Lecture 16.5

Tying up some loose ends, and answering your questions!

Announcements

- Final exam is a week from today!
  - ***Monday June 12, 3:30-6:30pm.***
- Answering a question from last time: yes, EthiCS content will be on the exam.
- The exam is technically cumulative, but with a heavy focus on the second half of the quarter (starting with graph stuff)
- Practice exam is out now! We hope that this is similar in terms of length/difficulty/format to the actual exam.
  - Disclaimer: different things are differently difficult for different people.
- Past exams to be posted soon.

How to study for exam?

- Review sessions!
  - Some have already happened, videos are available on Canvas
  - Ford-Fulkerson/Max-Flow/Min-Cut practice problem session!
    - Today (Monday June 5) 3-5pm with Robbie. (Check Ed for location updates)
  - Hashing and Universal Hash Families!
    - Tomorrow (Tuesday June 6) 3-5pm in 200-203 with Sophia.
  - Going over old exams!
    - Saturday June 10, 12-2pm AND 4-6pm in Hewlett 201 with Rishu.
  - Check Ed before you go for any location/time updates.

How to study for exam?

- Resources available (other than review sessions + OH + Ed):
  - Practice exam (and previous years' exams) on website
  - Textbook(s): Algs. Illuminated, CLRS
  - Problem-solving guide (posted on website, under Resources)
  - Concept-check questions from previous years (on website under Resources)

- Suggestions:
  - Do as many practice problems as you can!
  - Try to *come up* with practice problems!
  - Try to teach a concept or practice problem solution to a friend!
  - Making your cheat sheet is also a great study opportunity!
    - And, uh, maybe print a draft of your cheat sheet out the day before just in case....

Agenda

- Answering a few lingering questions about Ford-Fulkerson
- ... I got nothing. Answering your questions!

Question from last time

- Ford-Fulkerson (using BFS to find augmenting paths) runs in time $O(nm^2)$, which might be as bad as $O(n^5)$.
- Can we do better?

Answer 1: Yes!

- Ford-Fulkerson **also** runs in time $O(mf)$, where f is the max flow value, if all the capacities are integers.
  - We increase the flow at each step and start from 0, so there are at most $f + 1$ steps.
  - Each step takes time O(m).

Answer 1: Yes! (if the max flow is small)

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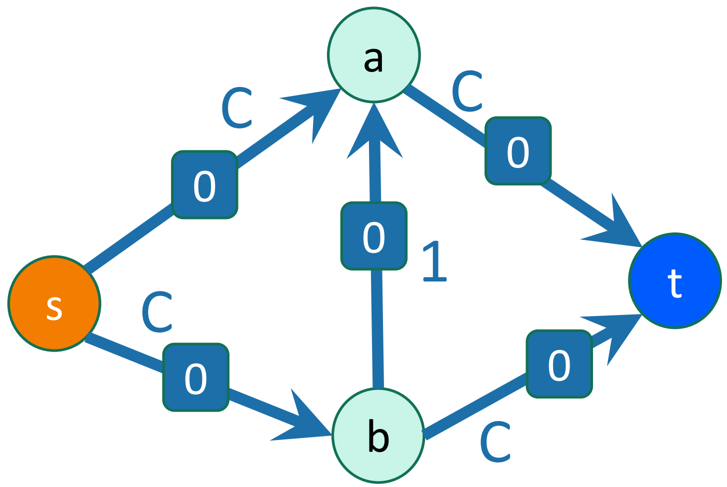

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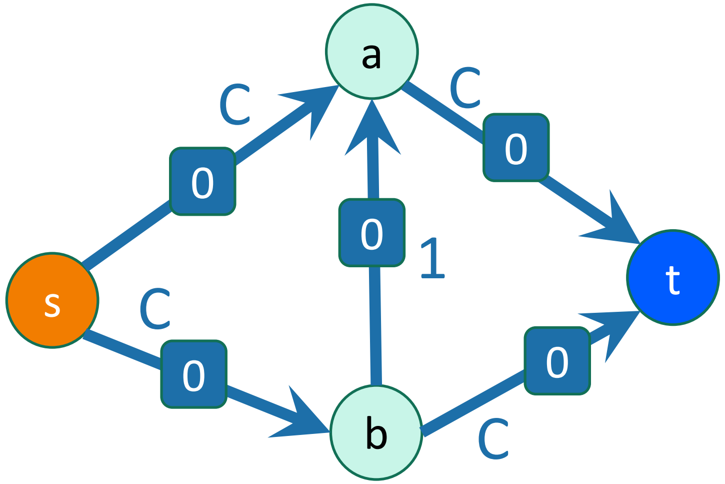

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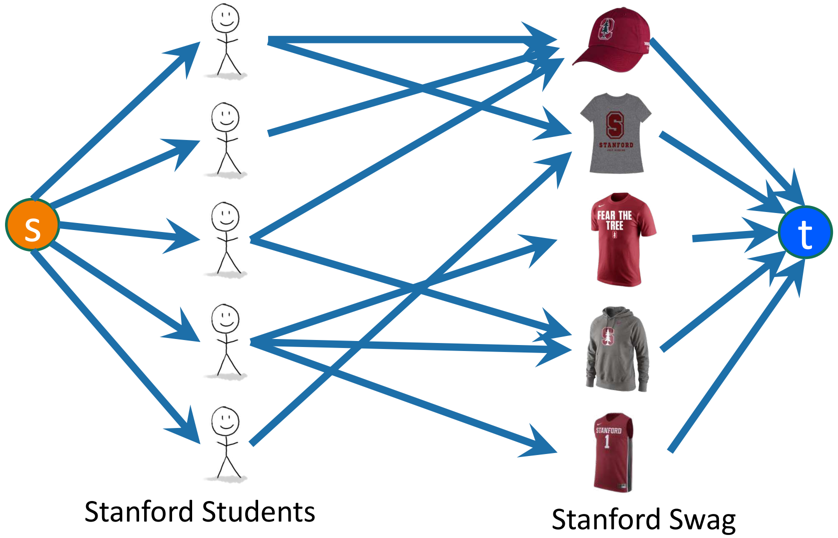


