# Maxent Models and Discriminative Estimation

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## Introduction

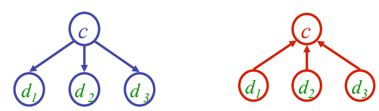
- So far we've looked at "generative models"
  - Language models, Naive Bayes, IBM MT
- In recent years there has been extensive use of *conditional* or *discriminative* probabilistic models in NLP, IR, Speech (and ML generally)
- Because:
  - They give high accuracy performance
  - They make it easy to incorporate lots of linguistically important features
  - They allow automatic building of language independent, retargetable NLP modules

## Joint vs. Conditional Models

- Joint (generative) models place probabilities over both observed data and the hidden stuff (generate the observed data from hidden stuff): P(c,d)
  - All the best known StatNLP models:
    - *n*-gram models, Naive Bayes classifiers, hidden Markov models, probabilistic context-free grammars
- Discriminative (conditional) models take the data as given, and put a probability over hidden structure given the data: P(c|d)
    - Logistic regression, conditional loglinear models, maximum entropy markov models, conditional random fields, (SVMs, perceptrons)

## Bayes Net/Graphical Models

- Bayes net diagrams draw circles for random variables, and lines for direct dependencies
- Some variables are observed; some are hidden
- Each node is a little classifier (conditional probability table) based on incoming arcs




Naive Bayes — Logistic Regression

Generative — Discriminative

## Conditional models work well: Word Sense Disambiguation

| Training Set | |
|---|---|
| Objective | Accuracy |
| Joint Like. | 86.8 |
| Cond. Like. | 98.5 |

| Test Set | |
|---|---|
| Objective | Accuracy |
| Joint Like. | 73.6 |
| Cond. Like. | 76.1 |

- Even with *exactly the same features*, changing from joint to conditional estimation increases performance
- That is, we use the same smoothing, and the same *word-class* features, we just change the numbers (parameters)

(Klein and Manning 2002, using Senseval-1 Data)

## Features

- In these slides and most maxent work: *features* are elementary pieces of evidence that link aspects of what we observe *d* with a category *c* that we want to predict.
- A feature has a (bounded) real value: $f: C \times D \rightarrow \mathbf{R}$
- Usually features specify an indicator function of properties of the input and a particular class (*every one we present is*). They pick out a subset.
  - $f_i(c, d) \equiv [\Phi(d) \wedge c = c_i]$ [Value is 0 or 1]
- We will freely say that $\Phi(d)$ is a feature of the data $d$, when, for each $c_i$, the conjunction $\Phi(d) \wedge c = c_i$ is a feature of the data-class pair $(c, d)$.

## Features

- For example:
  - $f_1(c, d) \equiv [c=\text{"NN"} \wedge \text{islower}(w_0) \wedge \text{ends}(w_0, \text{"d"})]$
  - $f_2(c, d) \equiv [c = \text{"NN"} \wedge w_{-1} = \text{"to"} \wedge t_{-1} = \text{"TO"}]$
  - $f_3(c, d) \equiv [c = \text{"VB"} \wedge \text{islower}(w_0)]$

IN NN *in bed* | TO NN *to aid* | TO VB *to aid* | IN JJ *in blue*

- Models will assign each feature a *weight*
- Empirical count (expectation) of a feature:
  $\text{empirical } E(f_i) = \sum_{(c,d)\in \text{observed}(C,D)} f_i(c,d)$
- Model expectation of a feature:
  $E(f_i) = \sum_{(c,d)\in(C,D)} P(c,d) f_i(c,d)$

## Feature-Based Models

- The decision about a data point is based only on the features active at that point.

| Text Categorization | Word-Sense Disambiguation | POS Tagging |
|---|---|---|
| Data: BUSINESS: Stocks hit a yearly low … | Data: … to restructure bank:MONEY debt. | Data: DT JJ NN … The previous fall … |
| Label: BUSINESS | Label: MONEY | Label: NN |
| Features: {…, stocks, hit, a, yearly, low, …} | Features: {…, P=restructure, N=debt, L=12, …} | Features: {W=fall, PT=JJ PW=previous} |

## Example: Text Categorization

(Zhang and Oles 2001)

- Features are a word in document and class (they do feature selection to use reliable indicators)
- Tests on classic Reuters data set (and others)
  - Naïve Bayes: 77.0% $F_1$
  - Linear regression: 86.0%
  - Logistic regression: 86.4%
  - Support vector machine: 86.5%
- Emphasizes the importance of *regularization* (smoothing) for successful use of discriminative methods (not used in most early NLP/IR work)

## Example: POS Tagging

- Features can include:
  - Current, previous, next words in isolation or together.
  - Previous (or next) one, two, three tags.
  - Word-internal features: word types, suffixes, dashes, etc.

