# An Introduction to Formal Computational Semantics

## CS224N/Ling 280

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# A first example

**Lexicon**

*Kathy*, NP : **kathy**

*Fong*, NP : **fong**

*respects*, V : $\lambda y.\lambda x.$**respect**$(x, y)$

*runs*, V : $\lambda x.$**run**$(x)$

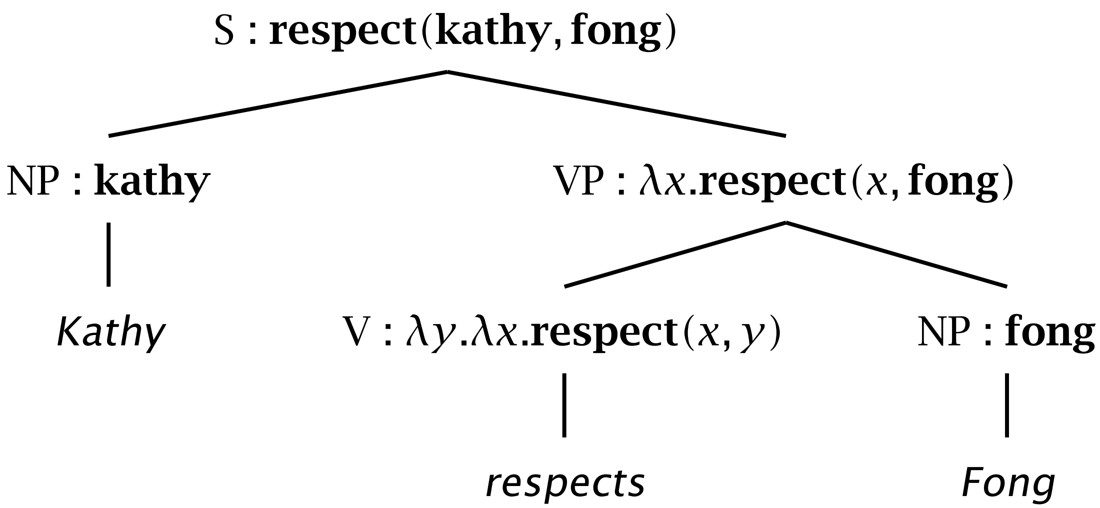
**Grammar**

S : $\beta(\alpha) \rightarrow$ NP : $\alpha$ VP : $\beta$

VP : $\beta(\alpha) \rightarrow$ V : $\beta$ NP : $\alpha$

VP : $\beta \rightarrow$ V : $\beta$

# A first example

- 


- [$_{VP}$ respects Fong] : $[\lambda y.\lambda x.\mathbf{respect}(x, y)](\mathbf{fong})$

  $= \lambda x.\mathbf{respect}(x, \mathbf{fong})$ [$\beta$ red.]

  [$_S$ Kathy respects Fong] : $[\lambda x.\mathbf{respect}(x, \mathbf{fong})](\mathbf{kathy})$

  $= \mathbf{respect(kathy,fong)}$

## Database/knowledgebase interfaces

- Assume that **respect** is a table Respect with two fields respecter and respected
- Assume that **kathy** and **fong** are IDs in the database: $k$ and $f$
- If we assert *Kathy respects Fong* we might evaluate the form **respect**(**fong**)(**kathy**) by doing an insert operation:

  insert into Respects(respecter, respected) values ($k$, $f$)

## Database/knowledgebase interfaces

- Below we focus on questions like *Does Kathy respect Fong* for which we will use the relation to ask:

  select 'yes' from Respects where Respects.respecter = $k$ and Respects.respected = $f$

- We interpret "no rows returned" as 'no' = **0**.

# Typed $\lambda$ calculus (Church 1940)

- Everything has a type (like Java!)
- **Bool** truth values (**0** and **1**)
  **Ind** individuals
  **Ind** → **Bool** properties
  **Ind** → **Ind** → **Bool** binary relations
- **kathy** and **fong** are **Ind**
  **run** is **Ind** → **Bool**
  **respect** is **Ind** → **Ind** → **Bool**
- Types are interpreted right associatively.
  **respect** is **Ind** → (**Ind** → **Bool**)
- We convert a several argument function into embedded unary functions. Referred to as *currying*.

## Typed $\lambda$ calculus (Church 1940)

- Once we have types, we don't need $\lambda$ variables just to show what arguments something takes, and so we can introduce another operation of the $\lambda$ calculus:

  $\eta$ reduction [abstractions can be contracted]

  $\lambda x.(P(x)) \Rightarrow P$

- This means that instead of writing:

  $\lambda y.\lambda x.\mathbf{respect}(x, y)$

  we can just write:

  **respect**

## Typed $\lambda$ calculus (Church 1940)

- $\lambda$ extraction allowed over any type (not just first-order)
- $\beta$ reduction [application]

  $(\lambda x.P(\cdots, x, \cdots))(Z) \Rightarrow P(\cdots, Z, \cdots)$
- $\eta$ reduction [abstractions can be contracted]

  $\lambda x.(P(x)) \Rightarrow P$
- $\alpha$ reduction [renaming of variables]

## Typed $\lambda$ calculus (Church 1940)

- The first form we introduced is called the $\beta, \eta$ long form, and the second more compact representation (which we use quite a bit below) is called the $\beta, \eta$ normal form. Here are some examples:

- | $\beta, \eta$ normal form | $\beta, \eta$ long form |
  |---|---|
  | **run** | $\lambda x.\mathbf{run}(x)$ |
  | $\mathbf{every}^2(\mathbf{kid}, \mathbf{run})$ | $\mathbf{every}^2((\lambda x.\mathbf{kid}(x)), (\lambda x.\mathbf{run}(x))$ |
  | $\mathbf{yesterday}(\mathbf{run})$ | $\lambda y.\mathbf{yesterday}(\lambda x.\mathbf{run}(x))(y)$ |

## Types of major syntactic categories

- nouns and verb phrases will be properties (**Ind** →**Bool**)
- noun phrases are **Ind** – though they are commonly type-raised to (**Ind** → **Bool**) → **Bool**
- adjectives are (**Ind** → **Bool**) → (**Ind** → **Bool**)
  This is because adjectives modify noun meanings, that is properties.
- Intensifiers modify adjectives: e.g, *very* in *a very happy camper*, so they're ((**Ind** → **Bool**) → (**Ind** → **Bool**)) → ((**Ind** → **Bool**) → (**Ind** → **Bool**)) [honest!].

