

# Suffix and LCP Arrays

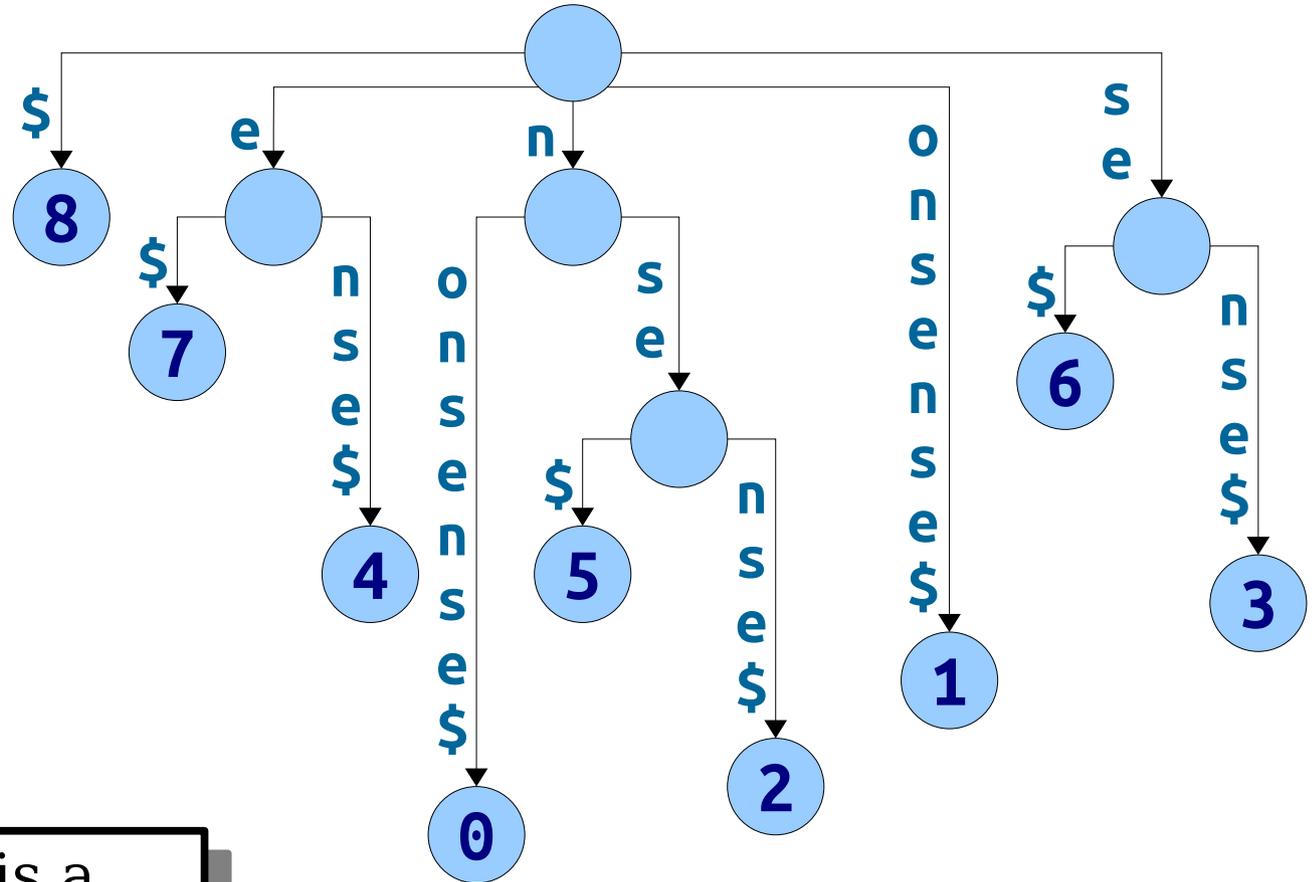
Recap from Last Time

# Suffix Trees

**nonsense**

# Suffix Trees

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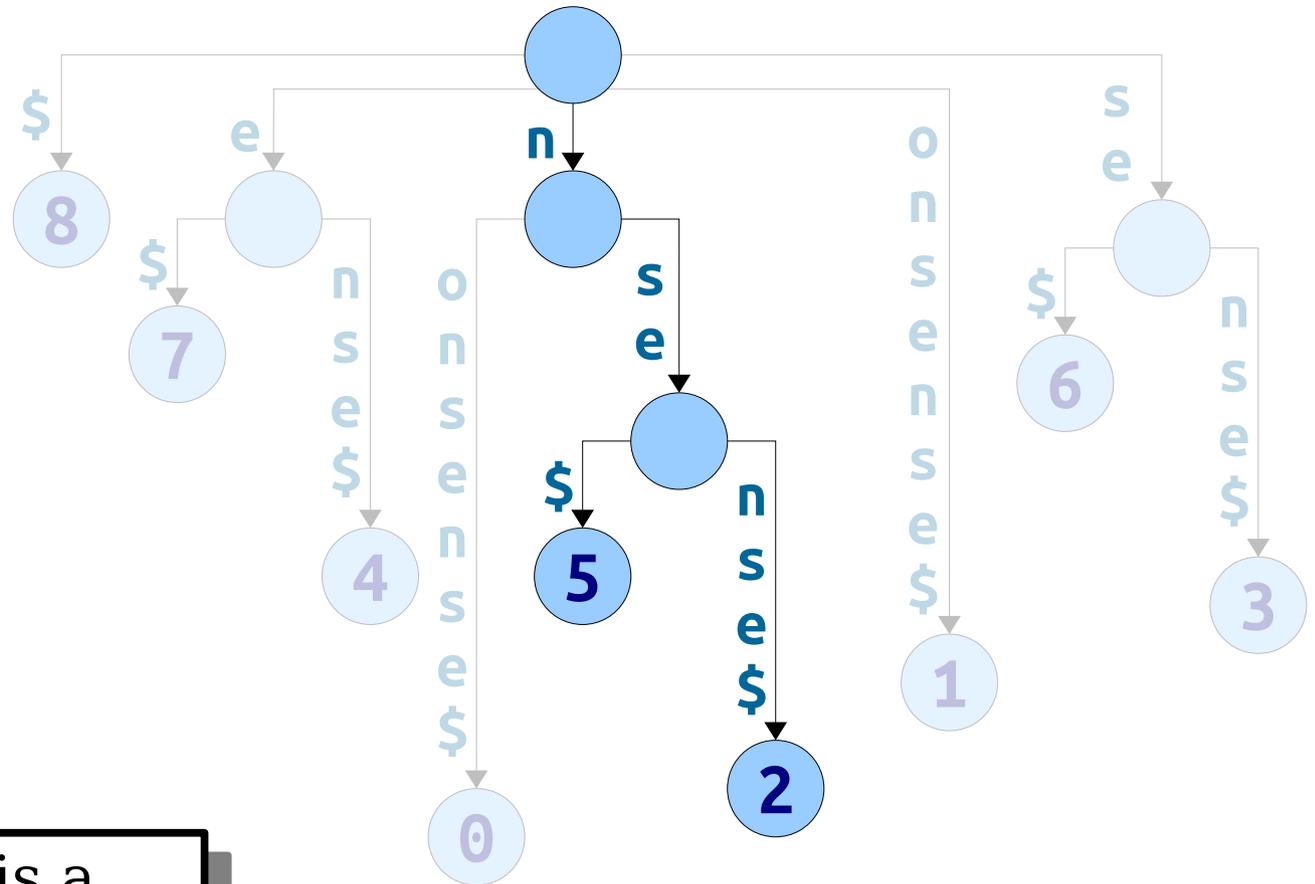


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**Theorem:**  $w$  is a substring of  $x$  if and only if  $w$  is a prefix of a suffix of  $x$ .

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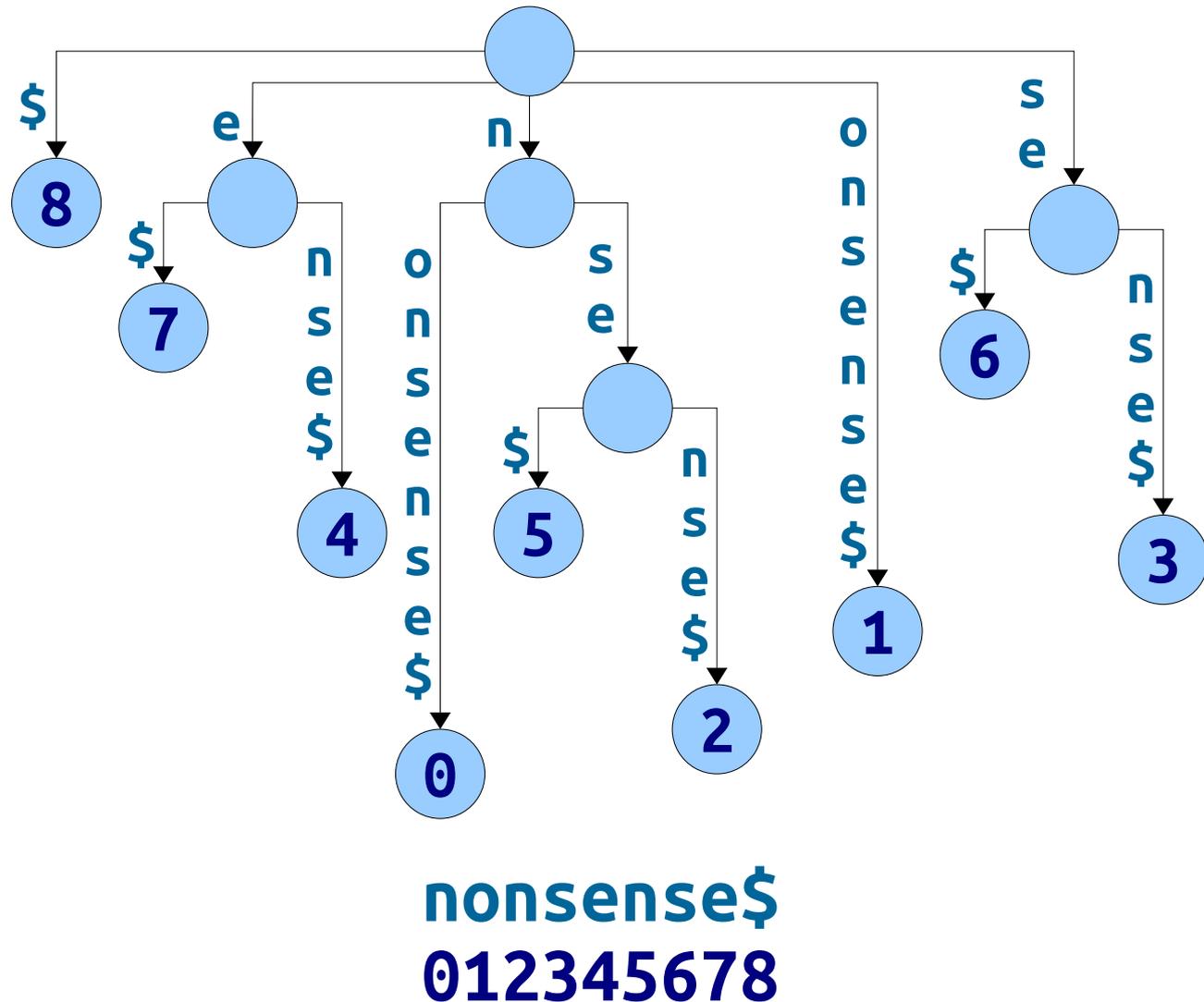
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New Stuff!