Local Context

Decision Point

| -3 | -2 | -1 | 0 | +1 |
|---|---|---|---|---|
| DT | NNP | VBD | ??? | ??? |
| The | Dow | fell | 22.6 | % |

Features

| | |
|---|---|
| $W_0$ | 22.6 |
| $W_{+1}$ | % |
| $W_{-1}$ | fell |
| $T_{-1}$ | VBD |
| $T_{-1}$-$T_{-2}$ | NNP-VBD |
| hasDigit? | true |
| … | … |

(Ratnaparkhi 1996; Toutanova et al. 2003, etc.)

## Other Maxent Examples

- Sentence boundary detection (Mikheev 2000)
  - Is period end of sentence or abbreviation?
- PP attachment (Ratnaparkhi 1998)
  - Features of head noun, preposition, etc.
- Language models (Rosenfeld 1996)
  - $P(w_0|w_{-n},\dots,w_{-1})$. Features are word n-gram features, and trigger features which model repetitions of the same word.
- Parsing (Ratnaparkhi 1997; Johnson et al. 1999, etc.)
  - Either: Local classifications decide parser actions or feature counts choose a parse.

## Conditional vs. Joint Likelihood

- We have some data $\{(d, c)\}$ and we want to place probability distributions over it.
- A *joint* model gives probabilities $P(d,c)$ and tries to maximize this likelihood.
  - It turns out to be trivial to choose weights: just relative frequencies.
- A *conditional* model gives probabilities $P(c|d)$. It takes the data as given and models only the conditional probability of the class.
  - We seek to maximize conditional likelihood.
  - Harder to do (as we'll see…)
  - More closely related to classification error.

## Feature-Based Classifiers

- "Linear" classifiers:
  - Classify from features sets $\{f_i\}$ to classes $\{c\}$.
  - Assign a weight $\lambda_i$ to each feature $f_i$.
  - For a pair $(c,d)$, features vote with their weights:
    - $\text{vote}(c) = \Sigma\lambda_i f_i(c,d)$

1.2 −1.8 TO NN to aid TO VB to aid 0.3

  - Choose the class $c$ which maximizes $\Sigma\lambda_i f_i(c,d) = \text{VB}$
  - There are many ways to chose weights
    - Perceptron: find a currently misclassified example, and nudge weights in the direction of a correct classification

## Feature-Based Classifiers

- Exponential (log-linear, maxent, logistic, Gibbs) models:
  - Use the linear combination $\Sigma\lambda_i f_i(c,d)$ to produce a probabilistic model:

$$P(c \mid d, \lambda) = \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

Makes votes positive.

Normalizes votes.

    - P(NN|*to, aid*, TO) = $e^{1.2}e^{-1.8}/(e^{1.2}e^{-1.8} + e^{0.3}) = 0.29$
    - P(VB|*to, aid*, TO) = $e^{0.3} /(e^{1.2}e^{-1.8} + e^{0.3}) = 0.71$
  - The weights are the parameters of the probability model, combined via a "soft max" function
  - Given this model form, we will choose parameters $\{\lambda_i\}$ that *maximize the conditional likelihood* of the data according to this model.

## Quiz question!

- Assuming exactly the same set up (2 class decision: NN or VB; 3 features defined as before, maxent model), what are:
  - $P(t = \text{"NN"} \mid w_{-1} = \text{"the"} \wedge w = \text{"aid"} \wedge t_{-1} = \text{"DT"})$
  - $P(t = \text{"VB"} \mid w_{-1} = \text{"the"} \wedge w = \text{"aid"} \wedge t_{-1} = \text{"DT"})$

  - 1.2 $f_1(c, d) \equiv [c = \text{"NN"} \wedge \text{islower}(w_0) \wedge \text{ends}(w_0, \text{"d"})]$
  - -1.8 $f_2(c, d) \equiv [c = \text{"NN"} \wedge w_{-1} = \text{"to"} \wedge t_{-1} = \text{"TO"}]$
  - 0.3 $f_3(c, d) \equiv [c = \text{"VB"} \wedge \text{islower}(w_0)]$

DT NN the aid DT VB the aid

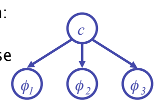
$$P(c \mid d, \lambda) = \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

## Other Feature-Based Classifiers

- The exponential model approach is one way of deciding how to weight features, given data.
- It constructs not only classifications, but probability distributions over classifications.
- There are other (good!) ways of discriminating classes: SVMs, boosting, even perceptrons – though these methods are not as trivial to interpret as distributions over classes.

