Game-playing: DeepBlue and AlphaGo

Brief history of gameplaying frontiers

- 1990s: Othello world champions refuse to play computers
- 1994: Chinook defeats Checkers world champion
- 1997: DeepBlue defeats world champion Gary Kasparov
- 2016: AlphaGo defeats world champion Lee Sedol

Today, we're going to talk about **DeepBlue** and **AlphaGo**.

DeepBlue

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DeepBlue

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- How?
  - **Minimax**
  - **Alpha-beta pruning**
  - **Evaluation function**
  - Sound familiar?

First, some review

Let's play a two-player game.

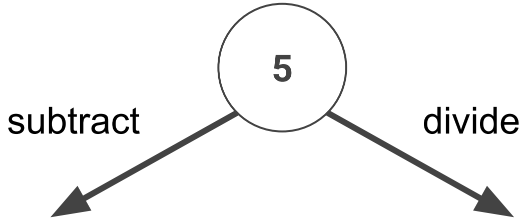
Start with **n=5**, and alternate turns.

- On every turn, player can either set **n = n - 1** or **n = floor(n/2)**
- The first player to set **n = 0** wins!

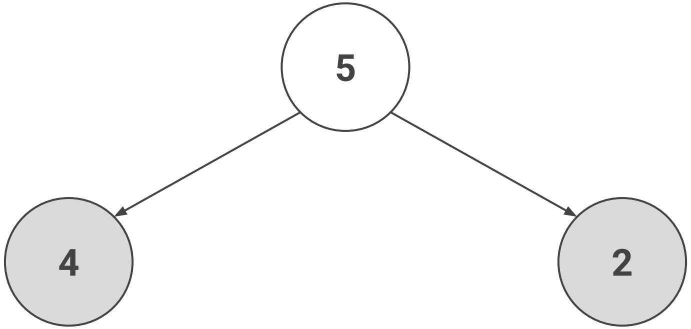
How can we model this?

Game trees

Game trees



Game trees



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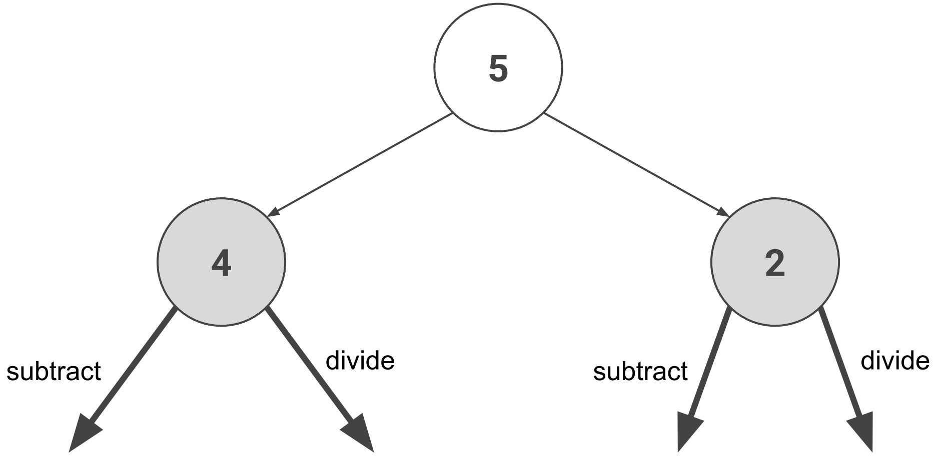

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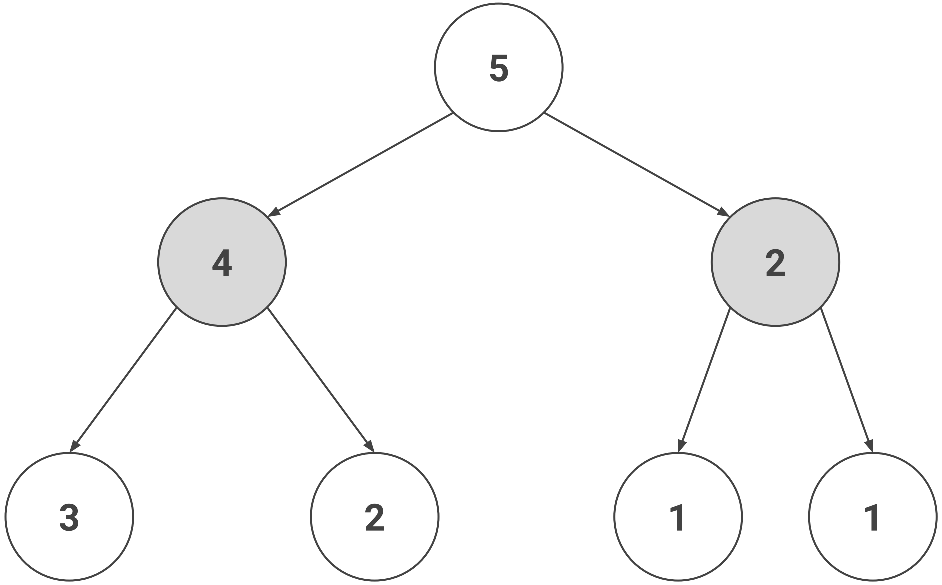

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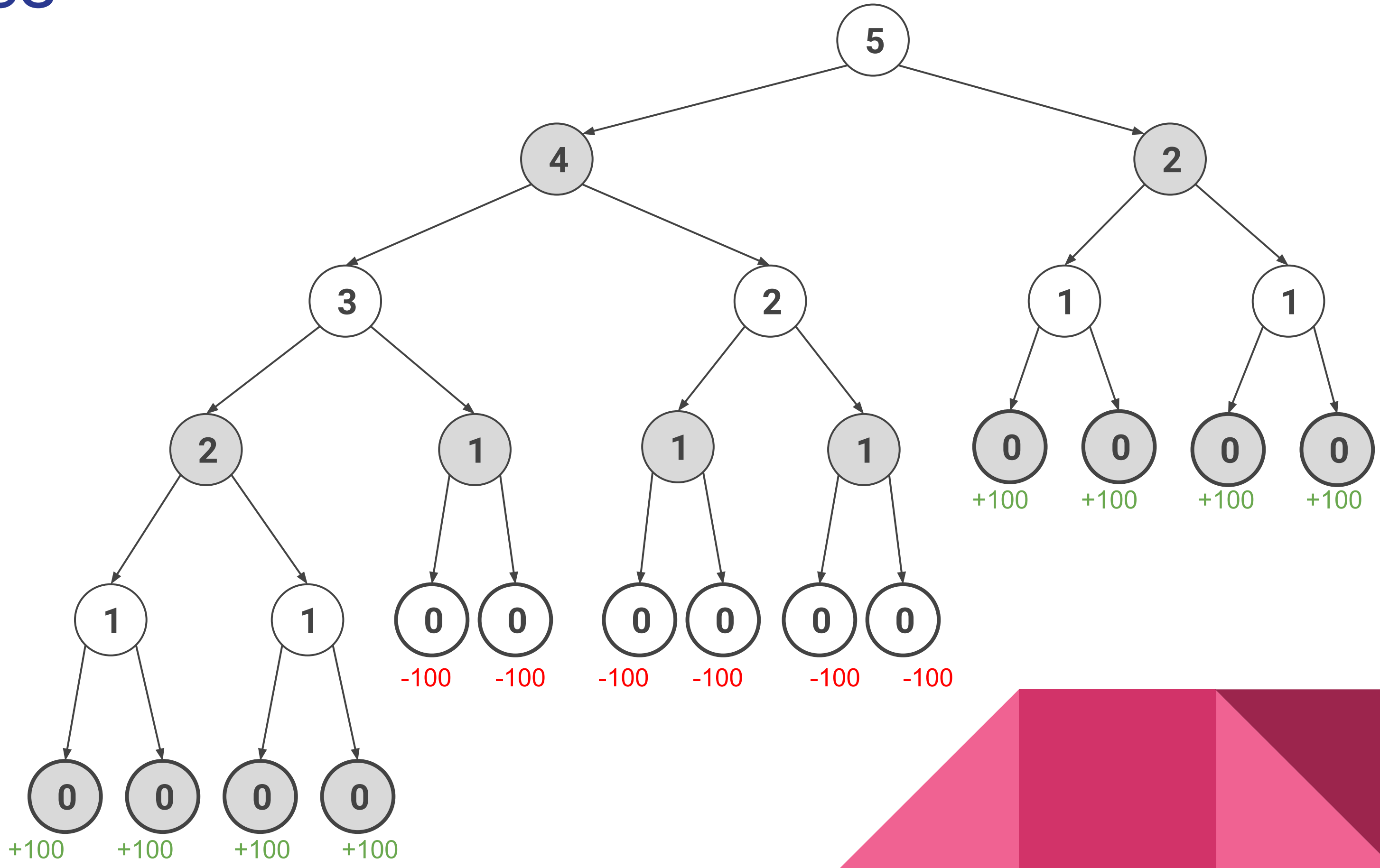

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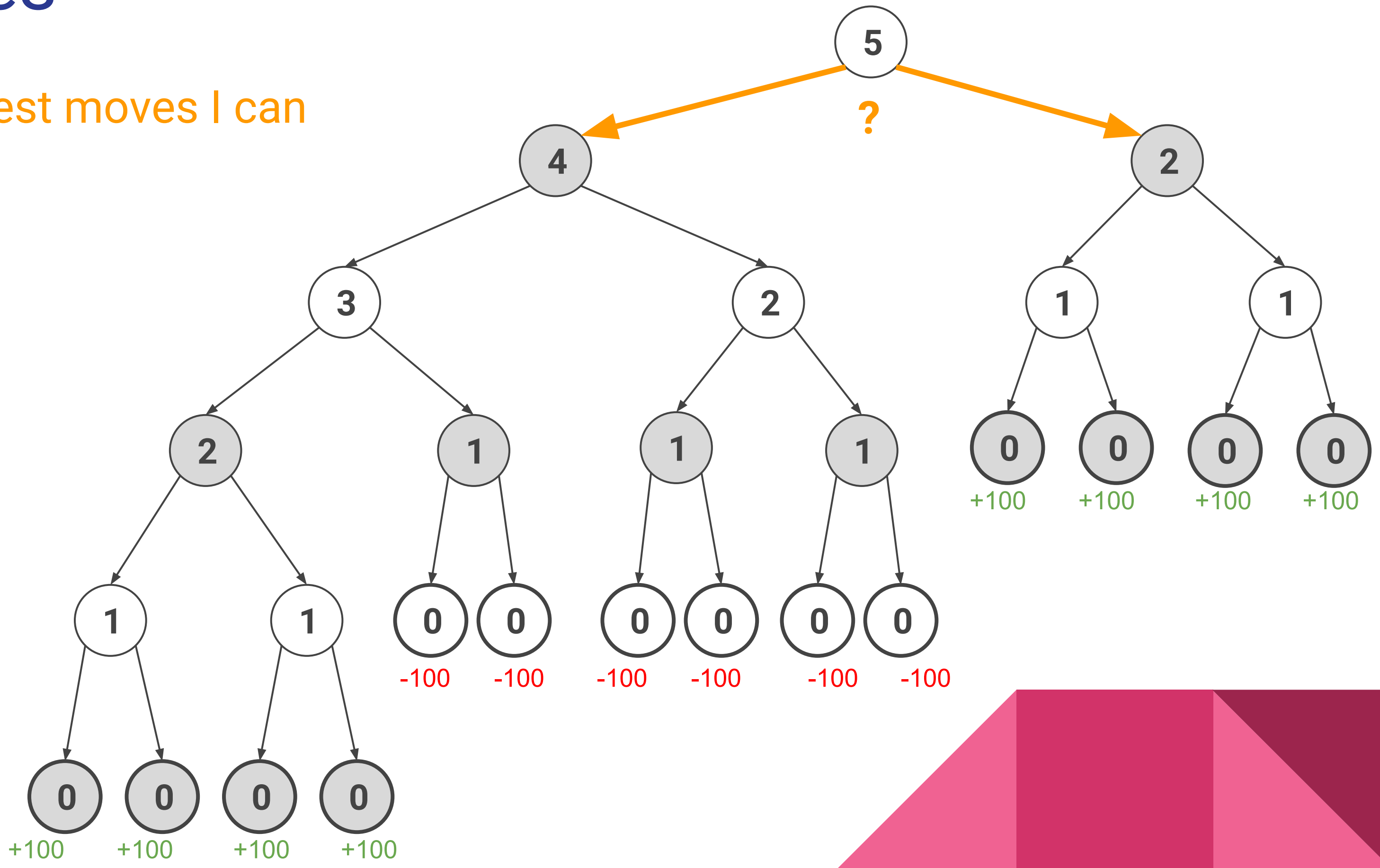
Game trees

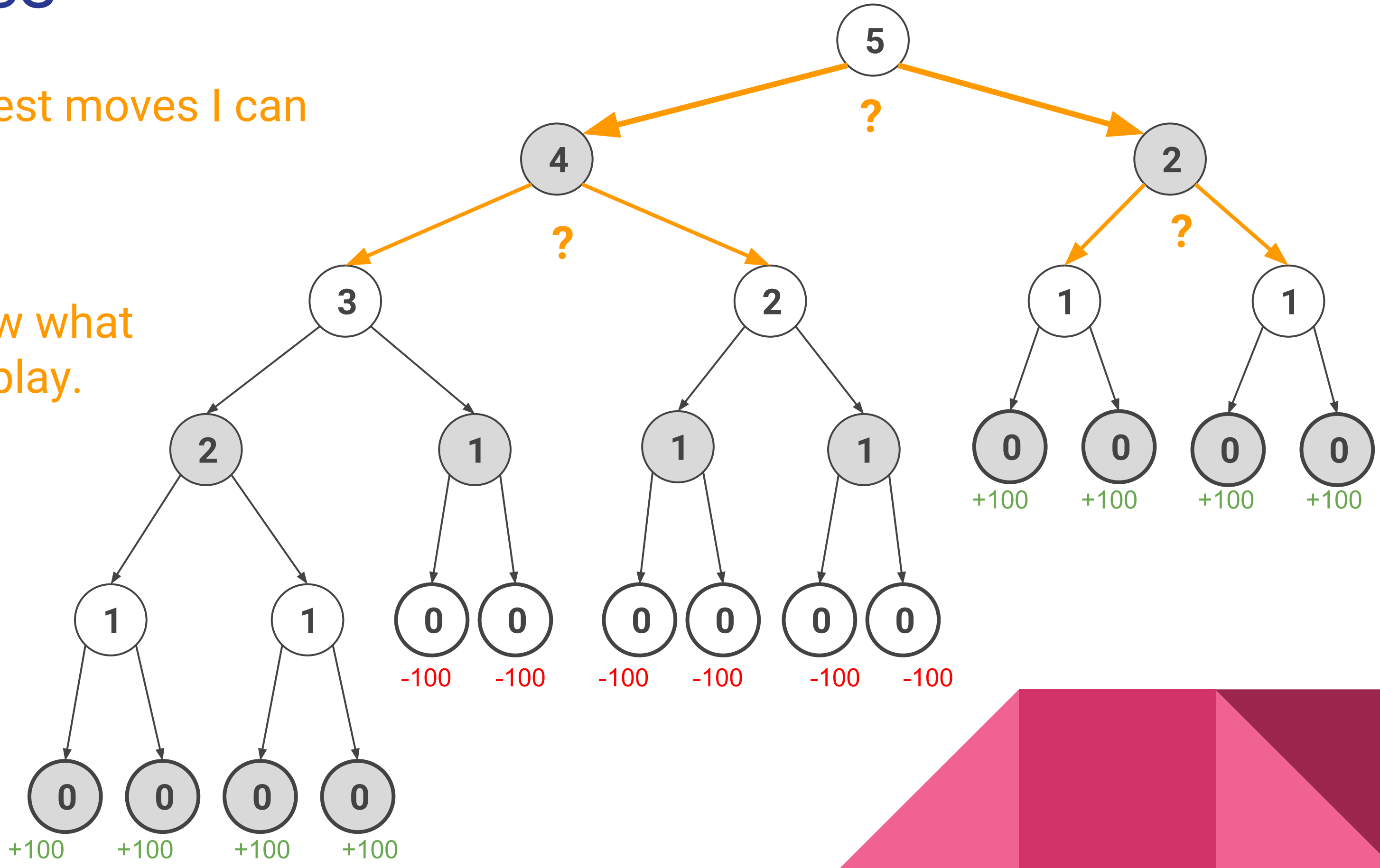
So what are the best moves I can play?



Game trees

So what are the best moves I can play?

Problem:
We also don't know what the opponent will play.



