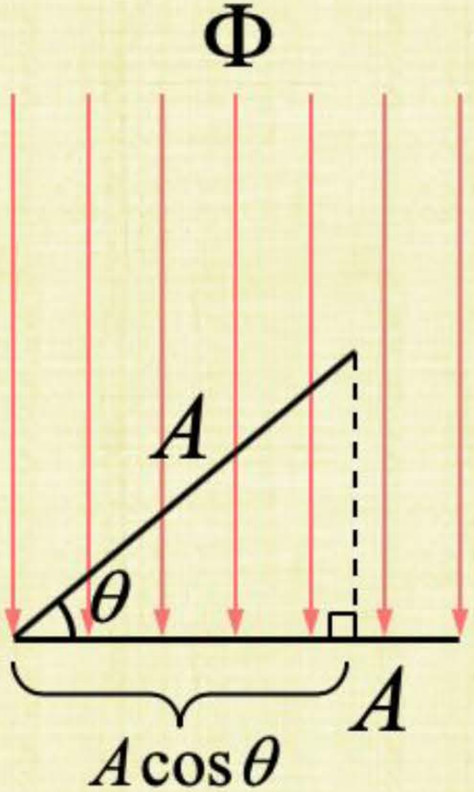
anisotropic light source

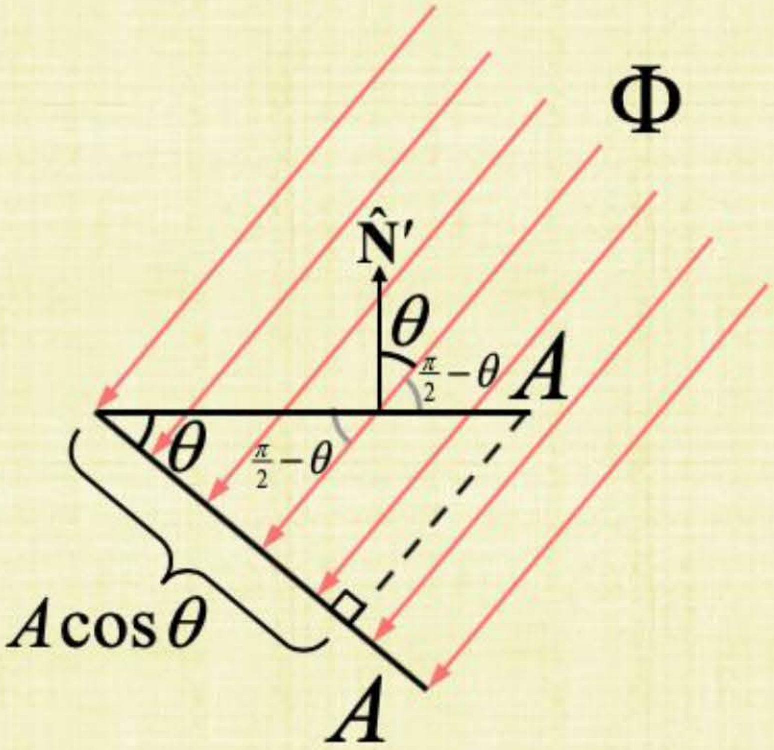


isotropic point light

Irradiance on a Surface

- Power per unit surface area $E = \frac{d\Phi}{dA}$
- Given $E = \frac{\Phi}{A}$, note that $E_{tilted} = \frac{\left(\frac{A\cos\theta}{A}\right)\Phi}{A} = E\cos\theta$
- Irradiance <u>decreases as you tilt</u> the surface, since less photons hit per unit surface area

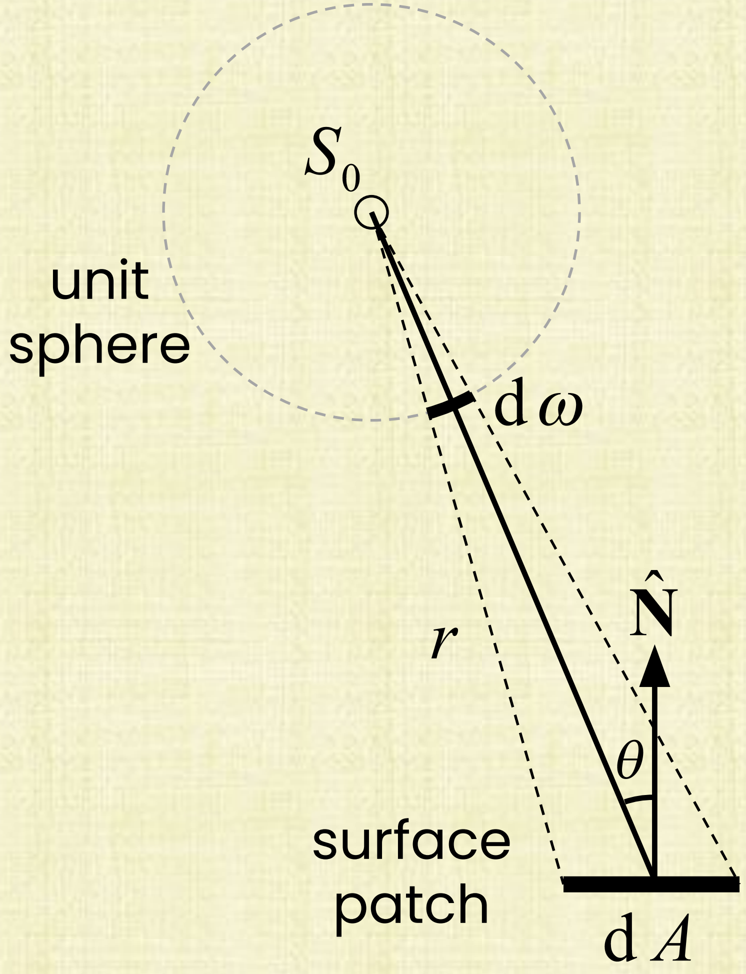




Solid Angle vs. Area

- The relationship between solid angle and cross-sectional area is $d\omega = \frac{dA_{sphere}}{r^2} = \frac{dA\, cos\theta}{r^2}$,

since the area of the orthogonal cross section is $dA\ cos\theta$ (see previous slide)