## Comparison to Naïve-Bayes

- Naïve-Bayes is another tool for classification:
  - We have a bunch of random variables (data features) which we would like to use to predict another variable (the class):




  - The Naïve-Bayes likelihood over classes is:

$$P(c \mid d, \lambda) = \frac{P(c)\prod_i P(\phi_i \mid c)}{\sum_{c'} P(c')\prod_i P(\phi_i \mid c')} \Rightarrow \frac{\exp\left[\log P(c) + \sum_i \log P(\phi_i \mid c)\right]}{\sum_{c'} \exp\left[\log P(c') + \sum_i \log P(\phi_i \mid c')\right]}$$

$$\Rightarrow \frac{\exp\left[\sum_i \lambda_{ic} f_{ic}(d,c)\right]}{\sum_{c'} \exp\left[\sum_i \lambda_{ic'} f_{ic'}(d,c')\right]}$$

Naïve-Bayes is just an exponential model.

## Comparison to Naïve-Bayes

- The primary differences between Naïve-Bayes and maxent models are:

| Naïve-Bayes | Maxent |
|---|---|
| Trained to maximize joint likelihood of data and classes. | Trained to maximize the conditional likelihood of classes. |
| Features assumed to supply independent evidence. | Features weights take feature dependence into account. |
| Feature weights can be set independently. | Feature weights must be mutually estimated. |
| Features must be of the conjunctive $\Phi(d) \wedge c = c_i$ form. | Features need not be of the conjunctive form (but usually are). |

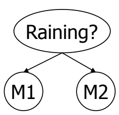
## Example: Sensors

**Reality**

Raining | Sunny

P(+,+,r) = 3/8   P(-,-,r) = 1/8   P(+,+,s) = 1/8   P(-,-,s) = 3/8

**NB Model**




NB FACTORS:
- P(s) = 1/2
- P(+|s) = 1/4
- P(+|r) = 3/4

PREDICTIONS:
- P(r,+,+) = (½)(¾)(¾)
- P(s,+,+) = (½)(¼)(¼)
- P(r|+,+) = 9/10
- P(s|+,+) = 1/10

## Example: Sensors

- Problem: NB multi-counts the evidence.

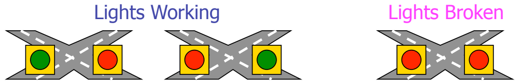
$$\frac{P(r\mid +\ldots+)}{P(s\mid +\ldots+)} = \frac{P(r)}{P(s)}\frac{P(+\mid r)}{P(+\mid s)}\cdots\frac{P(+\mid r)}{P(+\mid s)}$$

- Maxent behavior:
  - Take a model over $(M_1,\ldots M_n,R)$ with features:
    - $f_{ri}$: $M_i$=+, R=r   weight: $\lambda_{ri}$
    - $f_{si}$: $M_i$=+, R=s   weight: $\lambda_{si}$
  - $\exp(\lambda_{ri}-\lambda_{si})$ is the factor analogous to $P(+|r)/P(+|s)$
  - … but instead of being 3, it will be $3^{1/n}$
  - … because if it were 3, $E[f_{ri}]$ would be far higher than the target of 3/8!
- NLP problem: we often have overlapping features….