# Representing Suffix Trees

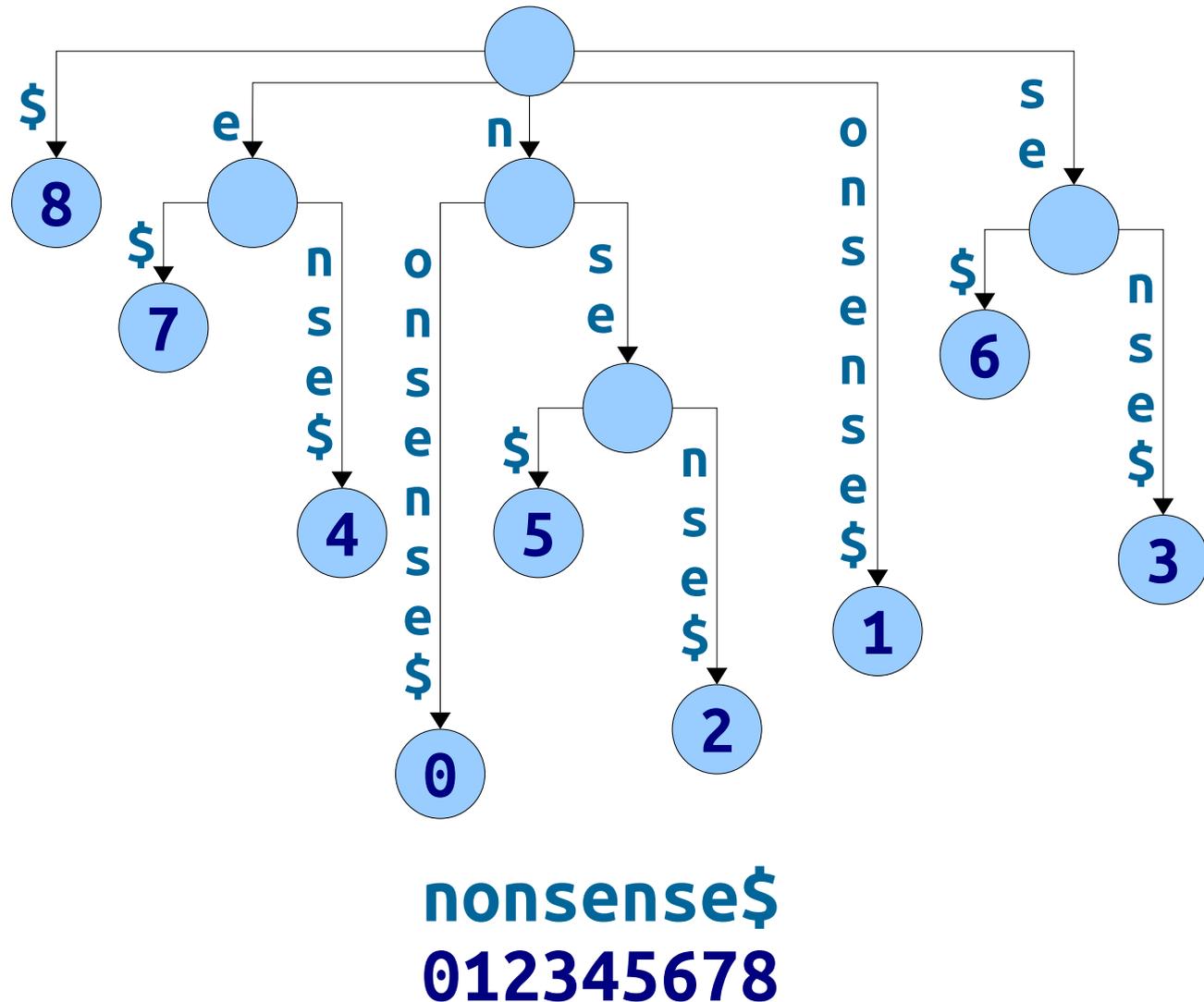
# Representing a Suffix Tree

- We know that a suffix tree has  $O(m)$  nodes, where  $m$  is the number of characters in the input string.
- This means that there are  $O(m)$  edges.
- **Question:** Why can't we immediately claim that the space usage of the suffix tree is  $O(m)$ ?



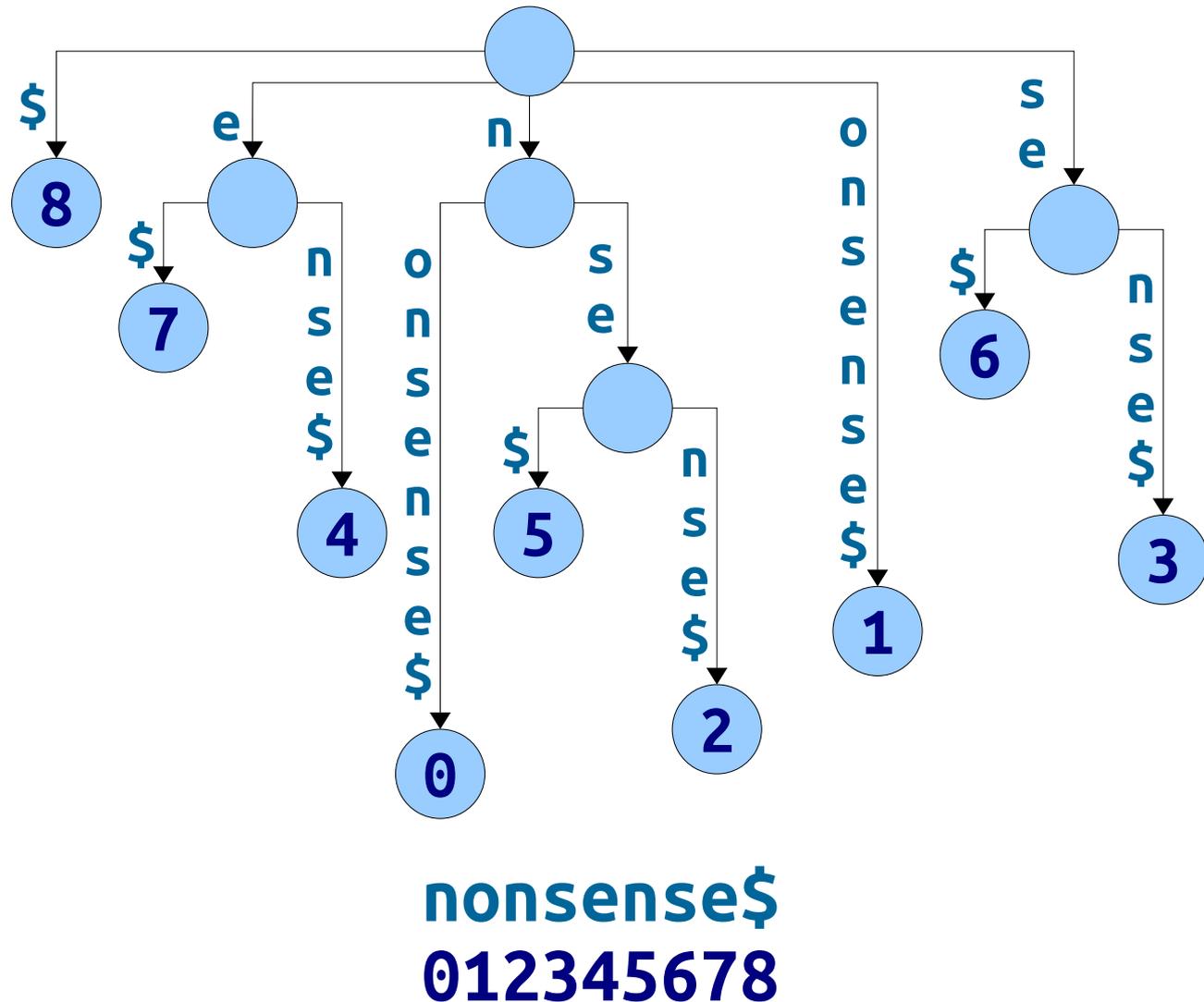
# Representing a Suffix Tree

- **Claim:** Writing out all suffixes of a string of length  $m$  requires  $\Theta(m^2)$  characters.
- **Proof idea:** Those suffixes have length  $1 + 2 + \dots + (m+1)$ , factoring in the special  $\$$  character.
- **Problem:** It is indeed possible to build a suffix tree with  $\Theta(m^2)$  total letters on the edges.

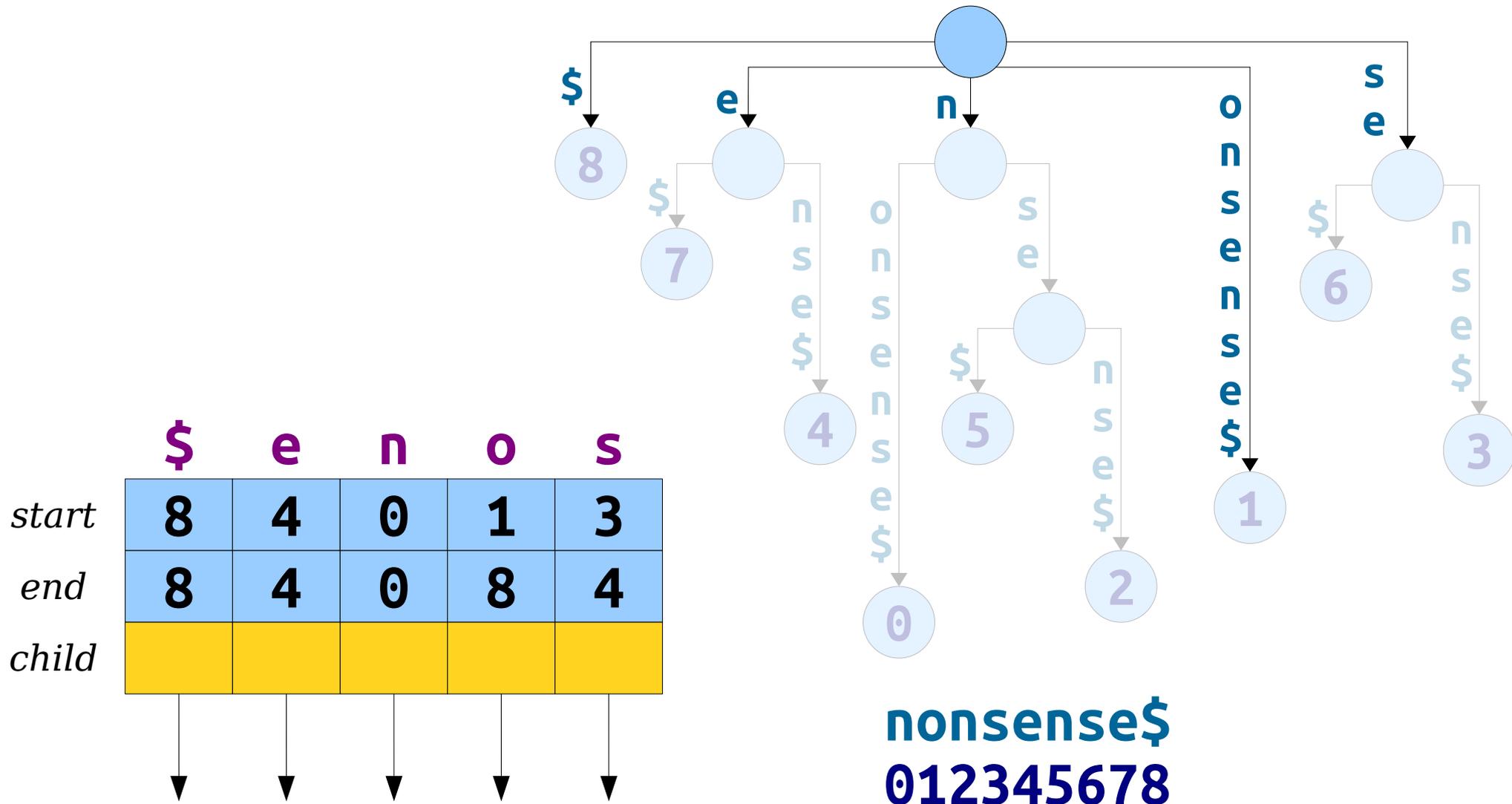


# Representing a Suffix Tree

- By being clever with our representation, we can guarantee that a suffix tree uses only  $\Theta(m)$  space, regardless of the input string.
- **Observation:** Each edge is labeled with a substring of the original input string.
- **Idea:** Don't actually write out the labels on the edges. Just write down the start and end index!