# A grammar fragment

- $S : \beta(\alpha) \rightarrow NP : \alpha \quad VP : \beta$
  $NP : \beta(\alpha) \rightarrow Det : \beta \quad N' : \alpha$
  $N' : \beta(\alpha) \rightarrow Adj : \beta \quad N' : \alpha$
  $N' : \beta(\alpha) \rightarrow N' : \alpha \quad PP : \beta$
  $N' : \beta \rightarrow N : \beta$
  $VP : \beta(\alpha) \rightarrow V : \beta \quad NP : \alpha$
  $VP : \beta(\gamma)(\alpha) \rightarrow V : \beta \quad NP : \alpha \quad NP : \gamma$
  $VP : \beta(\alpha) \rightarrow VP : \alpha \quad PP : \beta$
  $VP : \beta \rightarrow V : \beta$
  $PP : \beta(\alpha) \rightarrow P : \beta \quad NP : \alpha$

# A grammar fragment

- *Kathy*, NP : $\mathbf{kathy}_{\mathbf{Ind}}$
  *Fong*, NP : $\mathbf{fong}_{\mathbf{Ind}}$
  *Palo Alto*, NP : $\mathbf{paloalto}_{\mathbf{Ind}}$
  *car*, N : $\mathbf{car}_{\mathbf{Ind} \to \mathbf{Bool}}$
  *overpriced*, Adj : $\mathbf{overpriced}_{(\mathbf{Ind} \to \mathbf{Bool}) \to (\mathbf{Ind} \to \mathbf{Bool})}$
  *outside*, PP : $\mathbf{outside}_{(\mathbf{Ind} \to \mathbf{Bool}) \to (\mathbf{Ind} \to \mathbf{Bool})}$
  *red*, Adj : $\lambda P.(\lambda x.P(x) \wedge \mathbf{red}'(x))$
  *in*, P : $\lambda y.\lambda P.\lambda x.(P(x) \wedge \mathbf{in}'(y)(x))$
  *the*, Det : $\iota$
  *a*, Det : $\mathbf{some}^2{}_{(\mathbf{Ind} \to \mathbf{Bool}) \to (\mathbf{Ind} \to \mathbf{Bool}) \to \mathbf{Bool}}$
  *runs*, V : $\mathbf{run}_{\mathbf{Ind} \to \mathbf{Bool}}$
  *respects*, V : $\mathbf{respect}_{\mathbf{Ind} \to \mathbf{Ind} \to \mathbf{Bool}}$
  *likes*, V : $\mathbf{like}_{\mathbf{Ind} \to \mathbf{Ind} \to \mathbf{Bool}}$

# A grammar fragment

- **in′** is **Ind → Ind → Bool**
- **in** $\stackrel{\text{def}}{=}$ $\lambda y.\lambda P.\lambda x.(P(x) \wedge \textbf{in}'(y)(x))$ is **Ind → (Ind → Bool) →(Ind → Bool)**
- **red′** is **Ind → Bool**
- **red** $\stackrel{\text{def}}{=}$ $\lambda P.(\lambda x.(P(x) \wedge \textbf{red}'(x))$ is **(Ind → Bool) →(Ind → Bool)**

# Adjective and PP modification

- $\mathrm{N}' : \lambda x.\mathbf{car}(x) \wedge \mathbf{in'}(\mathbf{paloalto})(x) \wedge \mathbf{red'}(x)$
  - $\mathrm{Adj} : \lambda P.(\lambda x.P(x) \wedge \mathbf{red'}(x))$
    - *red*
  - $\mathrm{N}' : \lambda x.(\mathbf{car}(x) \wedge \mathbf{in'}(\mathbf{paloalto})(x))$
    - $\mathrm{N}' : \mathbf{car}$
      - $\mathrm{N} : \mathbf{car}$
        - *car*
    - $\mathrm{PP} : \lambda P.\lambda x.(P(x) \wedge \mathbf{in'}(\mathbf{paloalto})(x))$
      - $\mathrm{P} : \lambda y.\lambda P.\lambda x.(P(x) \wedge \mathbf{in'}(y)(x))$
        - *in*
      - $\mathrm{NP} : \mathbf{paloalto}$
        - *Palo Alto*
- $\mathrm{N}' : \lambda x.\mathbf{car}(x) \wedge \mathbf{in'}(\mathbf{paloalto})(x) \wedge \mathbf{red'}(x))$
  - $\mathrm{N}' : \lambda x.(\mathbf{car}(x) \wedge \mathbf{red'}(x))$
    - $\mathrm{Adj} : \lambda P.(\lambda x.P(x) \wedge \mathbf{red'}(x))$
      - *red*
    - $\mathrm{N}' : \mathbf{car}$
      - $\mathrm{N} : \mathbf{car}$
        - *car*
  - $\mathrm{PP} : \lambda P.\lambda x.(P(x) \wedge \mathbf{in'}(\mathbf{paloalto})(x))$
    - $\mathrm{P} : \lambda y.\lambda P.\lambda x.(P(x) \wedge \mathbf{in'}(y)(x))$
      - *in*
    - $\mathrm{NP} : \mathbf{paloalto}$
      - *Palo Alto*

## Intersective adjectives

- Syntactic ambiguity is spurious: you get the same semantics either way
- Database evaluation is possible via a table join

## Non-intersective adjectives

- For non-intersective adjectives get different semantics depending on what they modify
- **overpriced**(**in**(**paloalto**)(**house**))
- **in**(**paloalto**)(**overpriced**(**house**))
- But probably won't be able to evaluate it on database!