## Example: Stoplights

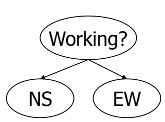
**Reality**

Lights Working | Lights Broken



P(g,r,w) = 3/7   P(r,g,w) = 3/7   P(r,r,b) = 1/7

**NB Model**




NB FACTORS:
- P(w) = 6/7
- P(r|w) = 1/2
- P(g|w) = 1/2
- P(b) = 1/7
- P(r|b) = 1
- P(g|b) = 0

## Example: Stoplights

- What does the model say when both lights are red?
  - P(b,r,r)   = (1/7)(1)(1)   = 1/7   = 4/28
  - P(w,r,r)   = (6/7)(1/2)(1/2)   = 6/28   = 6/28
  - P(w|r,r) = 6/10!
- We'll guess that (r,r) indicates lights are working!
- Imagine if P(b) were boosted higher, to 1/2:
  - P(b,r,r)   = (1/2)(1)(1)   = 1/2   = 4/8
  - P(w,r,r)   = (1/2)(1/2)(1/2)   = 1/8   = 1/8
  - P(w|r,r)   = 1/5!
- Changing the parameters bought conditional accuracy at the expense of data likelihood!

## Exponential Model Likelihood

- Maximum Likelihood (Conditional) Models :
  - Given a model form, choose values of parameters to maximize the (conditional) likelihood of the data.
- Exponential model form, for a data set (C,D):

$$\log P(C\mid D,\lambda) = \sum_{(c,d)\in(C,D)} \log P(c\mid d,\lambda) = \sum_{(c,d)\in(C,D)} \log \frac{\exp\sum_i \lambda_i f_i(c,d)}{\sum_{c'}\exp\sum_i \lambda_i f_i(c',d)}$$

## Building a Maxent Model

- Define features (indicator functions) over data points.
  - Features represent sets of data points which are distinctive enough to deserve model parameters.
    - Words, but also "word contains number", "word ends with *ing*"
  - Usually features are added incrementally to "target" errors.
- For any given feature weights, we want to be able to calculate:
  - Data (conditional) likelihood
  - Derivative of the likelihood wrt each feature weight
    - Use expectations of each feature according to the model
- Find the optimum feature weights (next part).

## The Likelihood Value

- The (log) conditional likelihood is a function of the iid data (C,D) and the parameters $\lambda$:

$$\log P(C \mid D,\lambda) = \log \prod_{(c,d)\in(C,D)} P(c \mid d,\lambda) = \sum_{(c,d)\in(C,D)} \log P(c \mid d,\lambda)$$

- If there aren't many values of *c*, it's easy to calculate:

$$\log P(C \mid D,\lambda) = \sum_{(c,d)\in(C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

- We can separate this into two components:

$$\log P(C \mid D,\lambda) = \sum_{(c,d)\in(C,D)} \log \exp \sum_i \lambda_i f_i(c,d) - \sum_{(c,d)\in(C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)$$

$$\log P(C \mid D,\lambda) = N(\lambda) - M(\lambda)$$

- The derivative is the difference between the derivatives of each component

## The Derivative I: Numerator

$$\frac{\partial N(\lambda)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \log \exp \sum_i \lambda_{ci} f_i(c,d)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{\partial \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} f_i(c,d)$$

Derivative of the numerator is: the empirical count($f_i$, $c$)

## The Derivative II: Denominator

$$\frac{\partial M(\lambda)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \frac{\partial \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c',d)}{1} \frac{\partial \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c',d)}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \frac{\partial \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \sum_{c'} P(c' \mid d,\lambda) f_i(c',d)$$

= predicted count($f_i$, $\lambda$)

## The Derivative III

$$\frac{\partial \log P(C \mid D,\lambda)}{\partial \lambda_i} = \text{actual count}(f_i, C) - \text{predicted count}(f_i, \lambda)$$

- The optimum parameters are the ones for which each feature's predicted expectation equals its empirical expectation. The optimum distribution is:
  - Always unique (but parameters may not be unique)
  - Always exists (if feature counts are from actual data).
- These models are also called maximum entropy models because we find the model having maximum entropy and satisfying the constraints:

$$E_p(f_j) = E_{\tilde{p}}(f_j), \forall j$$

## Fitting the Model

- To find the parameters $\lambda_1, \lambda_2, \lambda_3$ write out the conditional log-likelihood of the training data and maximize it

$$CLogLik(D) = \sum_{i=1}^{n} \log P(c_i \mid d_i)$$

- The log-likelihood is concave and has a single maximum; use your favorite numerical optimization package
- Good large scale techniques: conjugate gradient or limited memory quasi-Newton

