CS 161: Recitation 8 (Fall 2016)

Question 1

You are advising a growing startup that just outgrew their space in the building they have been for the last year. They are renting some additional space in a building a couple blocks away. The following principles have been agreed on for the move:

1. The company is arranged in groups, and no group should get split between two buildings.
2. We can think of the company Headquarters as its own group, and this group needs to remain in the old building.
3. There is one group who was most involved in negotiating rental of the new space, let's call them group A, they will move to the new rental.

Beyond these principles the move should be arranged to minimize total cost. While moving just group A is enough to have the rest fit in the old space, this may not be the optimal arrangement. Costs arise in a number of ways. If two groups collaborate a lot, there is a cost associated with breaking them up across buildings. For each pair of groups (B, C), we have an estimate c_{BC} for the cost incurred if B and C are not in the same building. Further, space in the new building needs to be rented, and depending on the size of each group C, there is a cost r_C for renting space for group C in the new building if they move. Give an algorithm to find the minimum cost way to split groups across the two buildings. You may assume that the numbers c_{BC} and r_C are non-negative integers, and the running time can depend polynomially on the number of groups and the maximum of the numbers c_{BC} and r_C.

Question 2

The university administration is working to set up a number of central committees for next year. There is a need for m committees $c_1, \ldots, c_m$, and assume the university has n departments. Not everyone would be equally good on any committee. A faculty member x is eligible to serve on a subset C_x for the committees, depending on their expertise. You are asked help assign faculty members to committees. As input you are given for each committee c_i the number of committee members n_i that the committee needs. Additionally, for each faculty member x you are given the set of committees C_x that x would be useful on as a member, and the department x belongs to (assume each faculty member is in only one department, i.e., x is either in the Department of Computer Science, or in Chemistry, etc., but there are no joint appointments). In addition, you are given a bound $B_d > 0$ for each department d. You need to find a committee assignment plan, that satisfies the following rules.

1. Any department can have at most one representatives on any committee (as otherwise the committee may favor the department too much).
2. No department may be overloaded with committee work, so for each department d we are given a bound B_d (depending on the size of the department), and members of department d should serve only on B_d committees in total.

3. A faculty member x can only serve on committees in the set C_x.

4. Each committee i needs n_i members.

Give a polynomial time algorithm to find a schedule of committees, satisfying the four requirements, if such a schedule is possible.

Question 3

In a particular network $G = (V, E)$ whose edges have integer capacities c_e, we have already found the maximum flow f from node s to node t. However, we now find out that one of the capacity values we used was wrong: for edge (u, v) we used c_{uv} whereas it should have been $c_{uv} - 1$. The flow f used edge (u, v) at full capacity though, and thus is no longer valid.

Give an algorithm for finding the new optimal flow in $O(|V| + |E|)$ time.