Summary

Radiant Intensity:

- total light power (exiting a light) per unit solid angle
- Measure of how strong a (point) light source is

Irradiance:

- total light power (hitting a surface) per unit surface area
- Measure of how much light is hitting a surface
- Varies based on distance from the light and the tilting angle of the surface (last two slides)
- The farther away and more tilted a surface area patch is, the smaller the solid angle of light that hits that surface area patch

Area Lights

- Light power is emitted per unit area (not from a single point)
- The emitted light goes in various directions, or solid angles

- Approximate an area light by breaking it up into small area chunks
- Each area chunk emits light into each of the solid angle directions
  - i.e. radiant intensity per area chunk
- Each direction has a cosine term similar to irradiance

- Radiance – radiant intensity per area chunk

$$L = \frac{dI}{dA\cos\theta} \quad \left(= \frac{d^2\Phi}{d\omega dA\cos\theta} = \frac{dE}{d\omega\cos\theta} \right)$$

Incoming Light

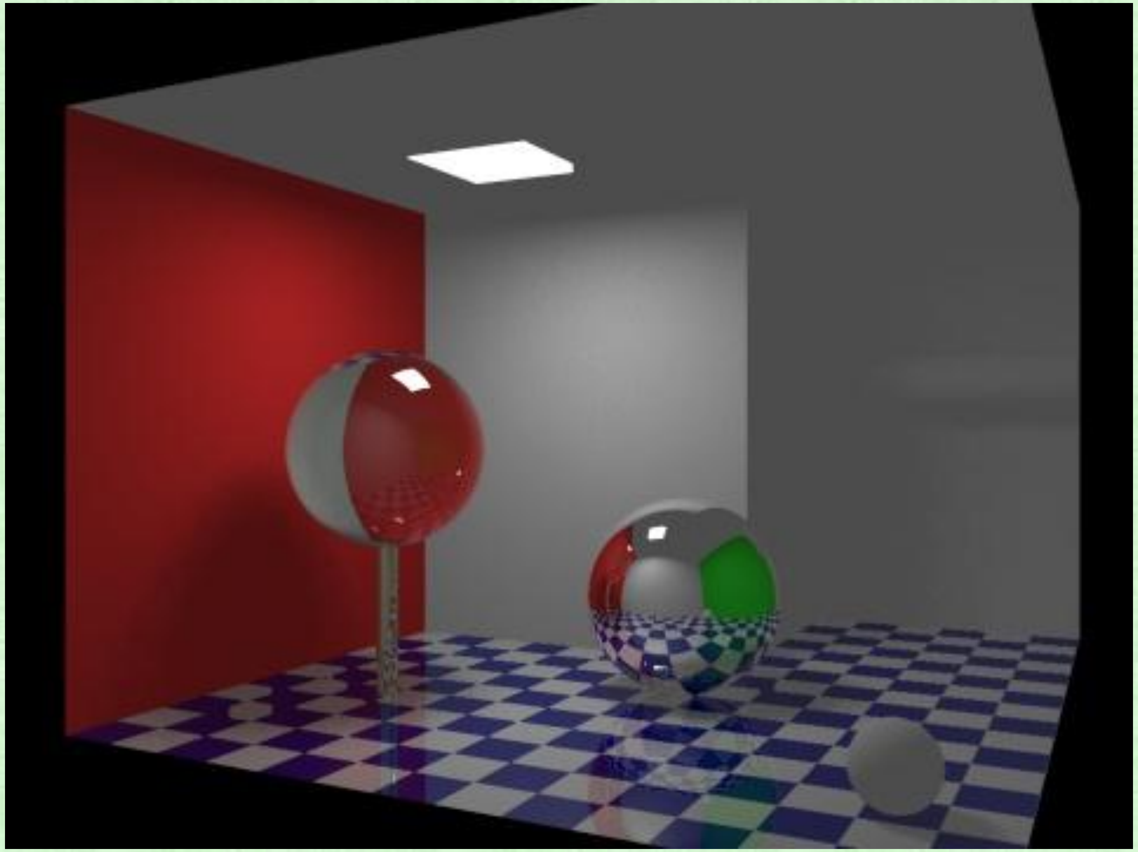
- Light doesn't just come from light sources, but from all visible objects in the world
- Each area chunk of each object acts as a source of light
- Here, the tree is shining light (with the color/brightness of the tree) onto the car

