The task of part of speech tagging

- A lightweight (usually linear time) processing task, which can usefully empower other applications:
  - Knowing how to pronounce a word: récord [noun] vs. record [verb]; lead as noun vs. verb
  - Matching small phrasal chunks or particular word class patterns for tasks such as information retrieval, information extraction or terminology acquisition (collocation extraction). E.g., just matching nouns, compound nouns, and adjective noun patterns:
    - {A[N]}* N
  - POS information can be used to lemmatize a word correctly (i.e., to remove inflections):
    - saw [n] → saw; saw [v] → see

(Hidden) Markov model tagger

- View sequence of tags as a Markov chain. Assumptions:
  - **Limited horizon.** \( P(X_{i+1} = t^j | X_1, \ldots, X_i) = P(X_{i+1} = t^j | X_i) \)
  - **Time invariant (stationary).** \( P(X_{i+1} = t^j | X_i) = P(X_2 = t^j | X_1) \)

We assume that a word’s tag only depends on the previous tag (limited horizon) and that this dependency does not change over time (time invariance)

- A state (part of speech) generates a word. We assume it depends only on the state

Standard HMM formalism

- \((X, O, \Pi, A, B)\)
- \(X\) is hidden state sequence; \(O\) is observation sequence
- \(\Pi\) is probability of starting in some state
  (can be folded into \(A\): let \(A' = [\Pi|A]\), i.e., \(a_{0j} = \pi_j\))
- \(A\) is matrix of transition probabilities (top row conditional probability tables (CPTs))
- \(B\) is matrix of output probabilities (vertical CPTs)

HMM is also a probabilistic (nondeterministic) finite state automaton, with probabilistic outputs (from vertices, not arcs, in the simplest case)

Most likely hidden state sequence

- Given \(O = (o_1, \ldots, o_T)\) and model \(\mu = (A, B, \Pi)\)
- We want to find:
  \[
  \arg \max_{X} P(X) \frac{P(X, O|\mu)}{P(O|\mu)} = \arg \max_{X} P(X, O|\mu)
  \]
  - \(P(O|X, \mu) = b_{X_1 o_1} b_{X_2 o_2} \cdots b_{X_T o_T}\)
  - \(P(X|\mu) = \pi_{X_1} a_{X_{1}X_2} a_{X_2X_3} \cdots a_{X_{T-1}X_T}\)
  - \(P(O, X|\mu) = P(O|X, \mu) P(X|\mu)\)
  - \(\arg \max_{X} P(O, X|\mu) = \arg \max_{X_1 \cdots X_T} \prod_{t=1}^{T} a_{X_{t-1}X_t} b_{X_t o_t}\)
- Problem: Exponential in sequence length!
Dynamic Programming

- Efficient computation of this maximum: Viterbi algorithm
- Intuition: Probability of the first $t$ observations is the same for all possible $t+1$ length state sequences.
- Define forward score
  $$\delta_i(t) = \max_{x_1 \cdots x_t} P(o_1 \cdots o_{t-1}, x_1 \cdots x_{t-1}, X_t = i | \mu)$$
- $$\delta_j(t+1) = \max_{i=1}^N \delta_i(t) b_{io_j} a_{ij}$$
- Compute it recursively from beginning
- Remember best paths
- A version of Bayes Net most likely state inference

Trellis algorithms

- Used to efficiently find the state sequence that gives the highest probability to the observed outputs
- A dynamic programming algorithm. Essentially the same except you do a max instead of a summation, and record the path taken:
  $$\delta_{i+1}(t^j) = \max_{1 \leq k \leq T} [\delta_i(t^k) \times P(w_i | t^k) \times P(t^j | t^k)]$$
  $$\psi_{i+1}(t^j) = \arg \max_{1 \leq k \leq T} [\delta_i(t^k) \times P(w_i | t^k) \times P(t^j | t^k)]$$
- This gives a best tag sequence for POS tagging
- (Note: this is different to finding the most likely tag for each time $t$!)

Closeup of the computation at one node

- $s_1 \delta_1(t)$
- $s_2 \delta_2(t)$
- $s_3 \delta_3(t)$
- $s_N \delta_N(t)$

- $t \rightarrow \delta_j(t+1)$

- $t+1 \delta_j(t+1) = \max_{1=1}^N \delta_i(t) b_{io_j} a_{ij}$

Viterbi algorithm (Viterbi 1967)

- Used to efficiently find the state sequence that gives the highest probability to the observed outputs
- A dynamic programming algorithm. Essentially the same except you do a max instead of a summation, and record the path taken:
  $$\delta_{i+1}(t^j) = \max_{1 \leq k \leq T} [\delta_i(t^k) \times P(w_i | t^k) \times P(t^j | t^k)]$$
  $$\psi_{i+1}(t^j) = \arg \max_{1 \leq k \leq T} [\delta_i(t^k) \times P(w_i | t^k) \times P(t^j | t^k)]$$
- This gives a best tag sequence for POS tagging
- (Note: this is different to finding the most likely tag for each time $t$!)