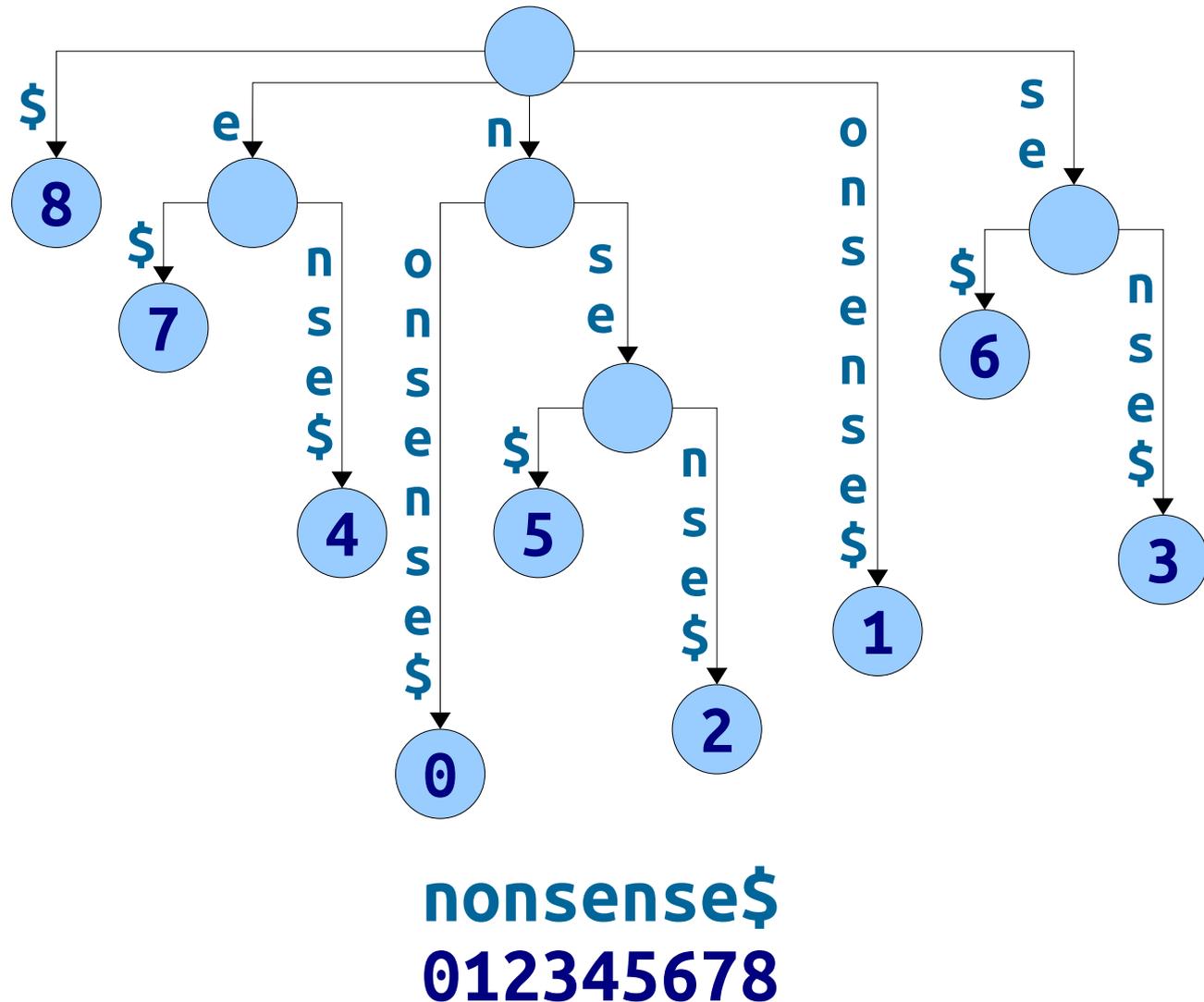


# Representing a Suffix Tree



# Representing a Suffix Tree

- Space usage required for a suffix tree:
  - $O(m)$  space for all the nodes.
  - $O(m)$  space for a copy of the original string.
  - $O(m)$  space for the edges.
- Total space:  $O(m)$ .

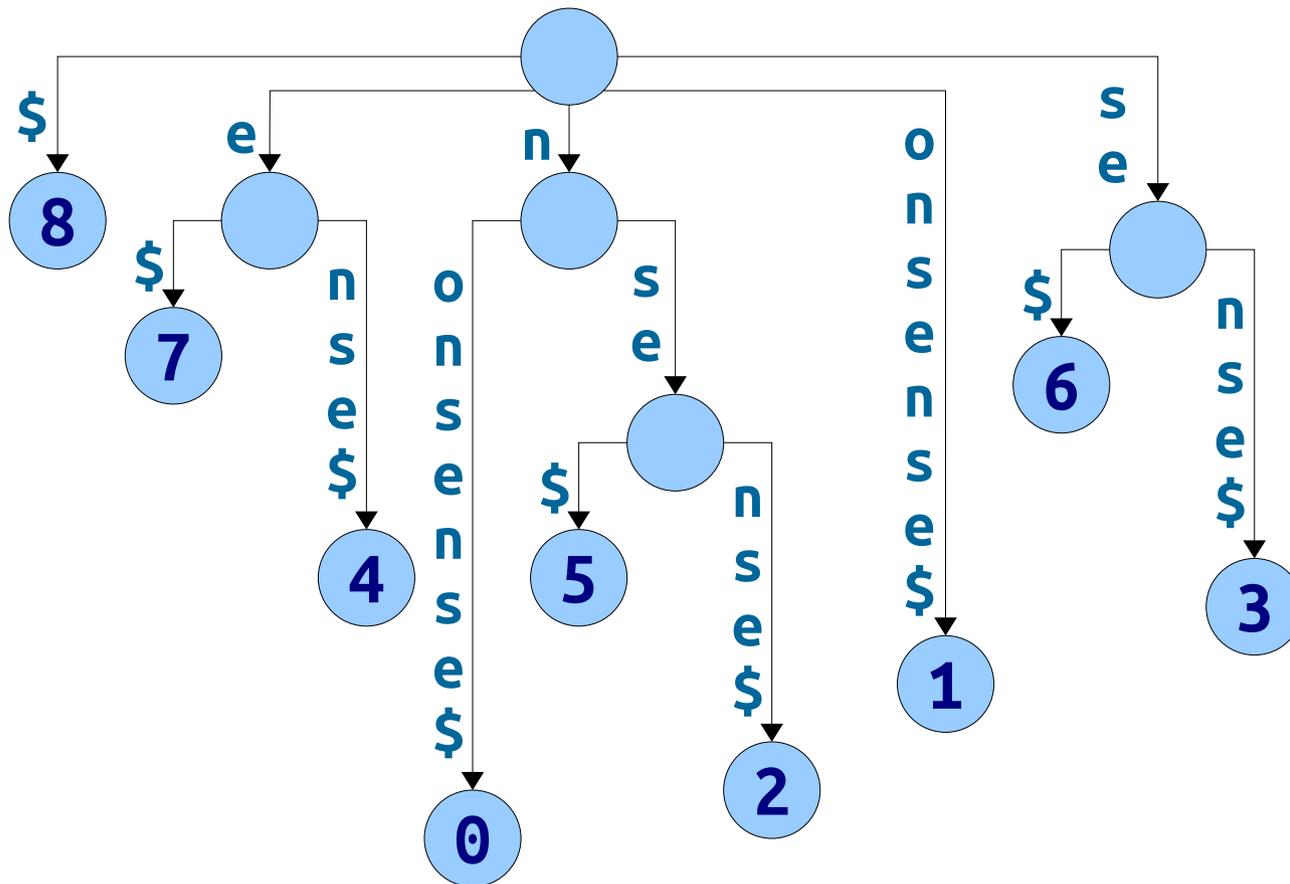


# Suffix Tree Space Usage

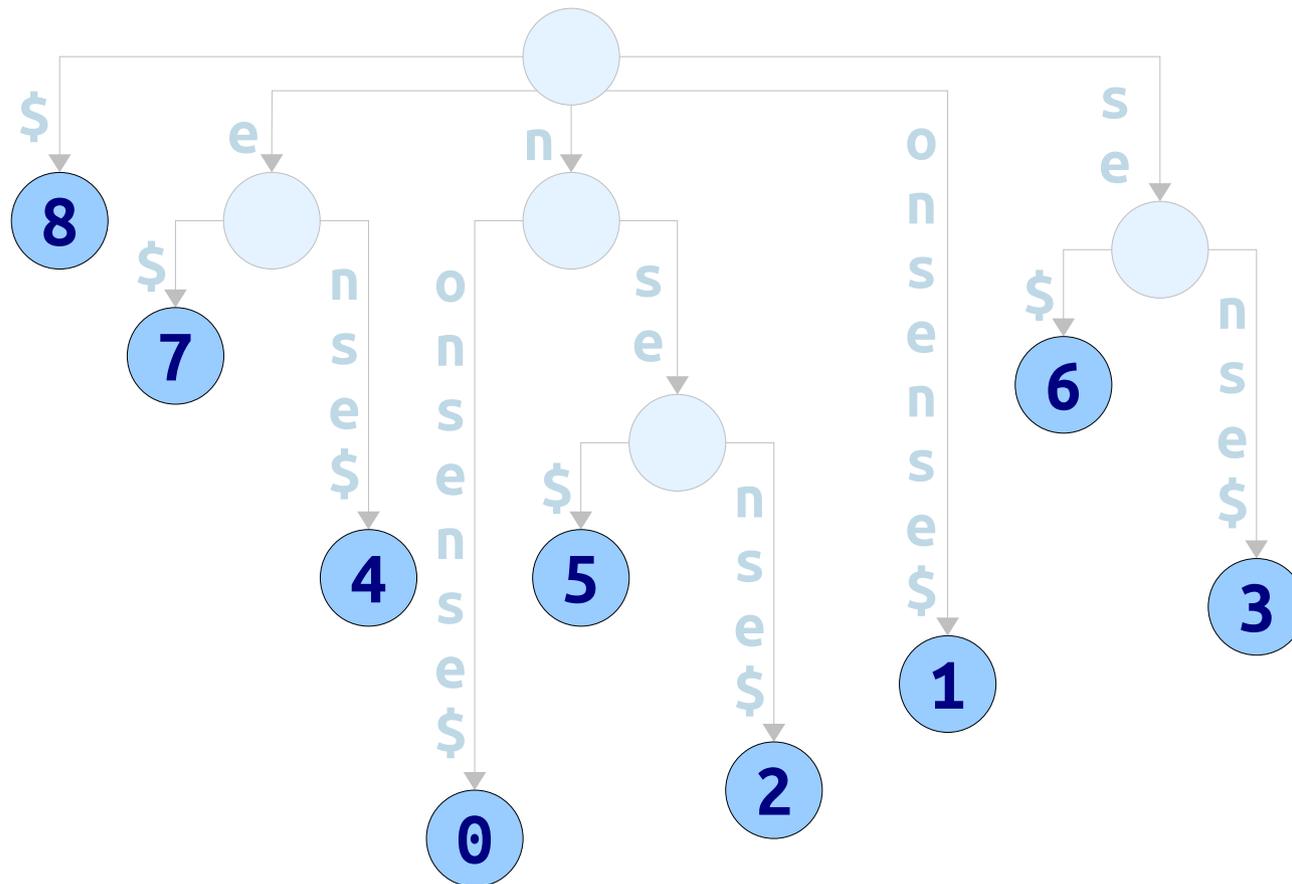
- Suffix tree edges take up a *lot* of space.
  - Two machine words per edge to denote the range of characters visited.
  - One machine word per edge for the pointer itself.
  - Number of edges ranges from  $m$  to  $2m - 1$ , so this is between  $3m$  and  $6m$  machine words for the whole string!
- Example: a human genome is about three billion characters long.
  - With clever techniques, that can be packed into about 800MB.
  - On a 32-bit machine, the suffix tree needs about 48GB – too big to fit into memory!
  - On a 64-bit machine, the suffix tree needs about 96GB – way more than a typical machine can hold!