Incoming Light

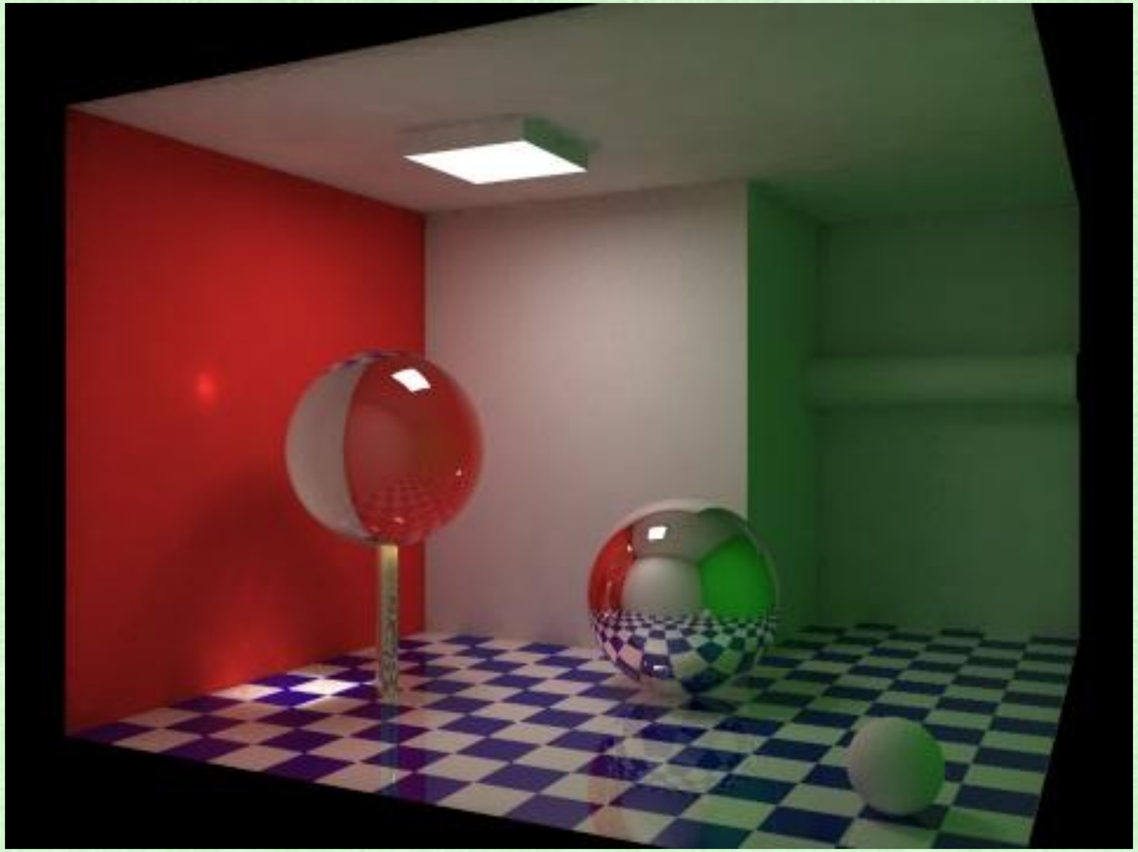
- Light doesn’t just come from light sources, but from all visible objects in the world
- Each area chunk of each object acts as a source of light
- Here, the red paper is shining red light onto the statue/face

Incoming Light

- Light doesn't just come from light sources, but from all visible objects in the world



using light only from the light source



using incoming light from all directions

Measuring Incoming Light

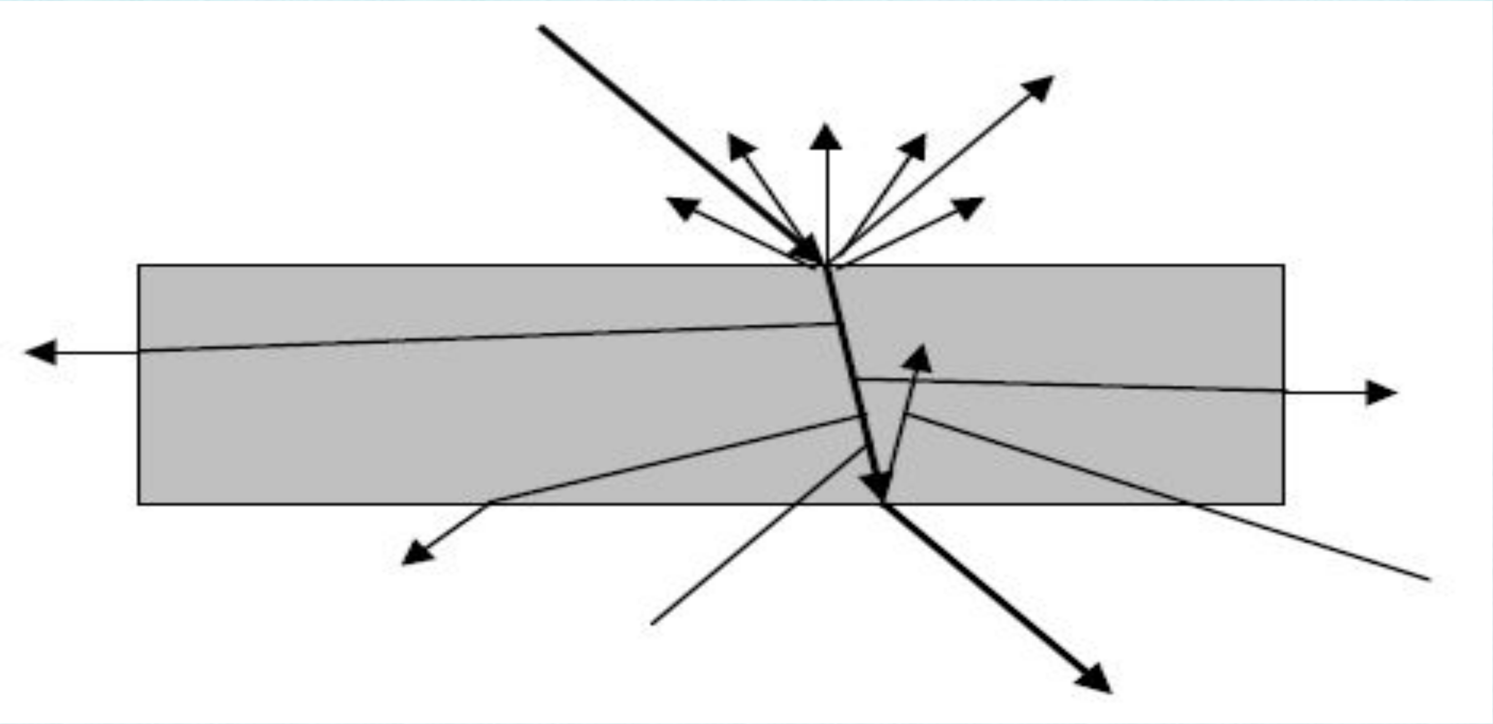
- Place a small reflective chrome sphere (a light probe) somewhere in the world
- Photograph it, in order to measure/record the intensity of light incoming from all directions