# Why things get more complex

- When doing predicate logic did you wonder why:
  - *Kathy runs* is **run**(**kathy**)
  - *no kid runs* is $\neg(\exists x)(\mathbf{kid}(x) \wedge \mathbf{run}(x))$
- Somehow the NP's meaning is wrapped around the predicate
- Or consider why this argument doesn't hold:
  - Nothing is better than a life of peace and prosperity.
    A cold egg salad sandwich is better than nothing.
    ---
    A cold egg salad sandwich is better than a life of peace and prosperity.
- The problem is that *nothing* is a quantifier

## Generalized Quantifiers

- We have a reasonable semantics for *red car in Palo Alto* as a property from **Ind** → **Bool**
- How do we represent noun phrases like *the red car in Palo Alto* or *every red car in Palo Alto*?
- $[\![\iota]\!](P) = a$ if ($P(b) = \mathbf{1}$ iff $b = a$)

  undefined, otherwise
- The semantics for *the* following Bertrand Russell, for whom *the* $x$ meant the unique item satisfying a certain description

## Generalized Quantifiers

- *red car in Palo Alto*

  select Cars.obj from Cars, Locations, Red where

  Cars.obj = Locations.obj AND

  Locations.place = ‘paloalto’ AND Cars.obj = Red.obj

  (here we assume the unary relations have one field, obj).

# Generalized Quantifiers

- *the red car in Palo Alto*

- NP : $\iota(\lambda x.\mathbf{car}(x) \wedge \mathbf{in'}(\mathbf{paloalto})(x) \wedge \mathbf{red'}(x))$

  Det : $\iota$      N$'$ : $\lambda x.\mathbf{car}(x) \wedge \mathbf{in'}(\mathbf{paloalto})(x) \wedge \mathbf{red'}(x)$

  *the*      *red car in Palo Alto*

- *the red car in Palo Alto*

  select Cars.obj from Cars, Locations, Red where

  Cars.obj = Locations.obj AND

  Locations.place = ‘paloalto’ AND Cars.obj = Red.obj

  having count(*) = 1

## Generalized Quantifiers

- What then of *every red car in Palo Alto*?
- A generalized determiner is a relation between two properties, one contributed by the restriction from the N′, and one contributed by the predicate quantified over:

  $$(\mathbf{Ind} \to \mathbf{Bool}) \to (\mathbf{Ind} \to \mathbf{Bool}) \to \mathbf{Bool}$$

- Here are some determiners

  $$\mathbf{some}^2(\mathbf{kid})(\mathbf{run}) \equiv \mathbf{some}(\lambda x.\mathbf{kid}(x) \wedge \mathbf{run}(x))$$

  $$\mathbf{every}^2(\mathbf{kid})(\mathbf{run}) \equiv \mathbf{every}(\lambda x.\mathbf{kid}(x) \to \mathbf{run}(x))$$

## Generalized Quantifiers

- Generalized determiners are implemented via the quantifiers:

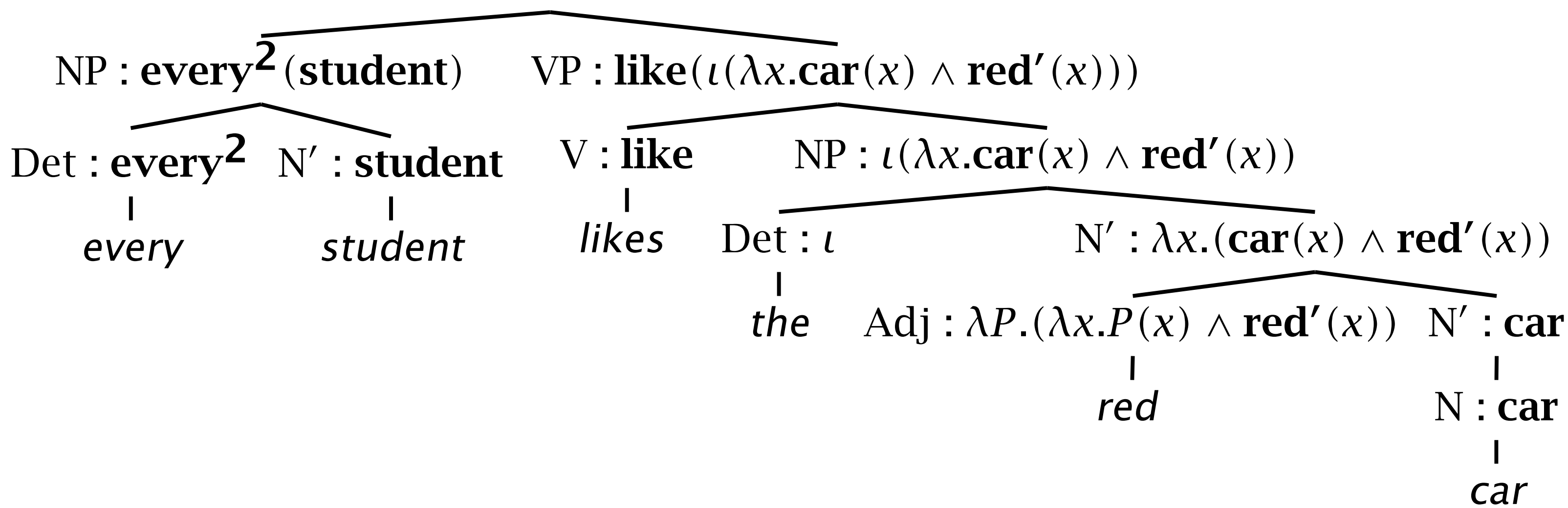
  **every**$(P)$ = **1** iff $(\forall x)P(x)$ = **1**;

  i.e., if $P = \mathbf{Dom_{Ind}}$

  **some**$(P)$ = **1** iff $(\exists x)P(x)$ = **1**; i.e., if $P \neq \emptyset$

# Generalized Quantifiers

- Every student likes the red car
- $\mathrm{S} : \mathbf{every^2}(\mathbf{student})(\mathbf{like}(\iota(\lambda x.\mathbf{car}(x) \wedge \wedge\mathbf{red'}(x))))$



## Representing proper nouns with quantifiers

- The central insight of Montague's PTQ was to treat individuals as of the same type as quantifiers (as type-raised individuals):
- *Kathy* : $\lambda P.P(\mathbf{kathy})$
- Both good and bad
- The main alternative (which we use) is flexible *type shifting* – you raise the type of something when necessary.