Expectimax

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$$V(s) = \max_{a \in Actions} V(Succ(s, a))$$

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Probability that our opponent will take action **a** from state **s**

Expectimax

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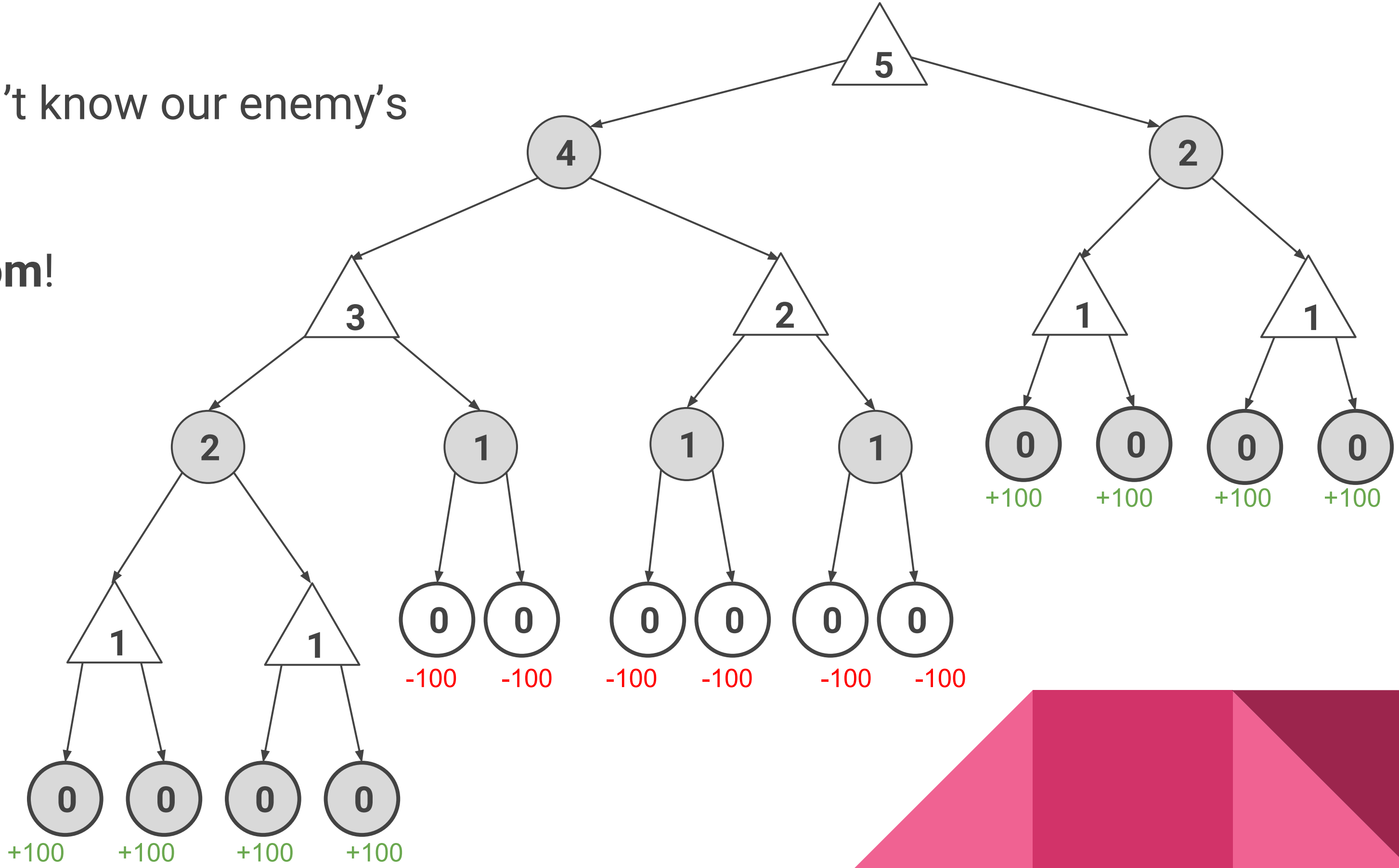
Expectimax

Let's say we don't know our enemy's policy at all.

Maybe it's **random**!



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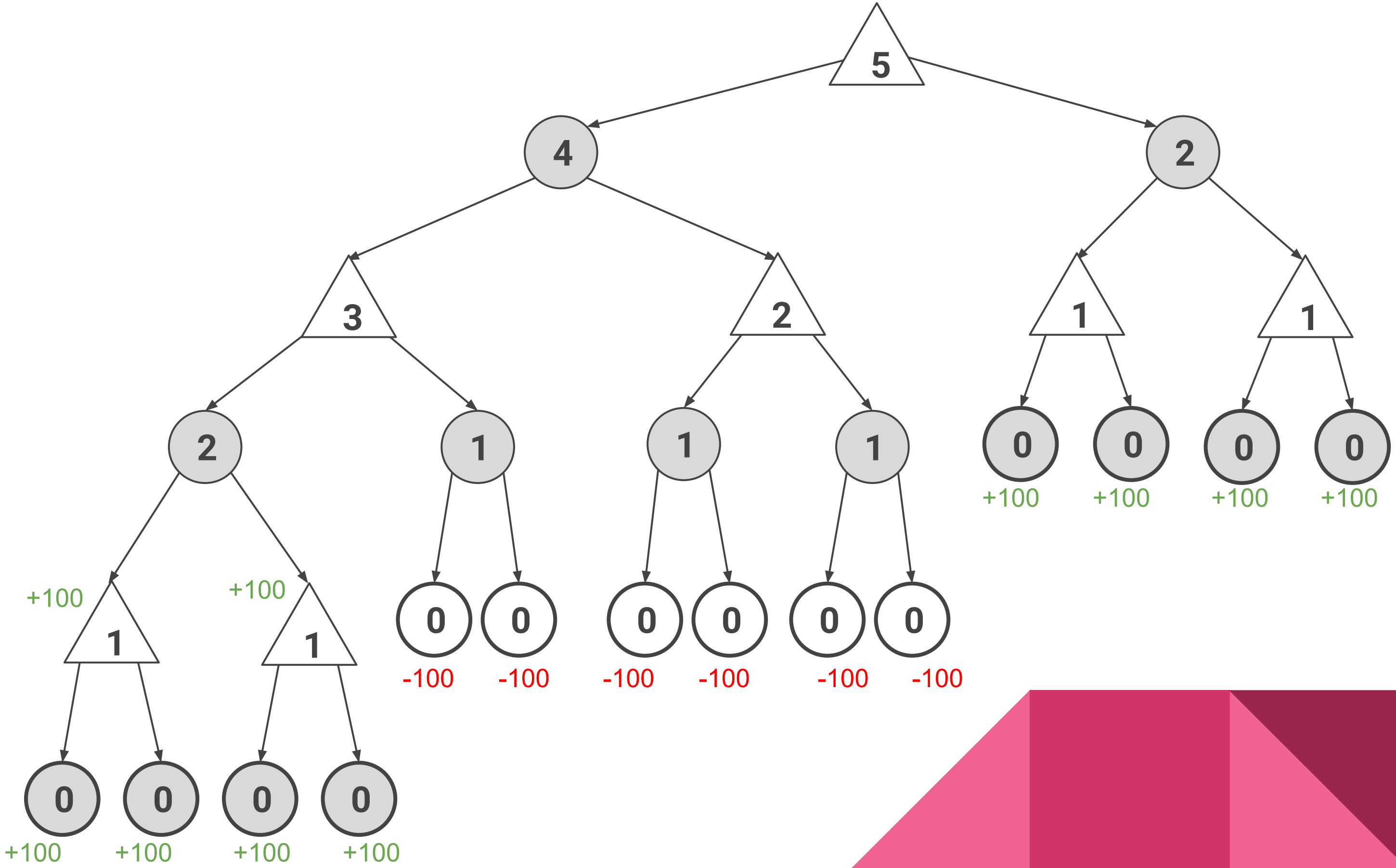

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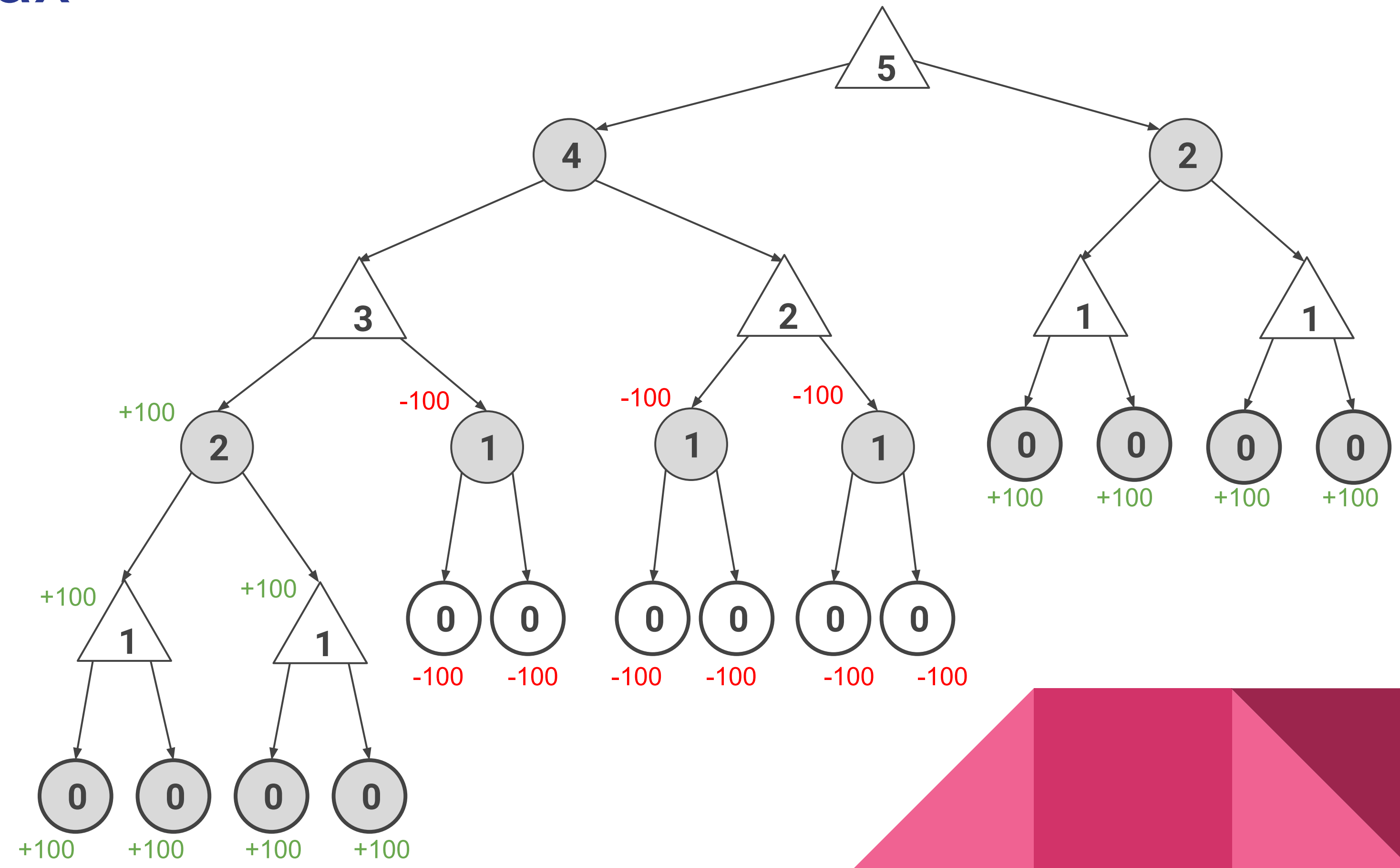
Expectimax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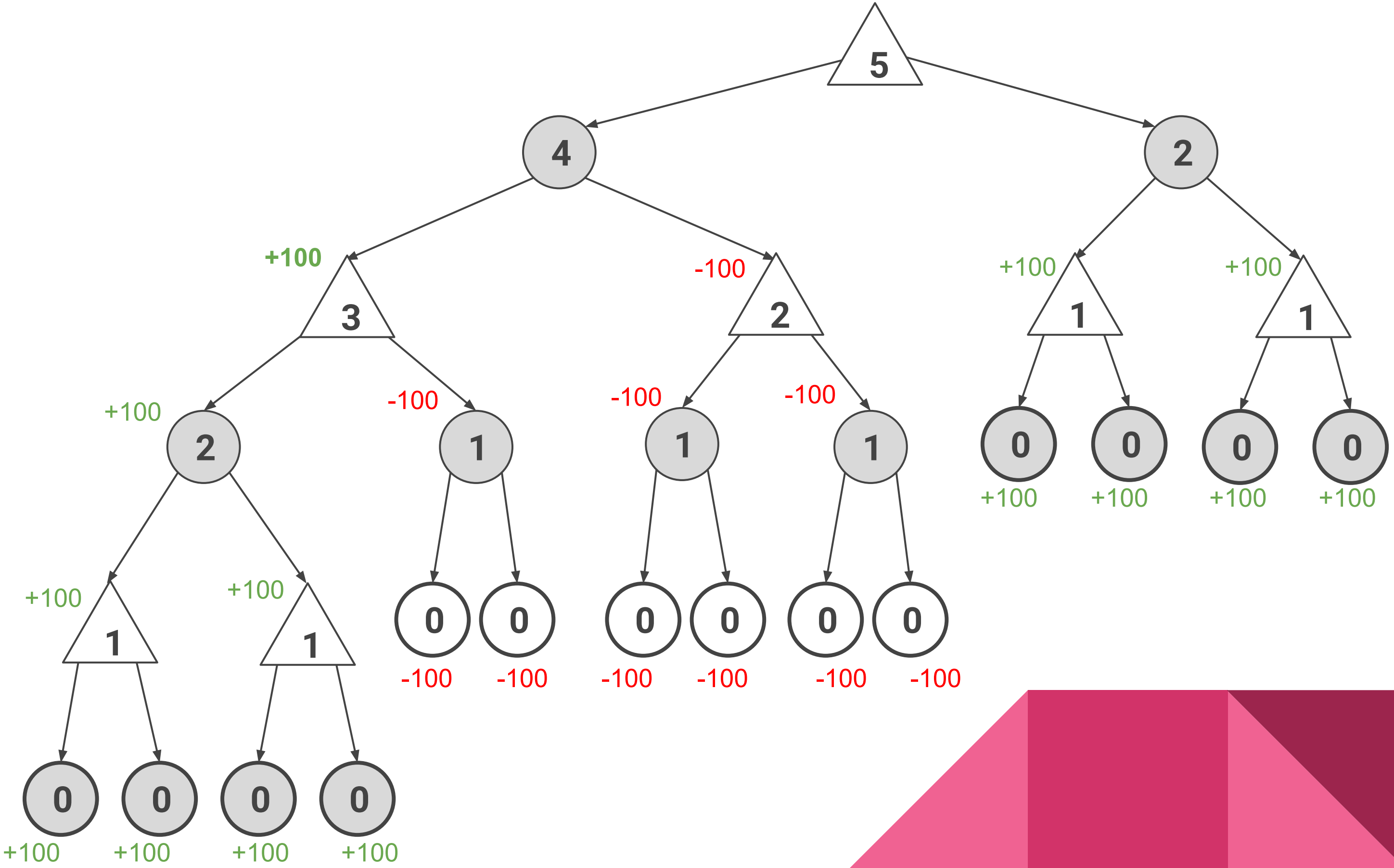
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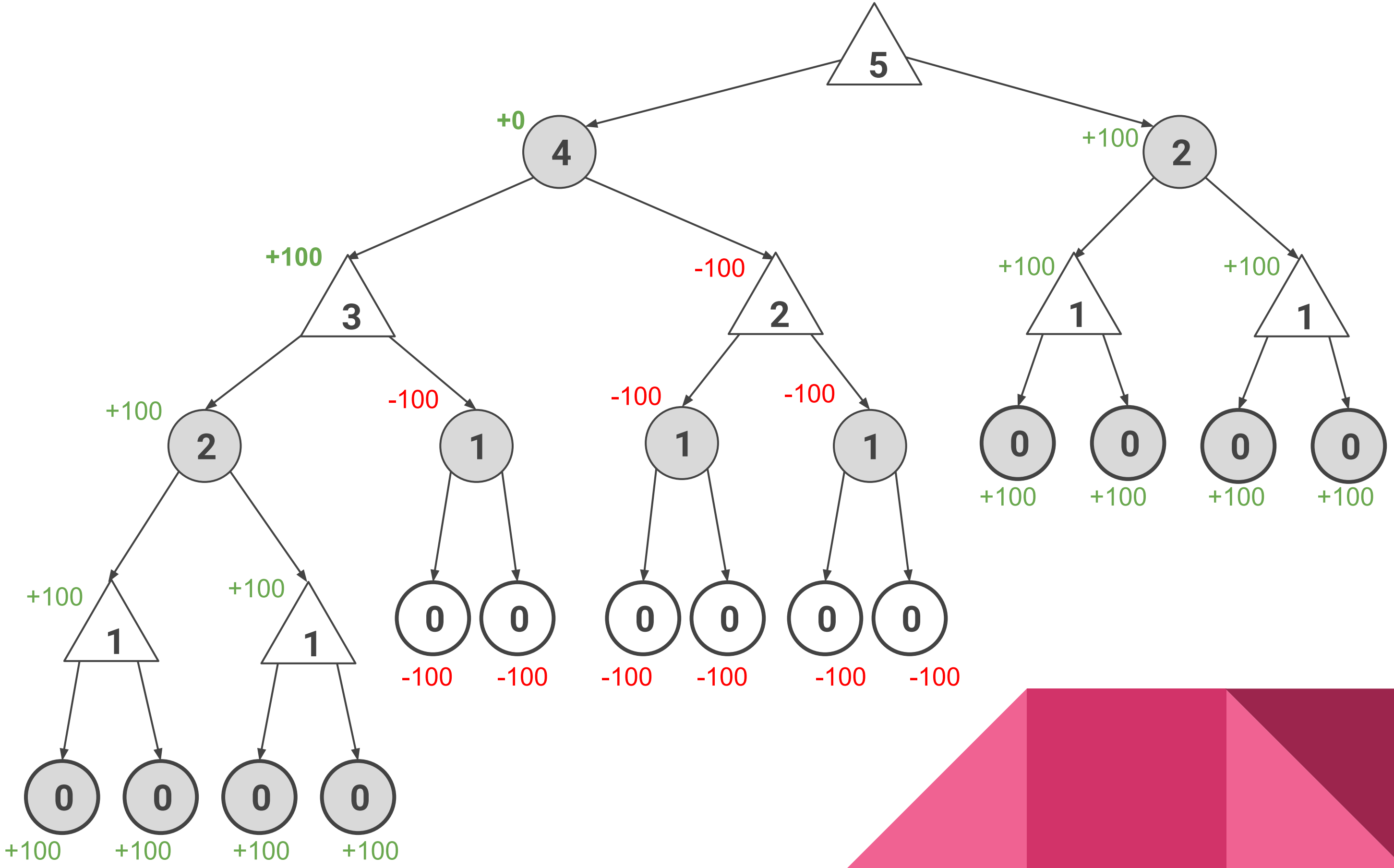

