CS 161

Design and Analysis of Algorithms

Lecture 1:

Logistics, introduction, and multiplication!

The big questions

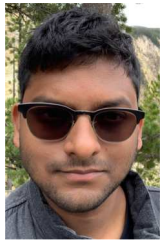
- Who are we?
  - Professor, TAs, students?
- Why are we here?
  - Why learn about algorithms?
- What is going on?
  - What is this course about?
  - Logistics?
- Can we multiply integers?
  - And can we do it quickly?

Who are we?


Karey


Luke


Adam


Pras


Nidhi


David


Anton


Brvce


Arjun


Albert


Kaylie


Amelie

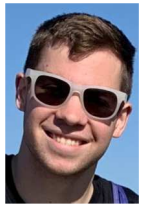
- Instructor:
  - Mary Wootters

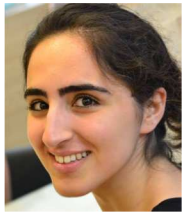
Course Coordinator:

- Amelie Byun

- Awesome CAs:
  - Karey Shi (Head CA)
  - Adam Stanford-Moore
  - Albert Zuo
  - Arjun Sawhney
  - Anastasiya Vitko
  - Anton de Leon
  - Bryan Cai
  - Bryce Cai
  - David Lin
  - Erik Jones
  - Geoff Ramseyer
  - Havin Hosgur
  - Jerry Qu
  - Joshua Spayd
  - Kaylie Zhu
  - Luke Miller
  - Milan Mosse
  - Nidhi Manoj
  - Prasanna Ramakrishnan
  - Vasco Portilheiro
  - William Zhuk

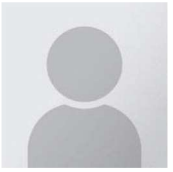

Bryan

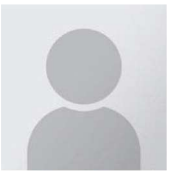

Erik

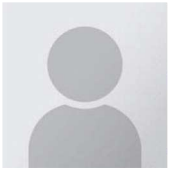

Havin


Milan

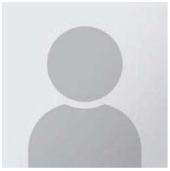

Josh


Vasco


William


Anastasiya


Geoff


Jerry

Who are you?

- Freshman
- Sophomores
- Juniors
- Seniors
- MA/MS Students
- PhD Students
- NDO Students

Concentrating in:

- Aero/Astro
- Applied Physics
- Art Practice
- Biomedical Informatics
- Chemical Eng.
- Chemistry
- Civil & Env. Eng.
- Classics
- CME
- Comparative Literature
- Computer Science
- Creative Writing
- Digital Humanities
- East Asian Studies
- Economics
- Education
- EE
- Energy Resources Eng.
- English
- Epidemiology
- Ethics in Society
- Fem, Gen, Sex Studies
- French
- Geological Sciences
- Global Studies
- History
- Human Biology
- Human Rights
- Immunology
- Linguistics
- Math
- Modern Languages
- Music
- MS&E
- Mech. Eng.
- Physics
- Philosophy
- Political Science
- Psychology
- Science, Tech. and Society
- Statistics
- Spanish
- Symbolic Systems
- Undeclared

Why are we here?

- I’m here because I’m super excited about algorithms!

Why are you here?

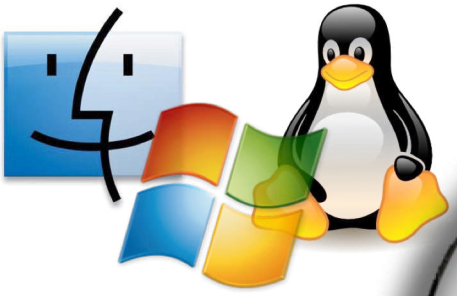
You are better equipped to answer this question than I am, but I'll give it a go anyway…

- Algorithms are fundamental.
- Algorithms are useful.
- Algorithms are fun!
- CS161 is a required course.

Why is CS161 required?

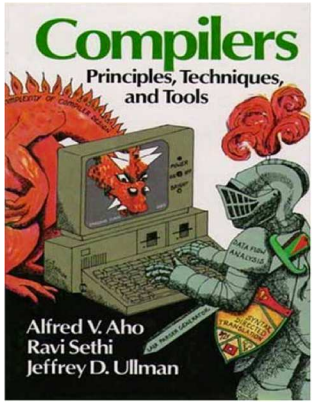
- Algorithms are fundamental.
- Algorithms are useful.
- Algorithms are fun!

Algorithms are fundamental



Operating Systems (CS 140)

Cryptography (CS 255)



Compilers (CS 143)

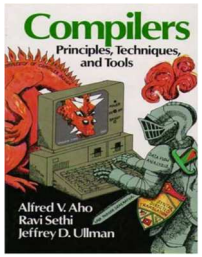
229)

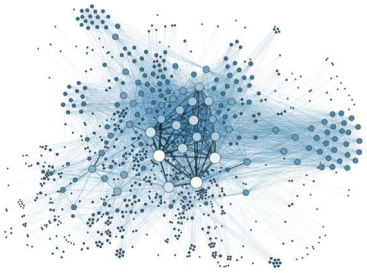
Networking (CS 144)

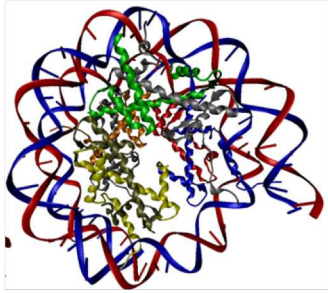
Computational Biology (CS 262)

Algorithms are useful

- All those things without the course numbers.
- As inputs get bigger and bigger, having good algorithms becomes more and more important!







Algorithms are fun!

- Algorithm design is both an art and a science.
- Many surprises!
- Many exciting research questions!

What’s going on?

- Course goals/overview
- Logistics

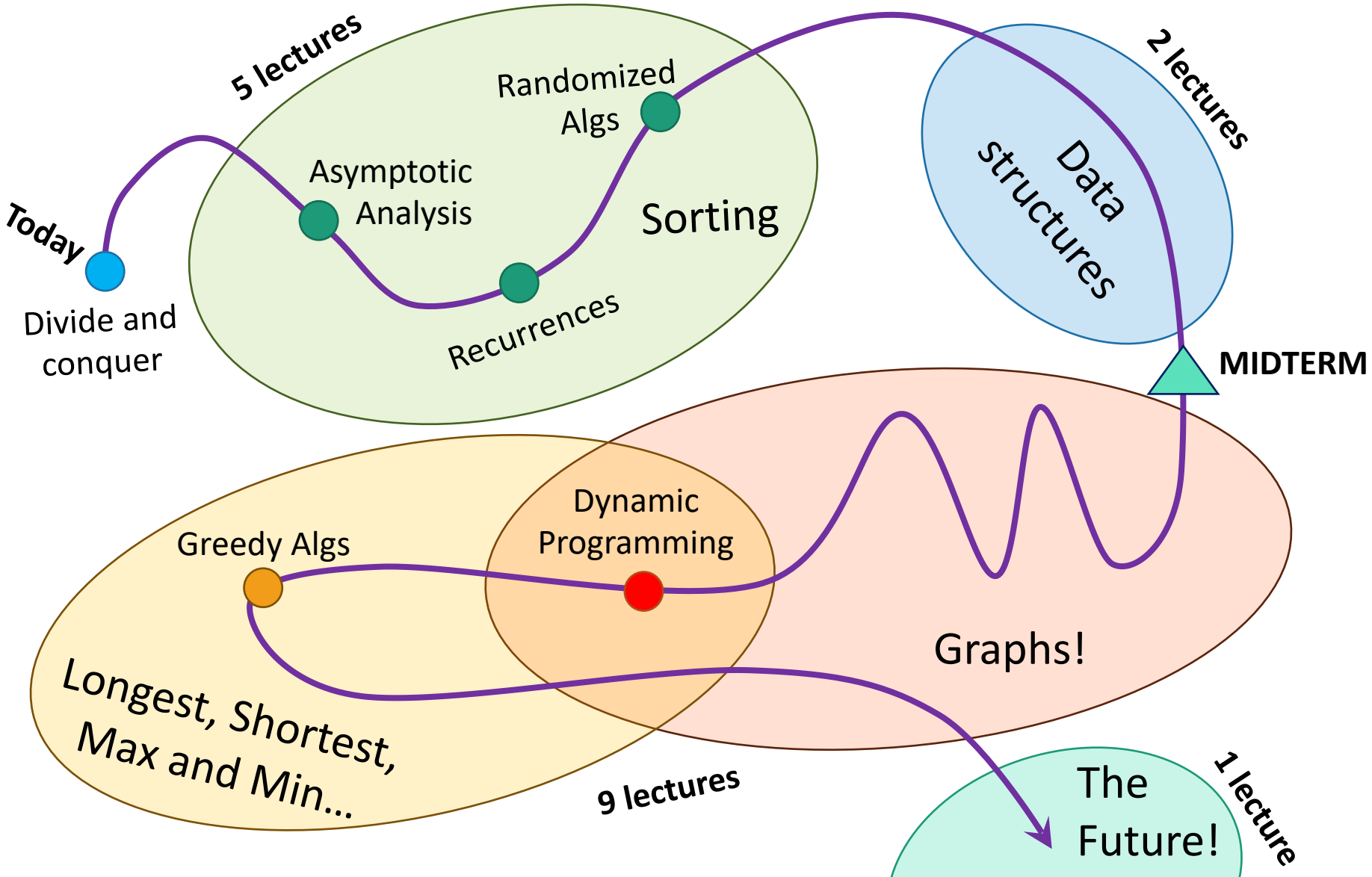
Course goals

- The design and analysis of algorithms
  - These go hand-in-hand

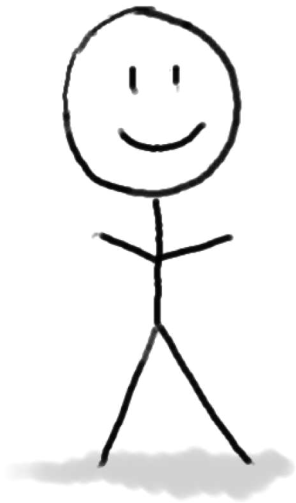
- In this course you will:
  - Learn to think analytically about algorithms
  - Flesh out an "algorithmic toolkit"
  - Learn to communicate clearly about algorithms

More detailed schedule on the website!