# Nominal type shifting

- Common patterns of nominal type shifting
- $$\begin{tikzcd}
\mathbf{Ind} \arrow[rr, shift left, "\mathbf{Q}"] \arrow[dr, "\mathbf{R}"'] & & \mathbf{Ind \to Bool} \arrow[ll, dashed, shift left, "\iota"] \arrow[dl, "\mathbf{some^2}"] \\
& \mathbf{(Ind \to Bool) \to Bool} &
\end{tikzcd}$$
- $\mathbf{R}(x) = \lambda P.P(x)$
  $\mathbf{some^2}(P) = \lambda Q.(Q \cap P) \neq \emptyset$
  $\mathbf{Q}(x) = \lambda y.x = y$
- In this diagram, **R** is exactly this basic type-raising function for individuals.

## Noun phrase scope – following Hendriks (1993)

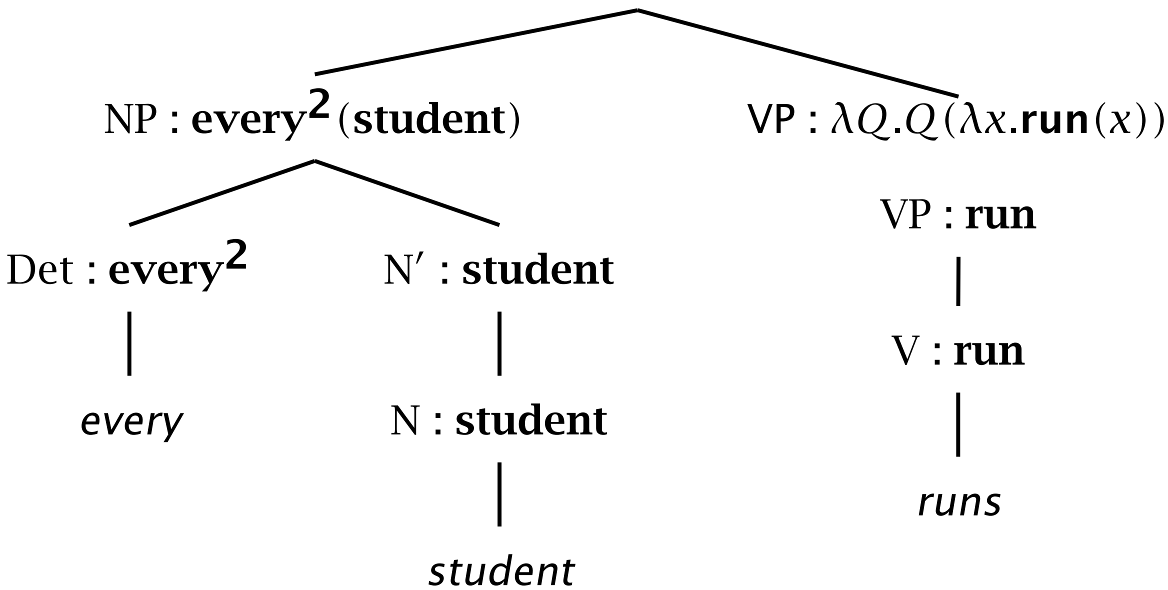
**Value raising** raises a function that produces an individual to one that produces a quantifer. If $\alpha : \sigma \to \mathbf{Ind}$ then $\lambda x.\lambda P.P(\alpha(x)) : \sigma \to (\mathbf{Ind} \to \mathbf{Bool}) \to \mathbf{Bool}$

**Argument raising** replaces an argument of a boolean function with a variable and applies the quantifier semantically binding the replacing variable. If $\alpha : \sigma \to \mathbf{Ind} \to \tau \to \mathbf{Bool}$ then $\lambda x_1.\lambda Q.\lambda x_3.Q(\lambda x_2.\alpha(x_1)(x_2)(x_3)) : \sigma \to (\mathbf{Ind} \to \mathbf{Bool}) \to \mathbf{Bool} \to \tau \to \mathbf{Bool}$

**Argument lowering** replaces a quantifier in a boolean function with an individual argument, where the semantics is calculated by applying the original function to the type raised argument. If $\alpha : \sigma \to ((\mathbf{Ind} \to \mathbf{Bool}) \to \mathbf{Bool}) \to \tau \to \mathbf{Bool}$ then $\lambda x_1.\lambda x_2.\lambda x_3.\alpha(x_1)(\lambda P.P(x_2))(x_3) : \sigma \to \mathbf{Ind} \to \tau \to \mathbf{Bool}$

# *Every student runs*

- S : $\textbf{every}^{\mathbf{2}}(\textbf{student})(\textbf{run}) \equiv \textbf{every}(\lambda x.\textbf{student}(x) \rightarrow \textbf{run}(x))$



# *Some kid broke every toy*

- $\mathrm{S} : \mathbf{every}^{2}(\mathbf{toy})(\lambda y_o.\mathbf{some}^{2}(\mathbf{kid})(\lambda x_s.\mathbf{break}(y_o)(x_s)))$

$\mathrm{NP} : \mathbf{some}^{2}(\mathbf{kid})$

$\mathrm{VP} : \lambda S'.\mathbf{every}^{2}(\mathbf{toy})(\lambda y_o.S'(\lambda x_s.\mathbf{break}(y_o)(x_s)))$