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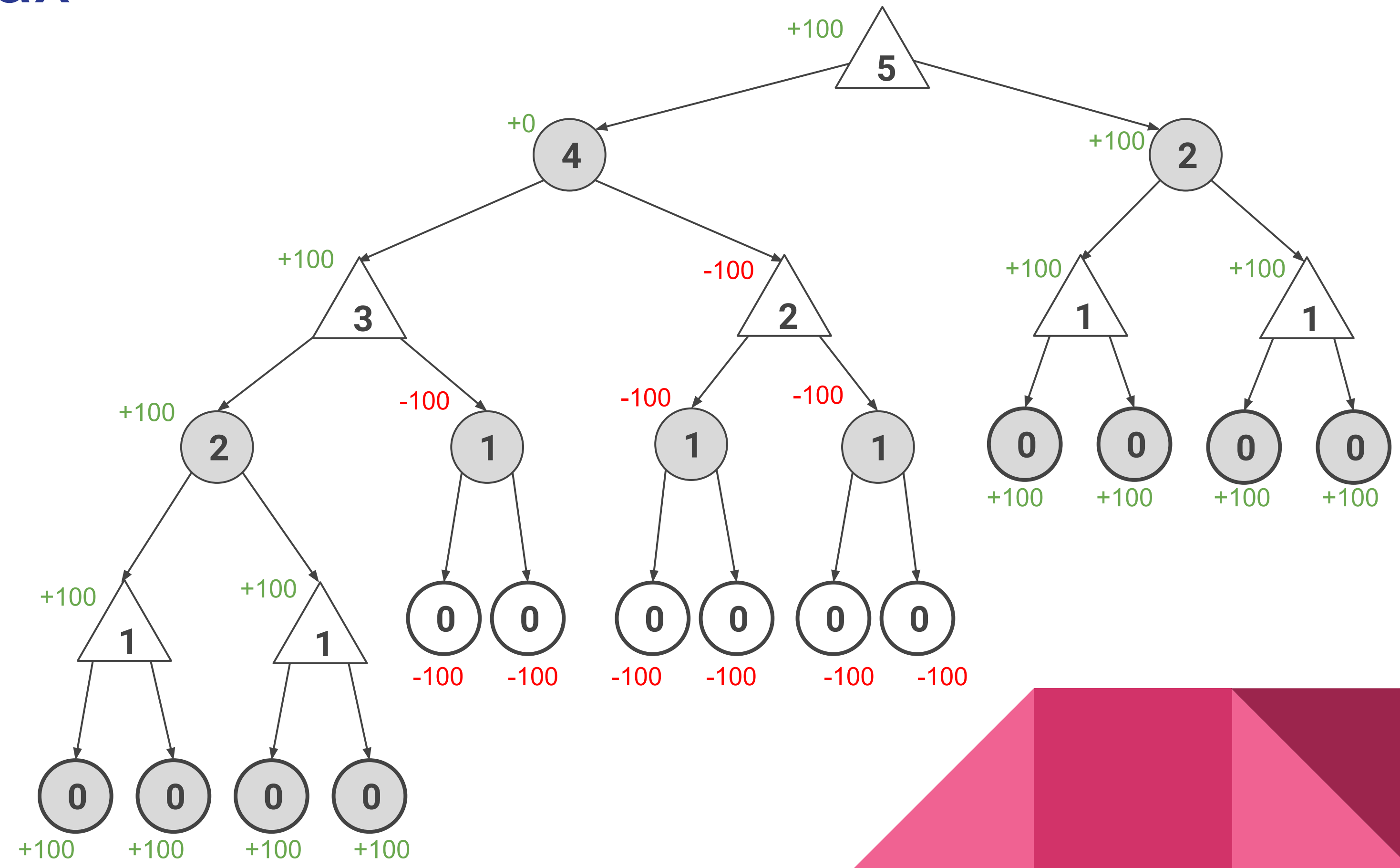

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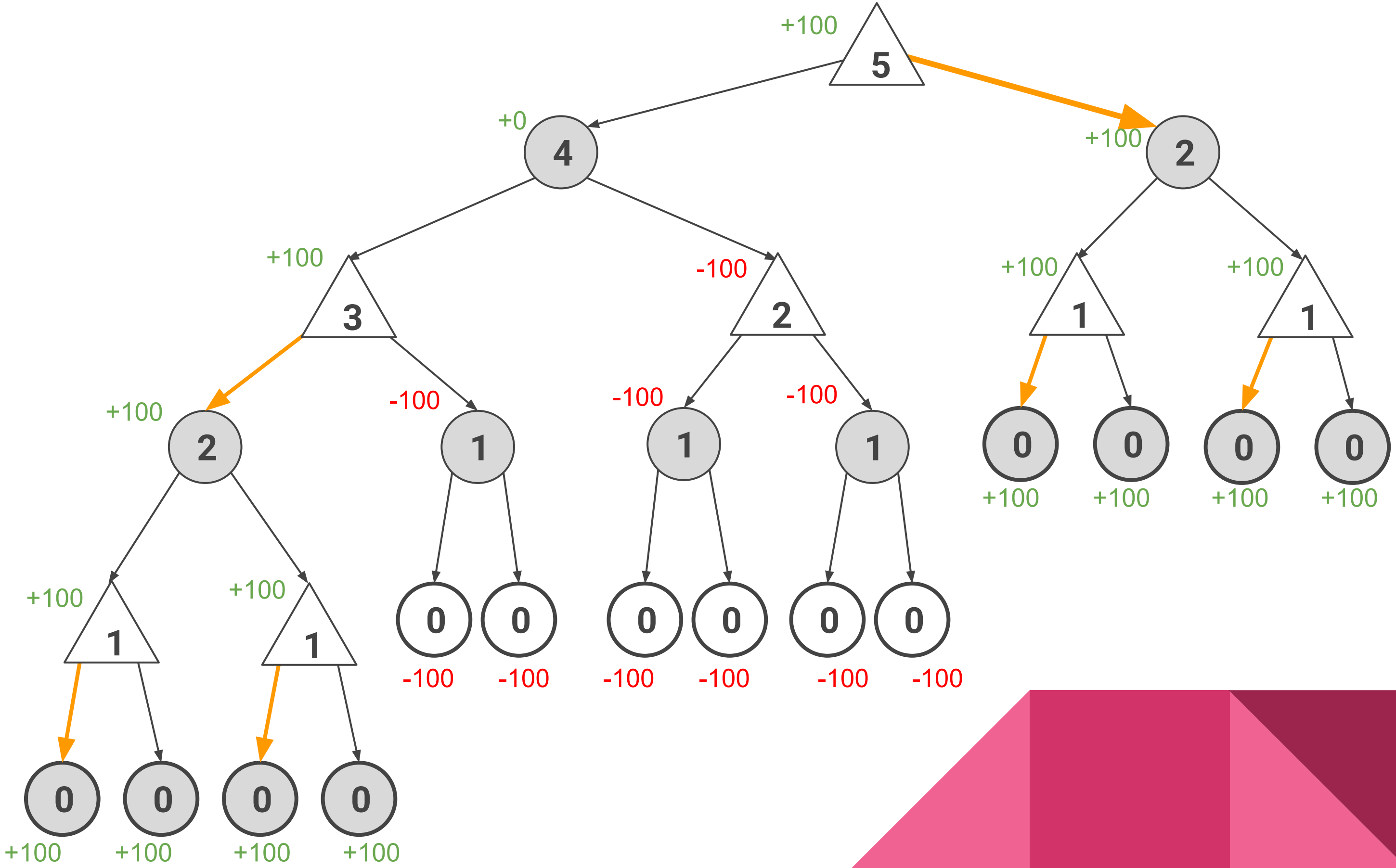
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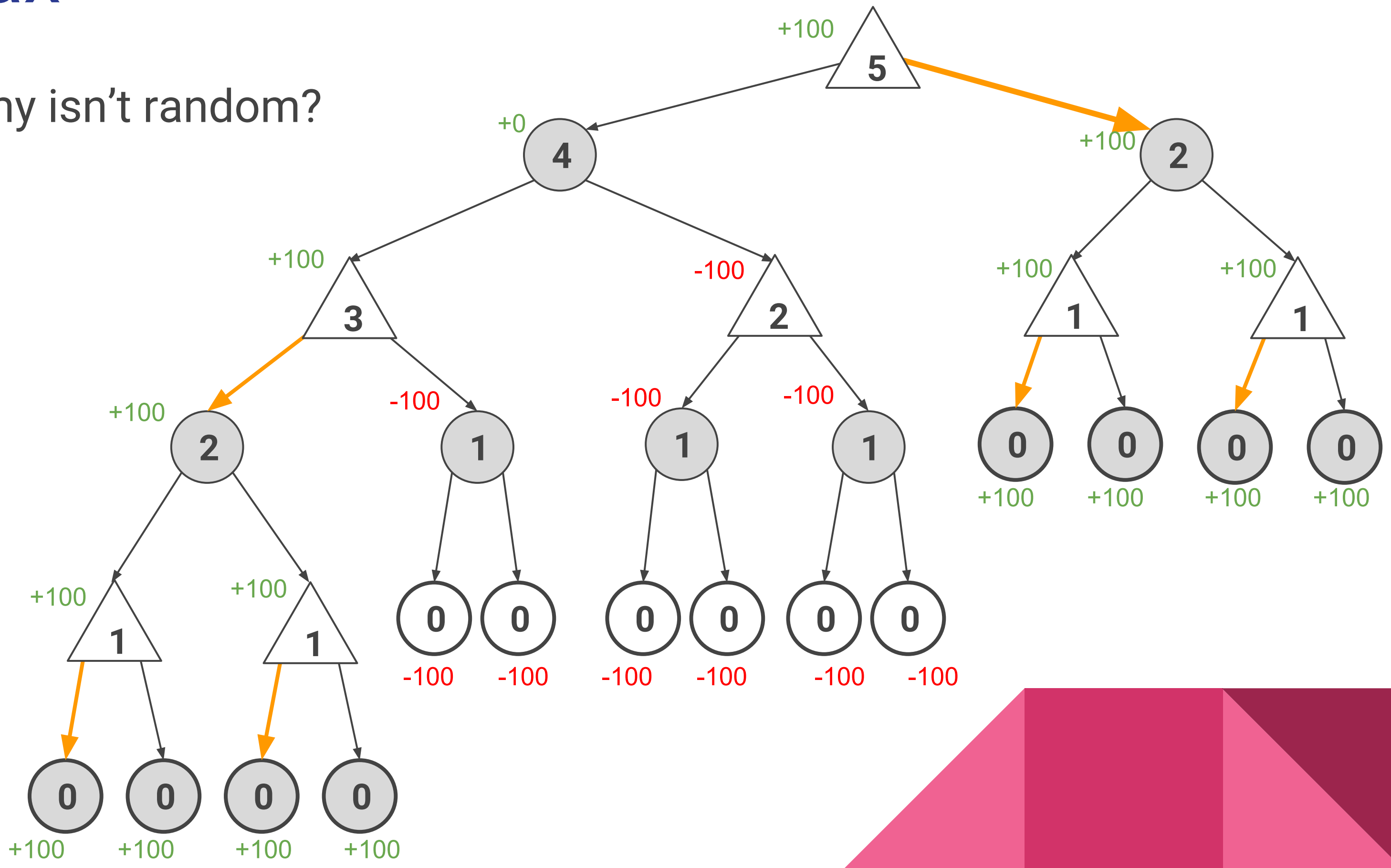
Expectimax

What if our enemy isn't random?



Minimax

- We know we want to **maximize our utility.**

$$V(s) = \max_{a \in Actions} V(Succ(s,a))$$

= Take the action **a** that maximizes the utility of the resulting state.

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$$V(s) = \max_{a \in Actions} V(Succ(s, a))$$

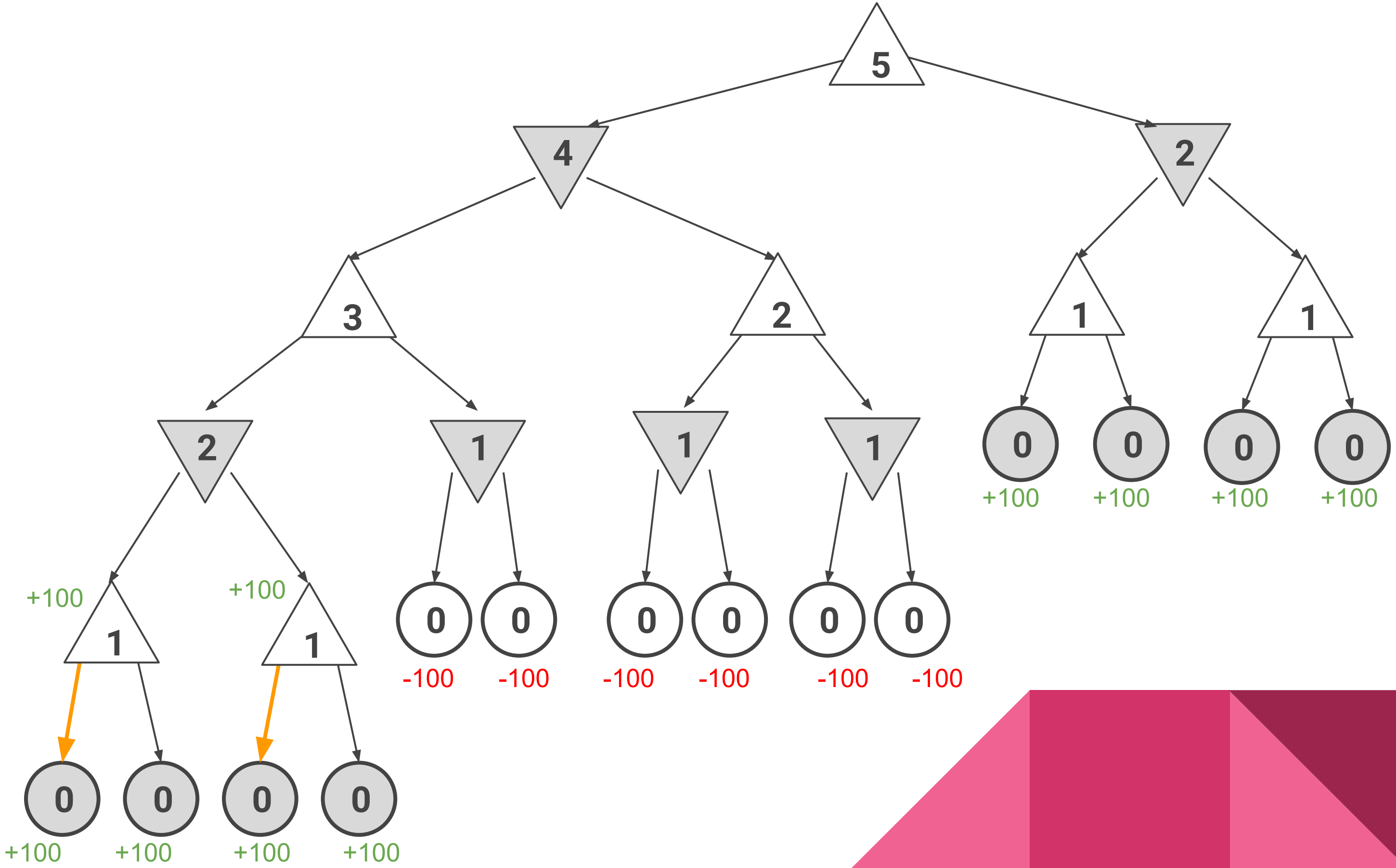
= Take the action **a** that maximizes the utility of the resulting state.

- Let's assume the enemy is adversarial, i.e. wants to **minimize our utility.**

$$V(s) = \min_{a \in Actions} V(Succ(s, a))$$

= Take the action **a** that minimizes the utility of the resulting state.