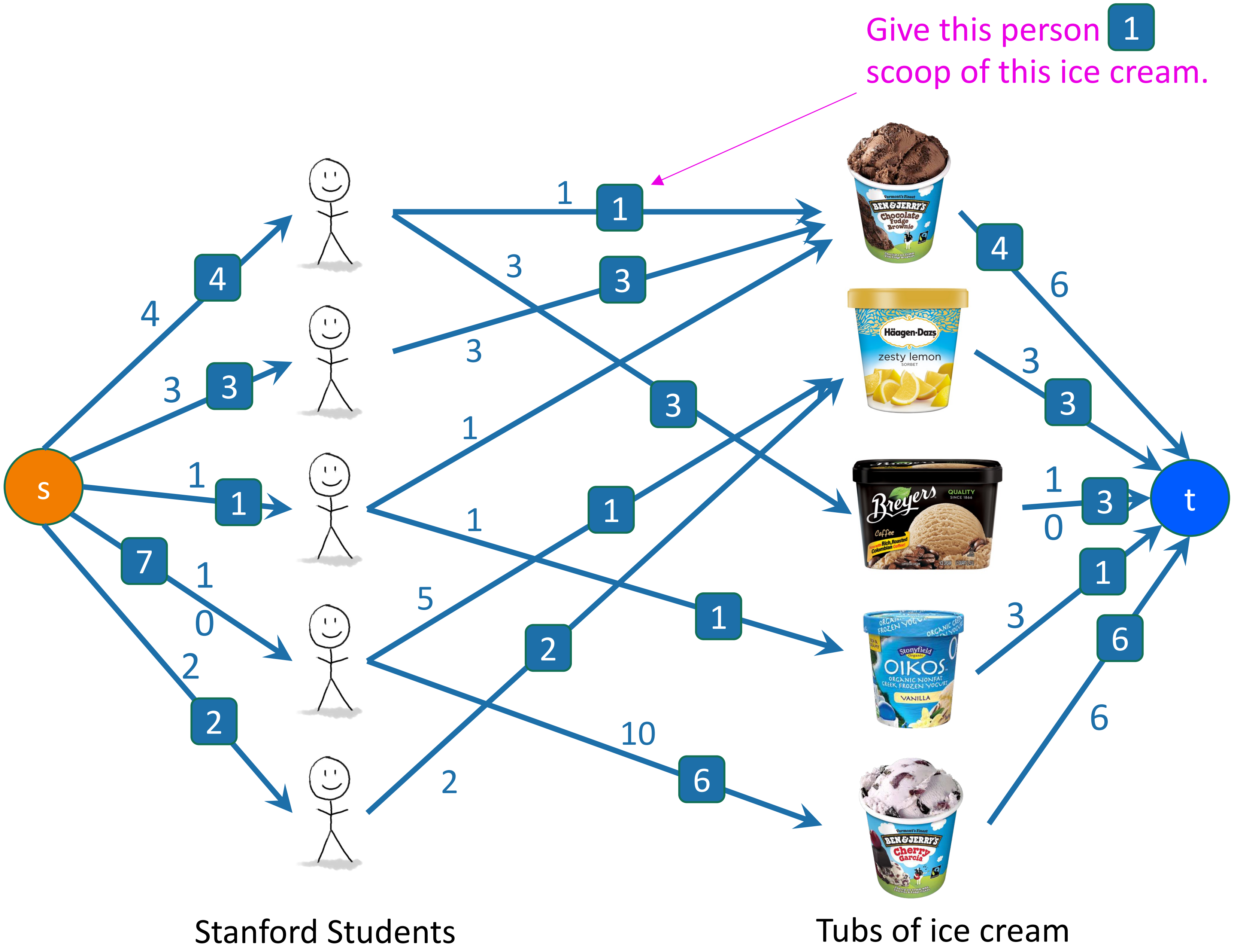
For this application, the max flow is $O(n)$ (number of swag items), so $O(mf) = O(n^3)$ is better than $O(n^5)$.

Answer 2: Yes! (no matter what the max flow is)

Authors	Year	Running time	Fine print
Ford-Fulkerson/Edmonds-Karp/Dinitz	1955 (and 1970/72)	$O(nm^2)$	
Goldberg-Tarjan	1986	$O(n^2m)$	
Goldberg-Rao	1997	$\tilde{O}\left(\min\{n^{2/3}, m^{1/2}\} \cdot m\right)$	There are some logarithmic factors suppressed.
Kelner, Lee, Orecchia, Sidford / Sherman	2013	$O(m^{1.001})$	Returns a flow that is within 0.9999 of maximum; only for undirected graphs; can make "0.001" arbitrarily close to 0.
Orlin	2013	$O(nm)$	
Chen, Kyng, Liu, Peng, Gutenberg, Sachdeva	2022	$O(m^{1.001})$	Can make "0.001" arbitrarily close to 0.

Question

- If this is the max flow (value 17), what's the min cut?