$\mathrm{Det} : \mathbf{some}^{2}$

*some*

$\mathrm{N}' : \mathbf{kid}$

$\mathrm{N} : \mathbf{kid}$

*kid*

$\mathsf{V} : \lambda O.\lambda S'.O(\lambda y_o.S'(\lambda x_s.\mathbf{break}(y_o)(x_s)))$

$\mathsf{V} : \lambda x_o.\lambda S.S(\lambda x_s.\mathbf{break}(x_o)(x_s))$

$\mathrm{V} : \lambda y.\lambda x.\mathbf{break}(y)(x)$

*broke*

$\mathrm{NP} : \mathbf{every}^{2}(\mathbf{toy})$

$\mathrm{Det} : \mathbf{every}^{2}$

*every*

$\mathrm{N}' : \mathbf{toy}$

$\mathrm{N} : \mathbf{toy}$

*toy*

# *Some kid broke every toy*

- S : $\mathbf{some^2}(\mathbf{kid})(\lambda y_s.\mathbf{every^2}(\mathbf{toy})(\lambda x_o.\mathbf{break}(x_o)(y_s)))$
  - NP : $\mathbf{some^2}(\mathbf{kid})$
    - Det : $\mathbf{some^2}$
      - *some*
    - N′ : $\mathbf{kid}$
      - N : $\mathbf{kid}$
        - *kid*
  - VP : $\lambda S.S(\lambda y_s.\mathbf{every^2}(\mathbf{toy})(\lambda x_o.\mathbf{break}(x_o)(y_s)))$
    - V : $\lambda O'.\lambda S.S(\lambda y_s.O(\lambda x_o.\mathbf{break}(x_o)(y_s)))$
      V : $\lambda x_s.O(\lambda x_o.\mathbf{break}(x_o)(x_s))$
      V : $\lambda y.\lambda x.\mathbf{break}(y)(x)$
      - *broke*
    - NP : $\mathbf{every^2}(\mathbf{toy})$
      - Det : $\mathbf{every^2}$
        - *every*
      - N′ : $\mathbf{toy}$
        - N : $\mathbf{toy}$
          - *toy*

## Questions with answers!

- A yes/no question (*Is Kathy running?*) will be something of type **Bool**, checked on database
- A content question (*Who likes Kathy?*) will be an *open proposition*, that is something semantically of the type *property* (**Ind** → **Bool**), and operationally we will consult the database to see what individuals will make the statement true.
- We use a grammar with a simple form of gap-threading for question words

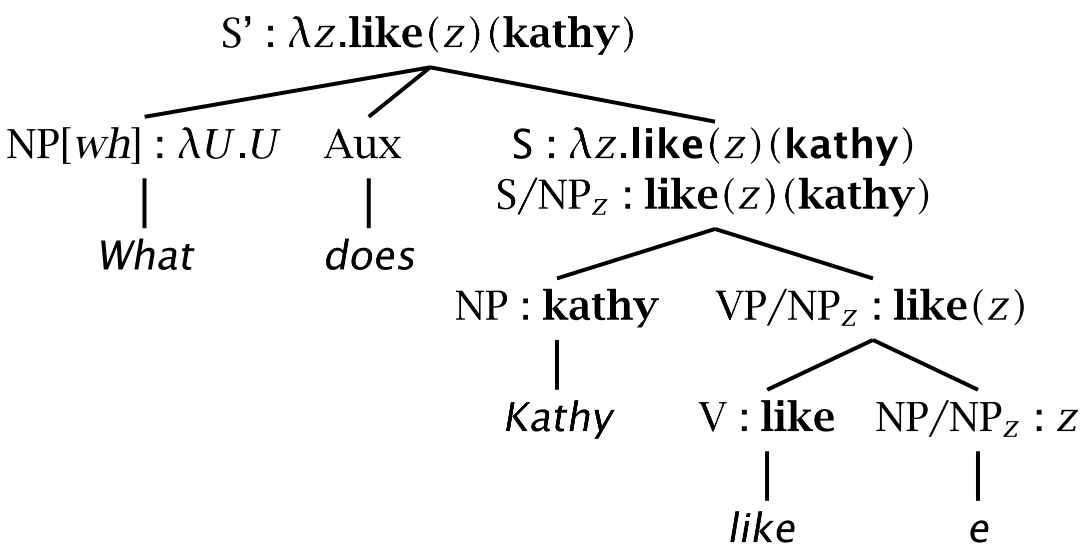
## Syntax/semantics for questions

- $\mathsf{S}' : \beta(\alpha) \to \mathsf{NP}[wh] : \beta \quad \mathsf{Aux} \quad \mathsf{S} : \alpha$
  $\mathsf{S}' : \alpha \to \mathsf{Aux} \quad \mathsf{S} : \alpha$
  $\mathsf{NP/NP}_z : z \to \mathsf{e}$
  $\mathsf{S} : \lambda z.F(\ldots z \ldots) \to \mathsf{S/NP}_z : F(\ldots z \ldots)$