Minimax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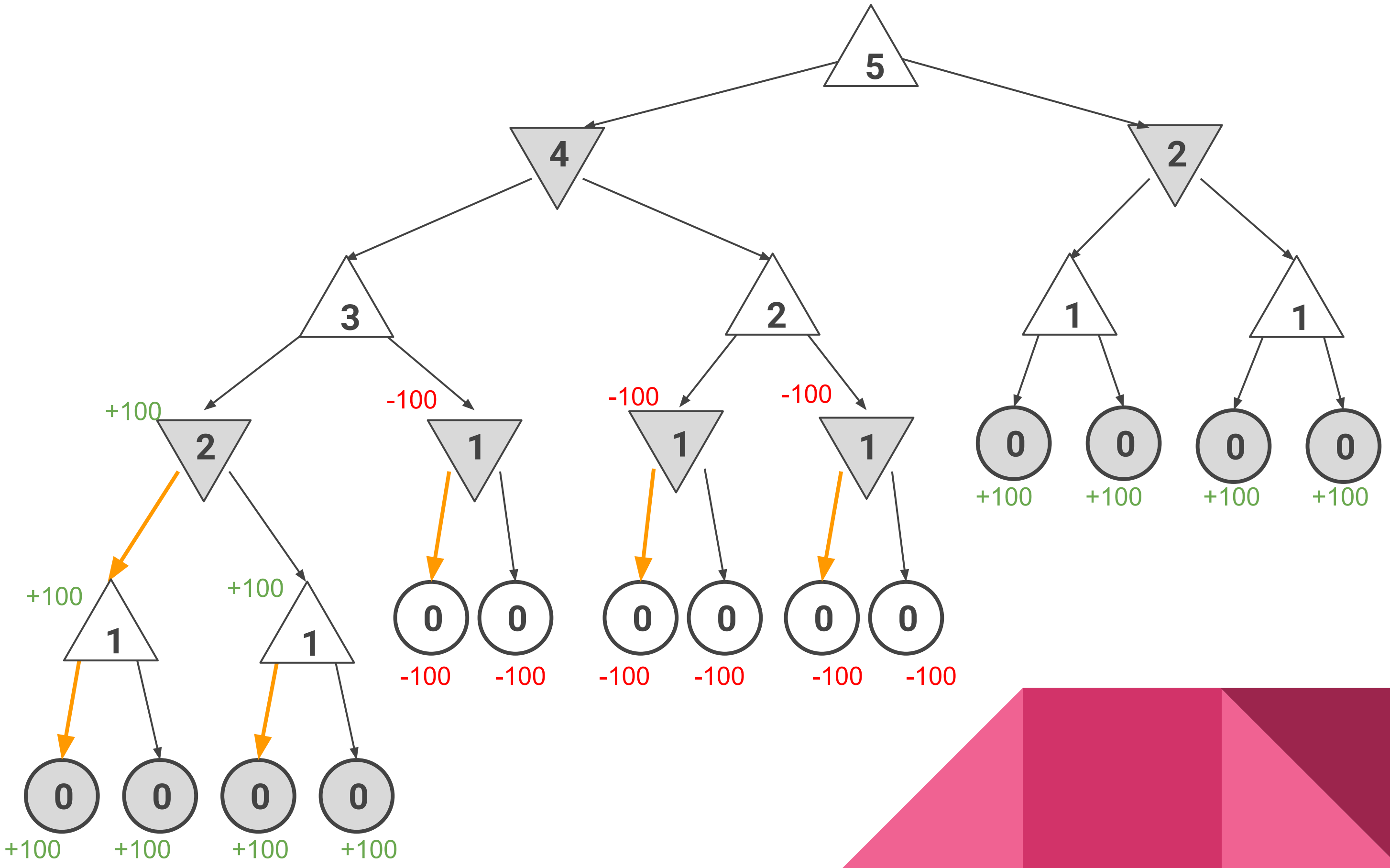

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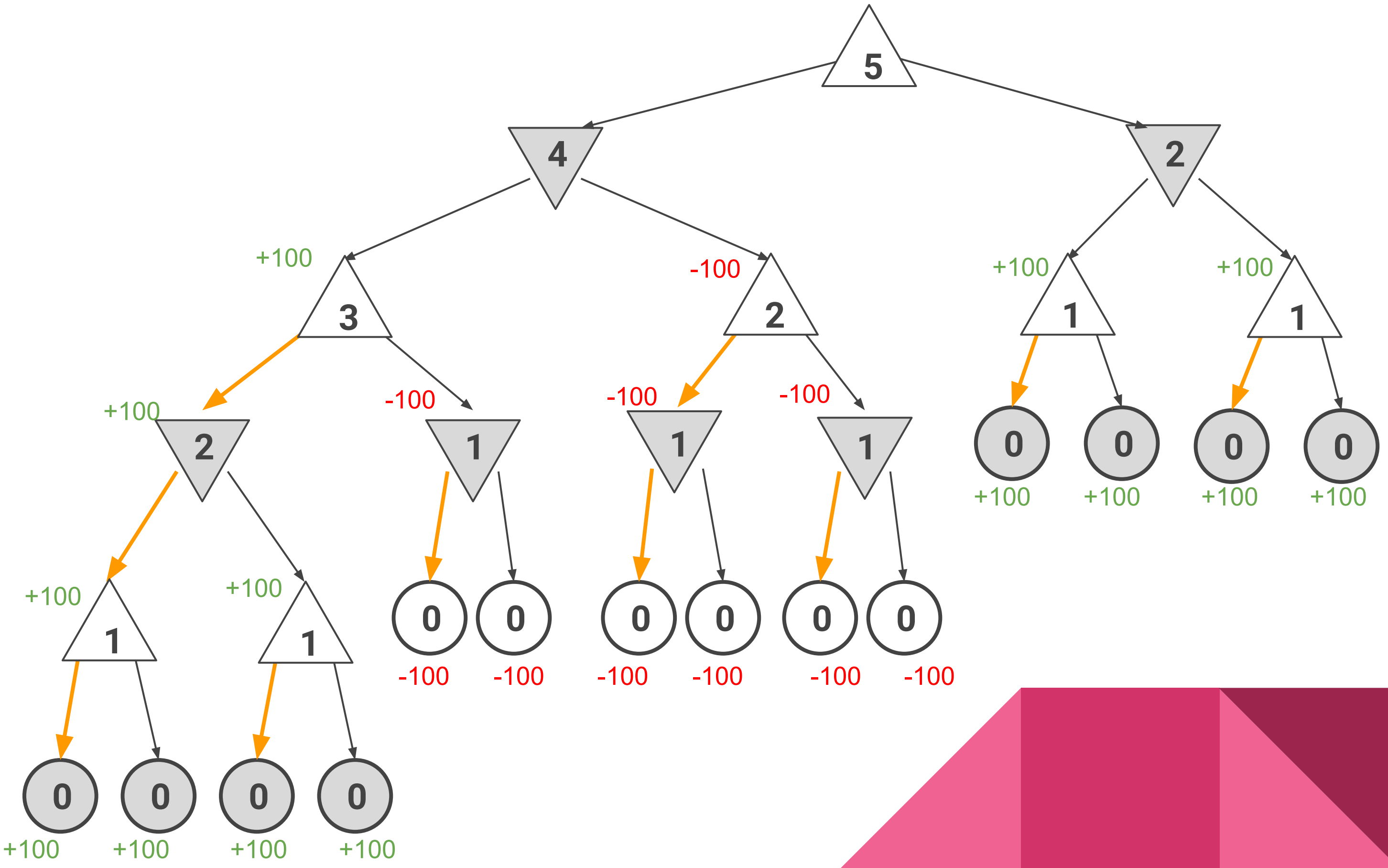
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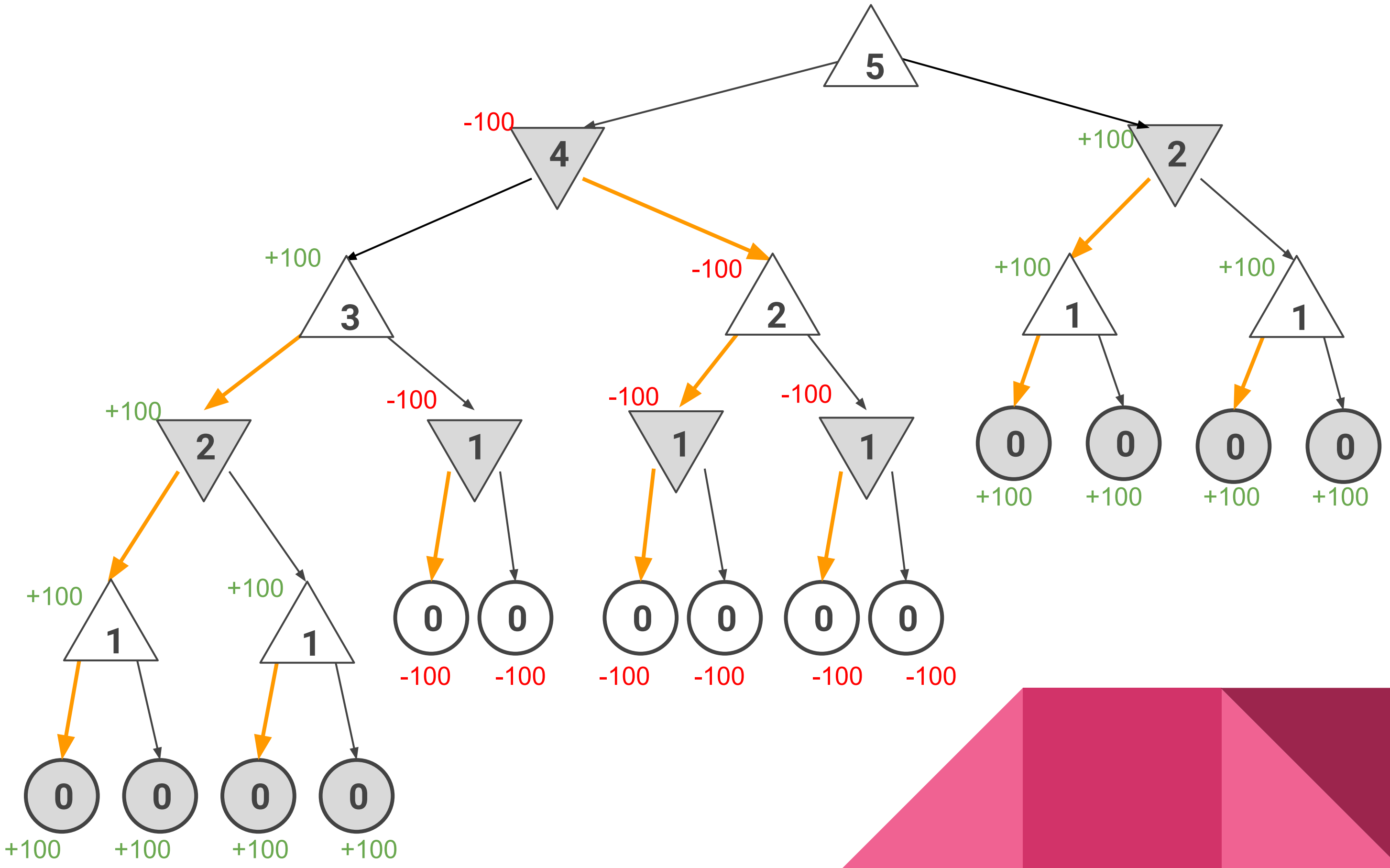

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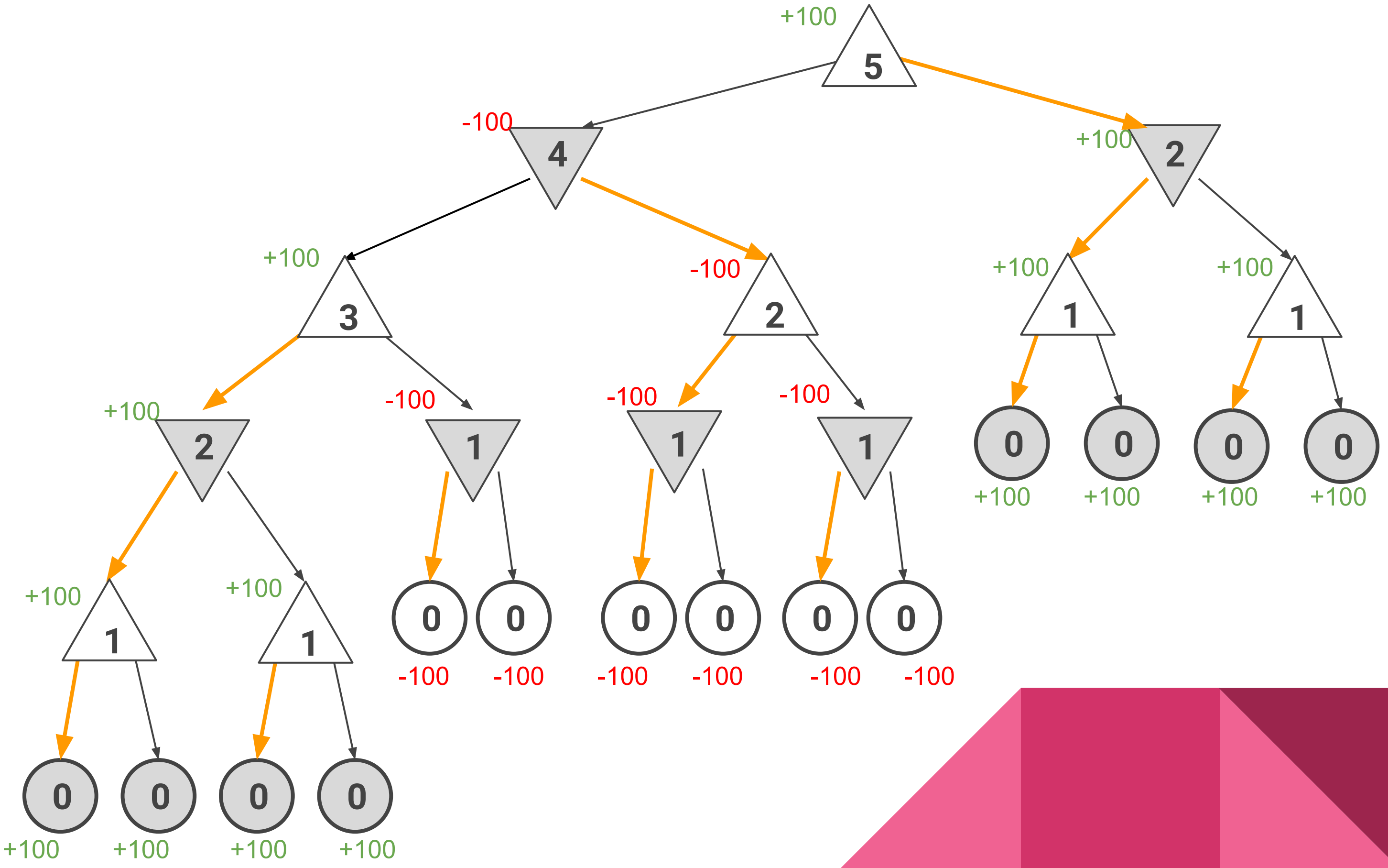
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- DeepBlue did not use vanilla MiniMax.
    - What’s wrong?

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- **Game trees are huge!!!**
- Can we do better?

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- **Game trees are huge!!!**

- Can we do better?
  - Idea: Prune the search space!