***Key Question:*** Can we get the benefits of a suffix tree without the space penalty?

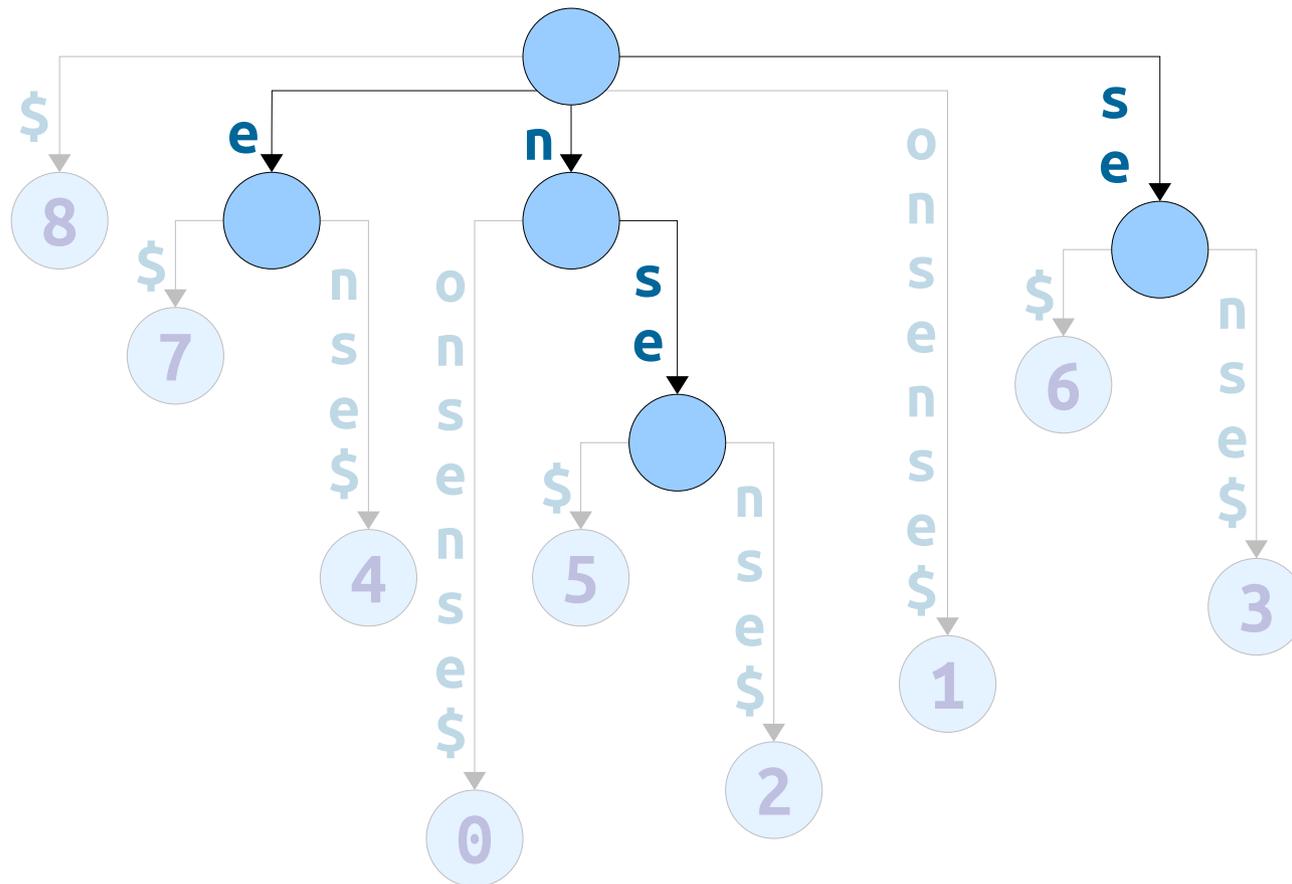
What is it about suffix trees that make them so useful algorithmically?



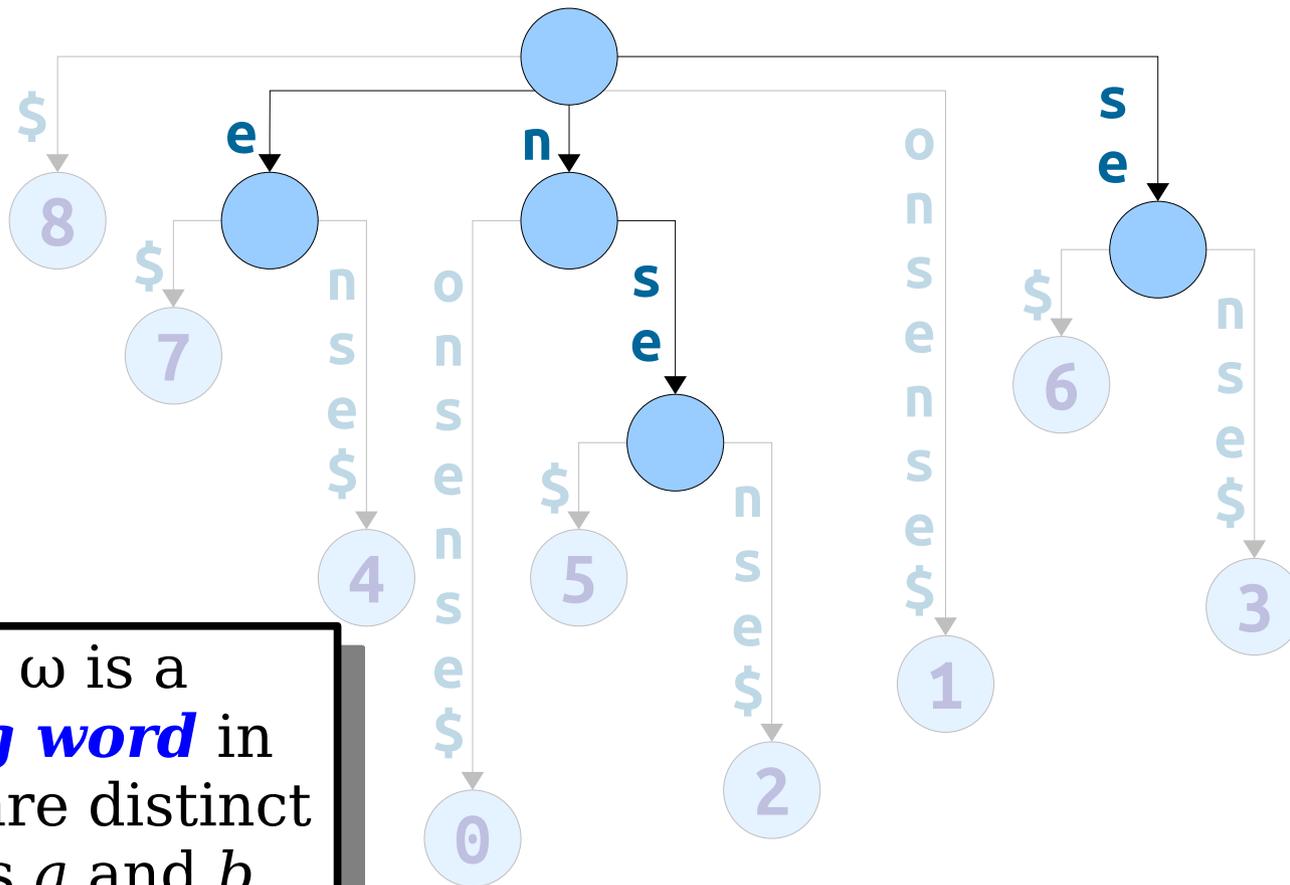
**Theorem:** There is a node labeled  $\omega$  in a suffix tree for  $T$   
*if and only if*  
 $\omega$  is a suffix of  $T\$$  or  $\omega$  is a branching word in  $T\$$ .



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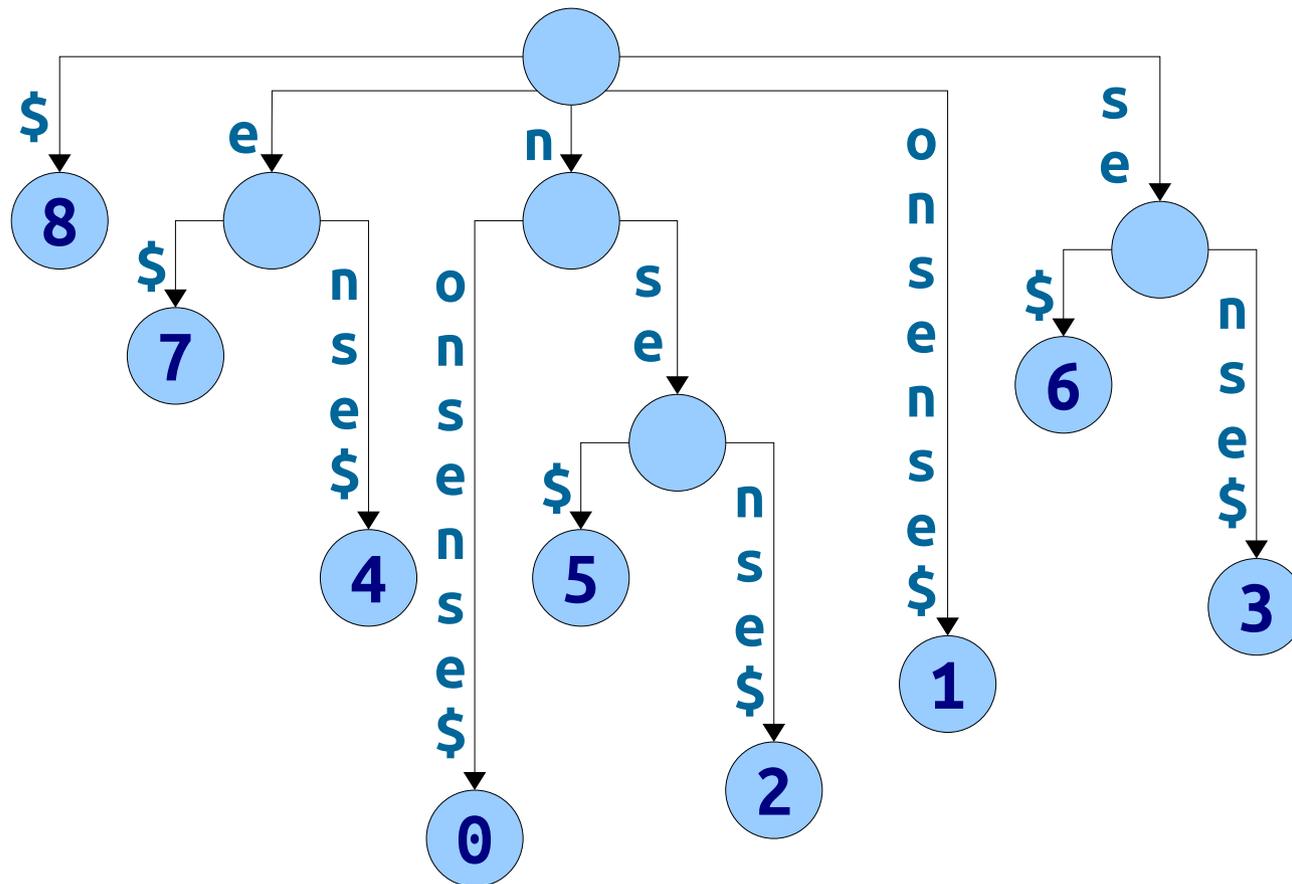
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A string  $\omega$  is a **branching word** in  $T\$$  if there are distinct characters  $a$  and  $b$  where  $\omega a$  and  $\omega b$  are substrings of  $T\$$ .