Measuring Incoming Light

- Understanding/measuring the incoming light allows one to render a new synthetic object in the original scene (with realistic lighting)

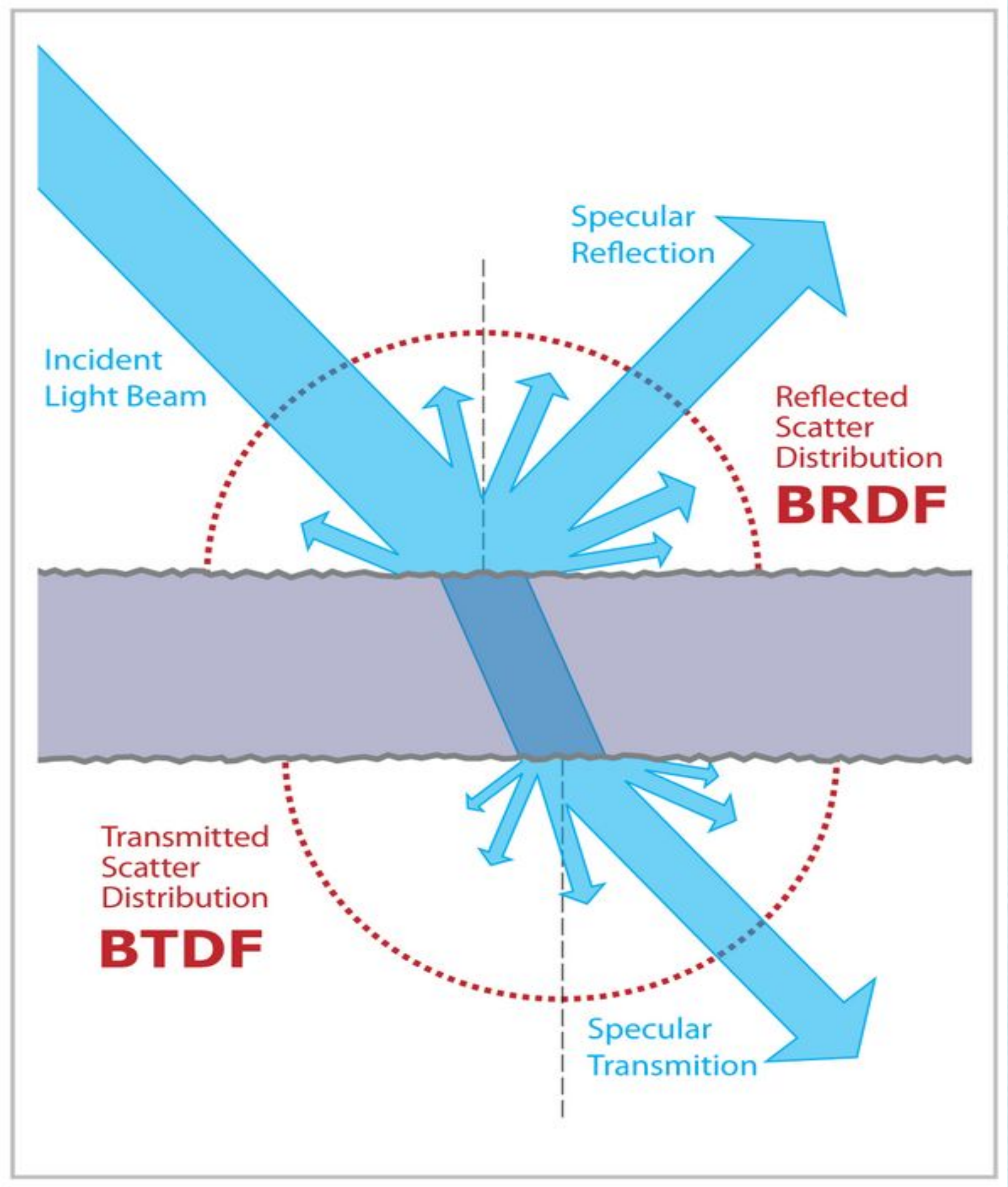
Light/Object Interaction

- When light hits a material it may be absorbed, reflected, or transmitted
- Light may be absorbed or scattered as it passes through a material
- Etc.