Alpha-Beta pruning

- From a max-node (our perspective):
  - If we know utility of action **a** is really high, we shouldn't have to evaluate other actions that we know will not be as good
- Inverse is true from a min-node (adversary's perspective)

Alpha-Beta pruning

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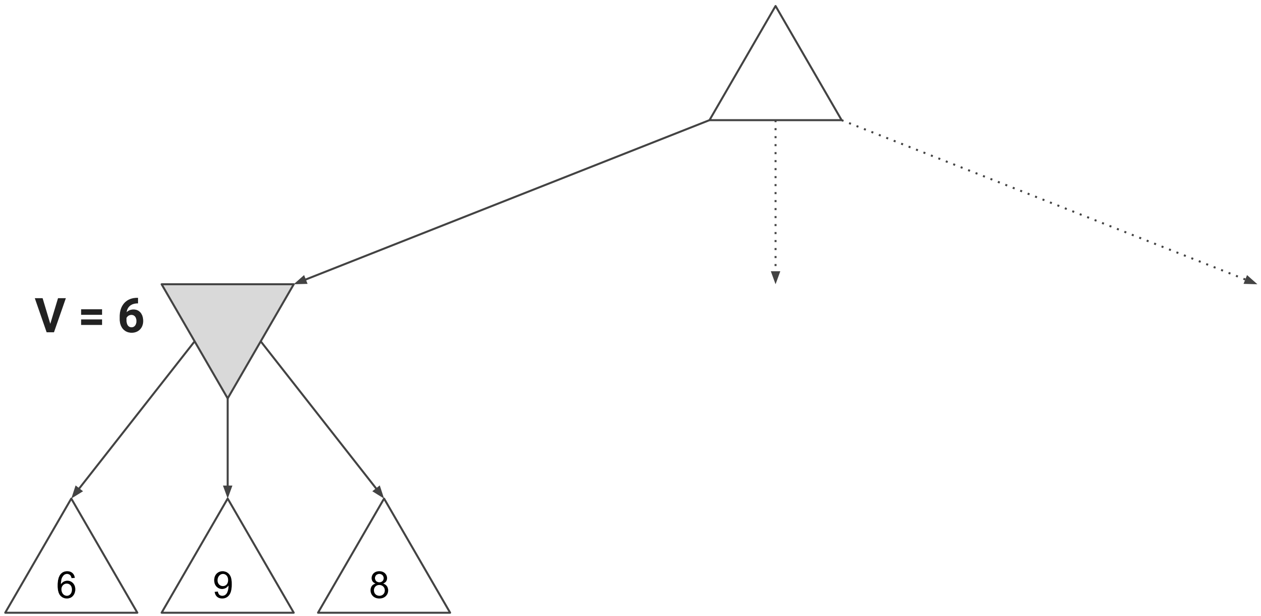
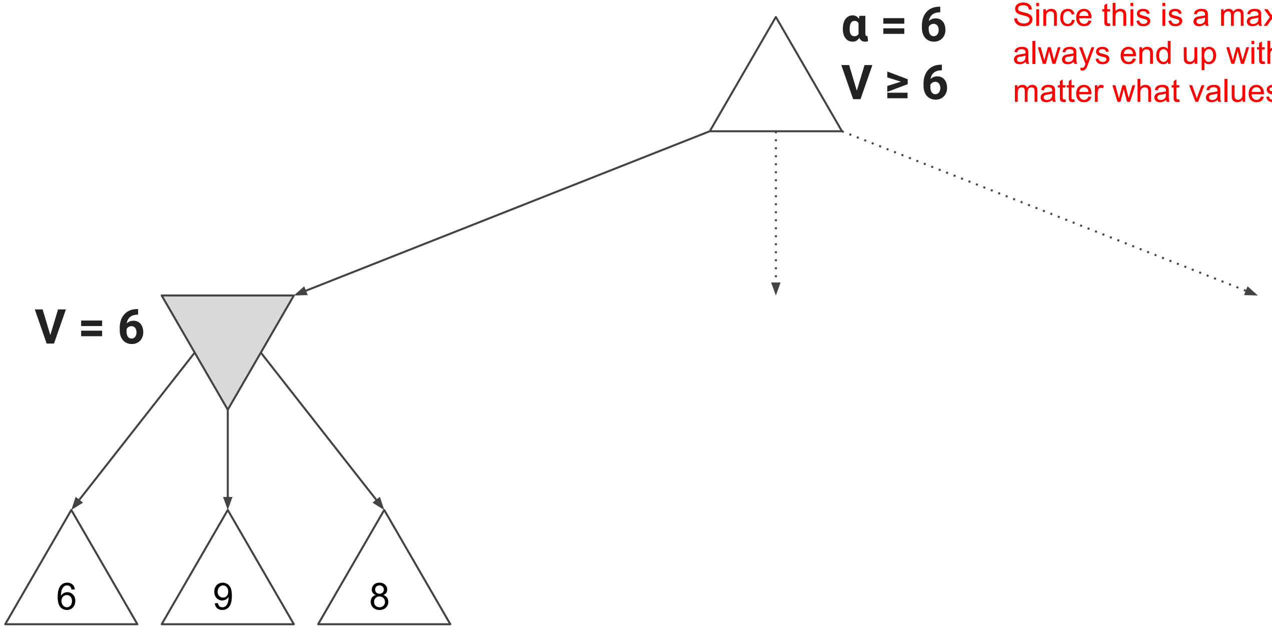
  - If we know utility of action **a** is really high, we shouldn't have to evaluate other actions that we know will not be as good
- Inverse is true from a min-node (adversary's perspective)

- Alpha: lower bound on the value that a max-node may ultimately be assigned
  - $v >= \alpha$
- Beta: upper bound on the value that a min-node may ultimately be assigned
  - $v <= \beta$

Alpha-Beta pruning

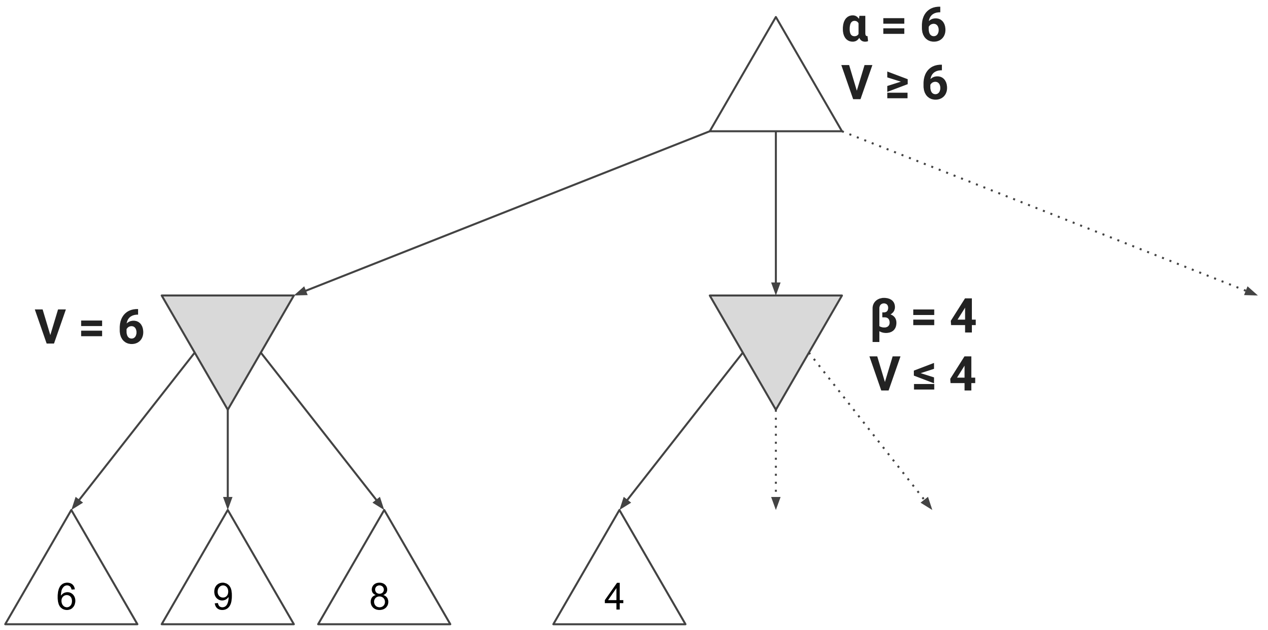


Alpha-Beta pruning

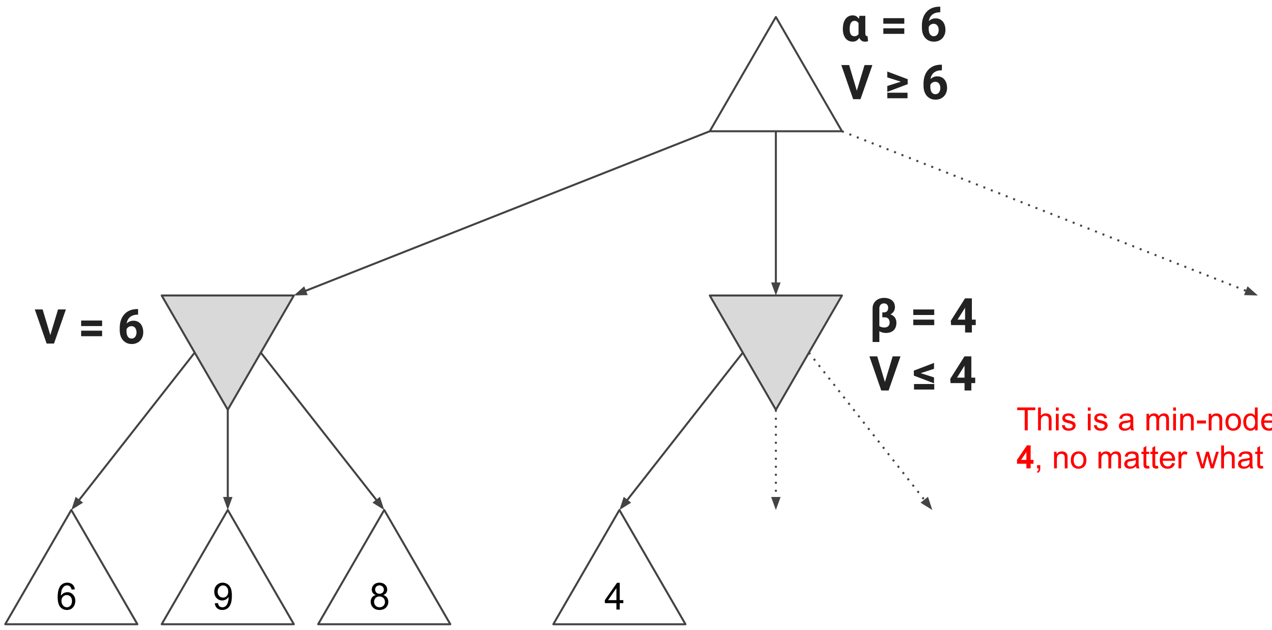


Since this is a max-node, the root node will always end up with a value of **at least 6**, no matter what values the children have

Alpha-Beta pruning

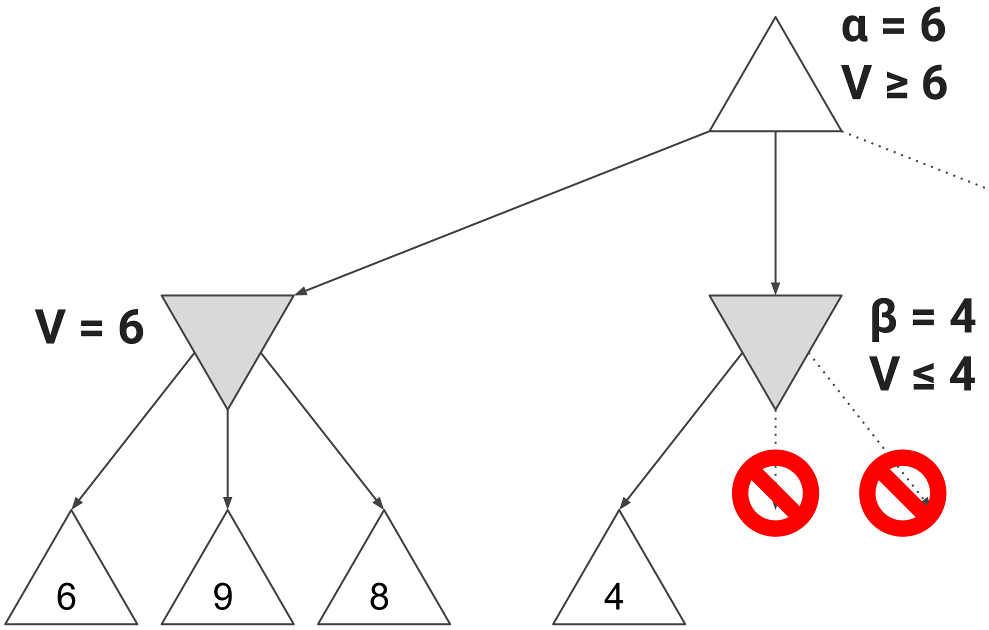


Alpha-Beta pruning



This is a min-node, so its value will be **at most 4**, no matter what values the children have.

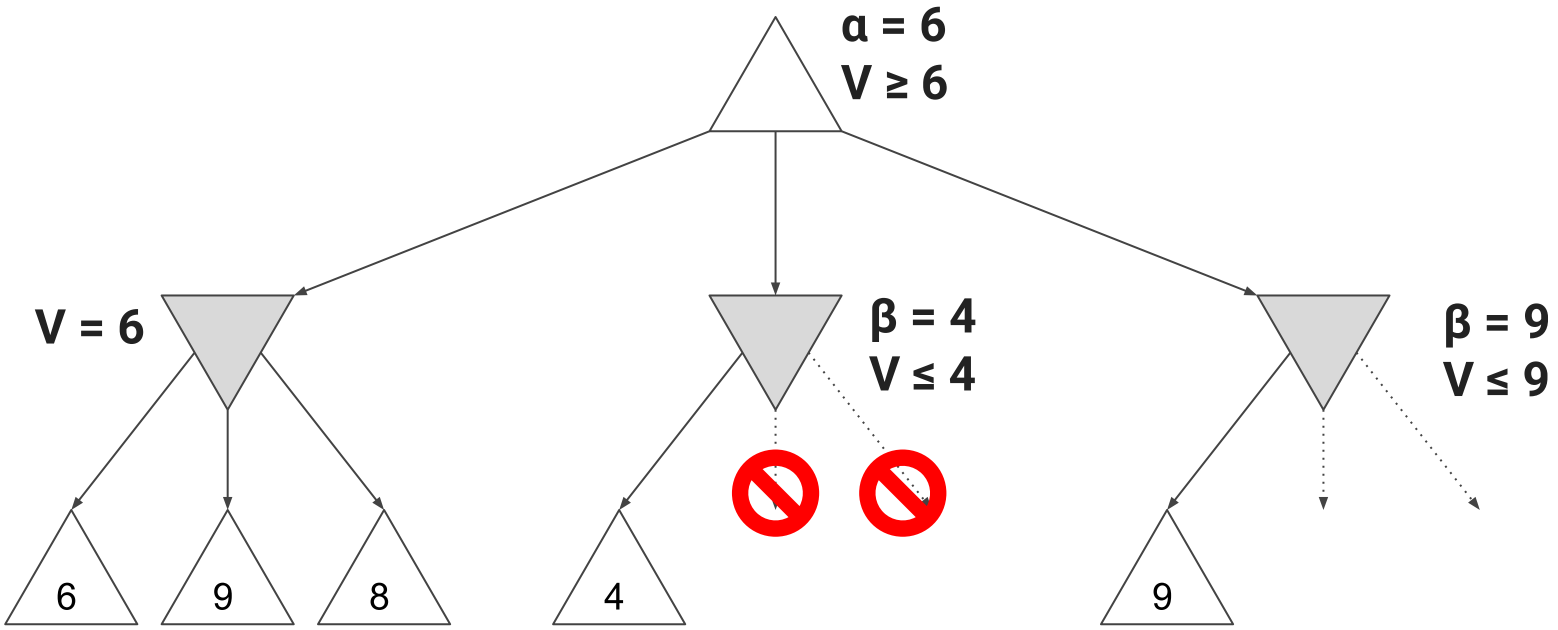
Alpha-Beta pruning



This is a min-node, so its value will be **at most 4**, no matter what values the children have.

4 < 6, so the value of this node is guaranteed to have no affect on the value of root node.