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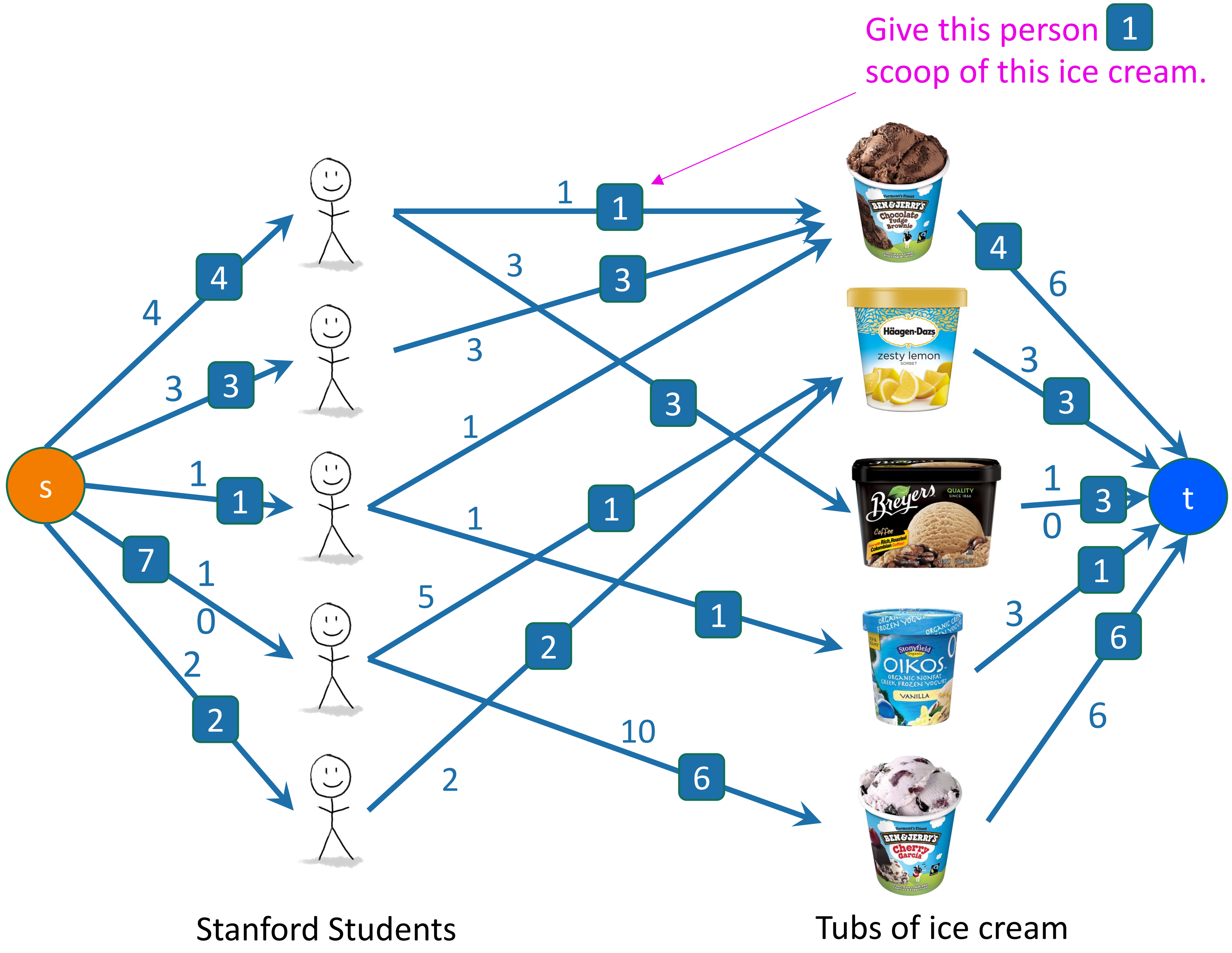
Let's look at the residual graph!



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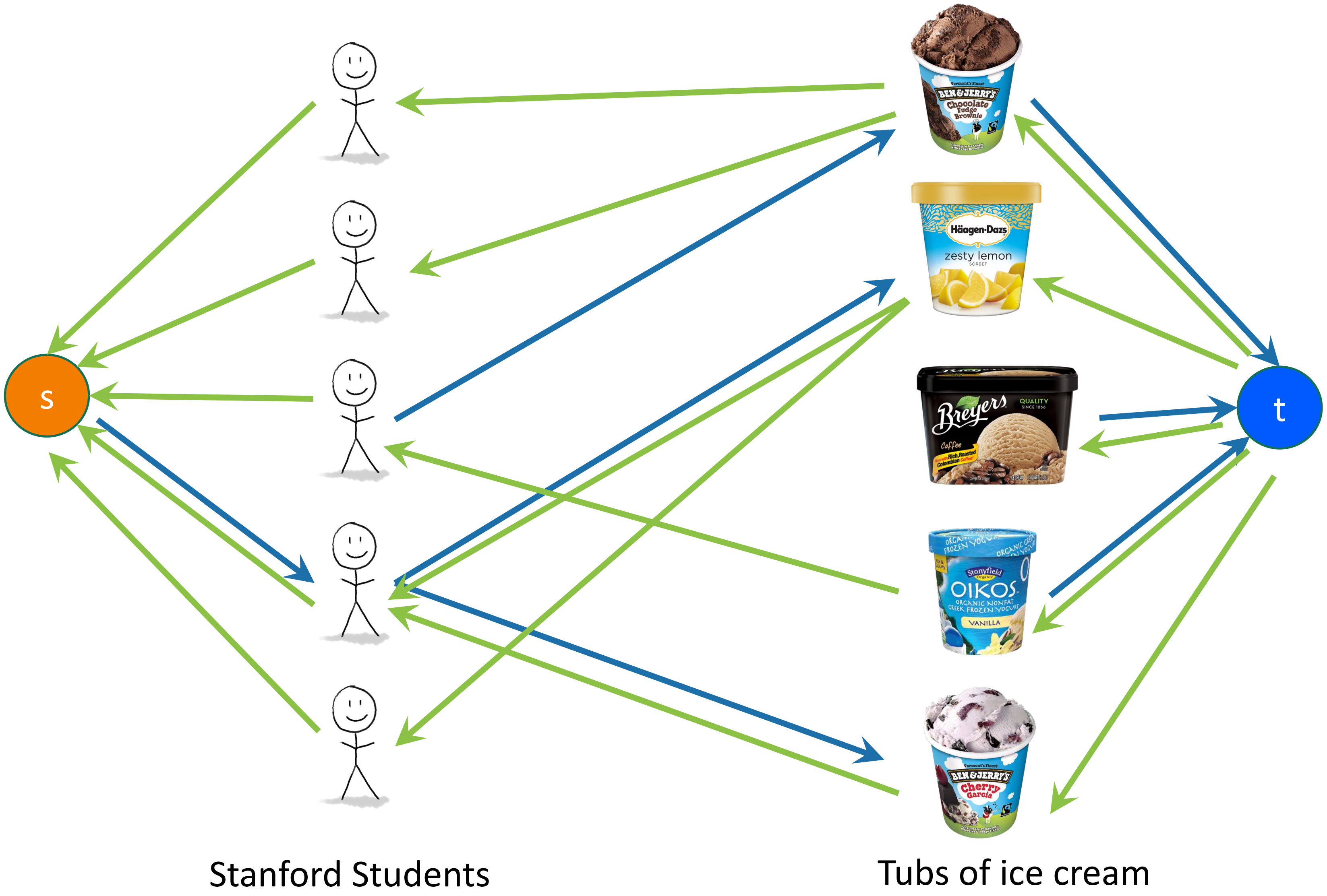
Weights omitted for readability