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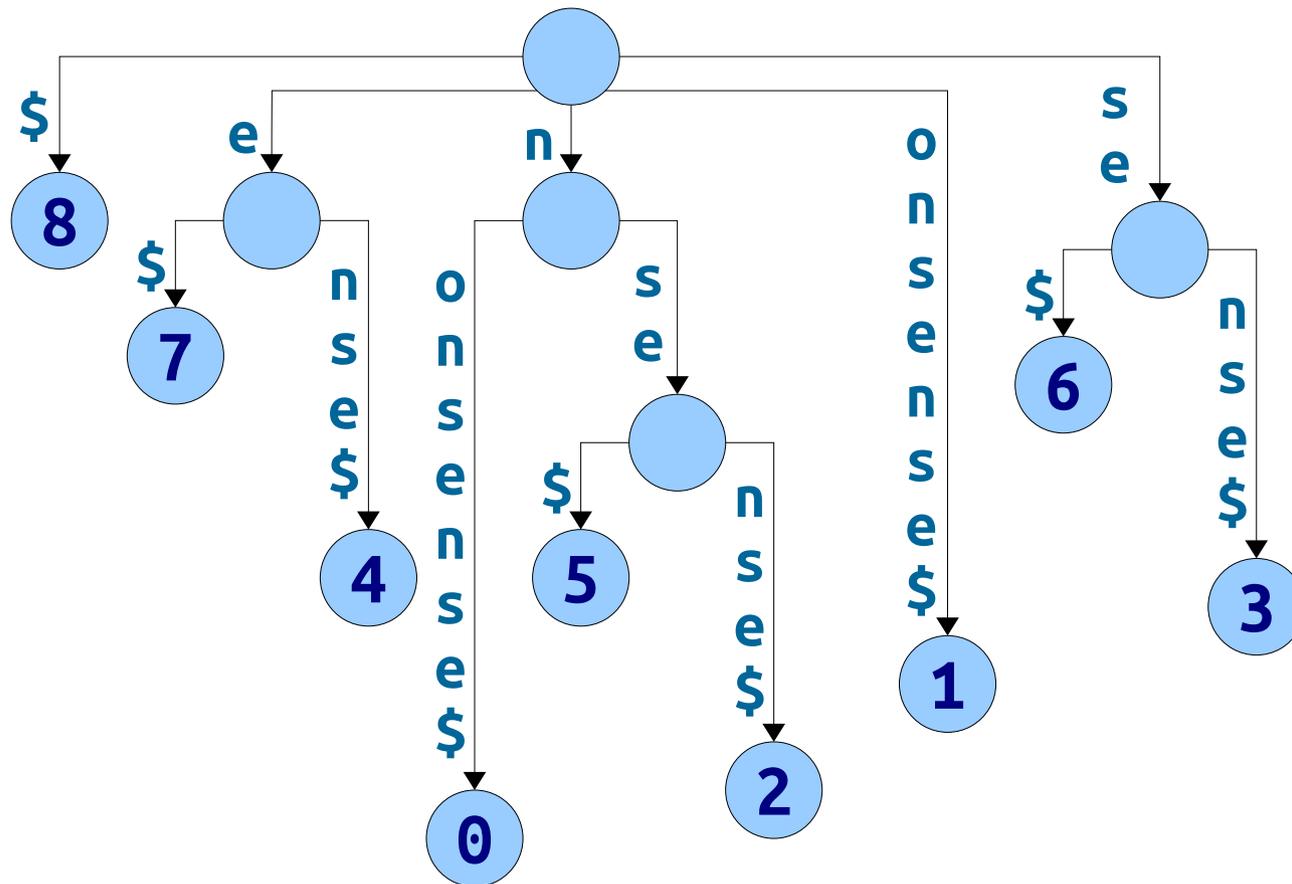


**Key Intuition:** The efficiency in a suffix tree is largely due to  
 1. keeping the suffixes in sorted order, and  
 2. exposing branching words.

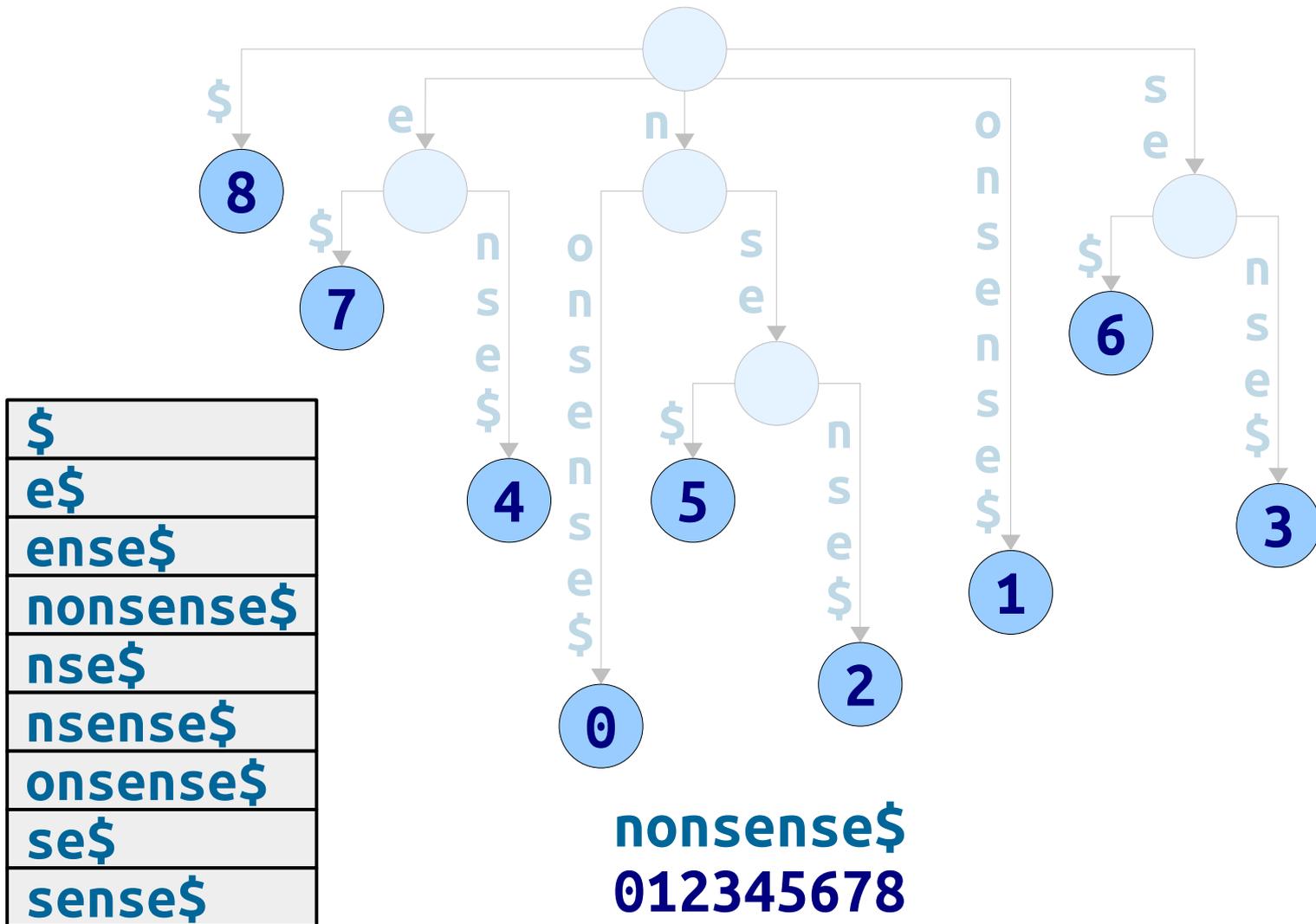
# Where We're Going

- Today, we'll see two data structures that encode much of the same information as suffix trees, but in much less space.
  - The *suffix array* stores information about the ordering of the suffixes of a string.
  - The *LCP array* stores information about the branching words of a string.
- Together, they'll provide algorithms that match or are comparable to the time bounds from last time.

# Suffix Arrays



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# Suffix Arrays

- A **suffix array** for a string  $T$  is a sorted array of the suffixes of the string  $T\$$ .
- Suffix arrays distill out just the first component of suffix trees: they store suffixes in sorted order.



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# Suffix Arrays

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- Suffix arrays distill out just the first component of suffix trees: they store suffixes in sorted order.
- **Non-obvious fact:** Suffix arrays can be built in time  $O(m)$ . Details next time!

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# Suffix Arrays

- The way we've drawn suffix arrays is terribly space-inefficient.
  - It always uses space  $\Theta(m^2)$ , since that's how many total characters occur in all suffixes.
- Can we do better?

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# Suffix Arrays

- We reduced the space usage of suffix trees by representing substrings, implicitly, as ranges within the original string.
- **Idea:** Don't store the suffixes themselves. Just store the starting positions of the suffixes.
- Space:  $\Theta(m)$ , and with only one machine word used per character of input.

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# Suffix Arrays

- Although the picture to the right is how we'd represent the suffix array in memory, for this lecture we'll draw things out the longer way.
- This is just to build intuition; we wouldn't actually do that in practice.

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# Using Suffix Arrays

- Last time, we saw how to find all instances of a pattern *P* in a text *T* using suffix trees.
- How could we do that with suffix arrays?

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# Using Suffix Arrays

- **Reminder:** Our text string  $T$  has length  $m$ . Our pattern string  $P$  has length  $n$ .
- **Claim:** With a suffix array, we can determine whether  $P$  appears in  $T$  in time  $O(n \log m)$ .

How?

Formulate a hypothesis!

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How?

Discuss with your neighbors!

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  - Binary search has  $O(\log m)$  rounds.
  - Each probe takes time  $O(n)$ .
- This bound can be made tight. (*How?*)
- Figure that  $m$  is often much bigger than  $n$ , so this is a huge win over a raw scan.

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# Using Suffix Arrays

- **Claim:** With a suffix array, we can find all matches of a pattern  $P$  in  $T$  in time  $O(n \log m + z)$ , where  $z$  is the number of matches.
- **Idea:** Binary search can be used to find a range of values equal to some key. Adapt that idea to find all suffixes beginning with the same prefix.