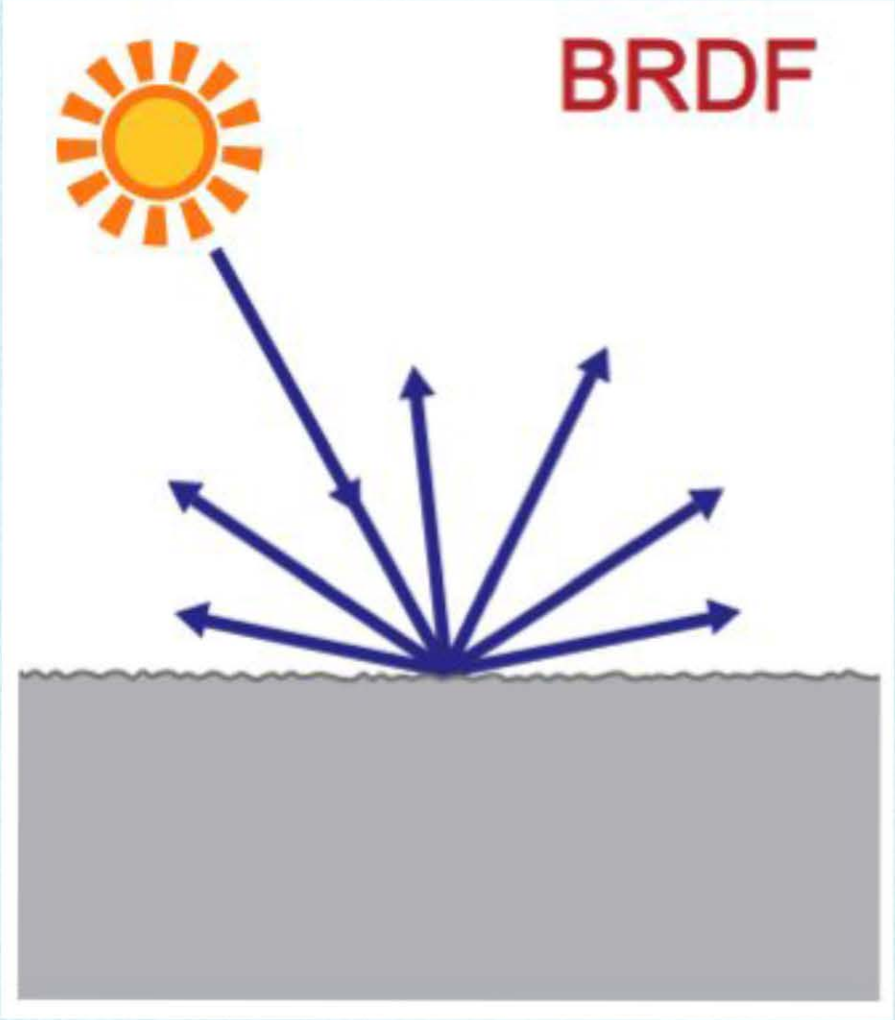


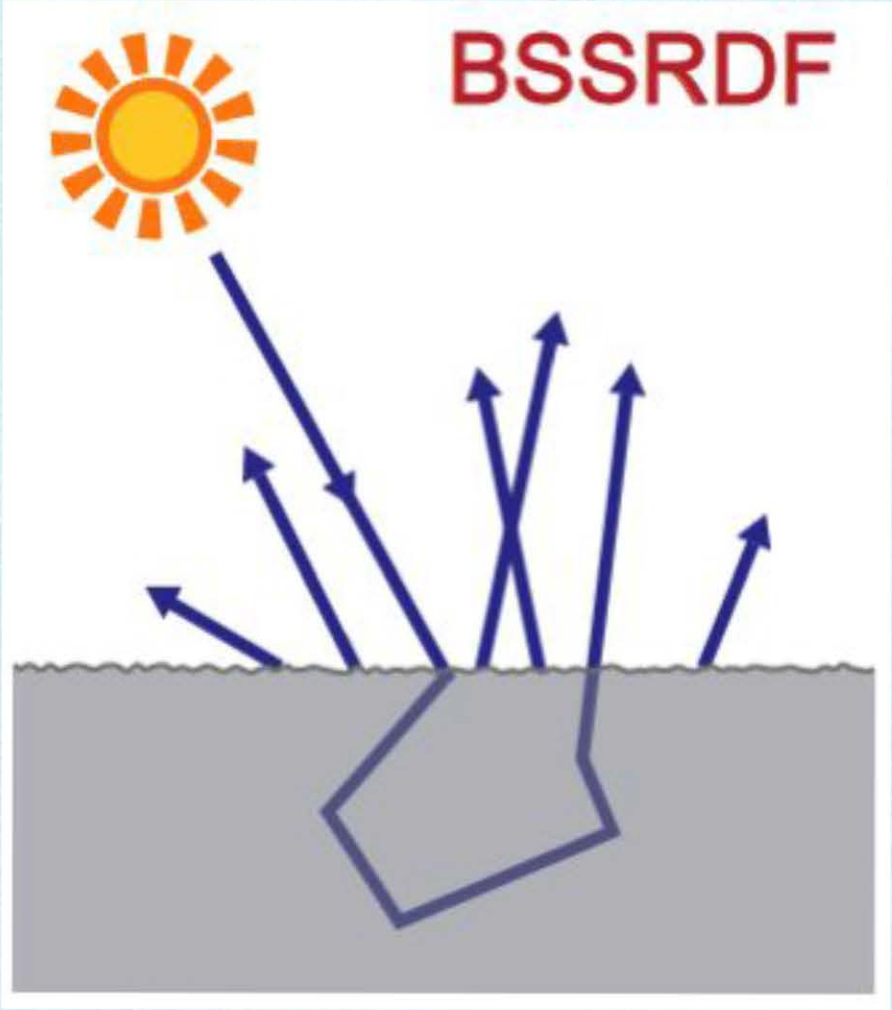
Engineering Approximations

- <u>BRDF</u>
  - Bidirectional Reflectance Distribution Function
  - models how much light is <u>reflected</u>
- <u>BTDF</u>
  - Bidirectional Transmittance Distribution Function
  - models how much light is <u>transmitted</u>
- <u>BSSRDF</u>
  - Bidirectional Surface Scattering Reflectance Distribution Function
  - <u>combined</u> reflection/transmission model



Opaque (BRDF) vs. Translucent (BSSRDF)