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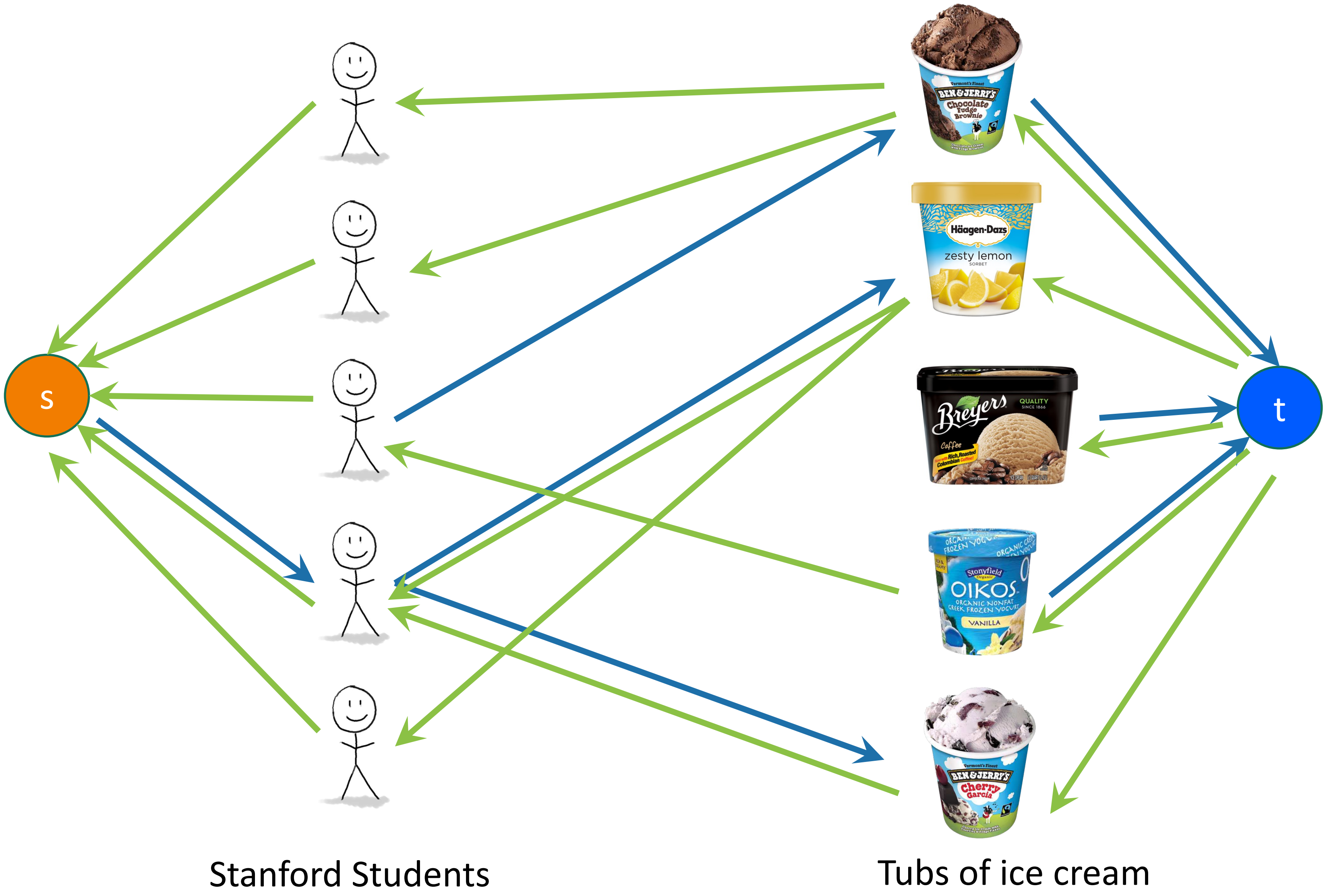
In the residual graph, consider the cut "things reachable from s" and "things not reachable from s"



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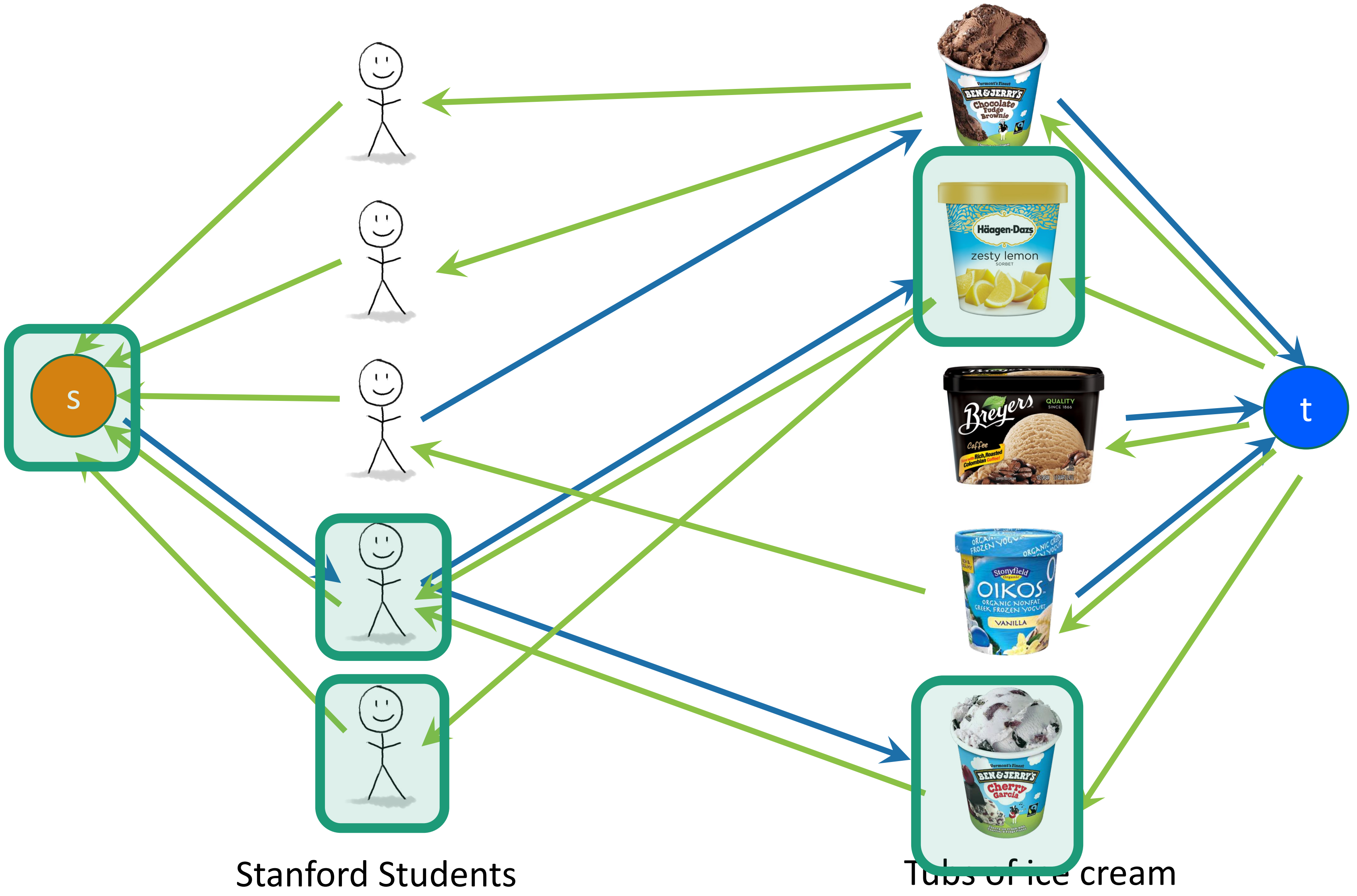