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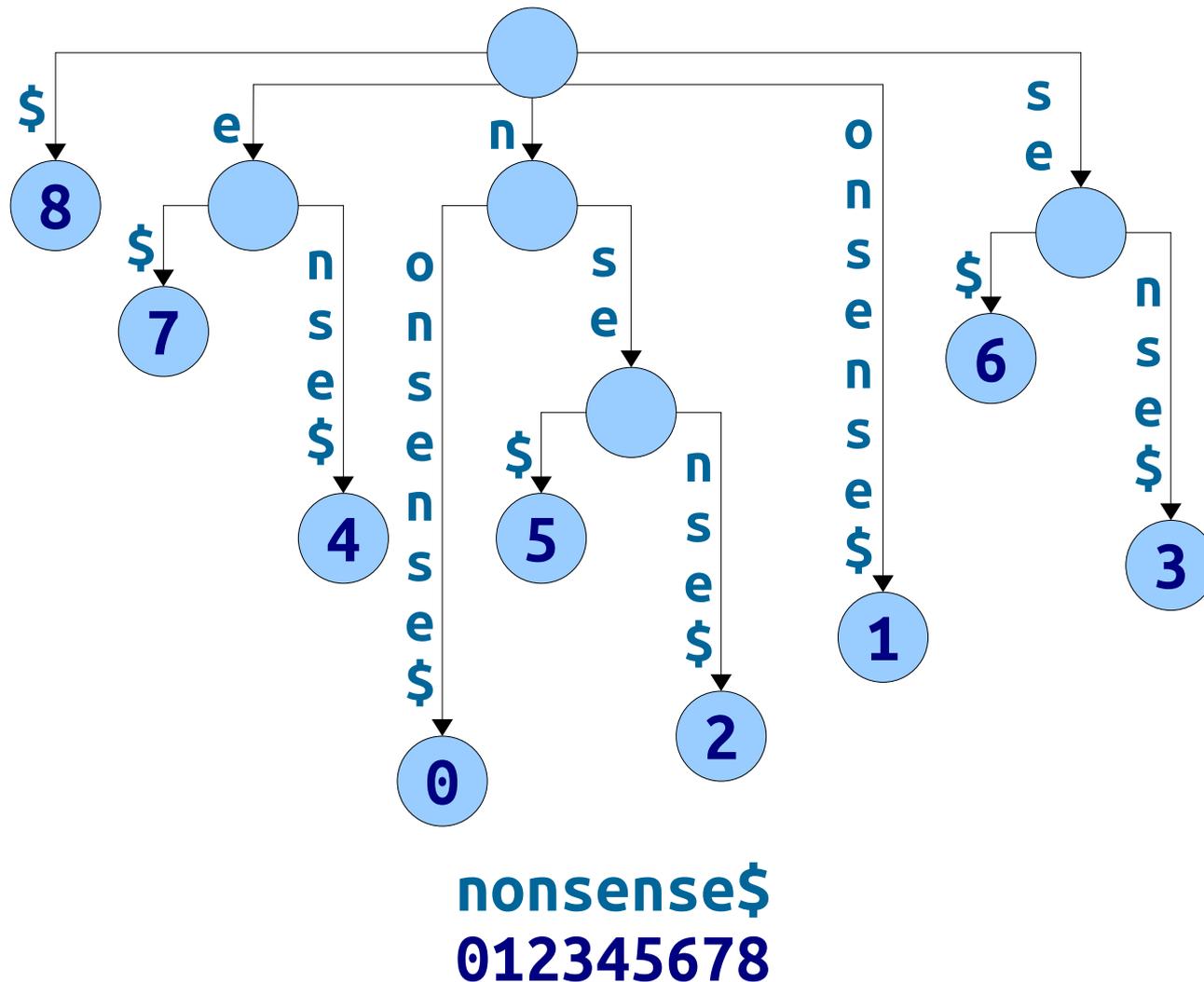
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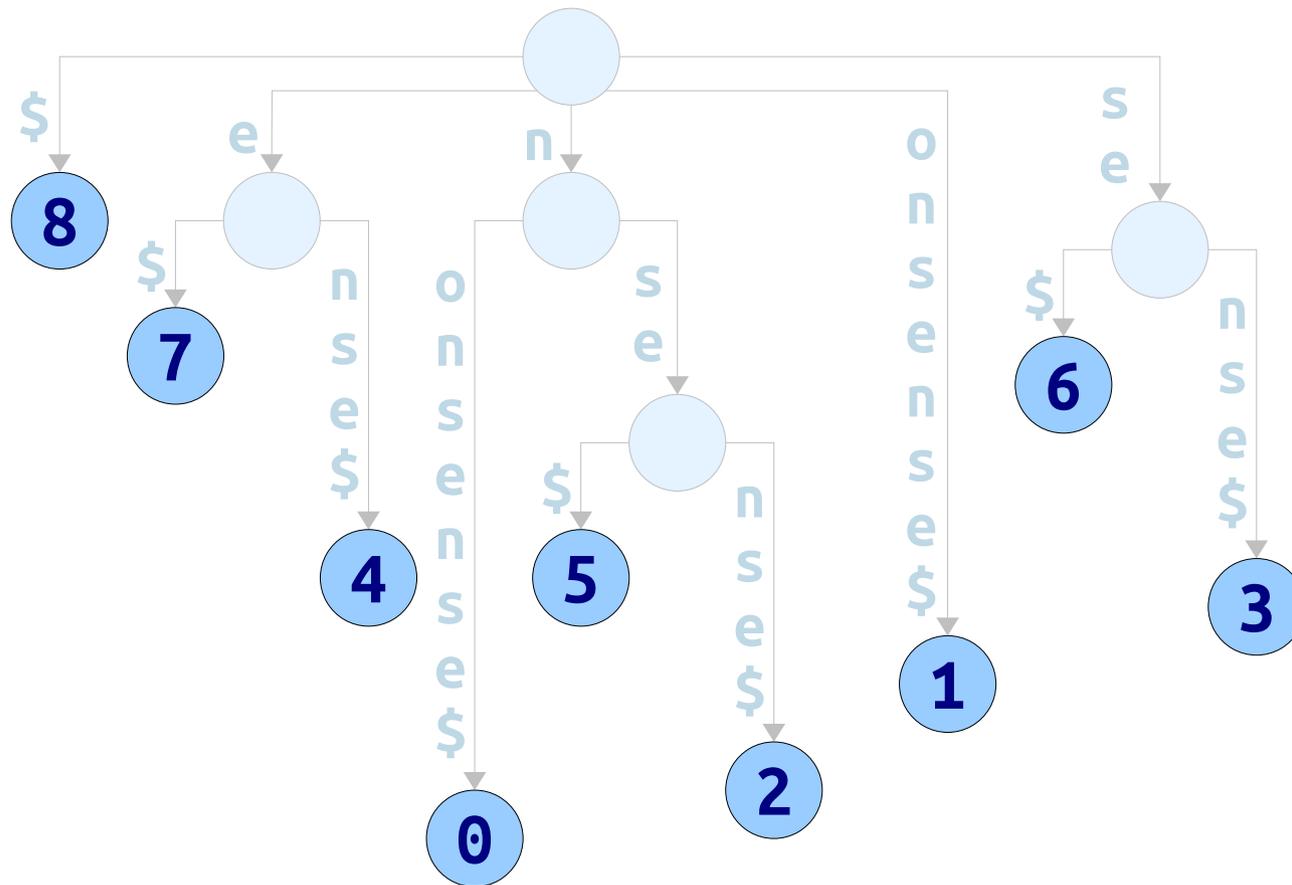
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# The Story So Far

- Suffix arrays store all the suffixes of a string in sorted order.
- They provide an  
 $\langle O(m), O(n \log m + z) \rangle$   
solution to the substring search problem.
- ***Intuition:*** Suffix trees are valuable in large part because they just keep the suffixes sorted.
- What else are suffix trees doing?

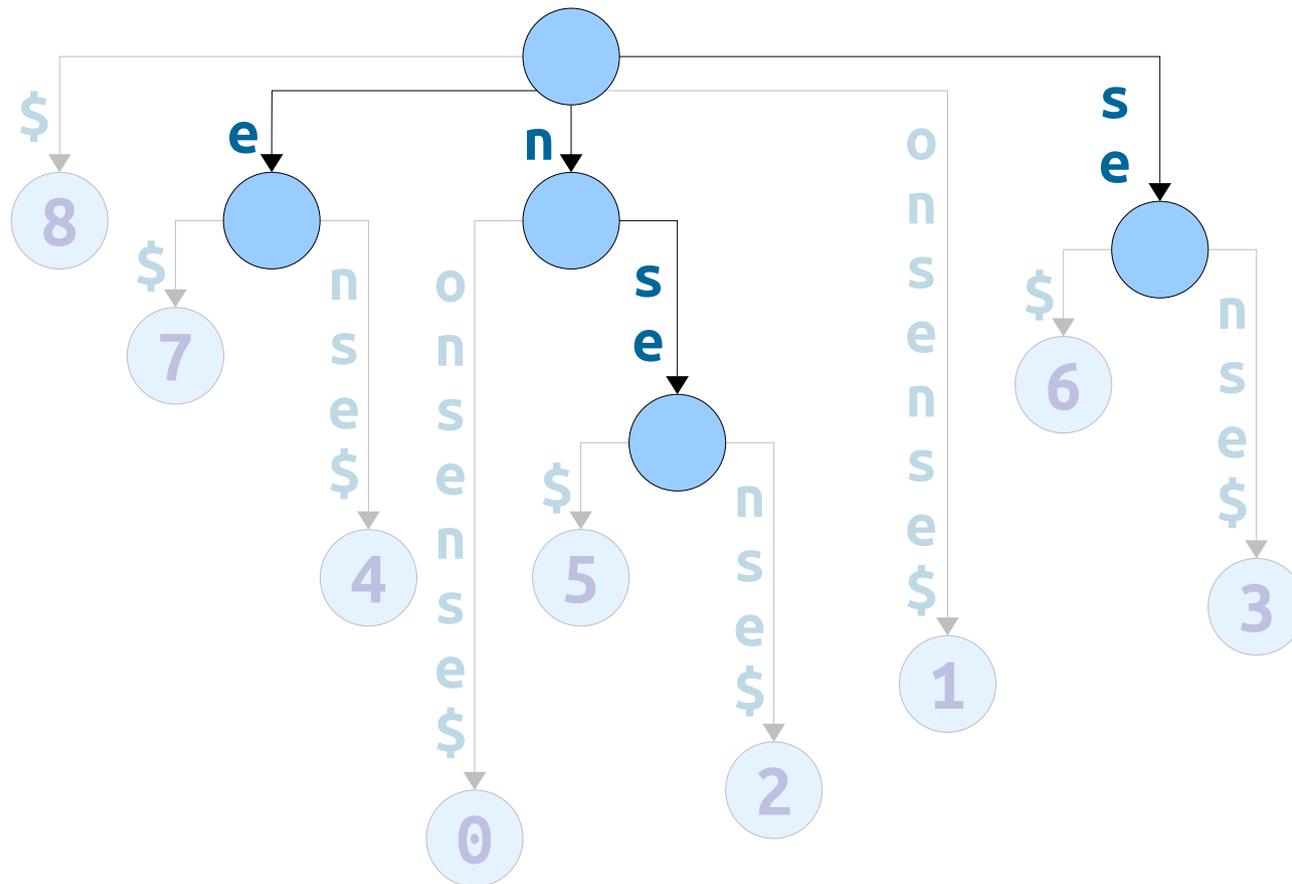


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# Branching Words

- **Recall:** If  $T$  is a string, then  $\omega$  is a **branching word** in  $T\$$  if there are characters  $a \neq b$  such that  $\omega a$  and  $\omega b$  are substrings of  $T\$$ .

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Although ABA is a repeated substring, it is not a branching word because all appearances are followed by N.

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# Branching Words

- **Recall:** If  $T$  is a string, then  $\omega$  is a **branching word** in  $T\$$  if there are characters  $a \neq b$  such that  $\omega a$  and  $\omega b$  are substrings of  $T\$$ .

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The substring ANANA only appears once, so it's not a branching word.

AB**ANANA**BANDANA\$

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NDANA\$

**ABANANABANDANA\$**

# Branching Words

- Notice that, by sorting suffixes, we've made it easier to spot branching words.
- Specifically, all suffixes starting with a branching word will be adjacent in the suffix array.
- The branching word will be the **longest common prefix** (or **LCP**) of those adjacent suffixes.

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\$
A\$
<b>ABAN</b> ANABANDANA\$
<b>ABAN</b> DANA\$
ANA\$
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NABANDANA\$
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NDANA\$

**ABANANABANDANA\$**

# Branching Words

- **Theorem:** A string  $\omega$  is a branching word in string  $T\$$  if and only if it's the longest common prefix of two adjacent suffixes in  $T$ 's suffix array.
- **Proof idea:** If  $\omega$  is the longest common prefix of two adjacent suffixes, let  $a$  and  $b$  be the characters immediately following  $\omega$  in those two suffixes. Then  $\omega a$  and  $\omega b$  are substrings of  $T\$$ .

If  $\omega$  is branching, choose the lexicographically smallest  $a$  and  $b$  making the definition work. Then the last suffix starting with  $\omega a$  and the first suffix starting with  $\omega b$  are adjacent in the suffix array. ■

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**ABANANABANDANA\$**



***Key Intuition:*** Adjacent suffixes with long shared prefixes correspond to subtrees of the suffix tree.

Harnessing this Connection



# Longest Repeated Substring

- We can list all branching words from a suffix array in time  $O(m^2)$ .
  - $O(m)$  pairs; each pair takes time  $O(m)$  to process.
- This worst-case bound can be realized.