Opaque (BRDF) vs. Translucent (BSSRDF)

Opaque (BRDF) vs. Translucent (BSSRDF)

Opaque (BRDF) vs. Translucent (BSSRDF)

BRDF

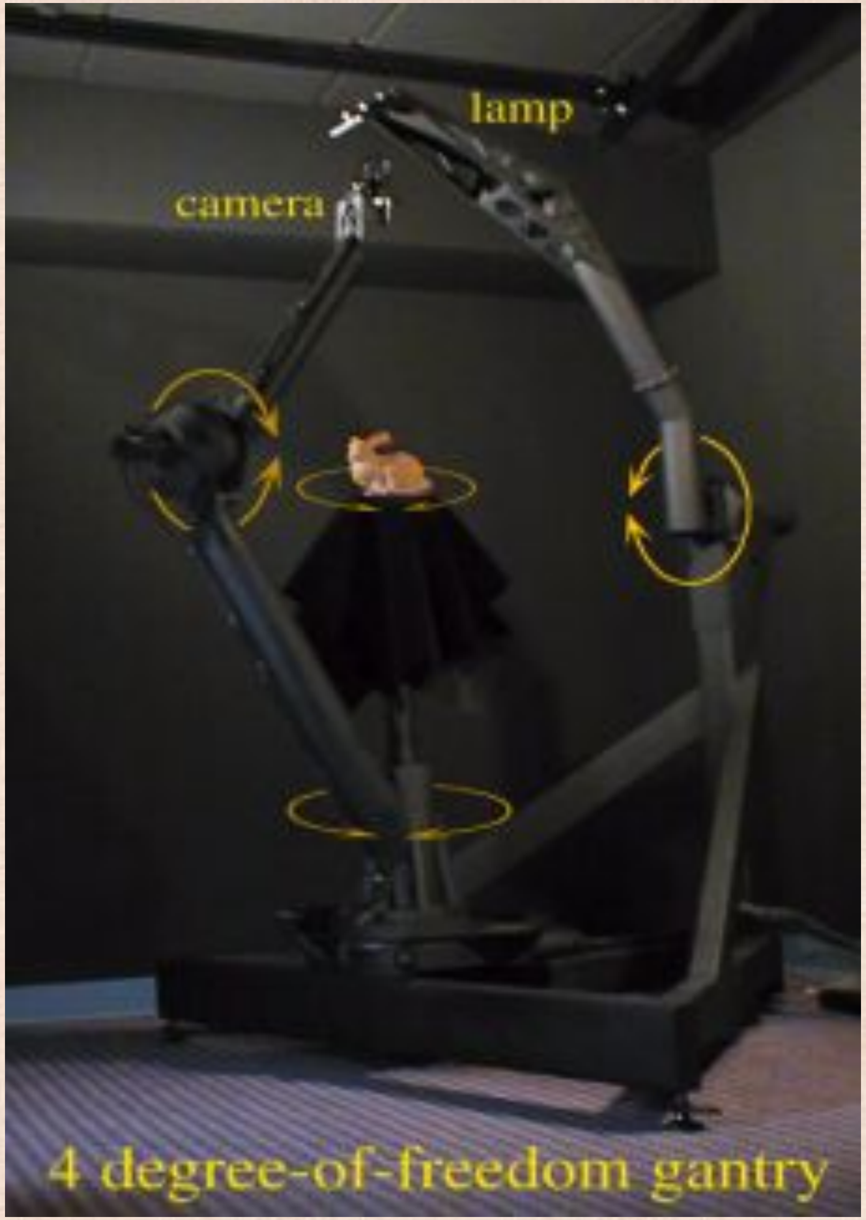
- $BRDF(\lambda, \omega_i, \omega_o, u, v)$
  - λ is the wavelength, but we will use R, G, B as usual (so 3 BRDFs, one for each channel)
  - (u, v) are the coordinates on the object's surface, but we will modulate with a texture
  - $\omega_i(\theta_i, \phi_i)$ and $\omega_o(\theta_o, \phi_o)$ are the incoming/outgoing light directions (parameterized by the 2D surface of a hemisphere)