Alpha-Beta pruning



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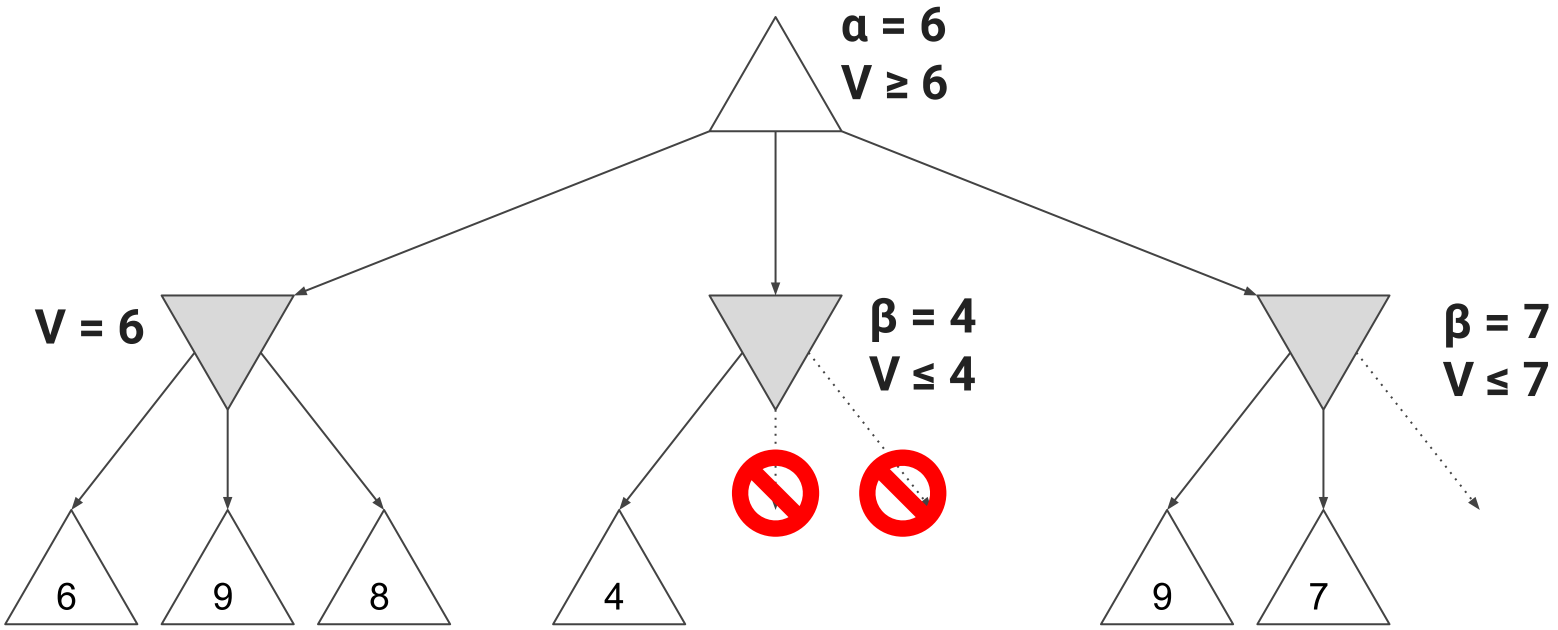

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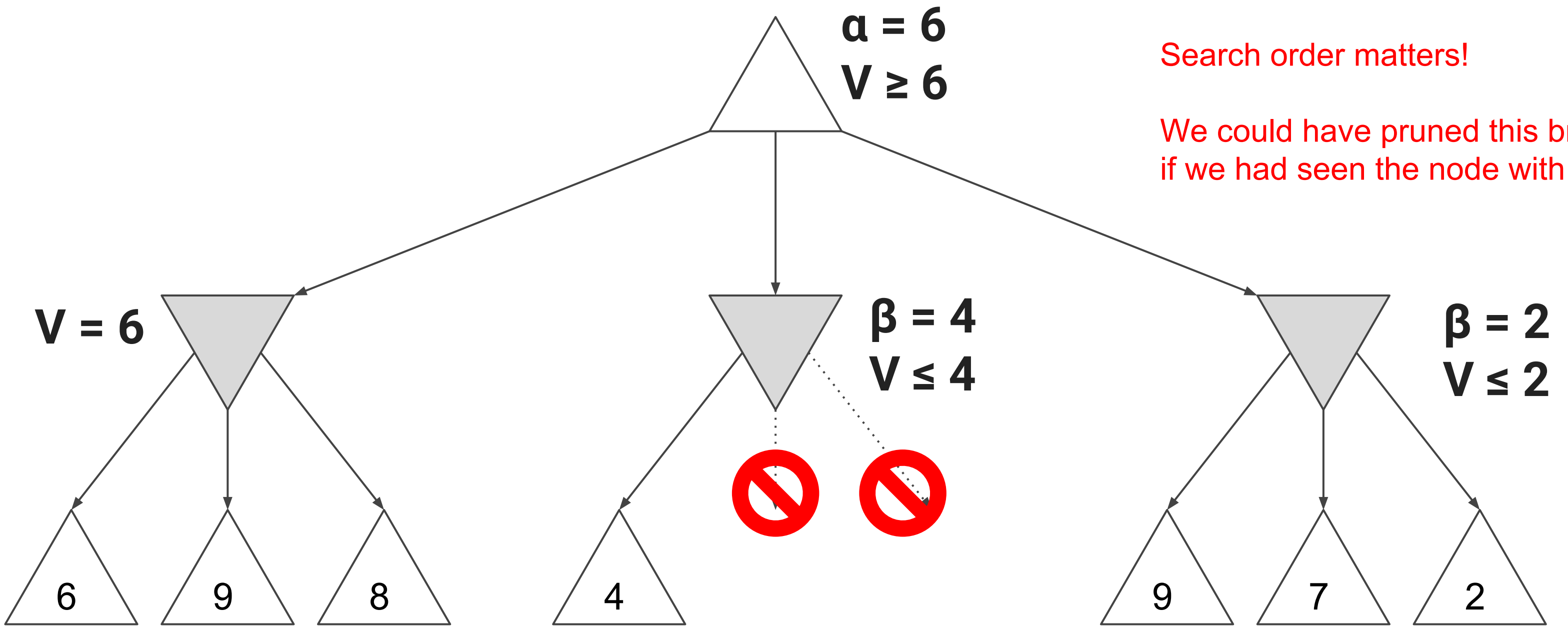


Search order matters!

We could have pruned this branch sooner if we had seen the node with value 2 first.

Alpha-Beta pruning

- DeepBlue did not just use MiniMax + Alpha-Beta pruning.
  - What’s wrong?

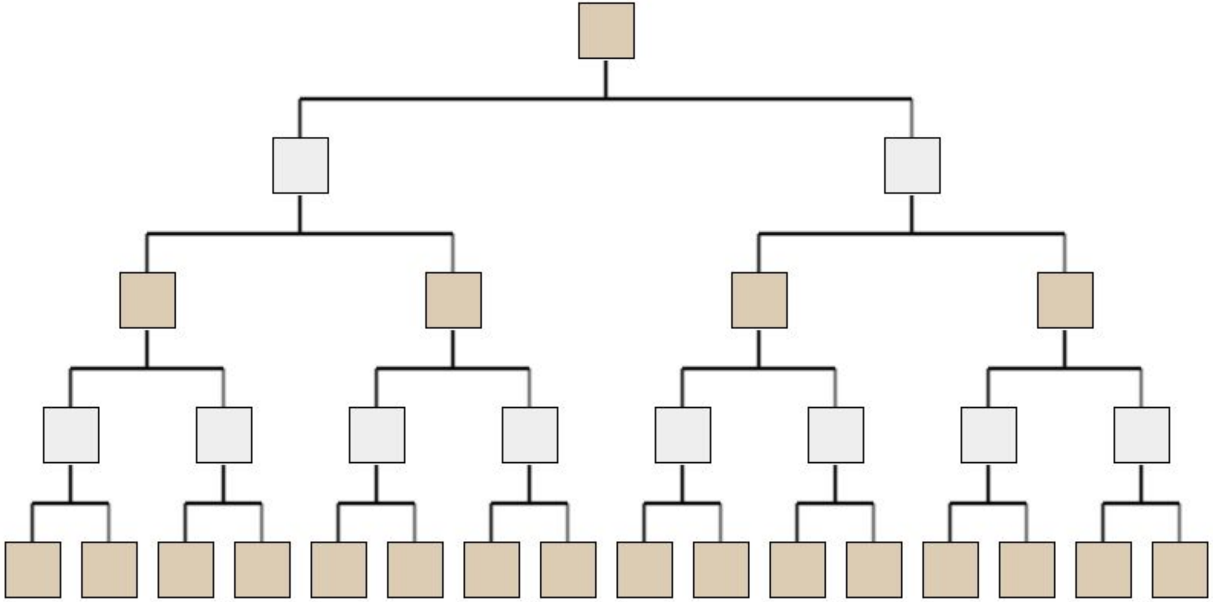
Alpha-Beta pruning

- Pretty cool, but DeepBlue did not just use MiniMax + Alpha-Beta pruning.
  - What’s wrong?
- **Game trees are too deep!!!**

- Can we do better?
  - Idea: Instead of playing the entire game, let’s guess how we’ll we’re doing after **d** moves.

Evaluation functions

Suppose we have finite computing resources and can't afford to compute this entire tree.



Evaluation functions

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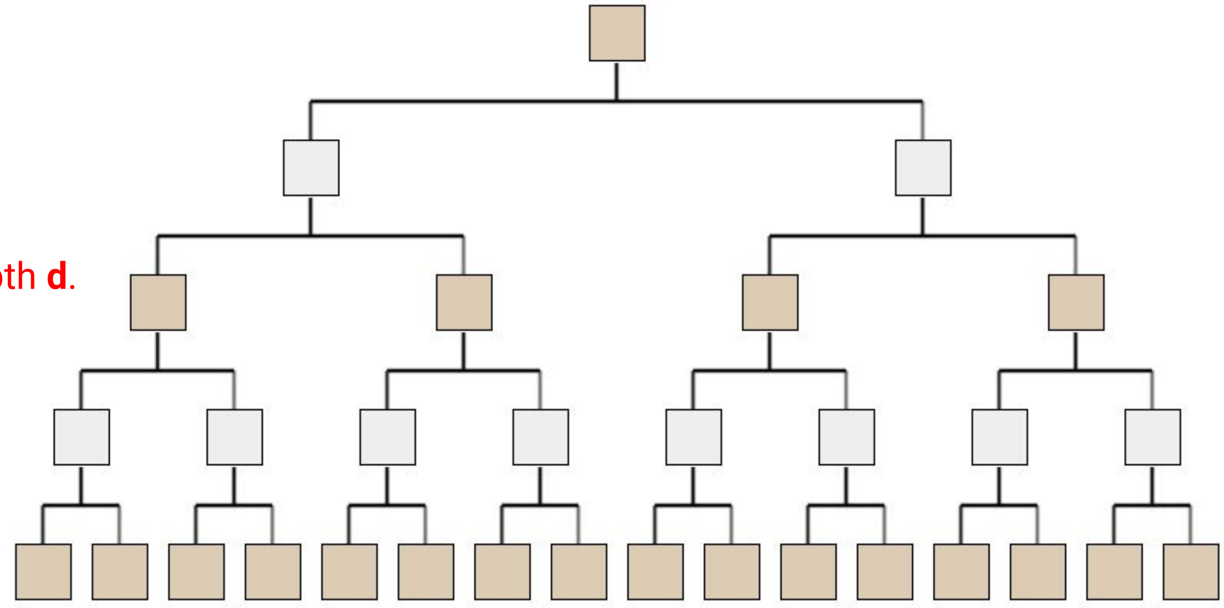
Let's stop our search at some fixed depth **d**.



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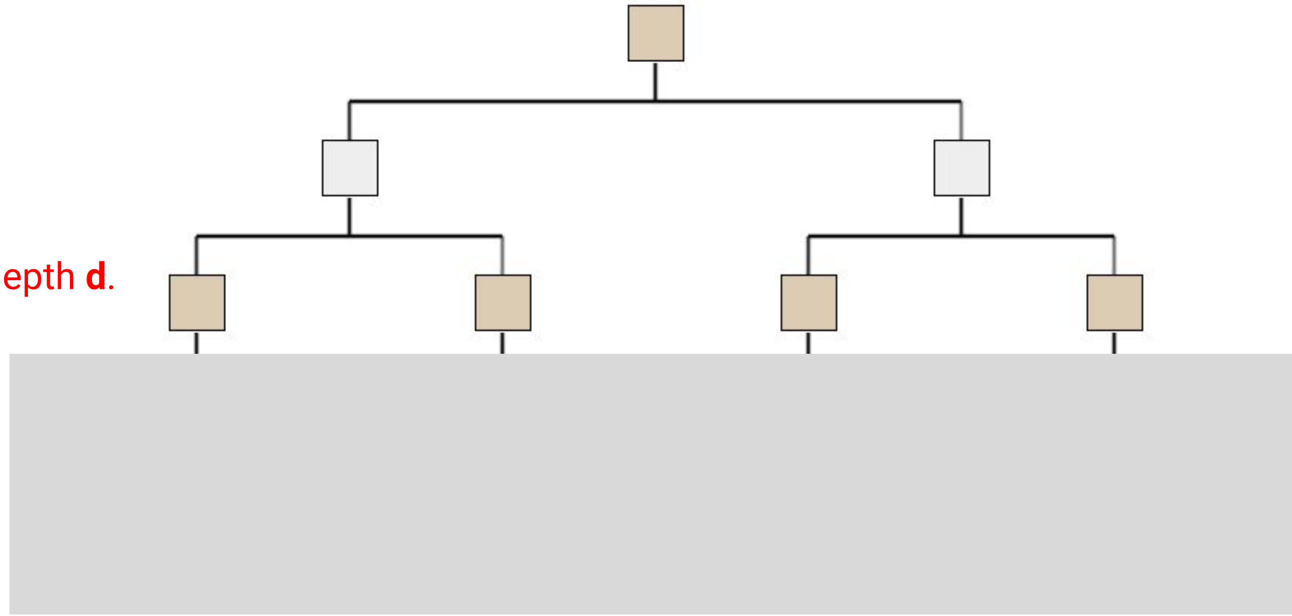

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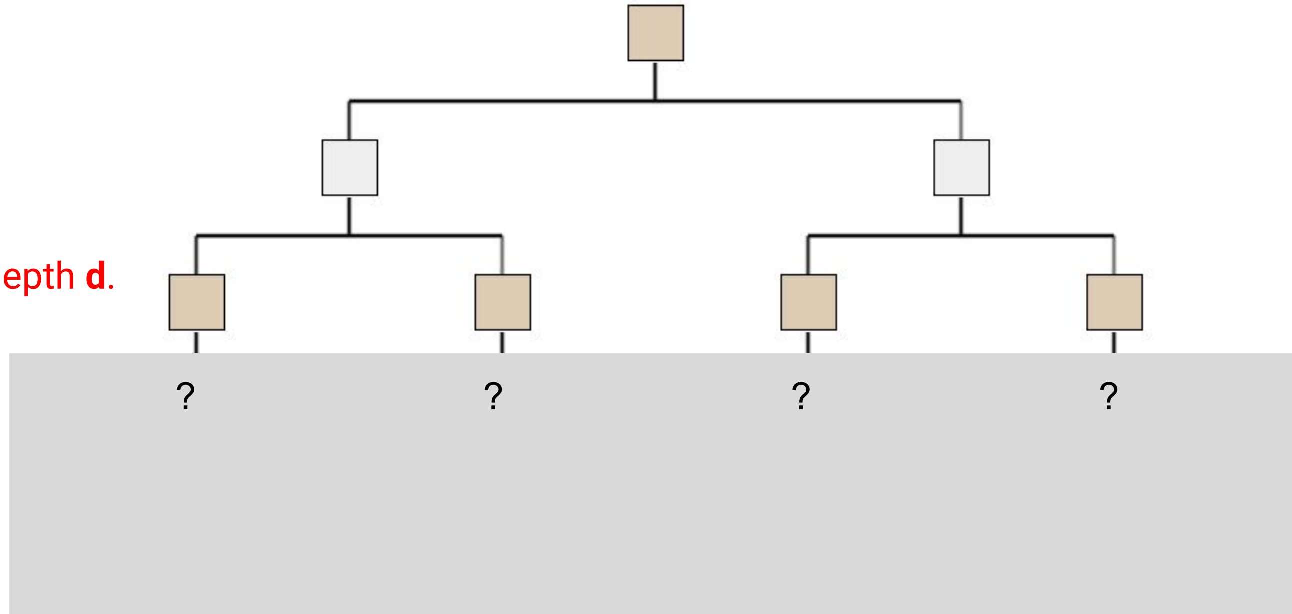
How do we know the utility of these new leaf nodes (to propagate up the game tree)?



Evaluation functions

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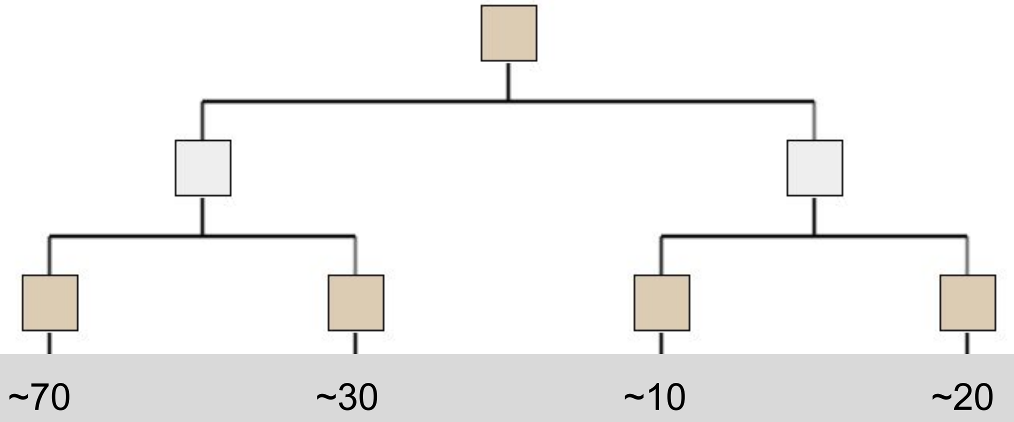
Guess! (use an **heuristic**)

From current game state, how likely am I to win?



Evaluation functions

- Connect-4:
    - How many “open” connect-3’s do I have?
    - How many “open” connect-2’s do I have?
- Chess (DeepBlue): “material, position, King safety and tempo”
    - Material: How many pieces do I have left? And what are they worth?
    - Position: How many empty/safe squares can I attack?
    - King safety: How in-danger of attack is my King?
    - Tempo: Have I been making progress recently?

- DeepBlue: MiniMax tree + Alpha-Beta pruning to a depth of ~13.
    - After that depth, used evaluation function to estimate utility.

Go

- Why wasn't DeepBlue's algorithm good for Go?

Go

- Why wasn't DeepBlue's algorithm good for Go?

- Go is way harder than chess.
  - ~300 possible actions for every game board (vs ~30 in chess)
  - ~150 moves per game (vs ~70 in chess)
  - Total number of possible games
    - ~10^761 (vs ~10^120) for chess
    - There's only 10^80 atoms in the universe?

Alpha Go’s Approach

- Monte Carlo Tree Search
- “Value network” as evaluation function
  - What’s the expected utility of this board state?
- “Policy network” as selection function
  - What moves are more likely to happen from this state?
- Fed data from seeing many expert games

Monte Carlo Tree Search

- I have limited resources to find the optimal policy for every game state.