## Syntax/semantics for questions

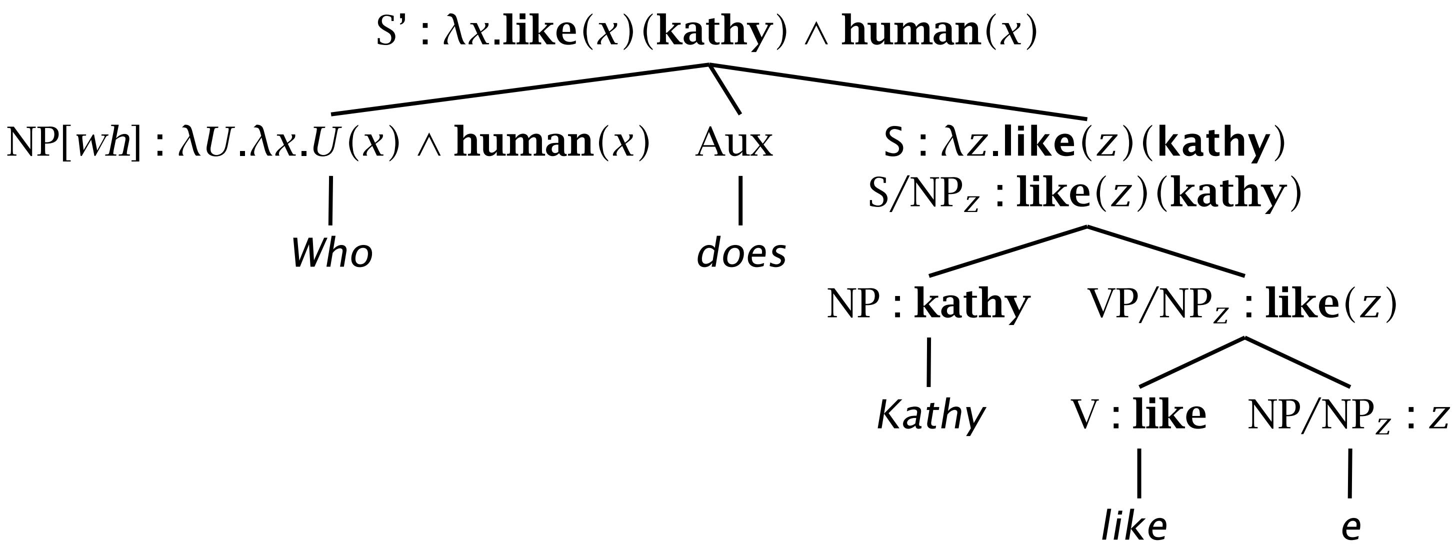
- *who*, NP[*wh*] : $\lambda U.\lambda x.U(x) \wedge \textbf{human}(x)$
  *what*, NP[*wh*] : $\lambda U.U$
  *which*, Det[*wh*] : $\lambda P.\lambda V.\lambda x.P(x) \wedge V(x)$
  *how_many*, Det[*wh*] : $\lambda P.\lambda V.|\lambda x.P(x) \wedge V(x)|$
- Where $|\cdot|$ is the operation that returns the cardinality of a set (count).

# Question examples

- 

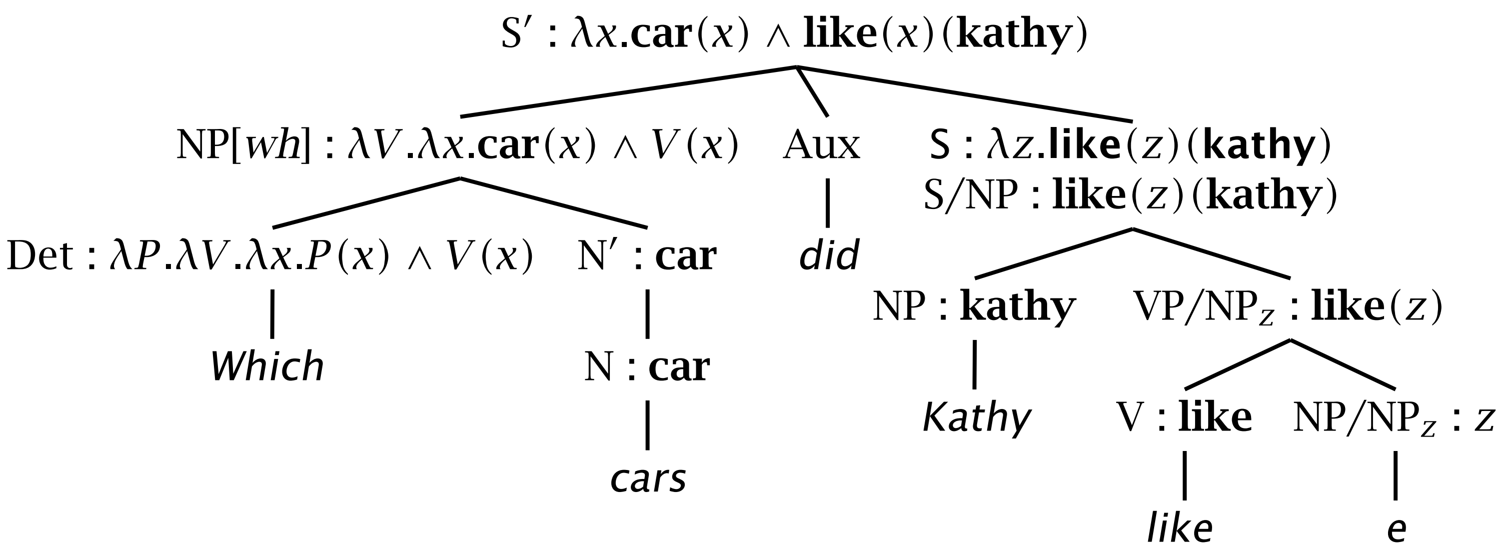

- select liked from Likes where Likes.liker='Kathy'