- Thus, we consider $BRDF_R(\omega_i, \omega_o)$, $BRDF_G(\omega_i, \omega_o)$, $BRDF_B(\omega_i, \omega_o)$ which are each functions of 4 variables $\theta_i, \phi_i, \theta_o, \phi_o$
  - i.e. 4D functions

- Relates the incoming light that hits a surface patch (irradiance E_i) to the outgoing light emitted from the surface patch acting as if it were an area light (radiance L_o)
- That is, $BRDF(\omega_i, \omega_o) = \frac{dL_o(\omega_o)}{dE_i(\omega_i)}$

Measuring/Approximating a BRDF

- 4D BRDF data can be acquired with a gonioreflectometer to obtain a 4D table of values
  - Alternatively, some simple analytical models:
    - Blinn-Phong Model – simplest and general purpose (plastic)
    - Cook-Torrance Model – better specular (metal)
    - Ward Model – anisotropic (brushed metal, hair)
    - Oren-Nayar Model – non-Lambertian (concrete, plaster, the moon)



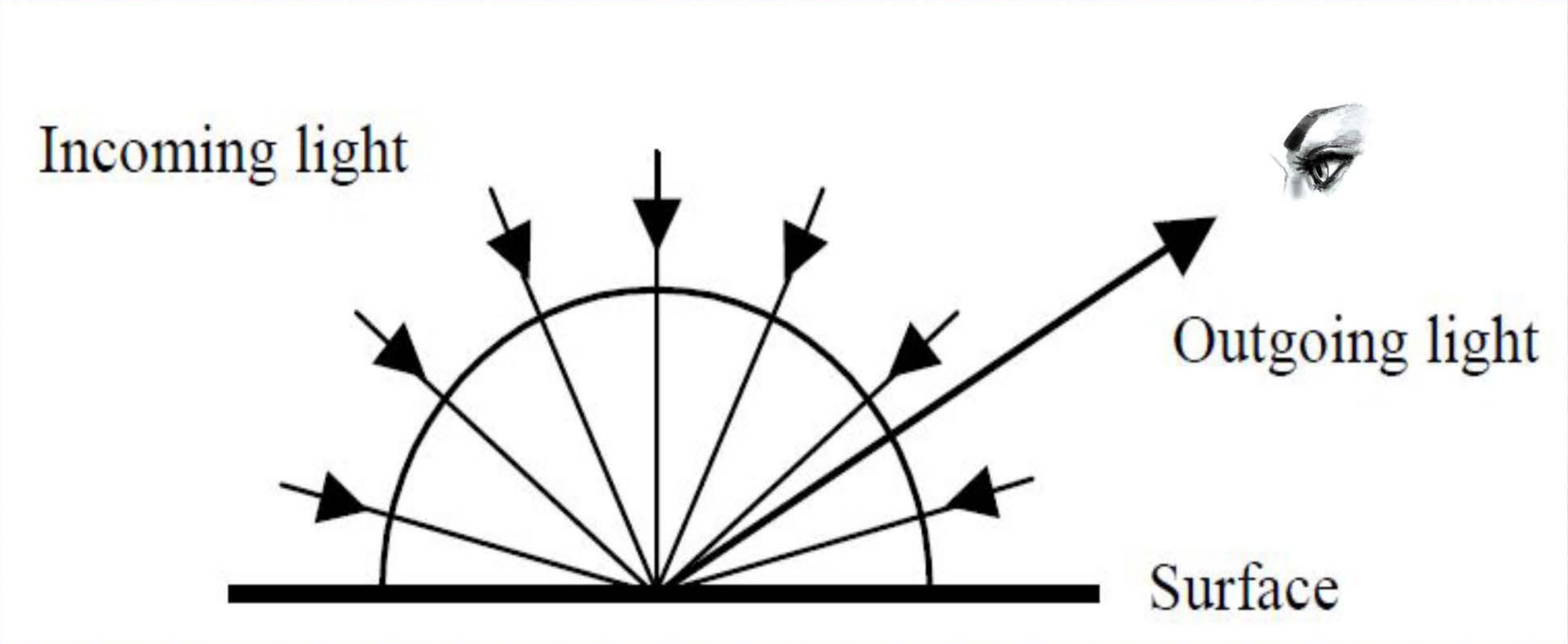
Lighting Equation

- Given a point on an object:
  - Light from every incoming direction ω_i hits that point
  - For each incoming direction ω_i, light is reflected outwards in every direction ω_o
  - The BRDF indicates how much light is reflected in each of the outgoing directions ω_o given an incoming direction ω_i

- Since light is reflected in all outgoing directions, we can all see that point on the object
- But we all see different light (so it can, and often does, look different to all of us)
- To render a synthetic scene, need to figure out what light each pixel of the camera's film sees