Roadmap



Our guiding questions:



Does it work?

Is it fast?

Can I do better?

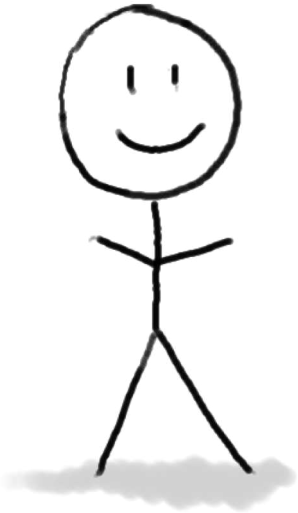
Our internal monologue…

What exactly do we mean by better? And what about that corner case? Shouldn't we be zero-indexing?

Plucky the Pedantic Penguin

Detail-oriented
Precise
Rigorous

Does it work?
Is it fast?
Can I do better?



Dude, this is just like that other time. If you do the thing and the stuff like you did then, it'll totally work real fast!

Lucky the Lackadaisical Lemur

Big-picture
Intuitive
Hand-wavey

Both sides are necessary!

Aside: the bigger picture

- Does it work?
- Is it fast?
- Can I do better?

- Should it work?
- Should it be fast?

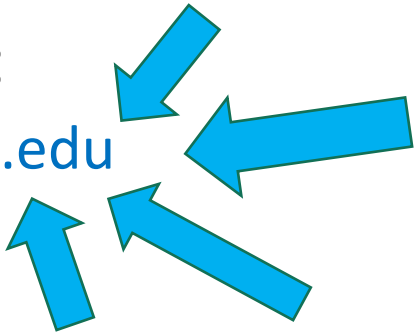
- We want to reduce crime.
- It would be more “efficient” to put cameras in everyone’s homes/cars/etc.

- We want advertisements to reach to the people to whom they are most relevant.
- It would be more “efficient” to make everyone’s private data public.

- We want to design algorithms, that work well, on average, in the population.
- It would be more “efficient” to focus on the majority population.

Course elements and resources

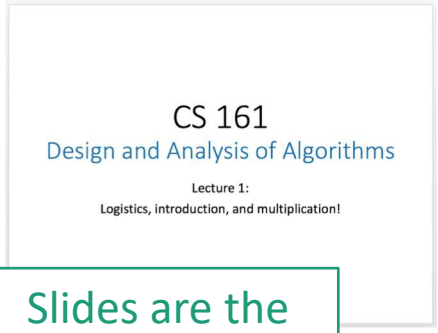
- Course website:
  - cs161.stanford.edu



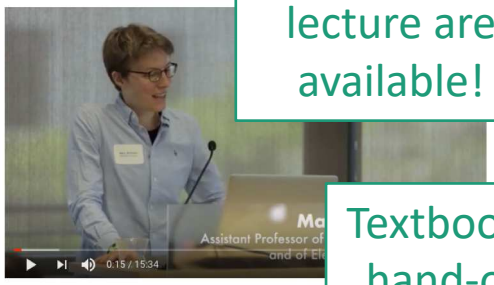
- Lectures
- Textbook
- IPython Notebooks
- Homework
- Exams
- Office hours, recitation sections, and Piazza

Lectures

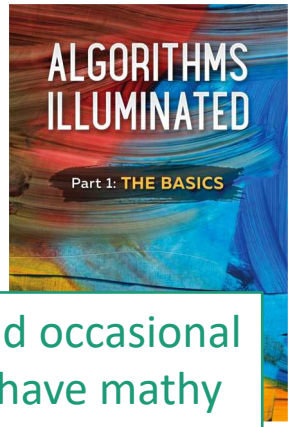
- Right here (NVIDIA Auditorium), M/W, 10:30-11:50!
- Resources available:
  - Slides, Videos, Book, IPython notebooks



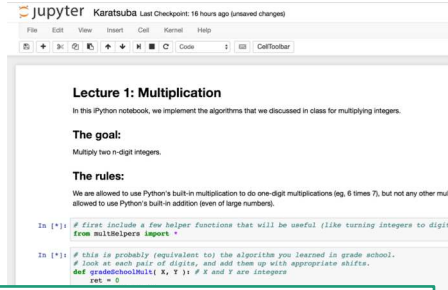
Slides are the slides from lecture.



Videos from lecture are available!



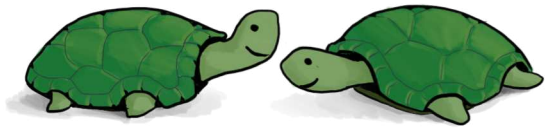
Textbook (and occasional hand-outs) have mathy details that slides may omit



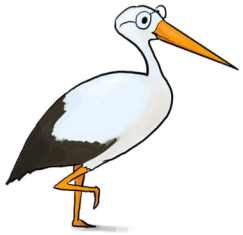
IPython notebooks have implementation details that slides may omit.

How to get the most out of lectures

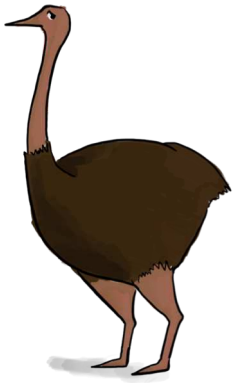
- **During lecture:**
  - Show up or tune in, ask questions.
  - Engage with in-class questions.
- **Before lecture:**
  - Do ***pre-lecture exercises*** on the website.
- **After lecture:**
  - Go through the exercises on the slides.



Think-Pair-Share Terrapins
(in-class questions)



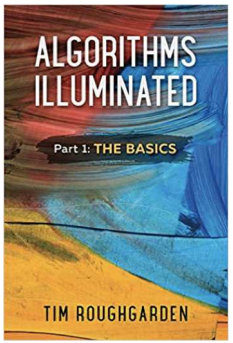
Siggi the Studious Stork
(recommended exercises)

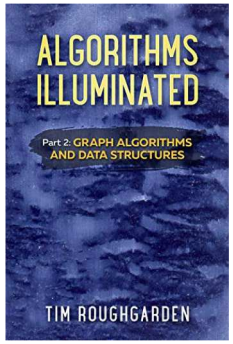


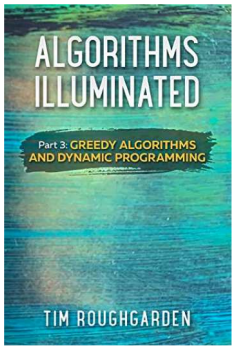
Ollie the Over-achieving Ostrich
(challenge questions)

- ***Do the reading***
  - either before or after lecture, whatever works best for you.
  - **do not wait to "catch up" the week before the exam.**

Textbook

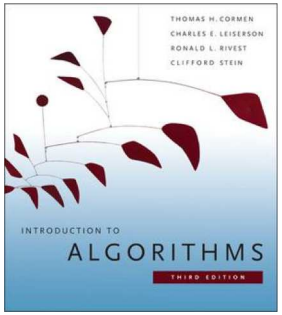




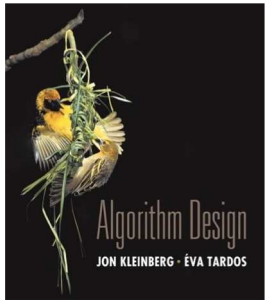


- **Algorithms Illuminated**, Vols 1,2 and 3 by Tim Roughgarden
- Additional resources at algorithmsilluminated.org

- We may also refer to to the following (optional) books:



”CLRS”: Introduction to Algorithms by Cormen, Leiserson, Rivest, and Stein. Available FOR FREE ONLINE through the Stanford library.



“Algorithm Design” by Kleinberg and Tardos

IPython Notebooks

- Lectures and homework will occasionally use IPython notebooks.
  - I will write Python code, you will read and maybe modify it.
- You will need to learn ***some*** Python to follow these.
  - For next lecture, the ***pre-lecture exercise*** is to get started with Jupyter Notebooks and with Python.
  - See course website for details.

- The goal is to make the algorithms (and their runtimes) more tangible.

Homework!

- Weekly assignments, posted Wednesday, due the next Wednesday (before class).
- First HW posted this Wednesday!

Exams

- There will be a **midterm** and a **final**.
  - **TAKE-HOME MIDTERM** on Tuesday 2/11 (during Week 6)
  - **FINAL:** 3/16, 3:30-6:30pm.

- We will release practice exams and hold review sessions before each.

- If you have a conflict with these exams, email cs161-win1920-staff@lists.stanford.edu ASAP!!!!!

Talk to us!

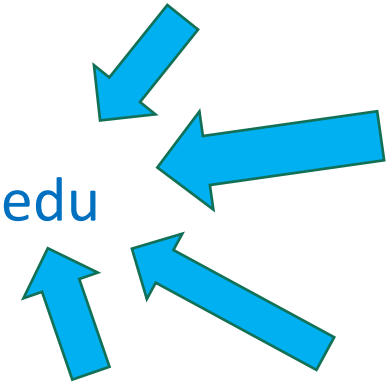
- Sign up for Piazza:
  - See course website for link
  - Course announcements will be posted there
  - Discuss material with TAs and your classmates
- Office hours:
  - See course website for schedule
- Recitation sections:
  - Thursdays and Fridays
  - See course website for schedule (to be posted soon)
  - Technically optional, but ***highly recommended***!
  - Extra practice with the material, example problems, etc.

Talk to each other!

- Answer your peers' questions on Piazza!
- We will host Homework Parties (room/time TBD) Thursday evenings.

Course elements and resources

- Course website:
  - cs161.stanford.edu

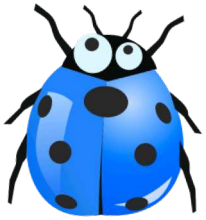


- Lectures
- Textbook
- IPython Notebooks
- Homework
- Exams
- Office hours, recitation sections, and Piazza

A note on course policies

- Course policies are listed on the website.
- Read them and adhere to them.
- That's all I'm going to say about course policies.