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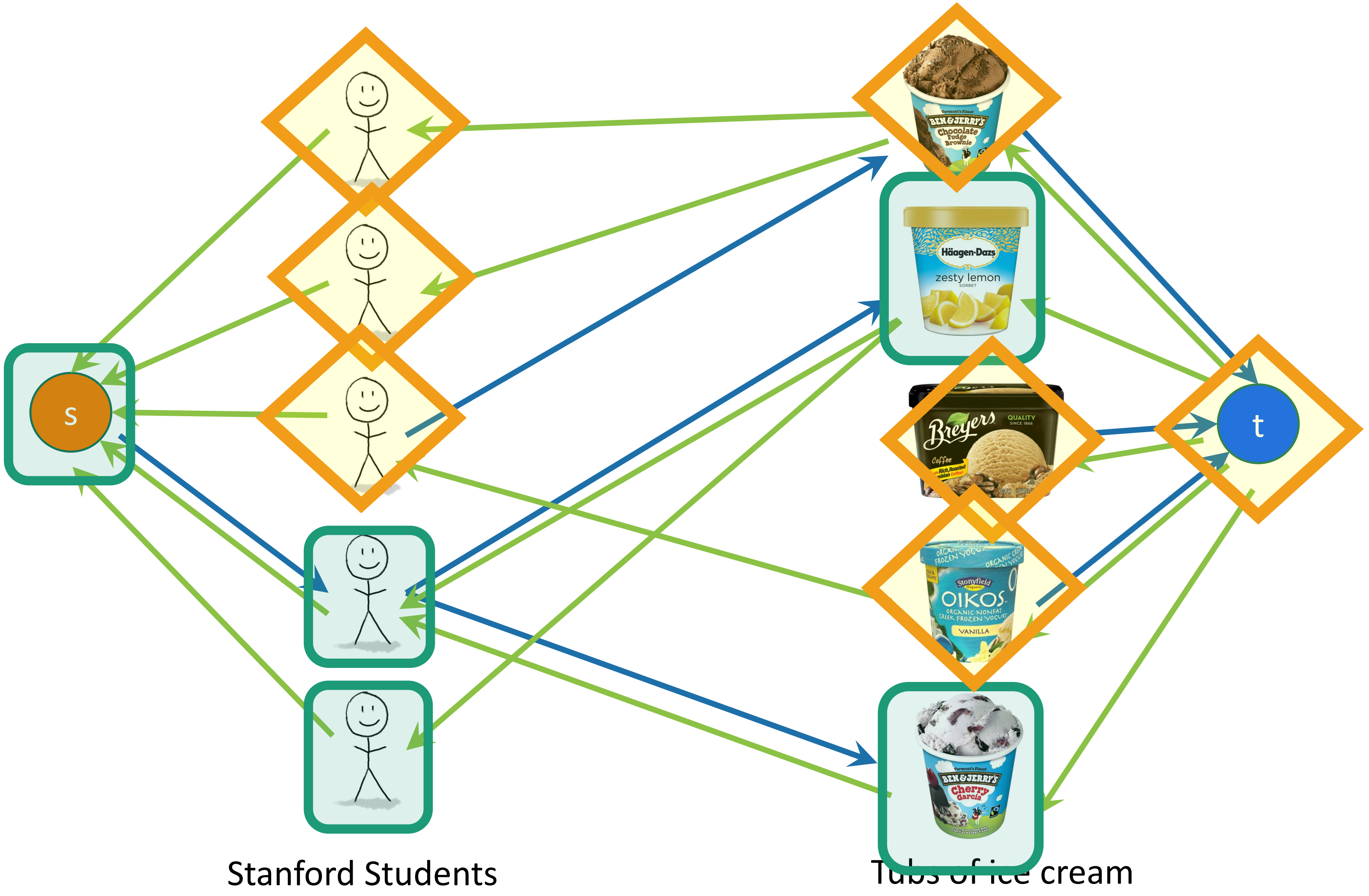