# Question examples

- 

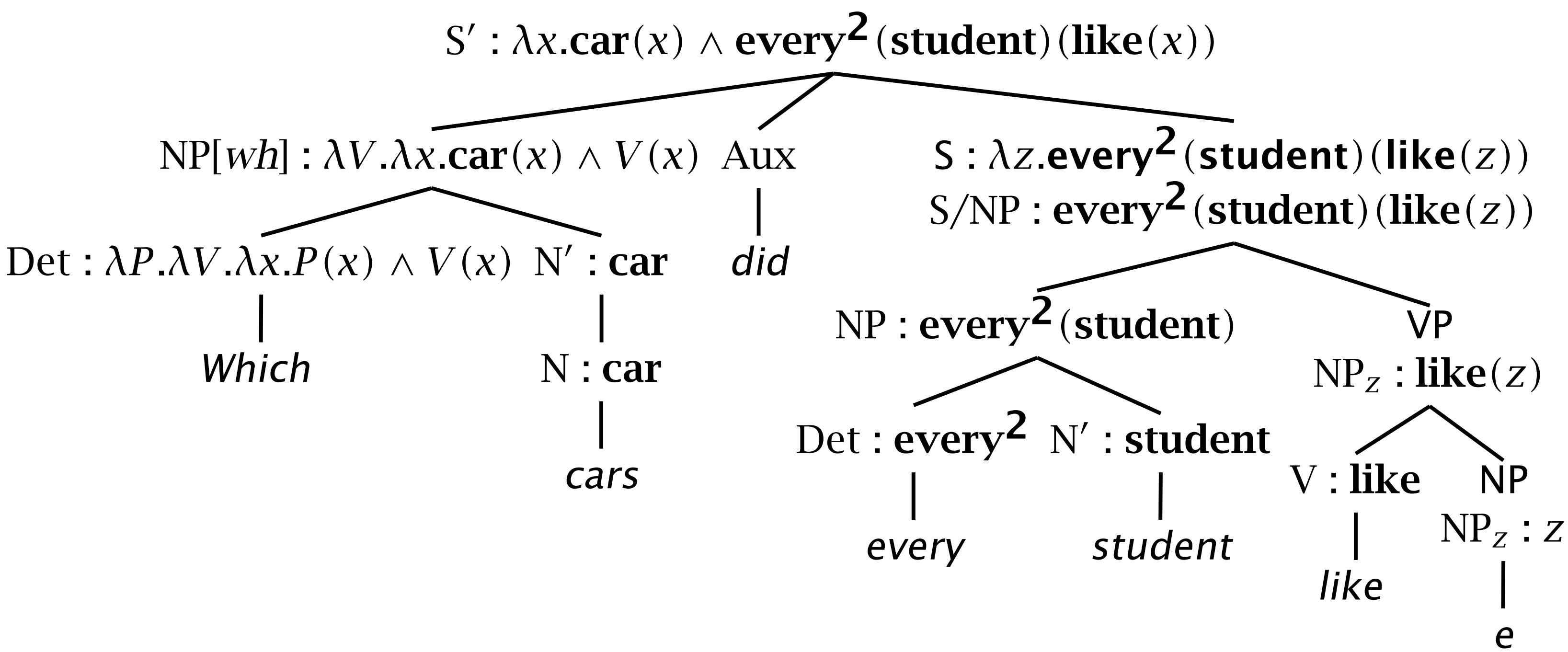

- select liked from Likes,Humans where Likes.liker='Kathy' AND Humans.obj = Likes.liked

# Question examples

- 


- select liked from Cars,Likes where Cars.obj=Likes.liked AND Likes.liker='Kathy'

# Question examples

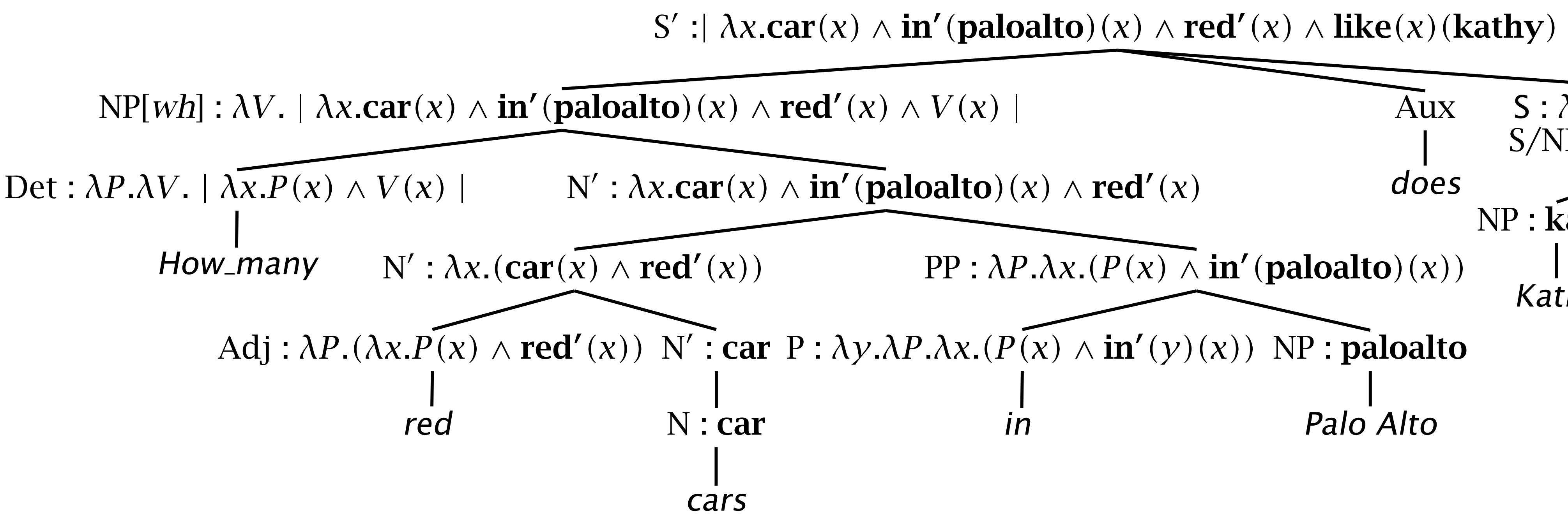
- 


- ???