Bug bounty!



- We hope all PSETs and slides will be bug-free.
- Howover, I sometmes maek typos.
- **If you find a typo** (that affects understanding*) on slides, IPython notebooks, Section material or PSETs:
  - Let us know! (Post on Piazza or tell a CA).
  - The first person to catch a bug gets a bonus point.

Bug Bounty Hunter

*So, typos lke thees onse don't count, although please point those out too. Typos like 2 + 2 = 5 do count, as does pointing out that we omitted some crucial information.

For SCPD Students

- There will be office hours held online.
- One of the recitation sections will be recorded.
- See the website for more details! (Coming soon…)

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OAE forms

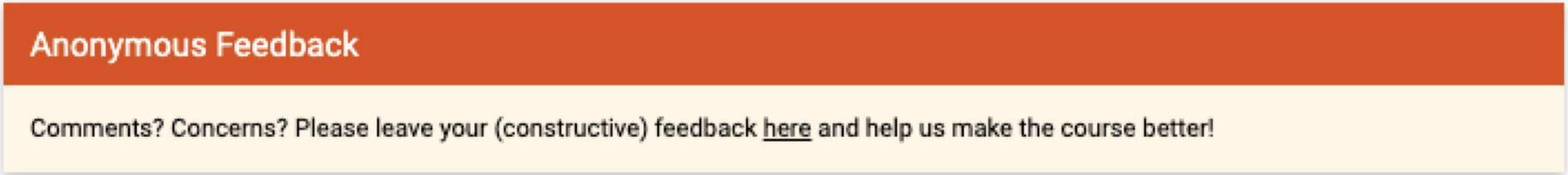
- Please send OAE forms to

  cs161-win1920-staff@lists.stanford.edu

- Note: this list is read only by me, Amelie Byun, Karey Shi, and Kaylie Zhu.

Feedback!

- There is an anonymous feedback form on the course website (bottom of the main page).
- Please help us improve the course!



Everyone can succeed in this class!

1. Work hard
2. Work smart
3. Ask for help

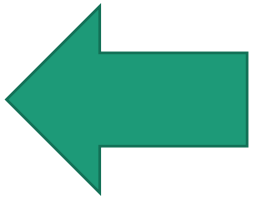
The big questions

- Who are we?
  - Professor, TA's, students?
- Why are we here?
  - Why learn about algorithms?
- What is going on?
  - What is this course about?
  - Logistics?
- Can we multiply integers?
  - And can we do it quickly?

Course goals

- Think analytically about algorithms
- Flesh out an "algorithmic toolkit"
- Learn to communicate clearly about algorithms

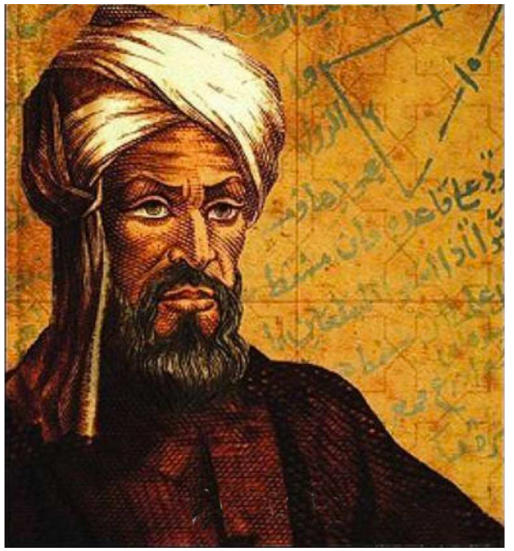
Today's goals

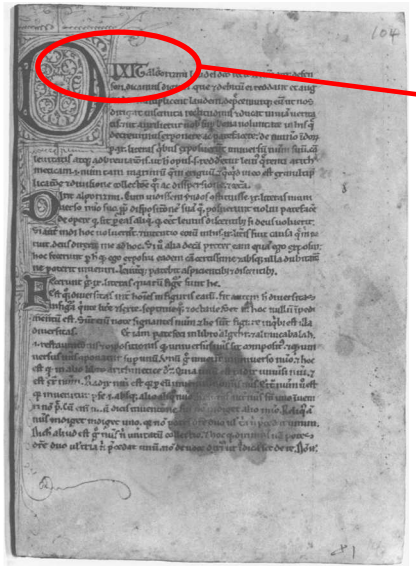
- Karatsuba Integer Multiplication 
- Algorithmic Technique:
  - Divide and conquer
- Algorithmic Analysis tool:
  - Intro to asymptotic analysis

Let’s start at the beginning

Etymology of "Algorithm"

- Al-Khwarizmi was a 9th-century scholar, born in present-day Uzbekistan, who studied and worked in Baghdad during the Abbassid Caliphate.
- Among many other contributions in mathematics, astronomy, and geography, he wrote a book about how to multiply with Arabic numerals.
- His ideas came to Europe in the 12th century.





Dixit algorizmi
(so says Al-Khwarizmi)

- Originally, "Algorisme" [old French] referred to just the Arabic number system, but eventually it came to mean "Algorithm" as we know today.

This was kind of a big deal

XLIV × XCVII = ?

$$\begin{array}{r} 44 \\ \times\ 97 \\ \hline \end{array}$$

Integer Multiplication

$$\begin{array}{r} 44 \\ \times\ 97 \\ \hline \end{array}$$

Integer Multiplication

$$
\begin{array}{rr}
 & 12345678959314 13 \\
\times & 4563823520395533 \\
\hline
\end{array}
$$

Integer Multiplication

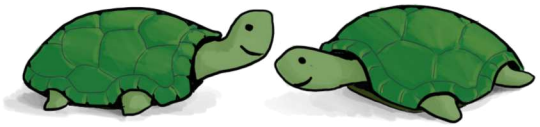
n

```
  12339257207527523846237642835683649183745238562 98
x 45623235823423952856234672350191307501335001375 3
```

???

How fast is the grade-school multiplication algorithm?

(How many one-digit operations?)



Think-pair-share Terrapins

About n^2 one-digit operations

Plucky the Pedantic Penguin

At most n^2 multiplications,
and then at most n^2 additions (for carries)
and then I have to add n different 2n-digit numbers…

Big-Oh Notation

- We say that Grade-School Multiplication

  "runs in time $O(n^2)$"

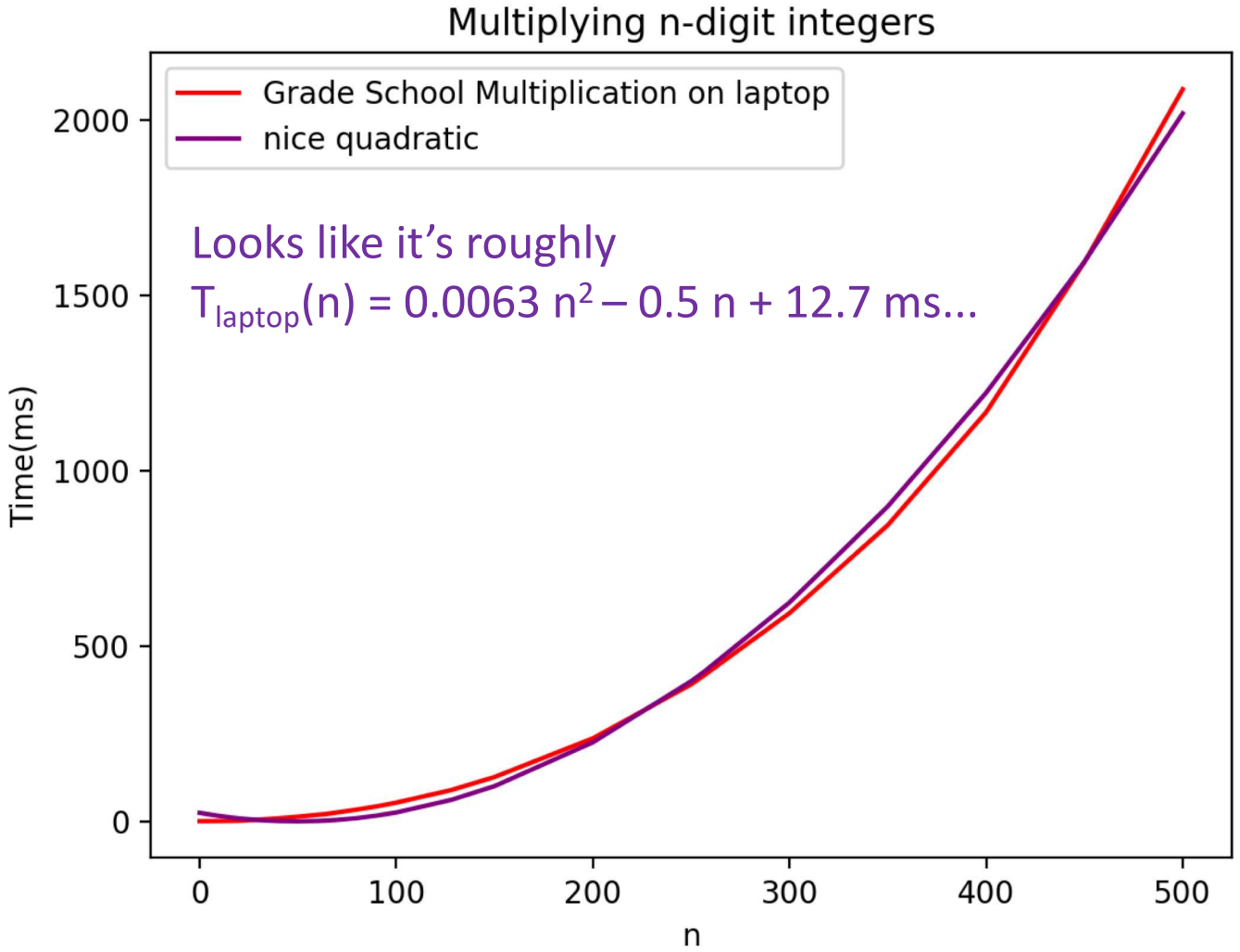
- Formal definition coming Wednesday!
- Informally, big-Oh notation tells us how the running time scales with the size of the input.