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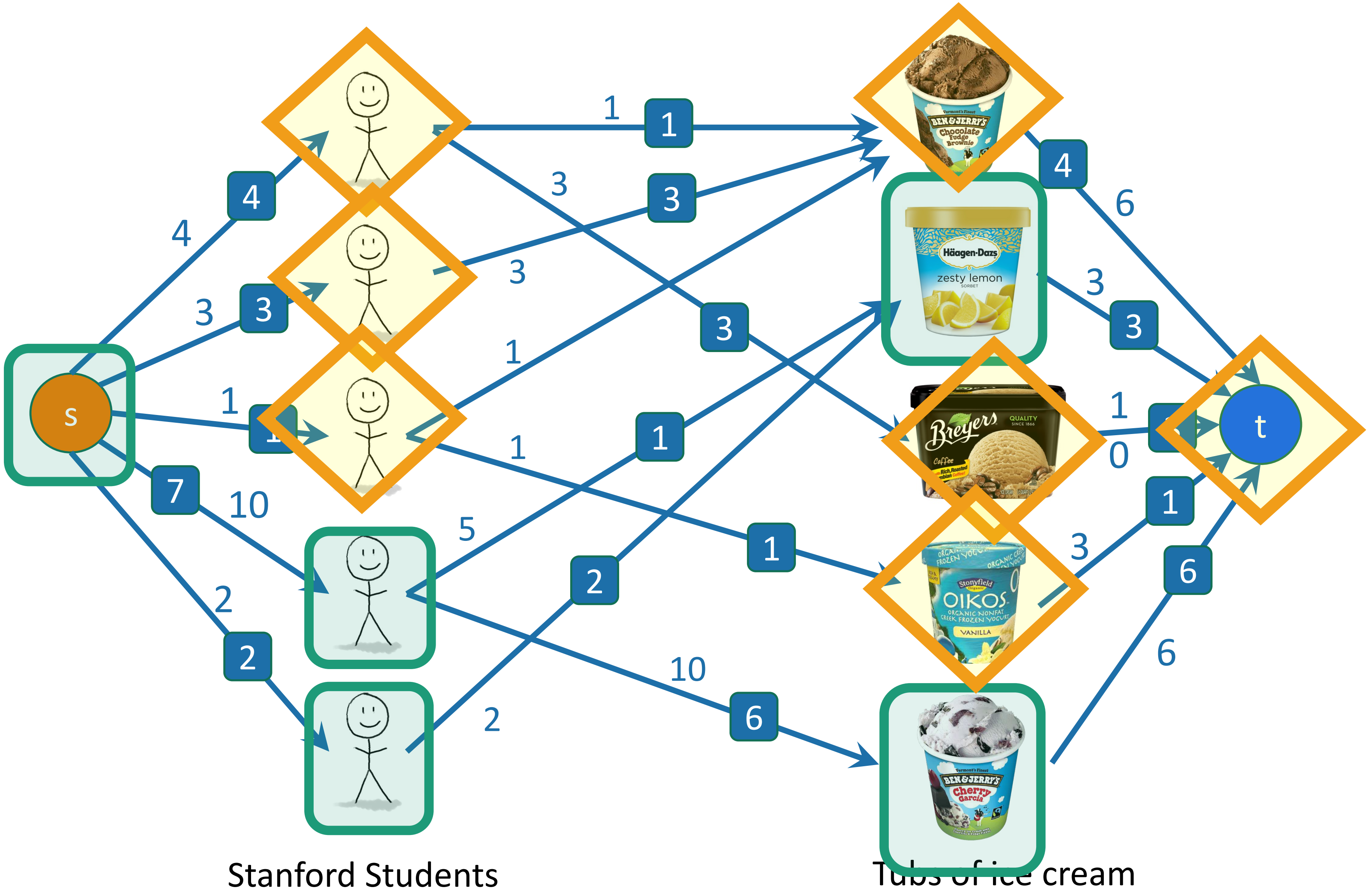

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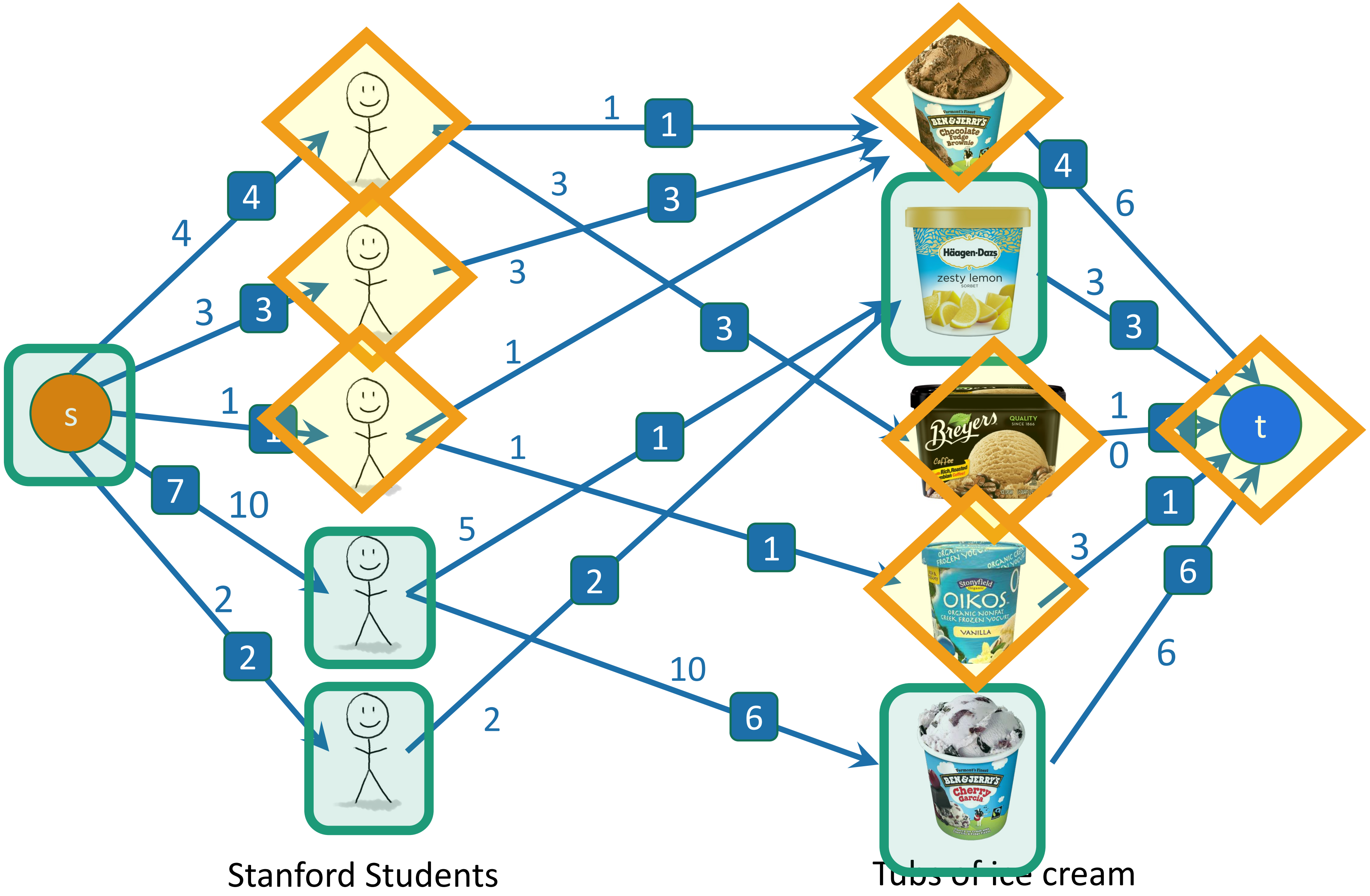
What edges cross this cut?