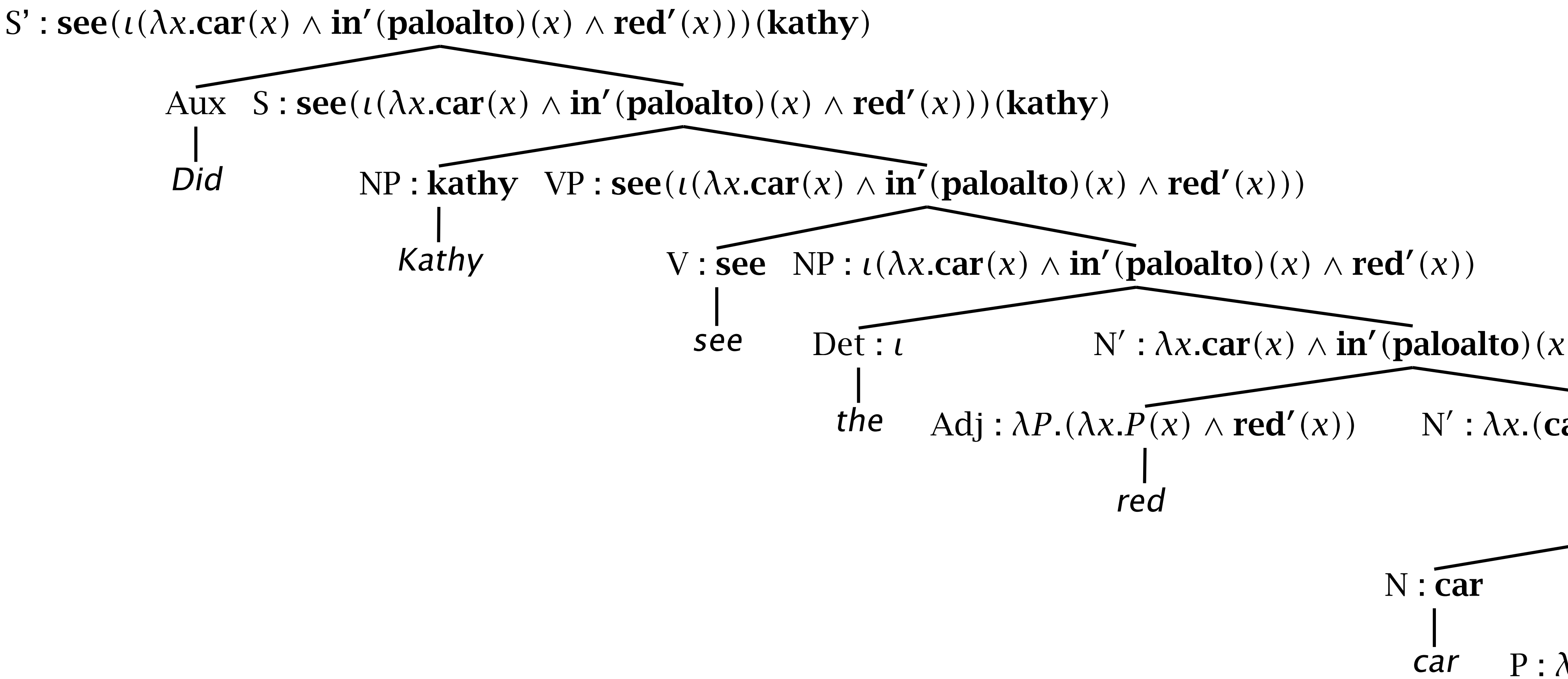
## Question examples

- *How many red cars in Palo Alto does Kathy like?*
- select count(*) from Likes,Cars,Locations,Reds where Cars.obj = Likes.liked AND Likes.liker = ’Kathy’ AND Red.obj = Likes.liked AND Locations.place = ’Palo Alto’ AND Locations.obj = Likes.liked
- *Did Kathy see the red car in Palo Alto?*
- select ‘yes’ where Seeings.seer = $k$ AND Seeings.seen = (select Cars.obj from Cars, Locations, Red where Cars.obj = Locations.obj AND Locations.place = ‘paloalto’ AND Cars.obj = Red.obj having count(*) = 1)

## *How many red cars in Palo Alto does Kathy like?*



## ***Did Kathy see the red car in Palo Alto?***



## How could we learn such representations?

- After disengagement for many years, there has started to be very interesting work in this area:
  - Luke S. Zettlemoyer and Michael Collins. 2005. Learning to Map Sentences to Logical Form: Structured Classification with Probabilistic Categorial Grammars. In *Proceedings of the 21st UAI*.
  - Yuk Wah Wong and Raymond J. Mooney. 2007. Learning Synchronous Grammars for Semantic Parsing with Lambda Calculus. In *Proceedings of the 45th ACL*, pp. 960–967.

## How could we learn such representations?

- General approach (ZC05): Start with initial lexicon, category templates, and paired sentences and meanings:

  What states border Texas?

  $\lambda x.\mathbf{state}(x) \wedge \mathbf{borders}(x, \mathbf{texas})$
- Learn lexical syntax/semantics for other words and learn to parse to logical form (parse structure is hidden).
- They successfully do iterative refinement of a lexicon and maxent parser

## How can we reason with such representations?

- Logical reasoning is practical for certain domains (business rules, legal code, etc.) and has been used (see Blackburn and Bos 2005 for background).
- But our knowledge of the world is in general incomplete and uncertain.
- There is various recent work on handling *restricted* fragments of first order logic in probabilistic models
  - Lise Getoor, Nir Friedman, Daphne Koller, Avi Pfeffer, Benjamin Taskar. 2007. Probabilistic Relational Models. In *An Introduction to Statistical Relational Learning*. MIT Press.

## How can we reason with such representations?

- Undirected model:
  - Pedro Domingos, Stanley Kok, Daniel Lowd, Hoifung Poon, Matthew Richardson, Parag Singla. 2008. Markov Logic. In L. De Raedt, P. Frasconi, K. Kersting and S. Muggleton (eds.), *Probabilistic Inductive Logic Programming*, pp. 92–117. Springer.
- A recent attempt to apply this to natural language inference:
  - Chloé Kiddon. 2008. Applying Markov Logic to the Task of Textual Entailment. Senior Honors Thesis, Computer Science. Stanford University.
- Logical formulae are given weights which are grounded out in an undirected markov network