highly non-optimized

Implemented in Python, on my laptop

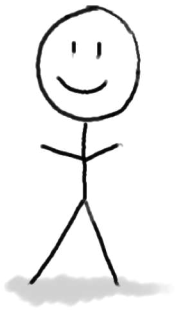
The runtime "scales like" n^2

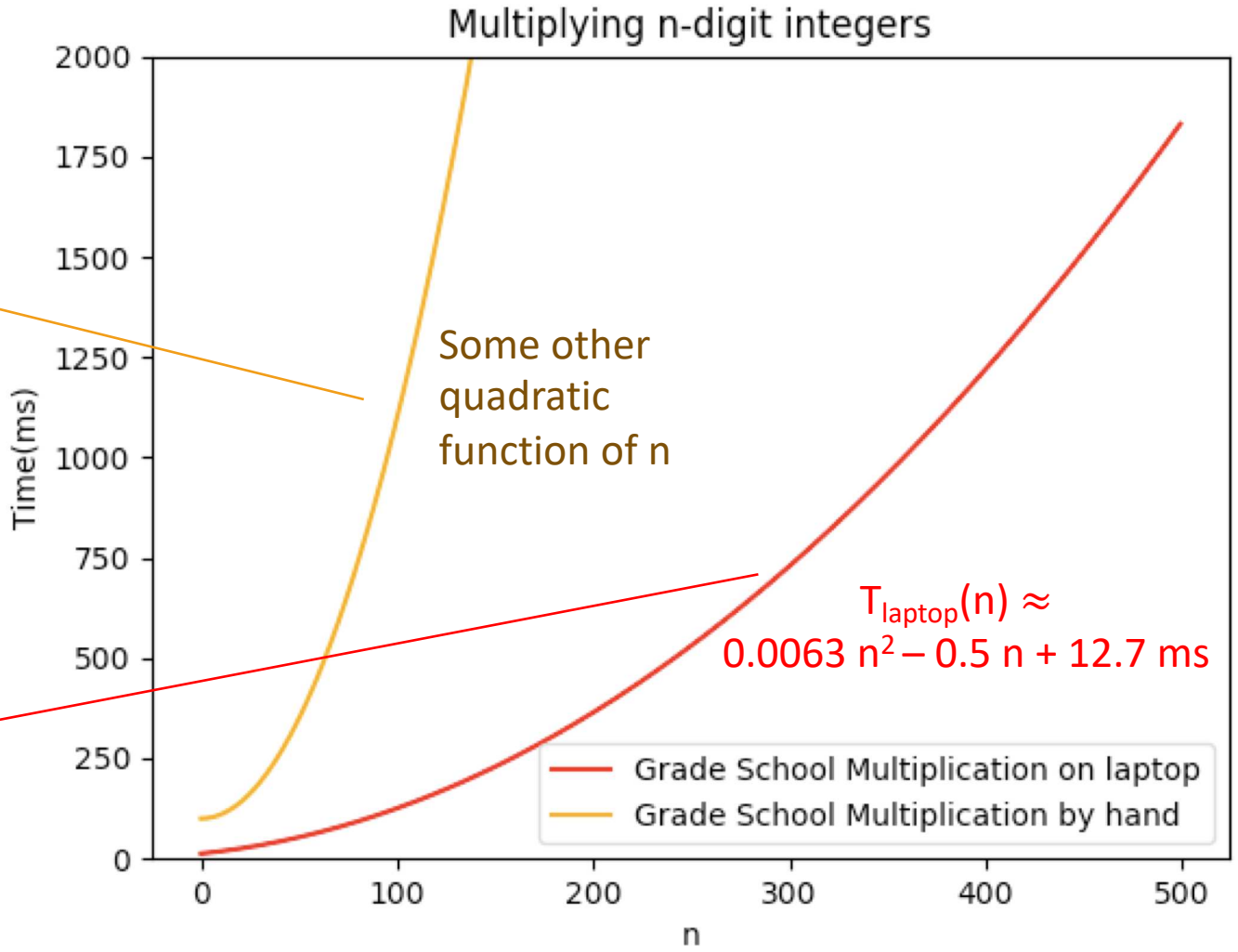
python™



Implemented by hand

The runtime still "scales like" n^2



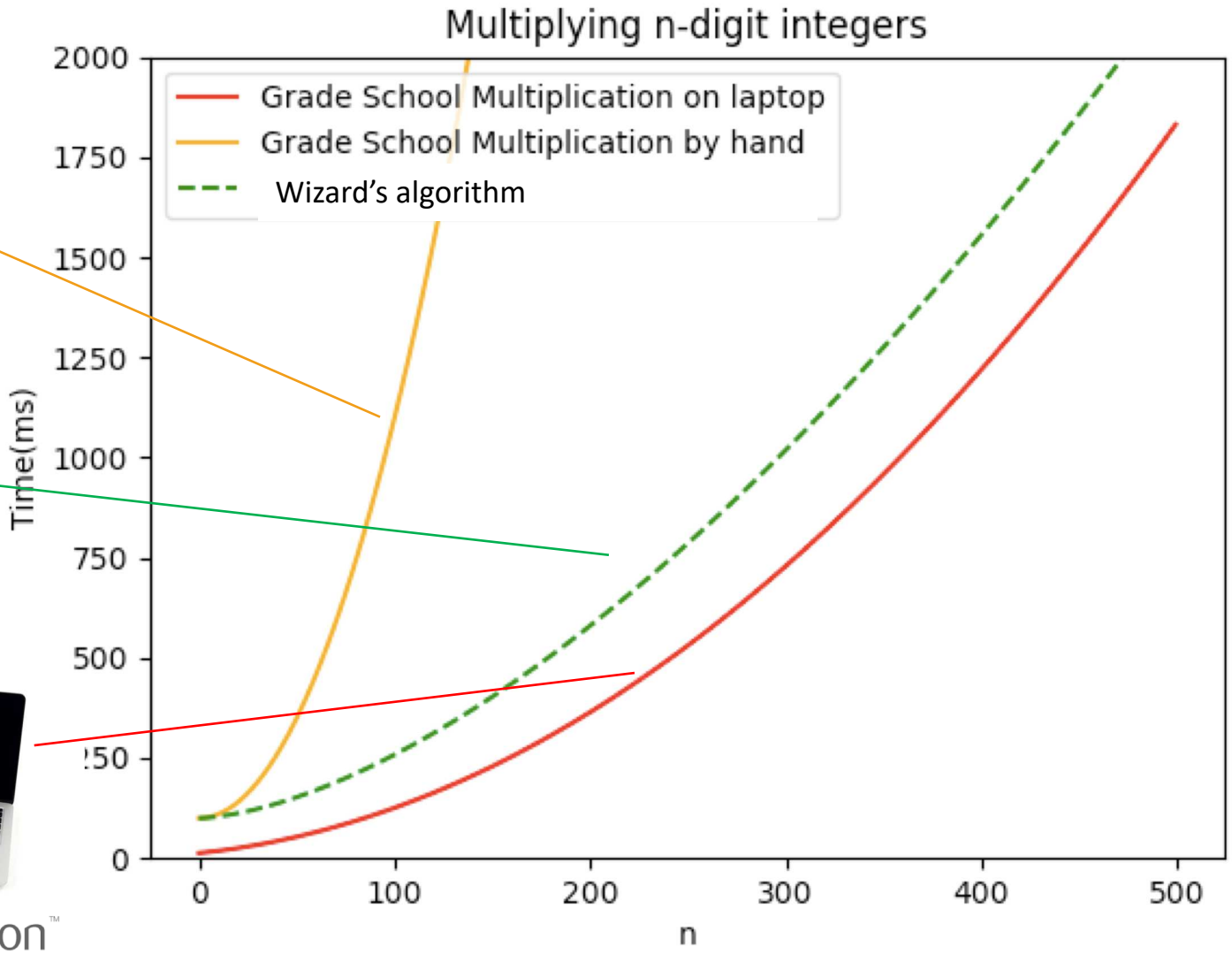


python™

Why is big-Oh notation meaningful?