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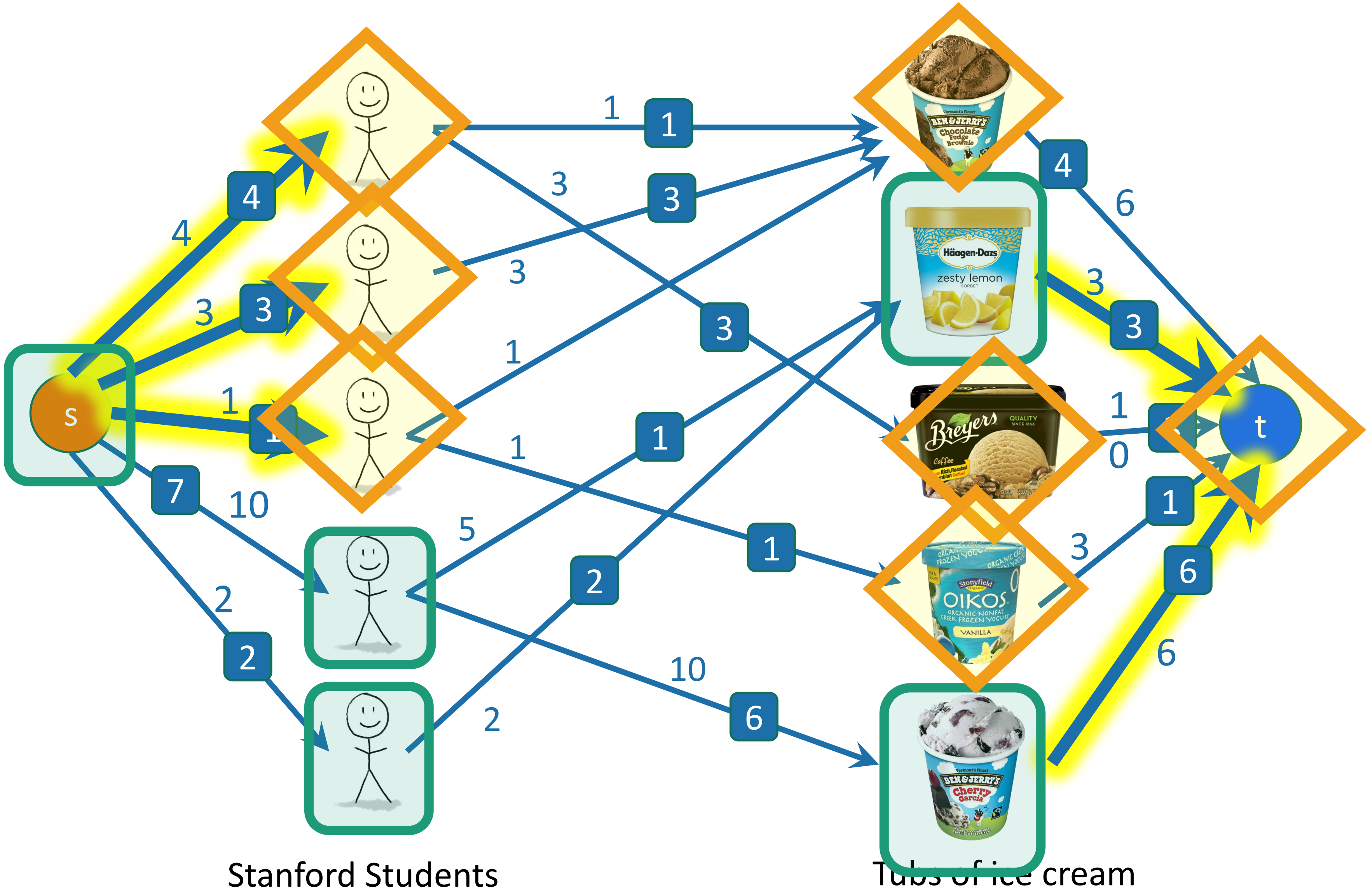

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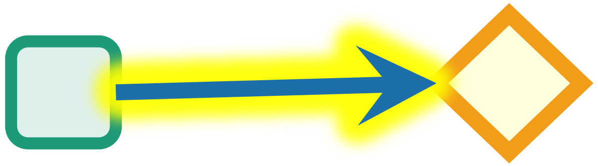
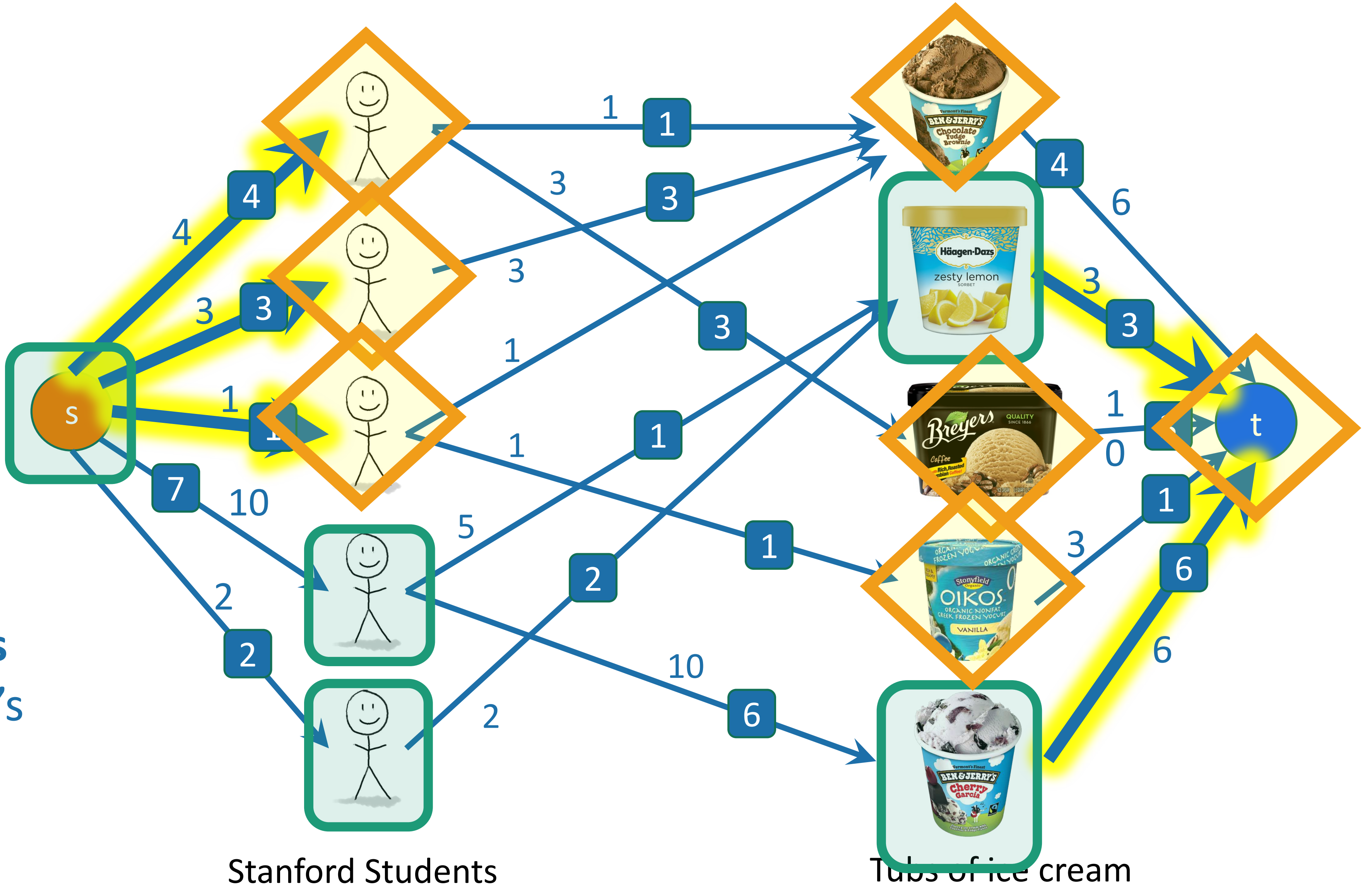
Recall, an edge **crosses the cut** if it goes from s's side to t's side.