## Fitting the Model Generalized Iterative Scaling

- A simple optimization algorithm which works when the features are non-negative
- We need to define a slack feature to make the features sum to a constant over all considered pairs from $D \times C$
- Define $M = \max_{i,c} \sum_{j=1}^{m} f_j(d_i, c)$
- Add new feature

$$f_{m+1}(d,c) = M - \sum_{j=1}^{m} f_j(d,c)$$

## Generalized Iterative Scaling

- Compute empirical expectation for all features
  $$E_{\tilde{p}}(f_j) = \frac{1}{N}\sum_{i=1}^{n} f_j(d_i, c_i)$$
- Initialize $\lambda_j = 0, j = 1 \ldots m+1$
- Repeat
  - Compute feature expectations according to current model
    $$E_{p^t}(f_j) = \frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{K} P(c_k \mid d_i) f_j(d_i, c_k)$$
  - Update parameters
    $$\lambda_j^{(t+1)} = \lambda_j^{(t)} + \frac{1}{M}\log\left(\frac{E_{\tilde{p}}(f_j)}{E_{p^t}(f_j)}\right)$$
- Until converged

## Maximum Entropy Models

- An equivalent approach:
  - Lots of distributions out there, most of them very spiked, specific, overfit.
  - We want a distribution which is uniform except in specific ways we require.
  - Uniformity means high entropy – we can search for distributions which have properties we desire, but also have high entropy.

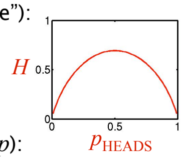
*Ignorance is preferable to error and he is less remote from the truth who believes nothing than he who believes what is wrong* – Thomas Jefferson (1781)

## (Maximum) Entropy

- Entropy: the uncertainty of a distribution.
- Quantifying uncertainty ("surprise"):
  - Event $x$
  - Probability $p_x$
  - "Surprise" $\log(1/p_x)$
- Entropy: expected surprise (over $p$):

$$\mathrm{H}(p) = E_p\left[\log_2 \frac{1}{p_x}\right]$$

$$\mathrm{H}(p) = -\sum_x p_x \log_2 p_x$$



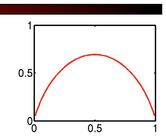

A coin-flip is most uncertain for a fair coin.

## Maxent Examples I

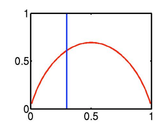
- What do we want from a distribution?
  - Minimize commitment = maximize entropy.
  - Resemble some reference distribution (data).
- Solution: maximize entropy H, subject to feature-based constraints:

$$E_p[f_i] = E_{\hat{p}}[f_i] \Leftrightarrow \sum_{x \in f_i} p_x = C_i$$

- Adding constraints (features):
  - Lowers maximum entropy
  - Raises maximum likelihood of data
  - Brings the distribution further from uniform
  - Brings the distribution closer to data

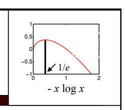


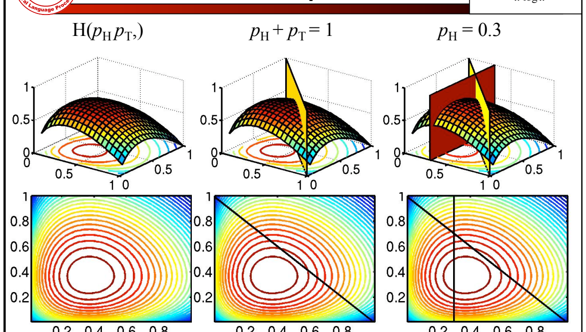
Unconstrained, max at 0.5



Constraint that $p_{\mathrm{HEADS}} = 0.3$

## Maxent Examples II




## Maxent Examples III

- Lets say we have the following event space:

| | | | | | |
|---|---|---|---|---|---|
| NN | NNS | NNP | NNPS | VBZ | VBD |

- … and the following empirical data:

| | | | | | |
|---|---|---|---|---|---|
| 3 | 5 | 11 | 13 | 3 | 1 |

- Maximize H:

| | | | | | |
|---|---|---|---|---|---|
| 1/e | 1/e | 1/e | 1/e | 1/e | 1/e |

- … want probabilities: E[NN,NNS,NNP,NNPS,VBZ,VBD] = 1

| | | | | | |
|---|---|---|---|---|---|
| 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 |