$\approx \frac{n^{1.6}}{10} + 100$

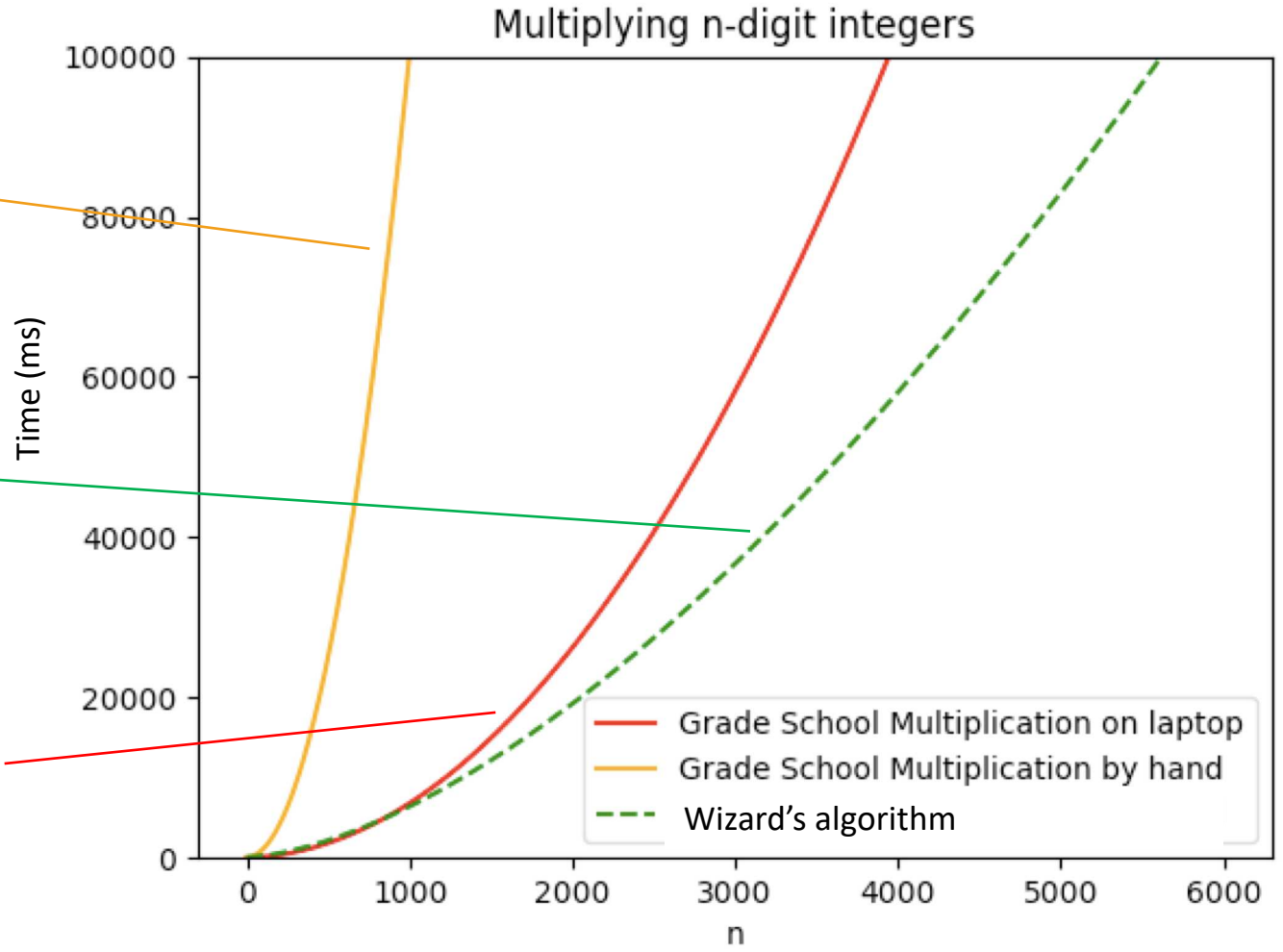
$\approx .0063n^2$



Let n get bigger…

$\approx \frac{n^{1.6}}{10} + 100$

$\approx .0063n^2$



Take-away

- An algorithm that runs in time $O(n^{1.6})$ is “better” than an an algorithm that runs in time $O(n^2)$.

- So the question is…

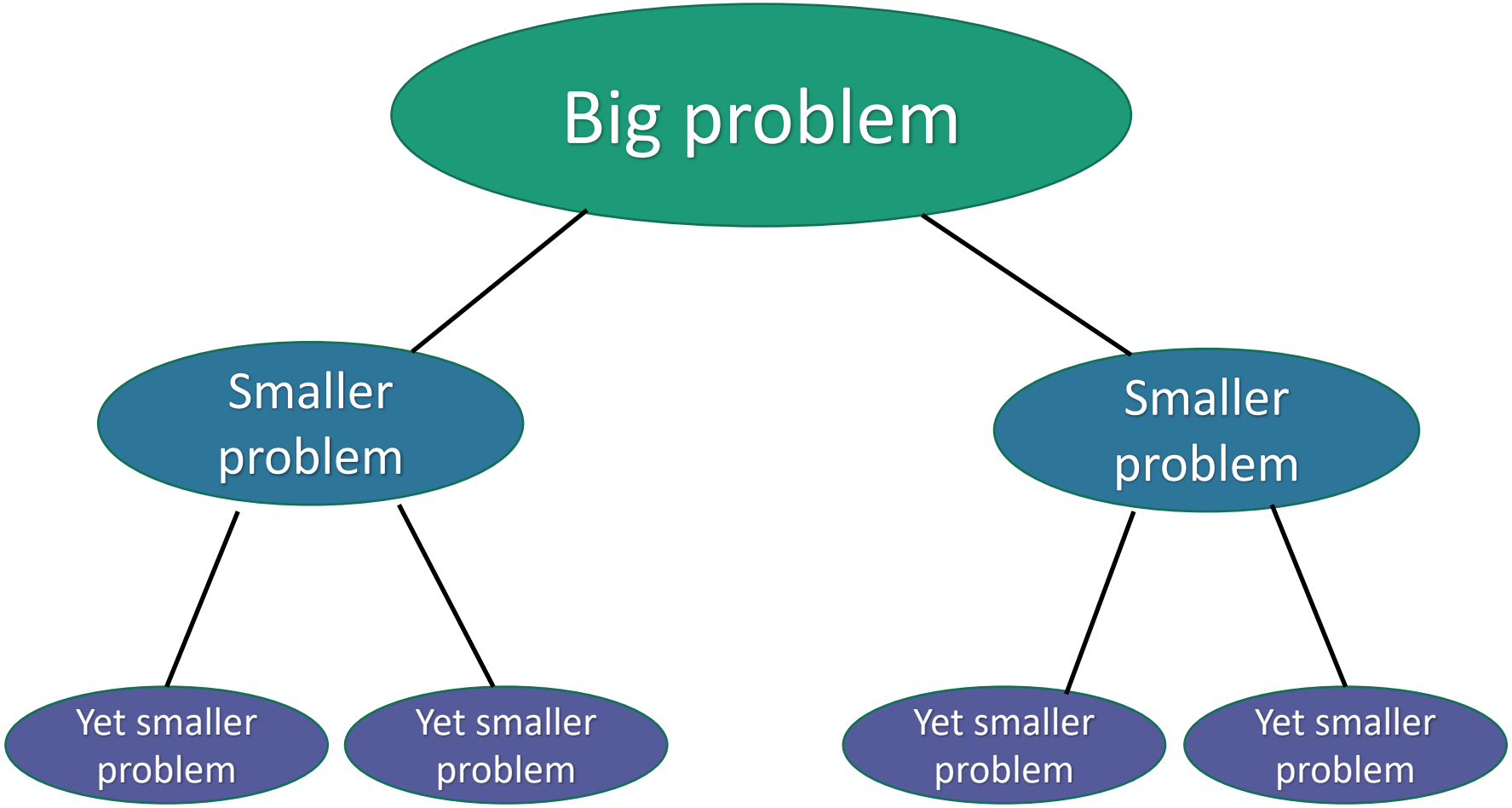
Can we do better?

Can we multiply n-digit integers faster than $O(n^2)$?

Let’s dig in to our algorithmic toolkit…

Divide and conquer

Break problem up into smaller (easier) sub-problems



Divide and conquer for multiplication

Break up an integer:

$$1234 = 12 \times 100 + 34$$

$$1234 \times 5678$$

$$= (12 \times 100 + 34) (56 \times 100 + 78)$$

$$= \underbrace{(12 \times 56)}_{1} 10000 + \underbrace{(34 \times 56}_{2} + \underbrace{12 \times 78)}_{3} 100 + \underbrace{(34 \times 78)}_{4}$$

One 4-digit multiply  Four 2-digit multiplies

More generally

Suppose n is even

Break up an n-digit integer:

$$[x_1 x_2 \cdots x_n] = [x_1 x_2 \cdots x_{n/2}] \times 10^{n/2} + [x_{n/2+1} x_{n/2+2} \cdots x_n]$$

$$x \times y = (a \times 10^{n/2} + b)(c \times 10^{n/2} + d)$$

$$= \underbrace{(a \times c)10^n}_{1} + \underbrace{(a \times d}_{2} \underbrace{+ c \times b)10^{n/2}}_{3} + \underbrace{(b \times d)}_{4}$$

One n-digit multiply → Four (n/2)-digit multiplies

Divide and conquer algorithm

not very precisely…

(Assume n is a power of 2…)

x,y are n-digit numbers

Multiply(x, y):

- **If** n=1:
  - **Return** xy
- Write $x = a\ 10^{\frac{n}{2}} + b$
- Write $y = c\ 10^{\frac{n}{2}} + d$
- Recursively compute ac, ad, bc, bd:
  - ac = **Multiply**(a, c), etc..
- Add them up to get xy:
  - xy = ac 10^n + (ad + bc) $10^{n/2}$ + bd

Base case: I've memorized my 1-digit multiplication tables…

a, b, c, d are n/2-digit numbers

Make this pseudocode more detailed! How should we handle odd n? How should we implement "multiplication by 10^n"?

Siggi the Studious Stork

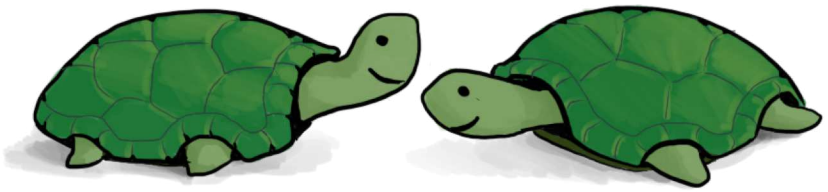
See the Lecture 1 Python notebook for actual code!