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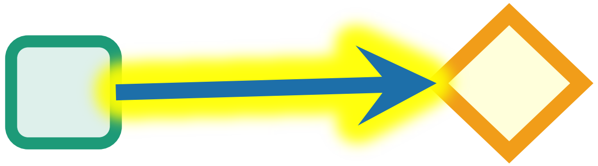
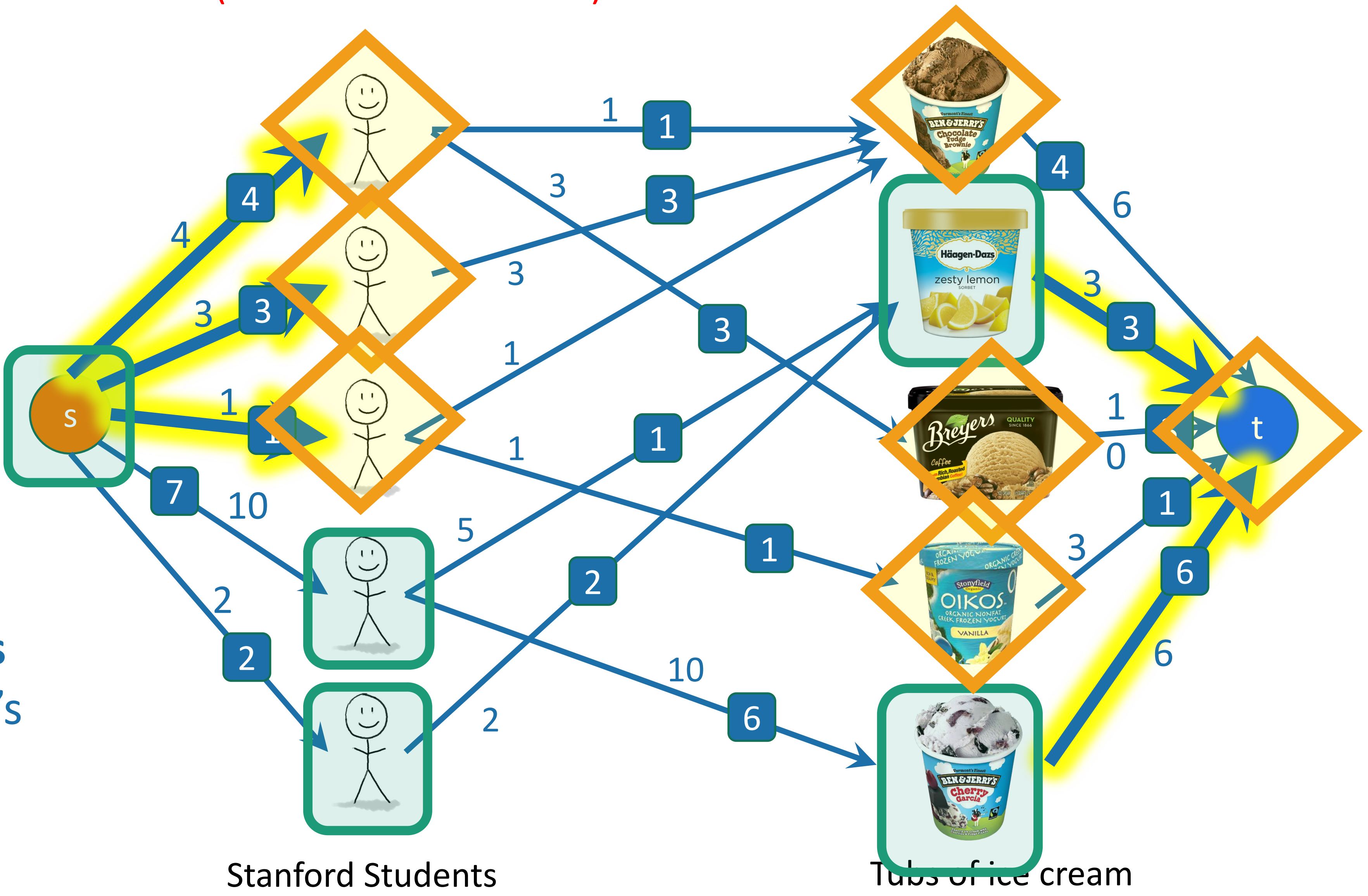


Cost of cut: 4+3+1+3+6 = 17

(Same as value of flow)



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Who has questions?

Technical, non-technical, whatever