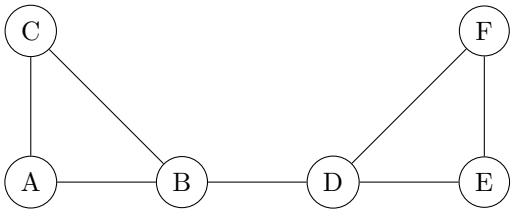
Some practice problems about Karger's algorithm

March 14, 2019

Note: While Karger's algorithm is fair game for the final, since we didn't have a proper homework assignment on it, it will not be weighted very heavily.

1. Consider the following graph:



   (a) What is the global mimimum cut of this graph?

   (b) What is the probability that Karger's algorithm chooses an edge of the minimum cut with its first choice?

   (c) What is the probability that one run of Karger's algorithm returns a minimum cut on this graph? How does it compare to the bound of $1/\binom{n}{2}$ that we saw in class? (You can either try to compute the probability exactly or else implement it and run it a bunch of times – note that you definitely wouldn't be asked this on an exam, but it's a great way to understand how Karger's algorithm works!)

2. Let n be some number. Suppose you have a magic button that, when pressed, outputs "Hooray!" with probability $1/n^3$. Otherwise it outputs "Hmmm..." Prove that if you push the button $T = n^3 \ln(100)$ times, that you will see at least one "Hooray" with probability at least 99/100.