Monte Carlo Tree Search

- I have limited resources to ~~find~~ the optimal policy for every game state.

approximate

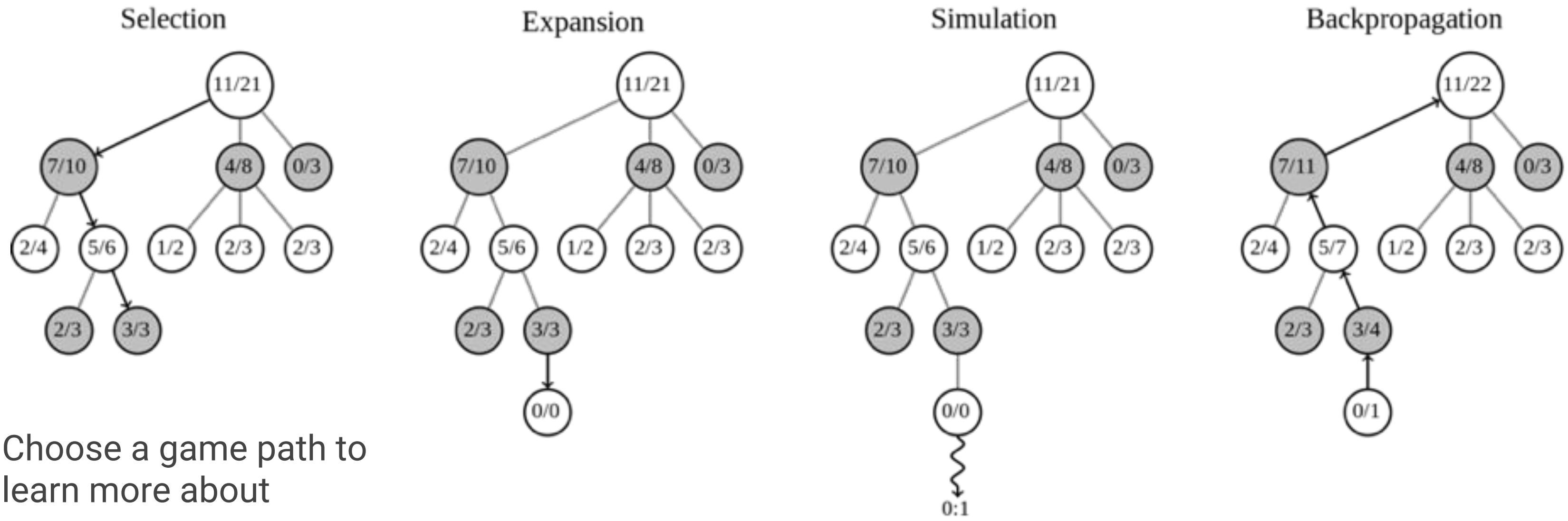
Monte Carlo Tree Search

- I have limited resources to ~~find~~ the optimal policy for ~~every game state~~.

approximate

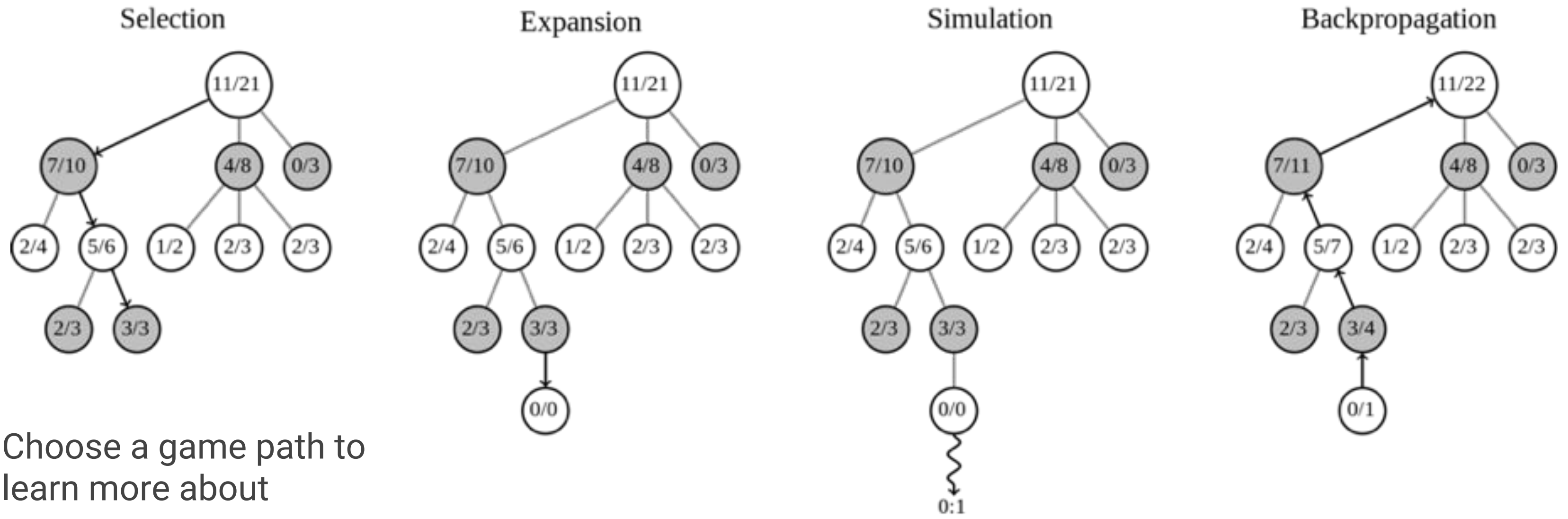
the most common
game states

Monte Carlo Tree Search: the core loop



Choose a game path to learn more about

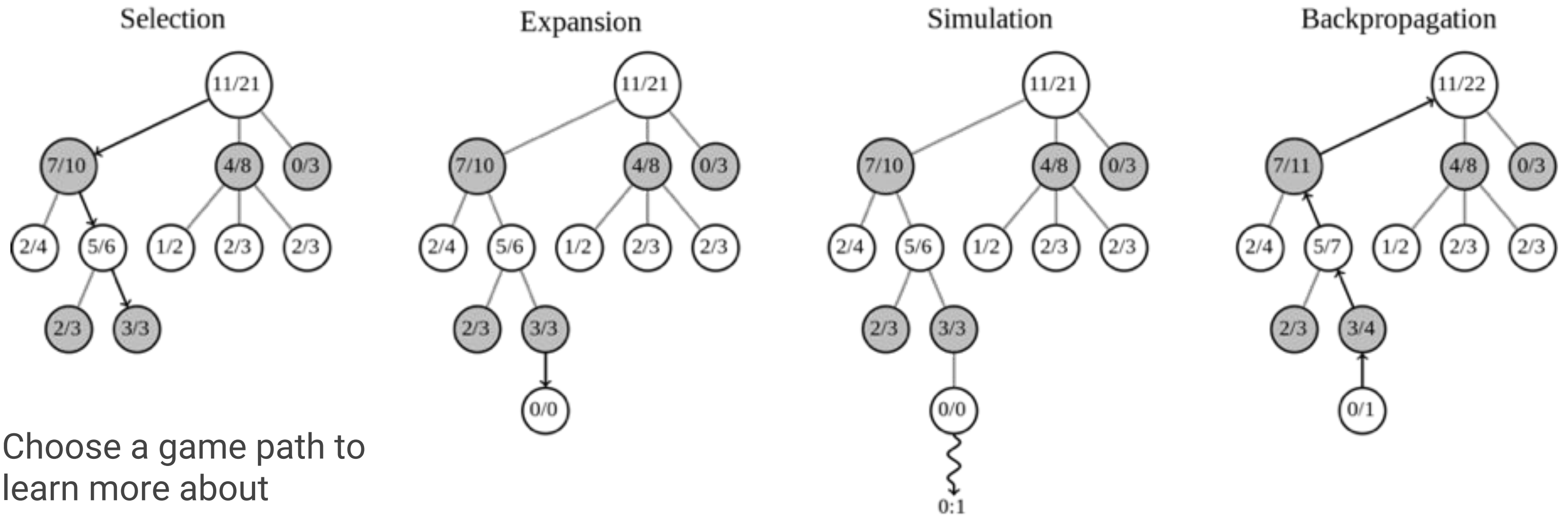
Monte Carlo Tree Search: the core loop



Choose a game path to learn more about

Add a MCTS node to our search tree

Monte Carlo Tree Search: the core loop

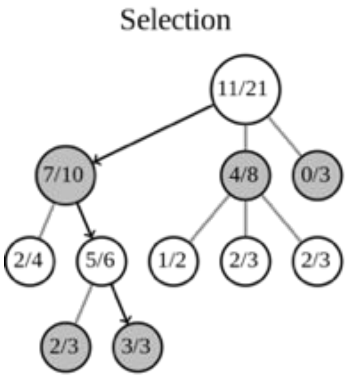


Choose a game path to learn more about

Add a MCTS node to our search tree

Play a game randomly: Did we win?

Monte Carlo Tree Search: the core loop



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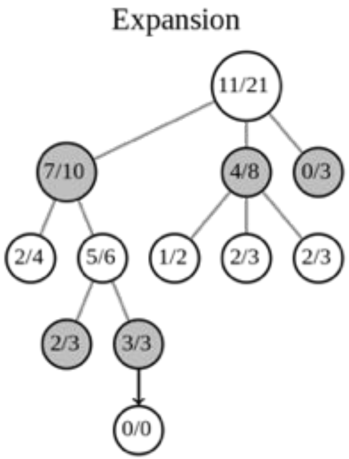

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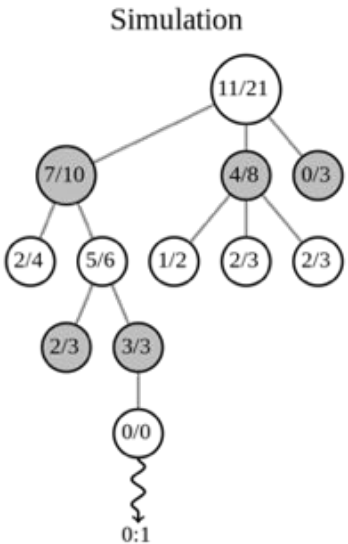

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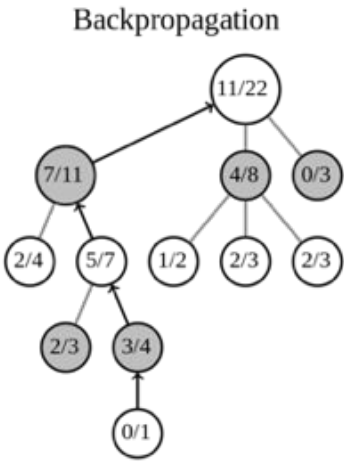
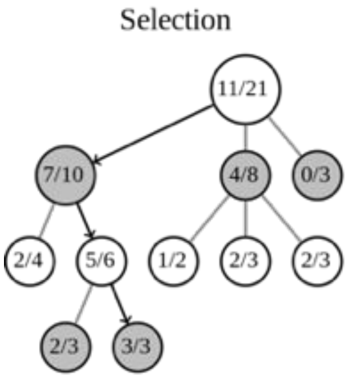


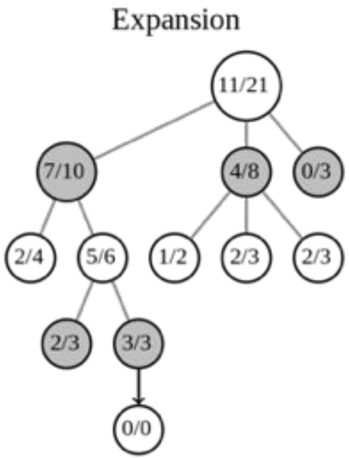
Propagate result up through path

Monte Carlo Tree Search: the core loop

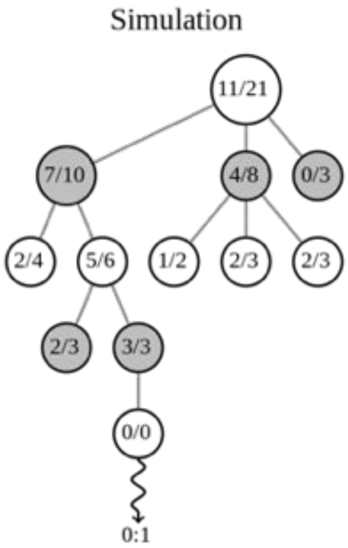


Choose a game path to learn more about

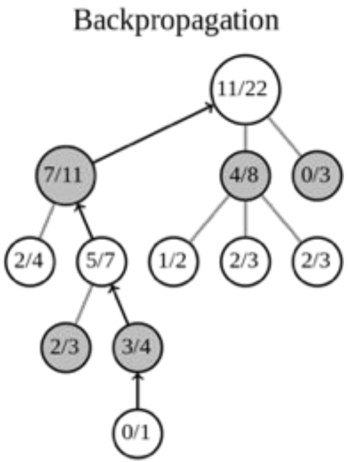
a good selection policy explores “common” game paths more often, while also exploring unknown states



Add a MCTS node to our search tree

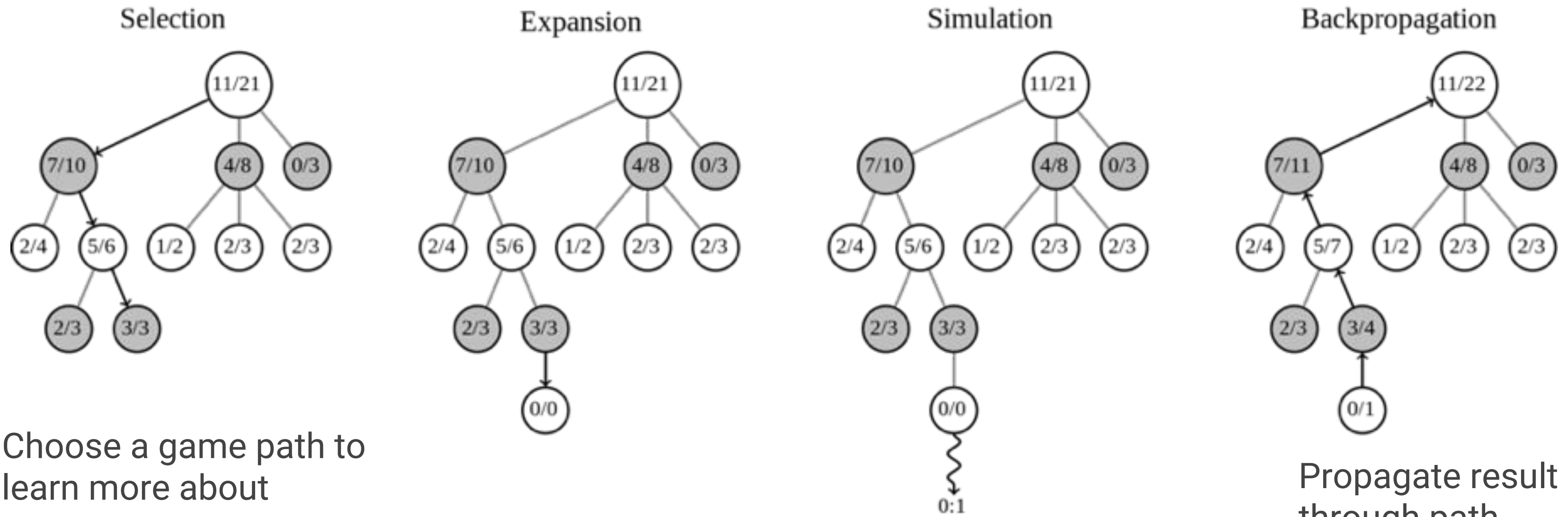


Play a game randomly: Did we win?



Propagate result up through path

Monte Carlo Tree Search: the core loop



Choose a game path to learn more about

Add a MCTS node to our search tree

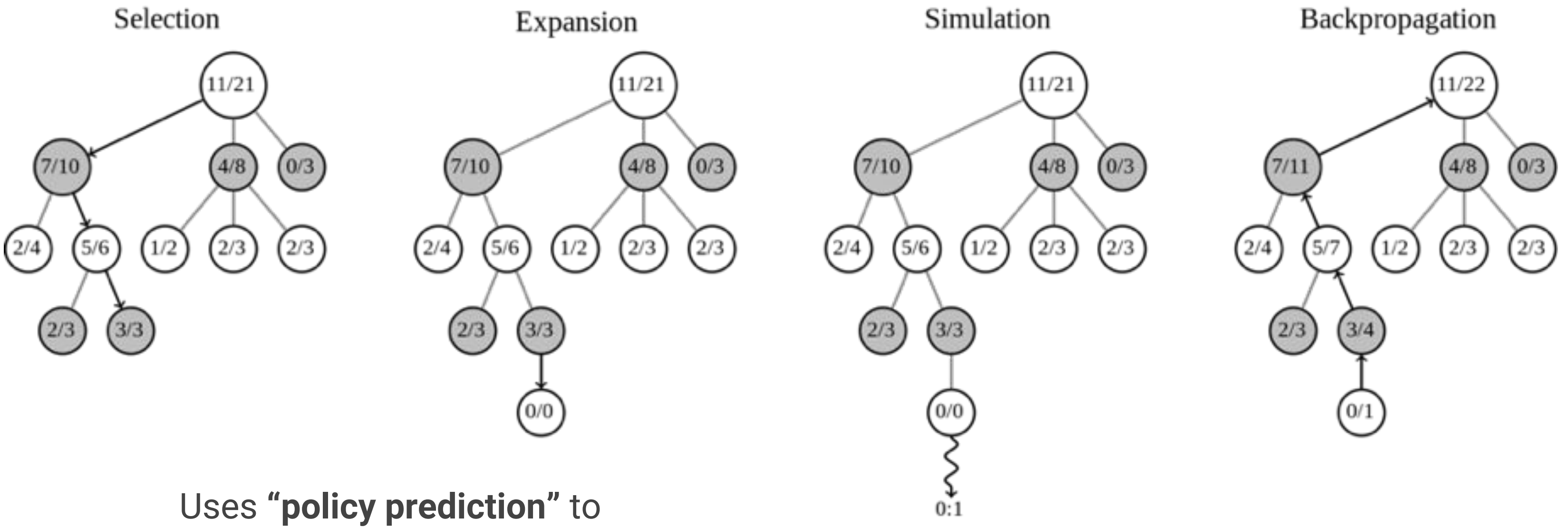
Play a game randomly: Did we win?