Think-Pair-Share

- We saw that this 4-digit multiplication problem broke up into four 2-digit multiplication problems

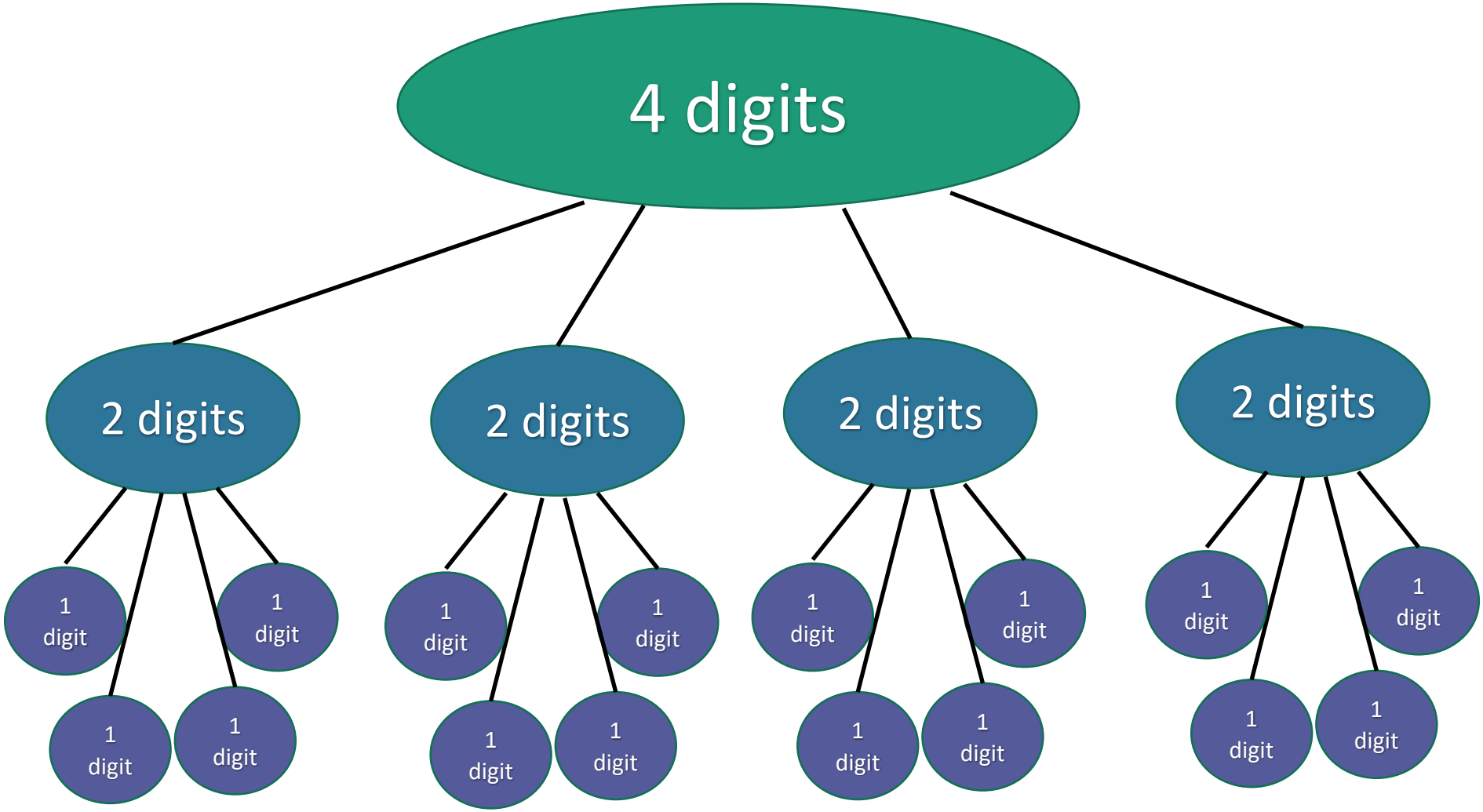
$$1234 \times 5678$$

- If you recurse on those 2-digit multiplication problems, how many 1-digit multiplications do you end up with total?



Recursion Tree

16 one-digit multiplies!



What is the running time?

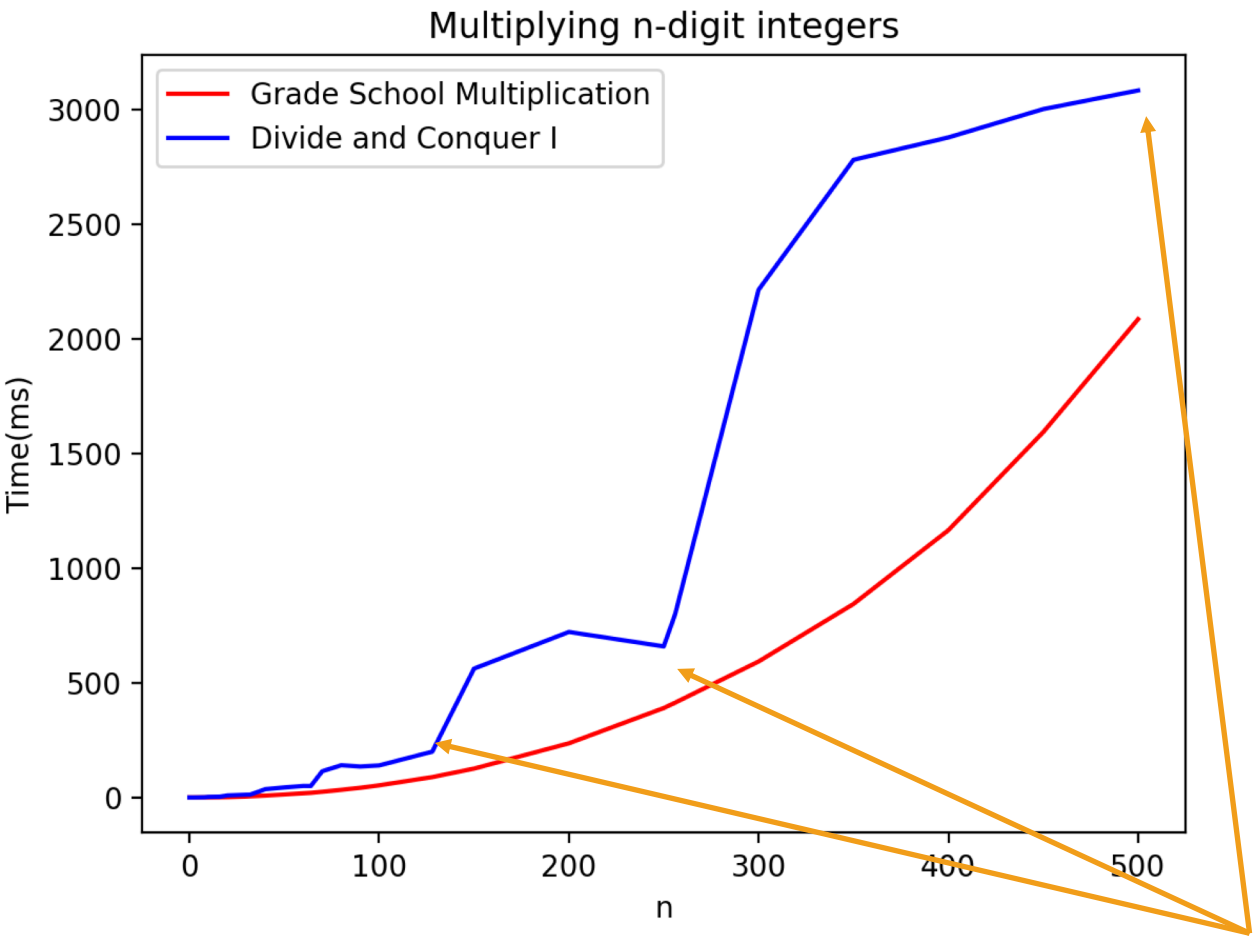
- Better or worse than the grade school algorithm?

- How do we answer this question?
  1. Try it.
  2. Try to understand it analytically.

1. Try it.

Check out the Lecture 1 IPython Notebook

Conjectures about running time?



Doesn't look too good but hard to tell…

Maybe one implementation is slicker than the other?

Maybe if we were to run it to n=10000, things would look different.

Something funny is happening at powers of 2…

2. Try to understand the running time analytically

- Proof by meta-reasoning:

It must be faster than the grade school algorithm, because we are learning it in an algorithms class.

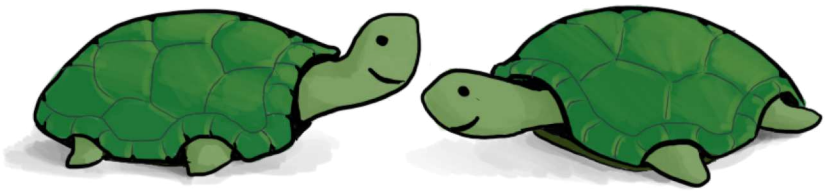
Not sound logic!

Plucky the Pedantic Penguin

2. Try to understand the running time analytically

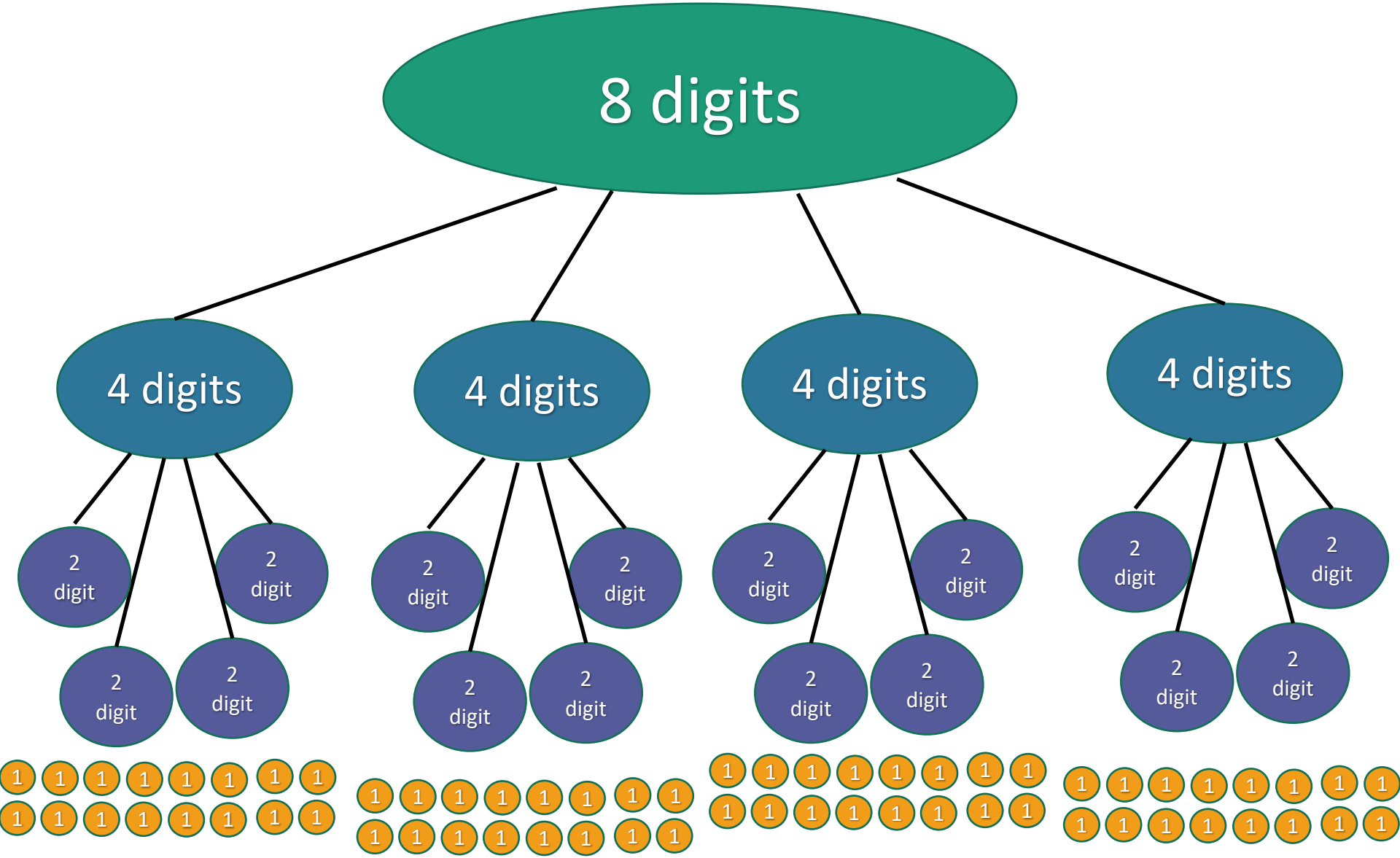
Think-Pair-Share:

- We saw that multiplying 4-digit numbers resulted in 16 one-digit multiplications.
- How about multiplying 8-digit numbers?
- What do you think about n-digit numbers?