Lighting Equation

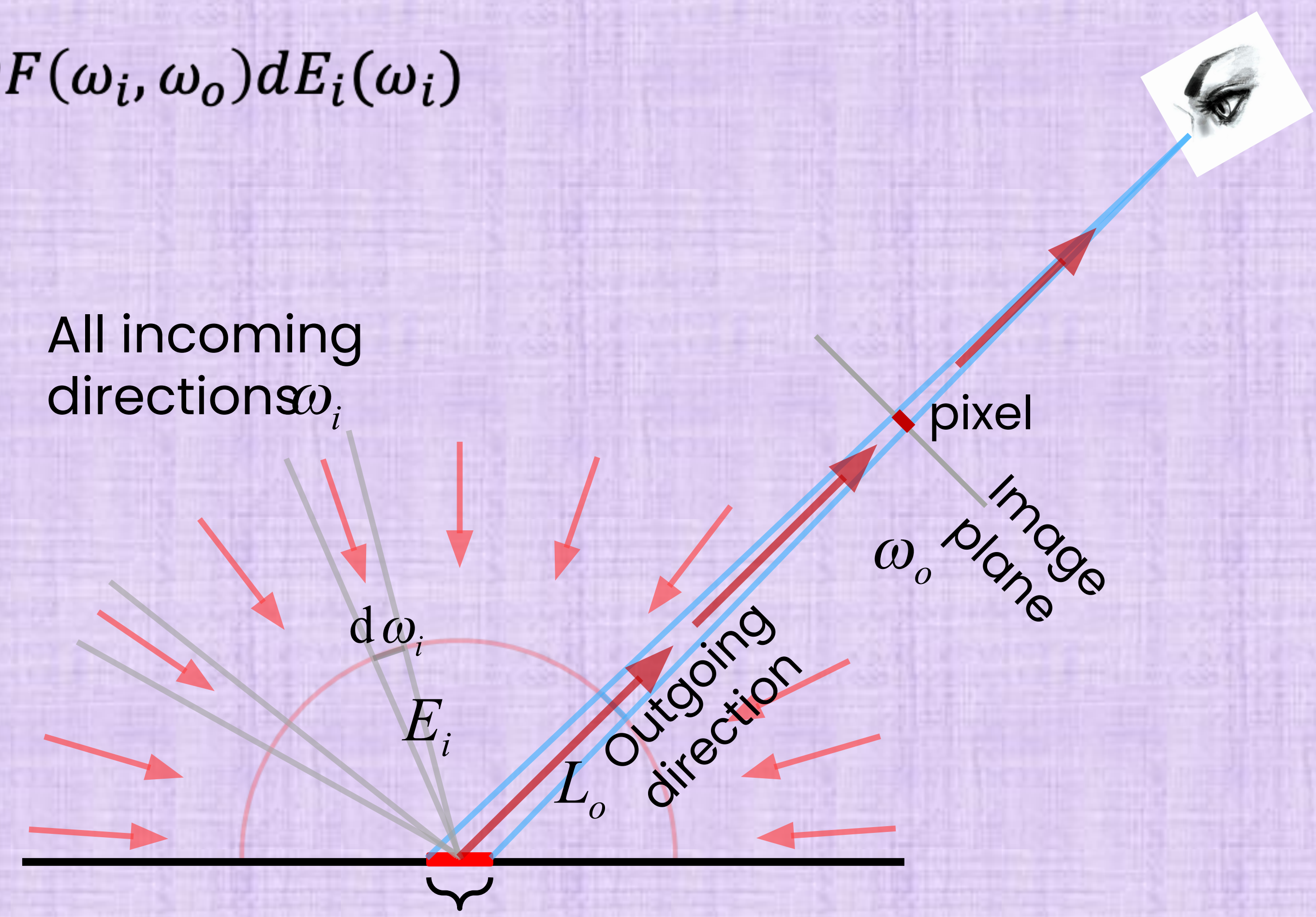
- The total amount of light reflected in an outgoing direction is the integral of the amount of light reflected in that direction due to light from every incoming direction: $L_o(\omega_o) = \sum_{i \in in} L_{o\ due\ to\ i}(\omega_i, \omega_o)$
- The BRDF gives each of the $L_{o\ due\ to\ i}(\omega_i, \omega_o)$



Lighting Equation

- For each pixel, integrate the BRDF across all incoming directions for every point in the projected area (which acts as an area light)

$$L_o(\omega_o) = \int_{i \in in} BRDF(\omega_i, \omega_o) dE_i(\omega_i)$$



Lighting Equation

- Multiplying the BRDF by an incoming irradiance gives the outgoing radiance

$$dL_{o\ due\ to\ i}(\omega_i, \omega_o) = BRDF(\omega_i, \omega_o) dE_i(\omega_i)$$

- For more realistic lighting, we bounce light all around the scene, and it is tedious to convert between irradiance and radiance, so we use $dE = Ld\omega \cos\theta$ to obtain:

$$dL_{o\ due\ to\ i}(\omega_i, \omega_o) = BRDF(\omega_i, \omega_o) L_i d\omega_i \cos\theta_i$$

- The outgoing radiance considering the light coming from all incoming directions is:

$$L_o(\omega_o) = \int_{i \in in} BRDF(\omega_i, \omega_o) L_i \cos\theta_i \, d\omega_i$$

Pixel Color

- Irradiance measures the power per unit area hitting a pixel:

$$E_i = \int L_i cos\theta_i \, d\omega_i$$

obtained from integrating
$dE = Ld\omega cos\theta$

- Since pixels are small, we approximate this via

$$E_{pixel} \approx L_{pixel,ave} \cos\theta_{pixel} \, \omega_{pixel}$$

- If the film is small, can approximate $\cos\theta_{pixel} \approx 1$ and $\omega_{pixel} = \frac{\omega_{film}}{\# \, pixels}$, so that the radiance and the irradiance differ only by a global constant
- Thus, we may store L instead of E and scale by this constant later:

$$E_{pixel} \approx \left(\frac{\omega_{film}}{\# \, pixels}\right) L_{pixel,ave}$$