Propagate result up through path

a good selection policy explores "common" game paths more often, while also exploring unknown states

Instead of doing a full playout, some MCTS use an evaluation function.

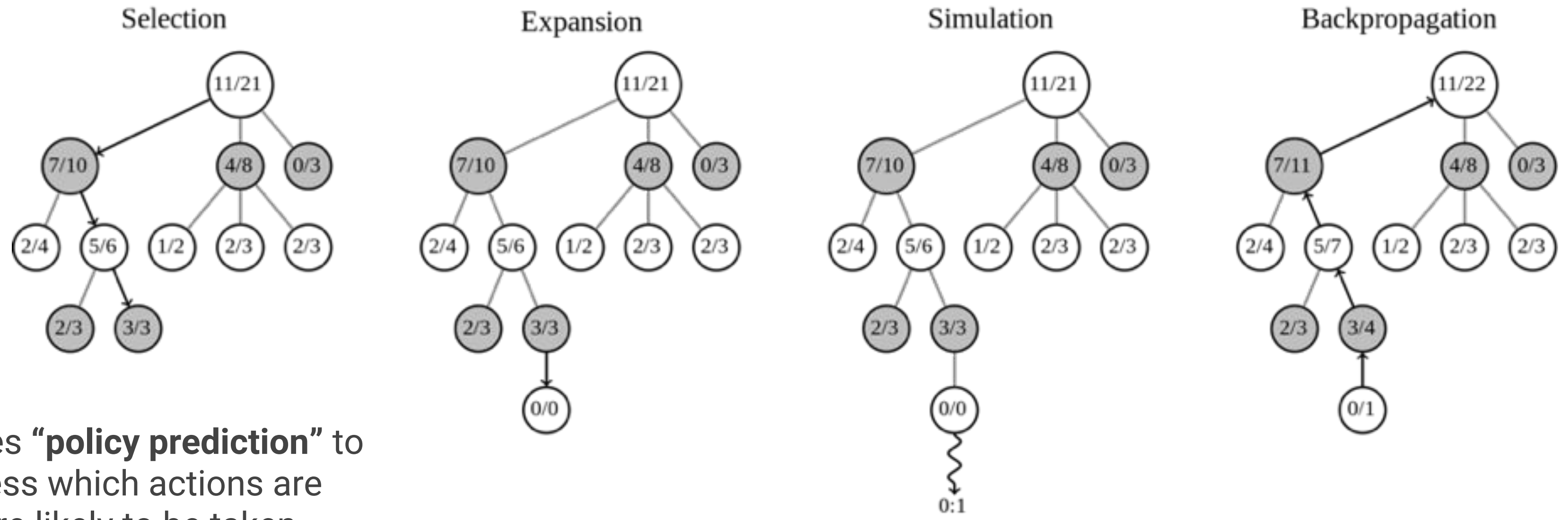
AlphaGo’s Monte Carlo Tree Search



Uses “**policy prediction**” to guess which actions are more likely to be taken.

Uses “**value prediction**” as an evaluation function instead of performing full playout.

AlphaGo’s Monte Carlo Tree Search

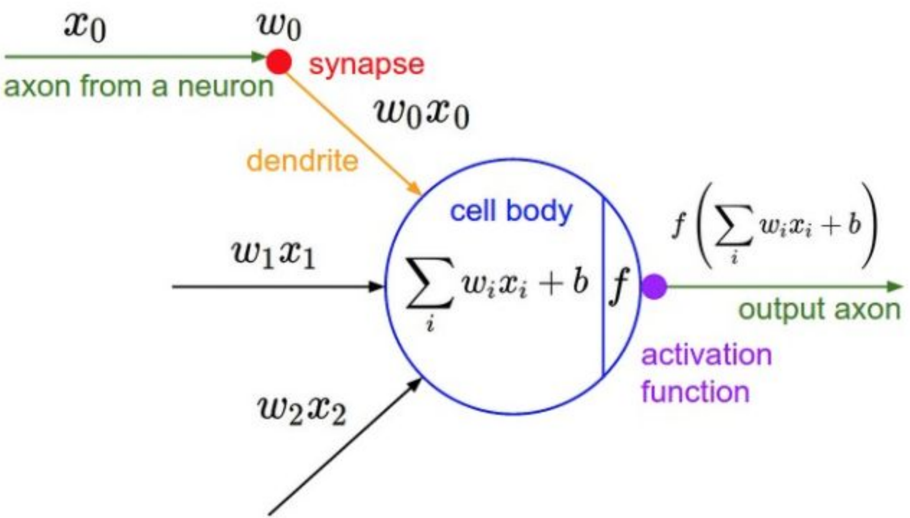


Uses “**policy prediction**” to guess which actions are more likely to be taken.

Uses “**value prediction**” as an evaluation function instead of performing full playout.

These predictions are trained using a **convolutional neural network.**

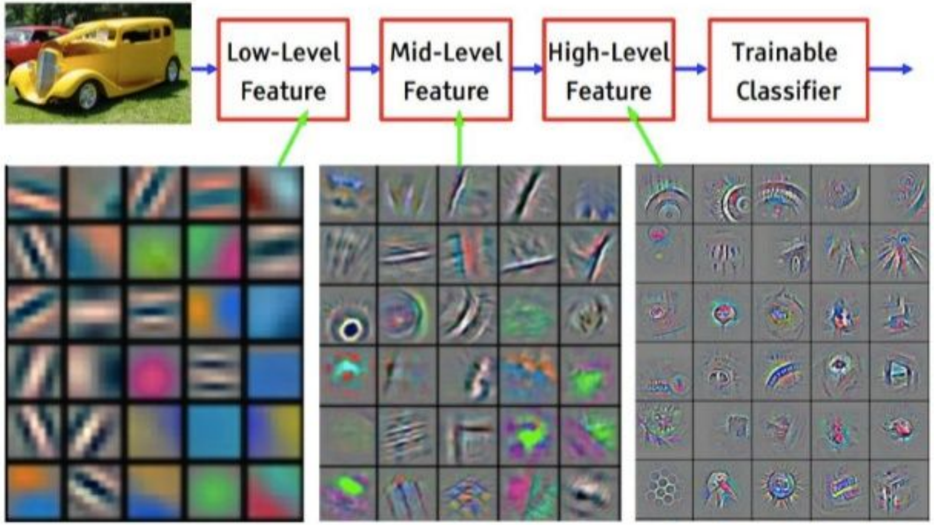
Convolutional Neural Networks



How does training work?

- Take an affine function of input (with weights)
- Pass this output through a nonlinear function -- activation function.

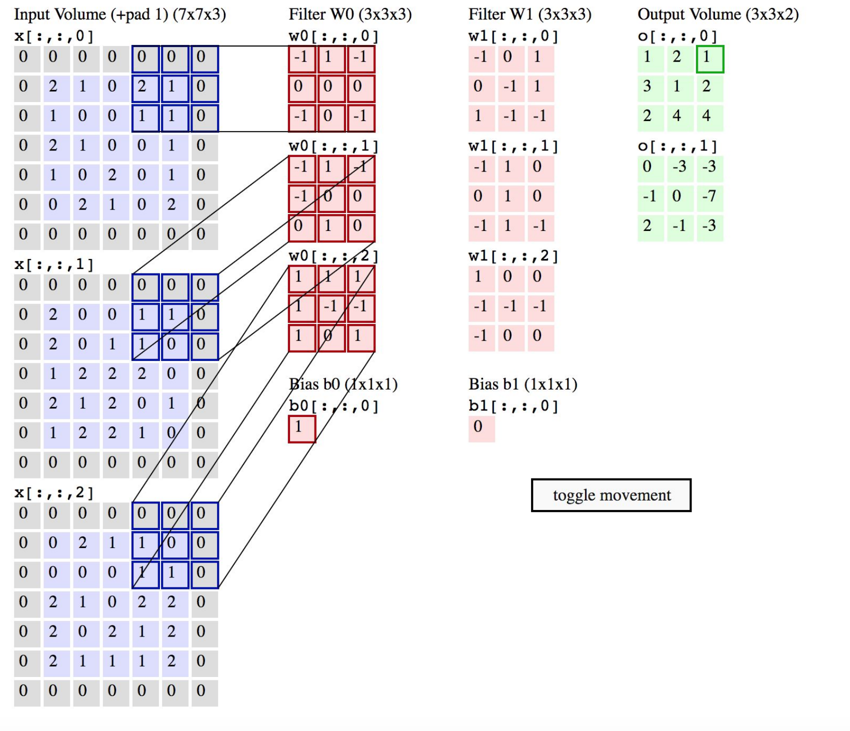
Convolutional Neural Networks



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

How do you train a classifier from these features

Convolutional Neural Networks

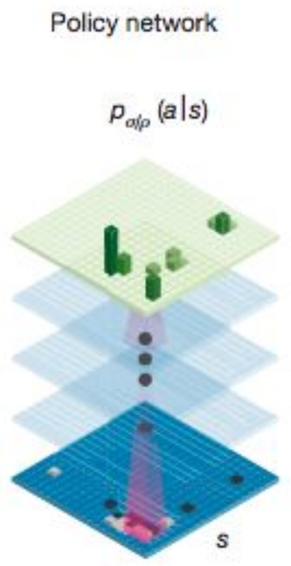


What are they doing mechanically?

- Finding local features in a picture
- Prioritizing features that help predict outcome of interest
- Value Network -> Predict Rewards
- Policy Network -> Predict Next Moves

Policy Network

- Given a 19x19 Go board, output **probability distribution over all legal moves**
- Data from 30 million positions, and data from "self-plays"
- 13 layers!



Value Network

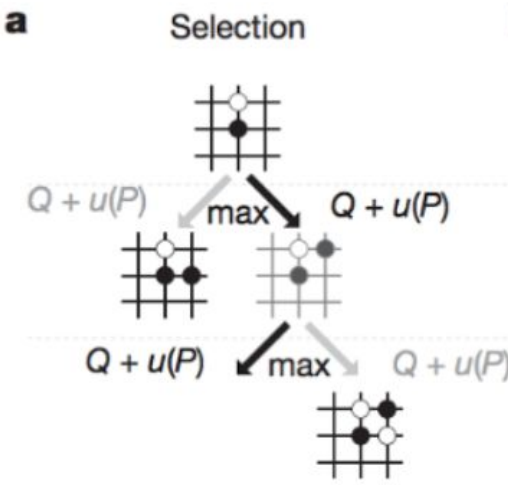
- Given a 19x19 Go board, output a **value.**
  - How likely am I to win?
- Learned on same games as policy network

MCTS in Alpha Go

Selection

We choose which path to “learn more” about by selecting paths with max “**Q + u(P)**”

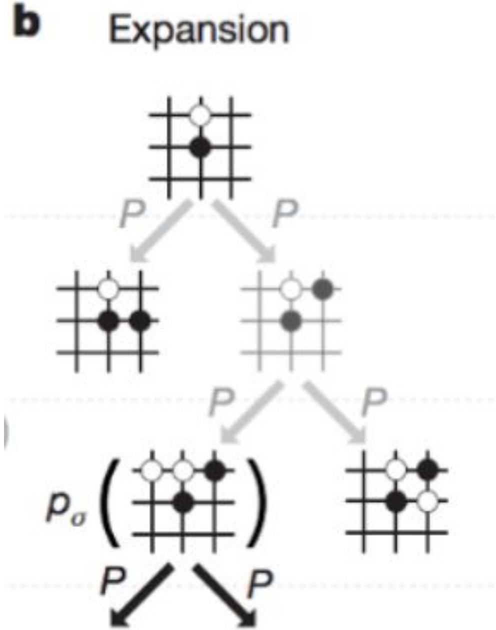
- **Q** trained by value network, **u(P)** samples probability of this action from policy network



MCTS in Alpha Go

Expansion

To choose a node to expand, randomly sample probability distribution from **policy network**.

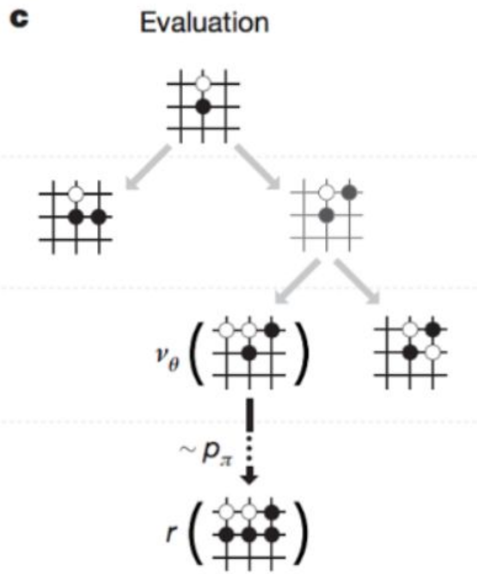


MCTS in Alpha Go

Evaluation

Heuristic is either:

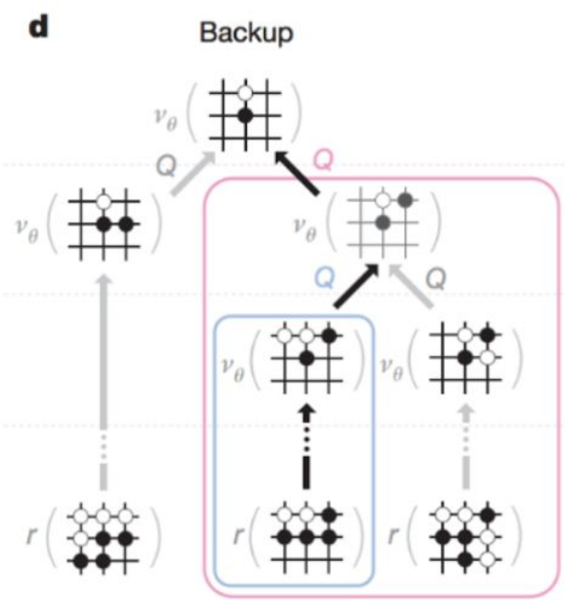
- **Q** from value network
- **r** from "fast rollout"
  - i.e. simulated game



MCTS in Alpha Go

Backpropagation

Q values in the entire path are backpropagated based on the evaluation result.



It’s not perfect

- Alpha Go’s only loss against Lee:
- White 78, Lee played an unexpected move
- AlphaGo failed to explore this in MCTS

Two possible reasons:

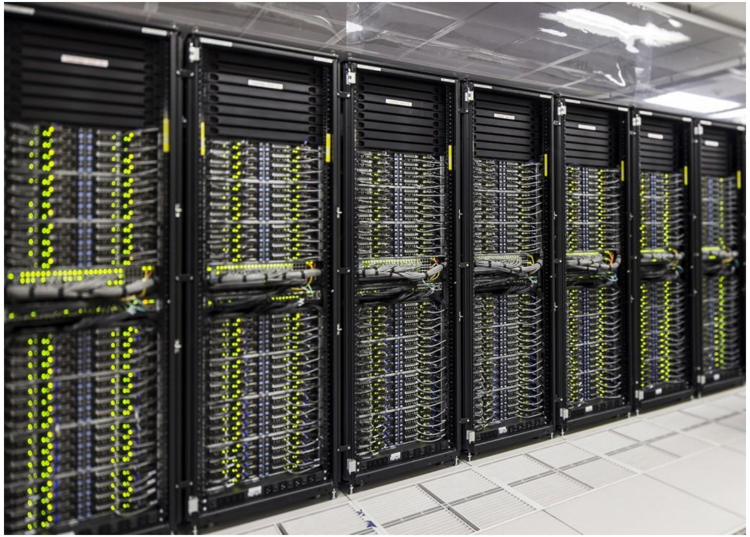
- Policy network hadn’t been trained for long enough
- Selection too aggressively chooses “common” game paths, not enough exploration

AlphaGo

- We just designed AlphaGo!
- … Almost

Computational Power

- 1202 CPUs!
- 176 GPUs!
- Specialized hardware against Lee Sedol



Summary

AlphaGo applied advanced versions of techniques in this class!

Name	ELO
Lee Sedol	3517
AlphaGo (2016)	~3594
Ke Jie (world champion)	3616