How?

Formulate a hypothesis!

\$
A\$
ABANANABANDANA\$
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ANANABANDANA\$
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BANANABANDANA\$
BANDANA\$
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NABANDANA\$
NANABANDANA\$
NDANA\$

**ABANANABANDANA\$**

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  - $O(m)$  pairs; each pair takes time  $O(m)$  to process.
- This worst-case bound can be realized.

How?

Discuss with your neighbors!

\$
A\$
ABANANABANDANA\$
ABANDANA\$
ANA\$
ANABANDANA\$
ANANABANDANA\$
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BANANABANDANA\$
BANDANA\$
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NABANDANA\$
NANABANDANA\$
NDANA\$

**ABANANABANDANA\$**

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ANANABANDANA\$
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BANANABANDANA\$
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NABANDANA\$
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- We can list all branching words from a suffix array in time  $O(m^2)$ .
  - $O(m)$  pairs; each pair takes time  $O(m)$  to process.
- This worst-case bound can be realized.
- Contrast this with  $O(m)$  for a suffix tree.
- Can we do better?

\$
A\$
ABANANABANDANA\$
ABANDANA\$
ANA\$
ANABANDANA\$
ANANABANDANA\$
ANDANA\$
BANANABANDANA\$
BANDANA\$
DANA\$
NA\$
NABANDANA\$
NANABANDANA\$
NDANA\$

**ABANANABANDANA\$**

# Longest Repeated Substring

- **Observation:** We don't actually need to know what all the branching words are to find the longest repeated substring.
- We just need to know how long they are.
- That way, we can figure out which is longest.
- Is there some nice way to do this?

\$
A\$
ABANANABANDANA\$
ABANDANA\$
ANA\$
ANABANDANA\$
ANANABANDANA\$
ANDANA\$
BANANABANDANA\$
BANDANA\$
DANA\$
NA\$
NABANDANA\$
NANABANDANA\$
NDANA\$

**ABANANABANDANA\$**

# LCP Arrays

# LCP Arrays

- The **LCP array**, often denoted **H**, is an array where  $H[i]$  is the length of the LCP of the  $i$ th and  $(i+1)$ st suffixes in the suffix array.
- (The letter  $H$  comes from “height.”)

	\$
0	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
1	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
0	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$

**ABANANABANDANA\$**



# Using LCP Arrays

- If you already have a suffix array and LCP array, you can solve longest repeated substring in time  $O(m)$ :
  - Find the largest element in the LCP array.
  - Return the string it corresponds to.
- **Question:** How fast can we construct an LCP array?

	\$
0	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
1	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
0	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$

**ABANANABANDANA\$**

# Building LCP Arrays

# Building LCP Arrays

- It never hurts to start with the naive algorithm and see what happens!
- **Algorithm:** For each consecutive pair of strings in the suffix array, compute the length of their longest common prefix.
- We can upper-bound the runtime at  $O(m^2)$ .
- **Question:** Can we realize this upper bound?

0	\$
1	A\$
4	ABANANABANDANA\$
1	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
0	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$

ABANANABANDANA\$

# Building LCP Arrays

- Why is our naive algorithm slow?
- **Intuition:** We aren't able to carry work from one suffix over to the next.

0	\$
1	A\$
4	ABANANABANDANA\$
1	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
2	ANANABANDANA\$
0	ANDANA\$
3	BANANABANDANA\$
0	BANDANA\$
0	DANA\$
0	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$

ABANANABANDANA\$

# Building LCP Arrays

- **Key intuition:** Suffixes overlap one another! It should be possible to share LCP information across suffixes.

\$
A\$
ABANABANDANA\$
ABANDANA\$
ANA\$
ANABANDANA\$
ANANABANDANA\$
ANDANA\$
BANANABANDANA\$
BANDANA\$
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NA\$
NABANDANA\$
NANABANDANA\$
NDANA\$

# Building LCP Arrays

- **Key intuition:** Suffixes overlap one another! It should be possible to share LCP information across suffixes.
- For example, suppose we compute the LCP entry shown here.

\$
A\$
ABANABANDANA\$
ABANDANA\$
ANA\$
ANABANDANA\$
ANANABANDANA\$
ANDANA\$
BANANABANDANA\$
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	\$
	A\$
4	ABANABANDANA\$
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	ANABANDANA\$
	ANANABANDANA\$
	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	NABANDANA\$
	NANABANDANA\$
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# Building LCP Arrays

- **Key intuition:** Suffixes overlap one another! It should be possible to share LCP information across suffixes.
- For example, suppose we compute the LCP entry shown here.
- Look at the suffixes formed by dropping the first letter of these two suffixes.

	\$
	A\$
4	ABANABANDANA\$
	ABANDANA\$
	ANA\$
	ANABANDANA\$
	ANANABANDANA\$
	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	NABANDANA\$
	NANABANDANA\$
	NDANA\$

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	\$
	A\$
4	<b>ABANANABANDANA\$</b>
	<b>ABANDANA\$</b>
	ANA\$
	ANABANDANA\$
	ANANABANDANA\$
	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	NABANDANA\$
	NANABANDANA\$
	NDANA\$

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**ABANDANA\$**

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	A\$
4	<b>ABANANABANDANA\$</b>
	<b>ABANDANA\$</b>
	ANA\$
	ANABANDANA\$
	ANANABANDANA\$
	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	NABANDANA\$
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	ANANABANDANA\$
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	<b>BANDANA\$</b>
	DANA\$
	NA\$
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	NANABANDANA\$
	NDANA\$

**ABANANABANDANA\$**  
**ABANDANA\$**

# Building LCP Arrays

- **Key intuition:** Suffixes overlap one another! It should be possible to share LCP information across suffixes.
- For example, suppose we compute the LCP entry shown here.
- Look at the suffixes formed by dropping the first letter of these two suffixes.
- What do we know about their LCP?

	\$
	A\$
4	<b>ABANANABANDANA\$</b>
	<b>ABANDANA\$</b>
	ANA\$
	ANABANDANA\$
	ANANABANDANA\$
	ANDANA\$
	<b>BANANABANDANA\$</b>
	<b>BANDANA\$</b>
	DANA\$
	NA\$
	NABANDANA\$
	NANABANDANA\$
	NDANA\$

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	\$
	A\$
4	<b>ABANABANDANA\$</b>
	<b>ABANDANA\$</b>
	ANA\$
	ANABANDANA\$
	ANANABANDANA\$
	ANDANA\$
3	<b>BANABANDANA\$</b>
	<b>BANDANA\$</b>
	DANA\$
	NA\$
	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**ABANABANDANA\$**  
**ABANDANA\$**

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	ANANABANDANA\$
	ANDANA\$
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	BANDANA\$
	DANA\$
	NA\$
	NABANDANA\$
	NANABANDANA\$
	NDANA\$

ABANANABANDANA\$  
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# Building LCP Arrays

- Let's do another example. Suppose we know the LCP of these suffixes.

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<b>ANA\$</b>
<b>ANABANDANA\$</b>
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	\$
	A\$
	ABANABANDANA\$
	ABANDANA\$
3	<b>ANA\$</b>
	<b>ANABANDANA\$</b>
	ANANABANDANA\$
	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	NABANDANA\$
	NANABANDANA\$
	NDANA\$

# Building LCP Arrays

- Let's do another example. Suppose we know the LCP of these suffixes.
- As before, drop the first letter from each suffix.

	\$
	A\$
	ABANABANDANA\$
	ABANDANA\$
3	<b>ANA\$</b>
	<b>ANABANDANA\$</b>
	ANANABANDANA\$
	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	NABANDANA\$
	NANABANDANA\$
	NDANA\$

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	\$
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	<b>ANABANDANA\$</b>
	ANANABANDANA\$
	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	NABANDANA\$
	NANABANDANA\$
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**ANA\$**

**ANABANDANA\$**

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	<b>ANABANDANA\$</b>
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	BANANABANDANA\$
	BANDANA\$
	DANA\$
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	NABANDANA\$
	NANABANDANA\$
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	<b>ANABANDANA\$</b>
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	BANANABANDANA\$
	BANDANA\$
	DANA\$
	<b>NA\$</b>
	<b>NABANDANA\$</b>
	NANABANDANA\$
	NDANA\$

ANA\$

ANABANDANA\$

# Building LCP Arrays

- Let's do another example. Suppose we know the LCP of these suffixes.
- As before, drop the first letter from each suffix.
- What can we say about the LCP of the resulting suffixes?

	\$
	A\$
	ABANABANDANA\$
	ABANDANA\$
3	<b>ANA\$</b>
	<b>ANABANDANA\$</b>
	ANANABANDANA\$
	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	<b>NA\$</b>
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	NANABANDANA\$
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ANA\$

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	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	<b>NA\$</b>
	<b>NABANDANA\$</b>
	NANABANDANA\$
	NDANA\$

ANA\$

ANABANDANA\$

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	\$
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	ABANABANDANA\$
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3	ANA\$
	ANABANDANA\$
	ANANABANDANA\$
	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	NA\$
	NABANDANA\$
	NANABANDANA\$
	NDANA\$

ANA\$

ANABANDANA\$

# Building LCP Arrays

- Sometimes, in dropping the first letter, two adjacent suffixes get spread out.

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A\$
ABANABANDANA\$
ABANDANA\$
ANA\$
ANABANDANA\$
ANANABANDANA\$
ANDANA\$
BANANABANDANA\$
BANDANA\$
DANA\$
NA\$
NABANDANA\$
NANABANDANA\$
NDANA\$

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A\$
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ABANDANA\$
ANA\$
ANABANDANA\$
ANANABANDANA\$
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# Building LCP Arrays

- Sometimes, in dropping the first letter, two adjacent suffixes get spread out.

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# Building LCP Arrays

- Sometimes, in dropping the first letter, two adjacent suffixes get spread out.
- **Claim:** Look at the second suffix in the pair. Its LCP with the suffix before it is at least the previous LCP minus one.

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# Building LCP Arrays

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Why?

Formulate a hypothesis!

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Why?

Discuss with your neighbor!

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# Building LCP Arrays

- Sometimes, in dropping the first letter, two adjacent suffixes get spread out.
- **Claim:** Look at the second suffix in the pair. Its LCP with the suffix before it is at least the previous LCP minus one.
- Think about the suffix tree. The two shorter suffixes are in the same subtree, so everything between them is also in that subtree.

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# Building LCP Arrays

- We know that these two new suffixes must have an LCP of at least 1, because the two old suffixes have an LCP of 2.
- However, the LCP may be longer than 1, since we've never seen one of these two suffixes.
- We still need to do some scanning, but we won't necessarily have to rescan the entire suffix.

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- For each suffix of the original string, except the last:
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- Use the insight from the previous slides to speed up step (★).

