CS 161: List of Summation Formulae

CS 161 Teaching team

Here are a few useful summation formulae! While we will try to remind you of these formulae wherever they are needed (especially on exams), we will expect that you are familiar with these expressions.

Contents

1 Geometric Progressions

$$1 + r + r^2 + r^3 + ... + r^n = \sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r} (\text{for } r \neq 1)$$

$$1 + r + r^2 + r^3 + ... = \sum_{i=0}^{\infty} r^i = \frac{1}{1 - r} \ (\text{for } |r| < 1)$$

2 Sum of First n Natural Number Powers

$$1 + 2 + 3 + ... + n = \sum_{i=1}^{n} i = n(n+1)/2$$

$$1^k + 2^k + ... + n^k = \sum_{i=1}^{n} i^k = \Theta(n^{k+1}) \quad (\text{for integers } k > 1)$$

3 Binomial Identities and Formulas

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$$

$$\binom{n}{0} + \binom{n}{1} + ... + \binom{n}{n} = \sum_{k=0}^{n} \binom{n}{k} = 2^n$$

$$(1 + x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

4 Harmonic Sum

$$1/1 + 1/2 + 1/3 + .. + 1/n = \sum_{i=1}^{n} 1/i = \Theta(\log n)$$