Recursion Tree

64 one-digit multiplies!

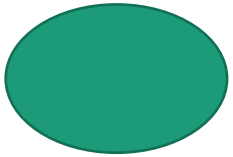


2. Try to understand the running time analytically

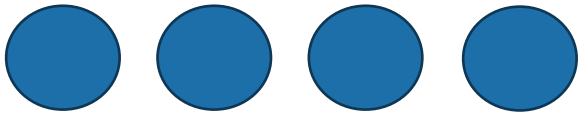
Claim:

The running time of this algorithm is AT LEAST n^2 operations.

There are n^2 1-digit problems

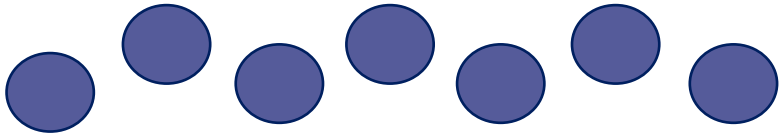


1 problem
of size n



4 problems
of size n/2

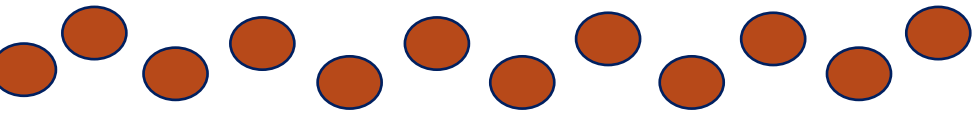
...



4^t problems
of size $n/2^t$

Note: this is just a cartoon – I'm not going to draw all 4^t circles!

...



n^2 problems
of size 1

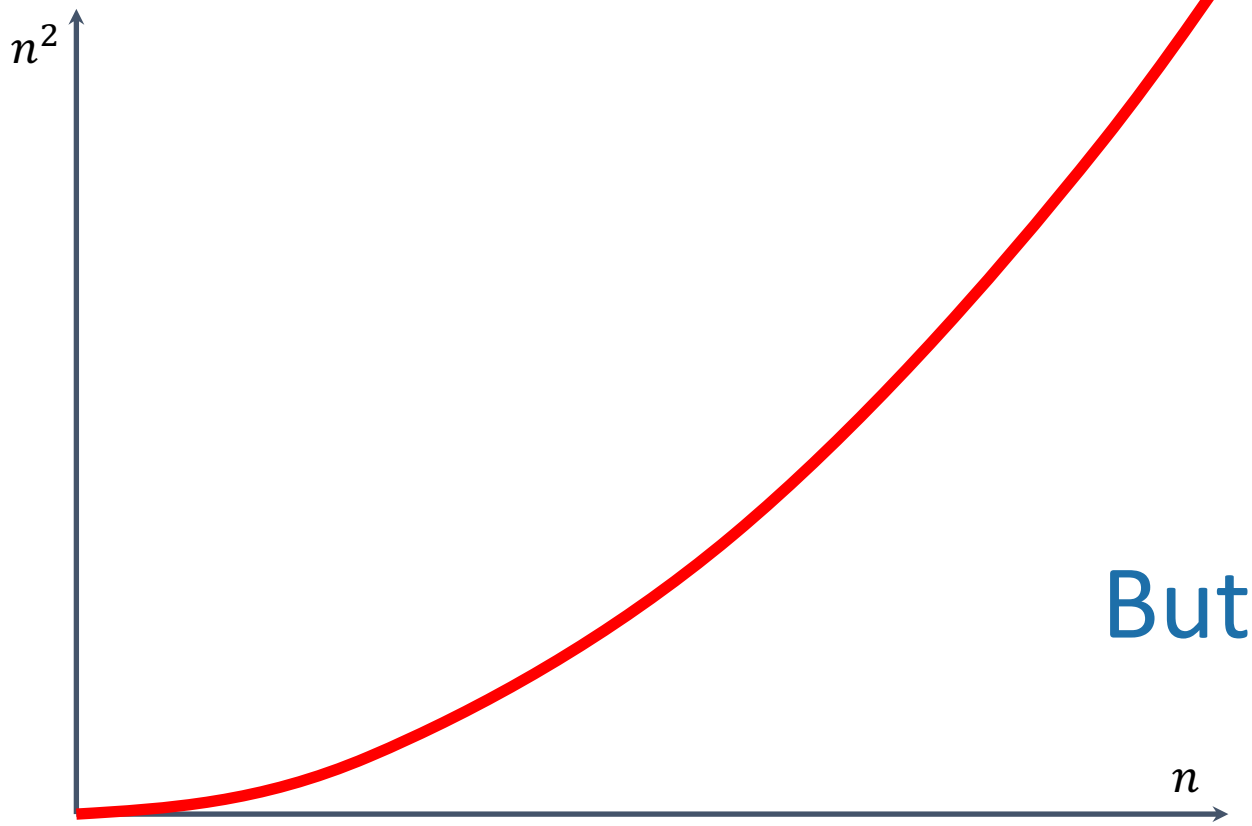
- If you cut n in half $\log_2(n)$ times, you get down to 1.
- So at level $t = \log_2(n)$ we get…

$$4^{\log_2 n} = n^{\log_2 4} = n^2$$

problems of size 1.

That's a bit disappointing

All that work and still (at least) $O(n^2)$…



But wait!!

Divide and conquer can actually make progress

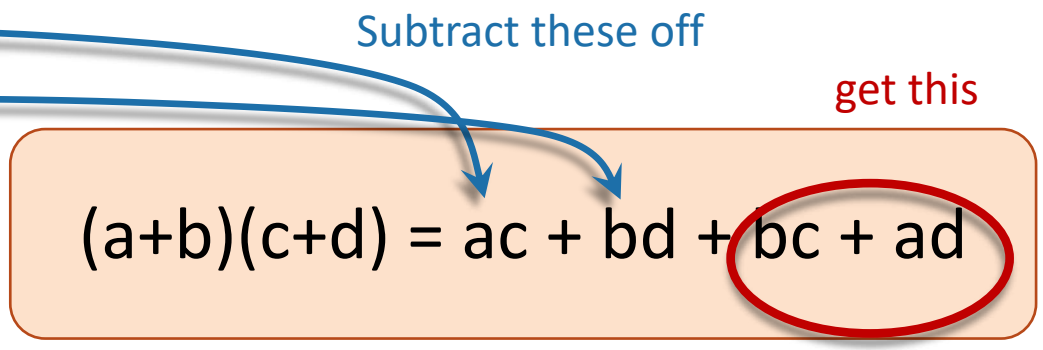
- Karatsuba figured out how to do this better!

$$xy = (a \cdot 10^{n/2} + b)(c \cdot 10^{n/2} + d)$$
$$= ac \cdot 10^{n} + (ad + bc)10^{n/2} + bd$$

- If only we could recurse on three things instead of four…

Karatsuba integer multiplication

- Recursively compute these THREE things:
  - ac
  - bd
  - (a+b)(c+d)



- Assemble the product:

$$xy = (a \cdot 10^{n/2} + b)(c \cdot 10^{n/2} + d)$$
$$= ac \cdot 10^{n} + (ad + bc)10^{n/2} + bd$$

✓ ✓ ✓

How would this work?

x,y are n-digit numbers

(Still not super precise, see IPython notebook for detailed code. Also, still assume n is a power of 2.)

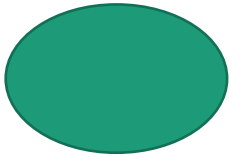
Multiply(x, y):

- **If** n=1:
  - **Return** xy
- Write $x = a\ 10^{\frac{n}{2}} + b$ and $y = c\ 10^{\frac{n}{2}} + d$

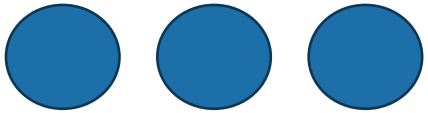
a, b, c, d are n/2-digit numbers

- ac = **Multiply**(a, c)
- bd = **Multiply**(b, d)
- z = **Multiply**(a+b, c+d)
- xy = ac 10^n + (z – ac - bd) $10^{n/2}$ + bd
- Return xy

What's the running time?

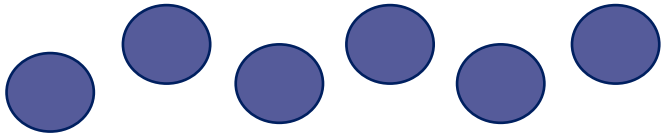


1 problem
of size n



3 problems
of size n/2

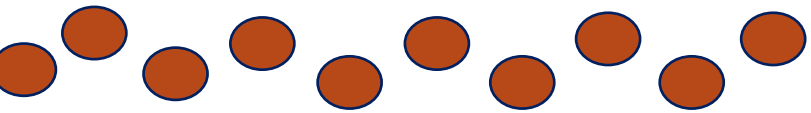
…



3^t problems
of size $n/2^t$

Note: this is just a cartoon – I'm not going to draw all 3^t circles!