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# Kasai's Algorithm

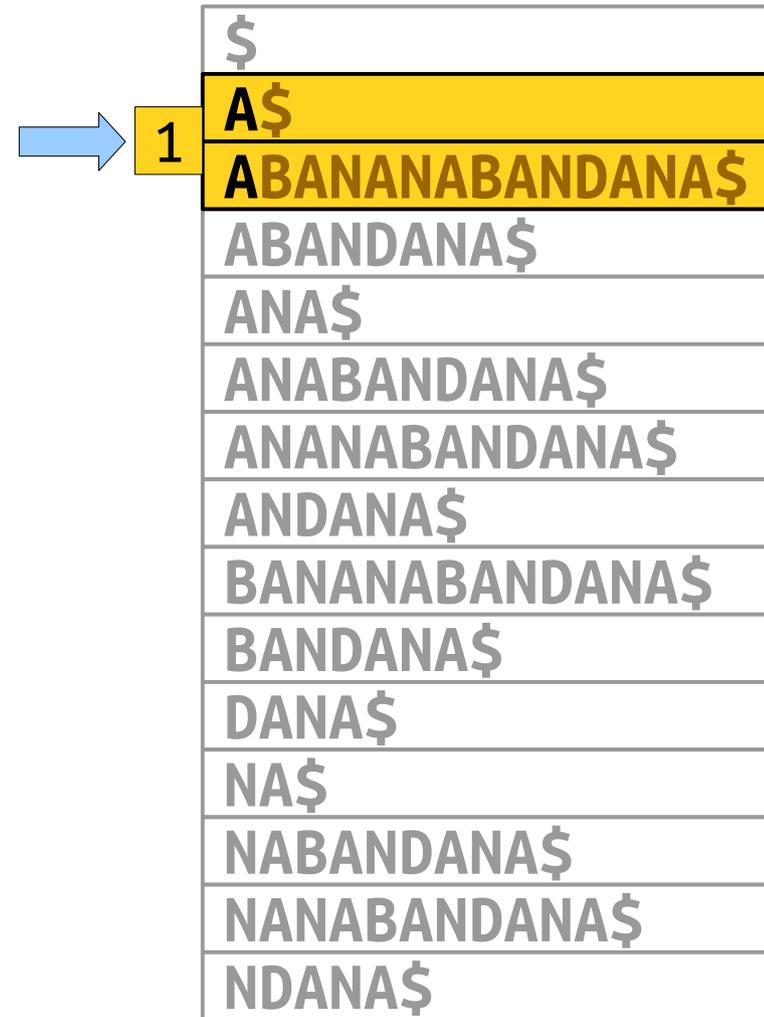
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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**NANABANDANA\$**

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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	NABANDANA\$
	<b>NANABANDANA\$</b>
	NDANA\$

**NANABANDANA\$**

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	\$
1	A\$
	ABANANABANDANA\$
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	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	<b>NABANDANA\$</b>
	<b>NANABANDANA\$</b>
	NDANA\$

**NANABANDANA\$**

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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
	<b>NABANDANA\$</b>
	<b>NANABANDANA\$</b>
	NDANA\$

**NANABANDANA\$**

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	\$
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	ABANANABANDANA\$
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	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**NANABANDANA\$**

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3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**NANABANDANA\$**

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3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**NANABANDANA\$**

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3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**ANABANDANA\$**

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	ANA\$
3	<b>ANABANDANA\$</b>
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**ANABANDANA\$**

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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
	<b>ANA\$</b>
3	<b>ANABANDANA\$</b>
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**ANABANDANA\$**

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	ABANDANA\$
	<b>ANA\$</b>
3	<b>ANABANDANA\$</b>
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**ANABANDANA\$**

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	ABANANABANDANA\$
	ABANDANA\$
	<b>ANA\$</b>
3	<b>ANABANDANA\$</b>
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
3	<b>ANA\$</b>
3	<b>ANABANDANA\$</b>
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

ANABANDANA\$

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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
3	<b>ANA\$</b>
3	<b>ANABANDANA\$</b>
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

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1	A\$
	ABANANABANDANA\$
	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

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1	A\$
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3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

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1	A\$
	ABANANABANDANA\$
	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	<b>NABANDANA\$</b>
	NANABANDANA\$
	NDANA\$

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3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
	<b>NA\$</b>
2	<b>NABANDANA\$</b>
	NANABANDANA\$
	NDANA\$

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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	<b>NA\$</b>
2	<b>NABANDANA\$</b>
	NANABANDANA\$
	NDANA\$

**NABANDANA\$**

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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$



**NABANDANA\$**

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	\$
1	A\$
	ABANANABANDANA\$
	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$



**ABANDANA\$**

# Kasai's Algorithm

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	\$
1	A\$
	ABANANABANDANA\$
	<b>ABANDANA\$</b>
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$



**ABANDANA\$**

# Kasai's Algorithm

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	\$
	A\$
1	ABANABANDANA\$
	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

ABANDANA\$

# Kasai's Algorithm

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	\$
	A\$
1	<b>ABANABANDANA\$</b>
	<b>ABANDANA\$</b>
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**ABANDANA\$**

# Kasai's Algorithm

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	\$
	A\$
1	<b>ABANABANDANA\$</b>
	<b>ABANDANA\$</b>
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
	ANDANA\$
0	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$

ABANDANA\$

# Kasai's Algorithm

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	\$
	A\$
1	<b>ABANABANDANA\$</b>
4	<b>ABANDANA\$</b>
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
	ANDANA\$
0	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$

**ABANDANA\$**

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	\$
	A\$
1	<b>ABANABANDANA\$</b>
4	<b>ABANDANA\$</b>
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**ABANDANA\$**

# Kasai's Algorithm

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	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
	ANDANA\$
0	BANANABANDANA\$
	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$

**ABANDANA\$**

# Kasai's Algorithm

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	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**BANDANA\$**

# Kasai's Algorithm

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	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	ANDANA\$
	BANANABANDANA\$
	<b>BANDANA\$</b>
	DANA\$
2	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$

**BANDANA\$**

# Kasai's Algorithm

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	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
	ANDANA\$
0	<b>BANANABANDANA\$</b>
	<b>BANDANA\$</b>
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$

**BANDANA\$**

# Kasai's Algorithm

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	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
	ANDANA\$
0	<b>BANANABANDANA\$</b>
	<b>BANDANA\$</b>
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$

**BANDANA\$**

# Kasai's Algorithm

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	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
	ANDANA\$
0	<b>BANANABANDANA\$</b>
3	<b>BANDANA\$</b>
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$

**BANDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
	ANDANA\$
0	<b>BANANABANDANA\$</b>
3	<b>BANDANA\$</b>
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$

**BANDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$



**BANDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$



**ANDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
1	A\$
4	ABANANABANDANA\$
	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
	ANANABANDANA\$
0	<b>ANDANA\$</b>
3	BANANABANDANA\$
	BANDANA\$
	DANA\$
2	NA\$
2	NABANDANA\$
	NANABANDANA\$
	NDANA\$



**ANDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	<b>ANANABANDANA\$</b>
	<b>ANDANA\$</b>
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$



**ANDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	<b>ANANABANDANA\$</b>
	<b>ANDANA\$</b>
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$



**ANDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	<b>ANANABANDANA\$</b>
2	<b>ANDANA\$</b>
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$



**ANDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	<b>ANANABANDANA\$</b>
2	<b>ANDANA\$</b>
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$

**ANDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$



**ANDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$



**NDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	<b>NDANA\$</b>



**NDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
	NDANA\$



**NDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	<b>NANABANDANA\$</b>
	<b>NDANA\$</b>

**NDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	<b>NANABANDANA\$</b>
1	<b>NDANA\$</b>



**NDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	<b>NANABANDANA\$</b>
1	<b>NDANA\$</b>



**NDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
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	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**NDANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
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	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**DANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
	<b>DANA\$</b>
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**DANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
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	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	<b>BANDANA\$</b>
	<b>DANA\$</b>
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**DANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	<b>BANDANA\$</b>
0	<b>DANA\$</b>
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**DANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	<b>BANDANA\$</b>
0	<b>DANA\$</b>
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**DANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**DANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**ANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
	<b>ANA\$</b>
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**ANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**ANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	<b>ABANDANA\$</b>
	<b>ANA\$</b>
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**ANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	<b>ABANDANA\$</b>
1	<b>ANA\$</b>
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**ANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
1	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**ANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
1	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$



**ANA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
1	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
	NA\$
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$

NA\$

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
1	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
2	<b>NA\$</b>
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$

NA\$

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
1	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	<b>DANA\$</b>
	<b>NA\$</b>
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$

NA\$

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
1	A\$
4	ABANANABANDANA\$
1	ABANDANA\$
3	ANA\$
3	ANABANDANA\$
2	ANANABANDANA\$
0	ANDANA\$
3	BANANABANDANA\$
0	BANDANA\$
0	<b>DANA\$</b>
0	<b>NA\$</b>
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$

**NA\$**

# Kasai's Algorithm

- For each suffix of the original string, except the last:
  - Find that suffix in the suffix array.
  - Look at the suffix that comes before it.
  - (★) Find the length of the longest common prefix of those suffixes.
  - Write that down in the  $H$  array.
- Use the insight from the previous slides to speed up step (★).

	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
1	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	<b>DANA\$</b>
0	<b>NA\$</b>
2	NABANDANA\$
2	NANABANDANA\$
1	NDANA\$

NA\$

# Kasai's Algorithm

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	\$
	A\$
1	ABANANABANDANA\$
4	ABANDANA\$
1	ANA\$
3	ANABANDANA\$
3	ANANABANDANA\$
2	ANDANA\$
0	BANANABANDANA\$
3	BANDANA\$
0	DANA\$
0	NA\$
2	NABANDANA\$
2	NANABANDANA\$
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NA\$

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A\$

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0	ANDANA\$
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**ABANANABANDANA\$**

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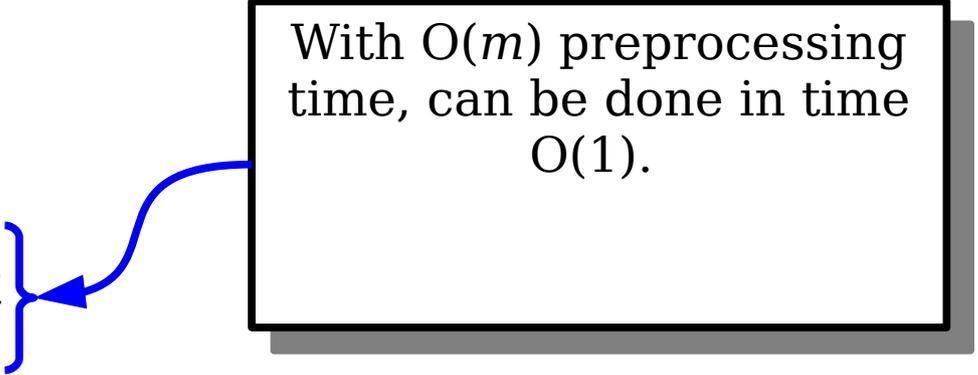
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With  $O(m)$  preprocessing time, can be done in time  $O(1)$ .

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How would you do this?

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The runtime of this step is proportional to how much the LCP increases on that step.

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**ABANDANA\$**

 *Already known to match*

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*Had to scan these characters*

**ABANANABANDANA\$**

**ABANDANA\$**

*Already known to match*

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The LCP value decreases by at most one per suffix. (*We saw this earlier.*)

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The LCP value maxes out at  $m$ . (*Can't match more than all the characters.*)

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The LCP value maxes out at  $m$ . (*Can't match more than all the characters.*)

Therefore, the LCP value can grow at most  $2m$  times. (*Prove this!*)

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How would you do this?

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The LCP value maxes out at  $m$ . (*Can't match more than all the characters.*)

Therefore, the LCP value can grow at most  $2m$  times. (*Prove this!*)

**Claim:** Across all iterations, this step takes a total of  $O(m)$  time.

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Total  
runtime:  
 $O(m)$ .

# More to Explore

- We could easily spend a whole quarter talking about suffix arrays. Here's what we didn't cover:
  - ***Bottom-up tree simulations:*** Using LCP arrays, you can simulate any  $O(m)$ -time suffix tree algorithm that works with a bottom-up DFS in time  $O(m)$ .
  - ***Faster substring searching:*** Using LCP arrays, plus RMQ, you can improve the cost of a substring search to  $O(n + z + \log m)$ .
  - ***Burrows-Wheeler transforms:*** Suffix arrays, plus LCP arrays, can be used to significantly improve the performance of text compressors.
- Check these out – they're super interesting!

# Next Time

- ***Building Suffix Trees***
  - ... is not that bad once you have a suffix array.
- ***Building Suffix Arrays***
  - ... with the mother of all divide-and-conquer algorithms!