…



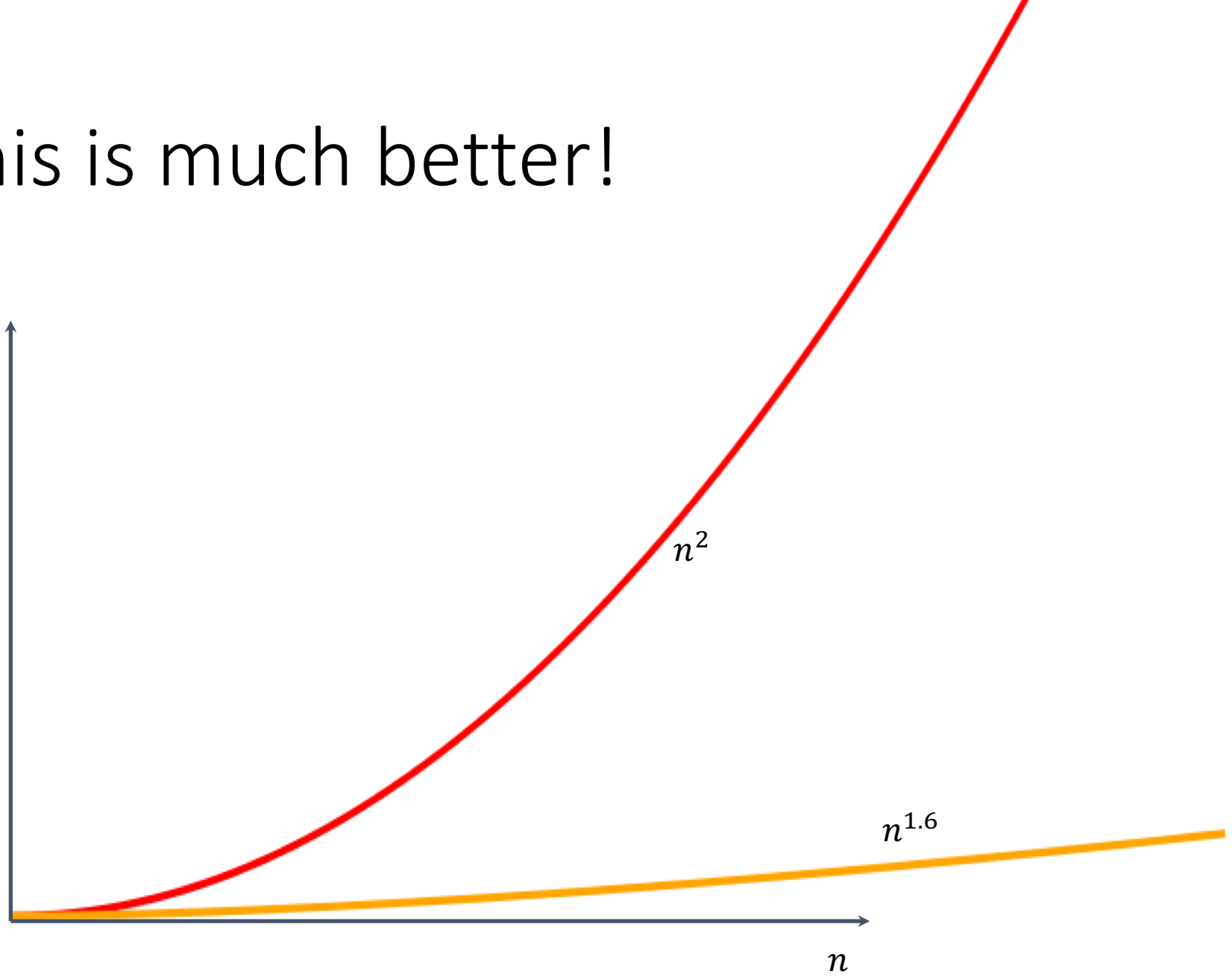
$\frac{n^{1.6}}{\_\_\_\_}$ problems
of size 1

- If you cut n in half $\log_2(n)$ times, you get down to 1.
- So at level $t = \log_2(n)$ we get…

$3^{\log_2 n} = n^{\log_2 3} \approx n^{1.6}$
problems of size 1.

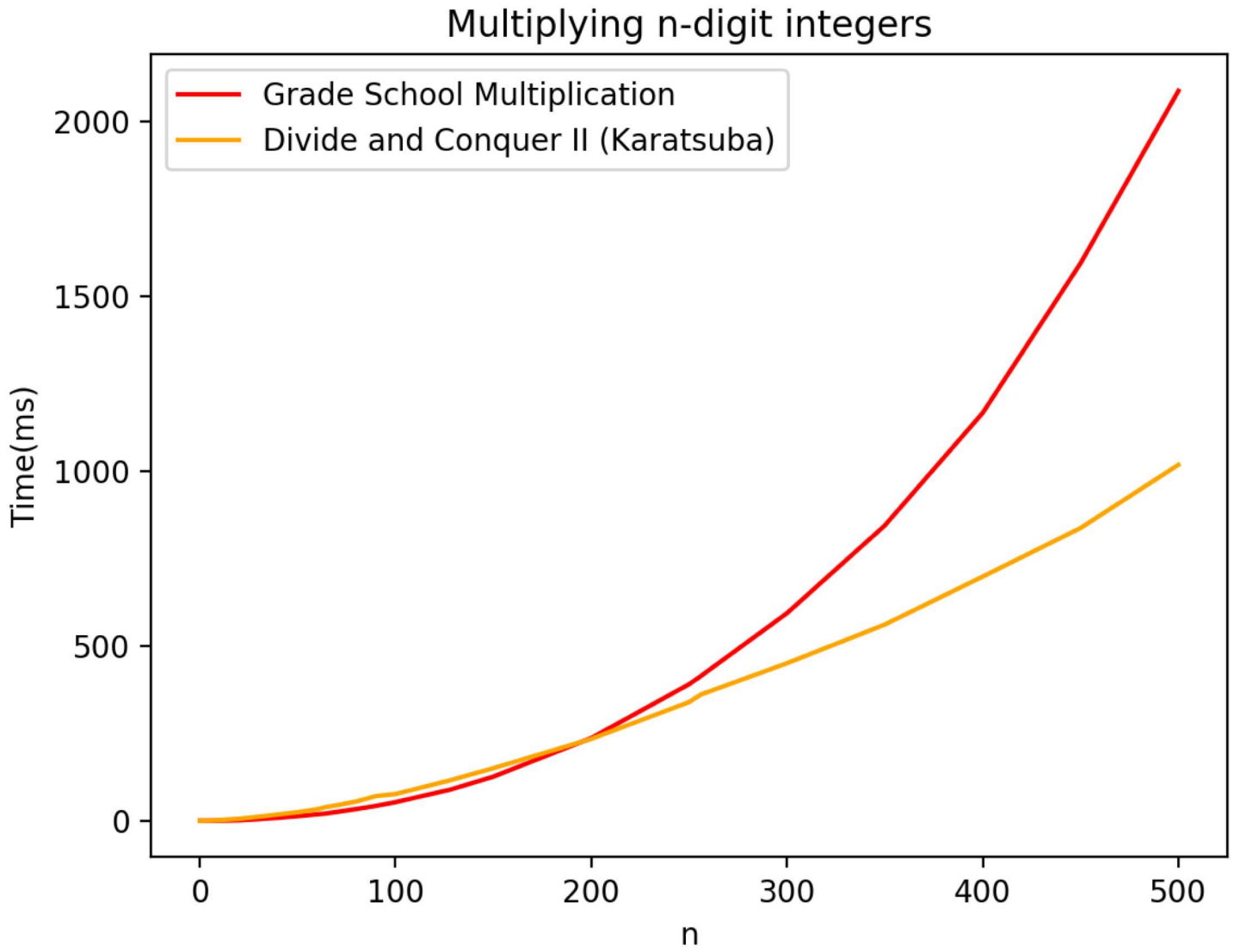
We aren't accounting for the work at the higher levels! But we'll see later that this turns out to be okay.

This is much better!



We can even see it in real life!

python™



Can we do better?

- **Toom-Cook** (1963): instead of breaking into three n/2-sized problems, break into five n/3-sized problems.
  - Runs in time $O(n^{1.465})$

Try to figure out how to break up an n-sized problem into five n/3-sized problems! **(Hint: start with nine n/3-sized problems).**

Ollie the Over-achieving Ostrich

Given that you can break an n-sized problem into five n/3-sized problems, where does the 1.465 come from?

Siggi the Studious Stork

- **Schönhage–Strassen** (1971):
  - Runs in time $O(n \log(n) \log\log(n))$
- **Furer** (2007)
  - Runs in time $n \log(n) \cdot 2^{O(\log^*(n))}$
- **Harvey and van der Hoeven** (2019)
  - Runs in time $O(n \log(n))$

[This is just for fun, you don't need to know these algorithms!]

Course goals

- Think analytically about algorithms
- Flesh out an "algorithmic toolkit"
- Learn to communicate clearly about algorithms

Today's goals

- Karatsuba Integer Multiplication
- Algorithmic Technique:
  - Divide and conquer
- Algorithmic Analysis tool:
  - Intro to asymptotic analysis

The big questions

- Who are we?
  - Professor, TA's, students?
- Why are we here?
  - Why learn about algorithms?
- What is going on?
  - What is this course about?
  - Logistics?
- Can we multiply integers?
  - And can we do it quickly?
- Wrap-up

Wrap up

- cs161.stanford.edu
- Algorithms are fundamental, useful and fun!
- In this course, we will develop both algorithmic intuition and algorithmic technical chops

- Karatsuba Integer Multiplication:
  - You can do better than grade school multiplication!
  - Example of divide-and-conquer in action
  - Informal demonstration of asymptotic analysis

Next time

- Sorting!
- Asymptotics and (formal) Big-Oh notation
- Divide and Conquer some more

BEFORE Next time

- ***Pre-lecture exercise!*** On the course website!
- Join Piazza!