## Maxent Examples IV

- Too uniform!
- N* are more common than V*, so we add the feature $f_N$ = {NN, NNS, NNP, NNPS}, with E[$f_N$] =32/36

| NN | NNS | NNP | NNPS | VBZ | VBD |
|---|---|---|---|---|---|
| 8/36 | 8/36 | 8/36 | 8/36 | 2/36 | 2/36 |

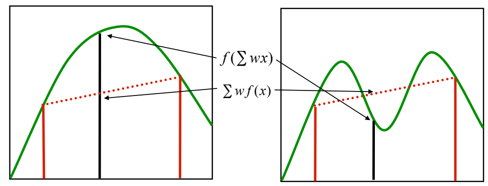
- … and proper nouns are more frequent than common nouns, so we add $f_P$ = {NNP, NNPS}, with E[$f_P$] =24/36

| | | | | | |
|---|---|---|---|---|---|
| 4/36 | 4/36 | 12/36 | 12/36 | 2/36 | 2/36 |

- … we could keep refining the models, e.g. by adding a feature to distinguish singular vs. plural nouns, or verb types.

## Convexity

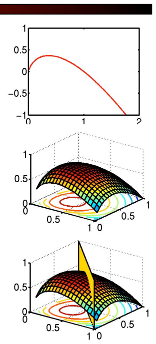
$$f(\sum_i w_i x_i) \geq \sum_i w_i f(x_i) \qquad \sum_i w_i = 1$$




Convexity guarantees a single, global maximum because any higher points are greedily reachable.

## Convexity II

- Constrained $H(p) = -\sum x \log x$ is convex:
  - $-x \log x$ is convex
  - $-\sum x \log x$ is convex (sum of convex functions is convex).
  - The feasible region of constrained H is a linear subspace (which is convex)
  - The constrained entropy surface is therefore convex.
- The maximum likelihood exponential model (dual) formulation is also convex.



## Feature Overlap

- Maxent models handle overlapping features well.
- Unlike a NB model, there is no double counting!

Empirical

| | A | a |
|---|---|---|
| B | 2 | 1 |
| b | 2 | 1 |

| | A | a |
|---|---|---|
| B | | |
| b | | |

| | A | a |
|---|---|---|
| B | | |
| b | | |

| | A | a |
|---|---|---|
| B | | |
| b | | |

All = 1

| | A | a |
|---|---|---|
| B | 1/4 | 1/4 |
| b | 1/4 | 1/4 |

| | A | a |
|---|---|---|
| B | | |
| b | | |

A = 2/3

| | A | a |
|---|---|---|
| B | 1/3 | 1/6 |
| b | 1/3 | 1/6 |

| | A | a |
|---|---|---|
| B | $\lambda_A$ | |
| b | $\lambda_A$ | |

A = 2/3

| | A | a |
|---|---|---|
| B | 1/3 | 1/6 |
| b | 1/3 | 1/6 |

| | A | a |
|---|---|---|
| B | $\lambda'_A+\lambda''_A$ | |
| b | $\lambda'_A+\lambda''_A$ | |

## Example: NER Overlap

Grace is correlated with PERSON, but does not add much evidence on top of already knowing prefix features.

Local Context

| | Prev | Cur | Next |
|---|---|---|---|
| State | Other | ??? | ??? |
| Word | at | Grace | Road |
| Tag | IN | NNP | NNP |
| Sig | x | Xx | Xx |

Feature Weights

| Feature Type | Feature | PERS | LOC |
|---|---|---|---|
| Previous word | at | -0.73 | 0.94 |
| Current word | Grace | 0.03 | 0.00 |
| Beginning bigram | <G | 0.45 | -0.04 |
| Current POS tag | NNP | 0.47 | 0.45 |
| Prev and cur tags | IN NNP | -0.10 | 0.14 |
| Previous state | Other | -0.70 | -0.92 |
| Current signature | Xx | 0.80 | 0.46 |
| Prev state, cur sig | O-Xx | 0.68 | 0.37 |
| Prev-cur-next sig | x-Xx-Xx | -0.69 | 0.37 |
| P. state - p-cur sig | O-x-Xx | -0.20 | 0.82 |
| … | | | |
| Total: | | -0.58 | 2.68 |

## Feature Interaction

- Maxent models handle overlapping features well, but do not automatically model feature interactions.

Empirical

| | A | a |
|---|---|---|
| B | 1 | 1 |
| b | 1 | 0 |

| | A | a |
|---|---|---|
| B | | |
| b | | |

| | A | a |
|---|---|---|
| B | | |
| b | | |

| | A | a |
|---|---|---|
| B | | |
| b | | |

All = 1

| | A | a |
|---|---|---|
| B | 1/4 | 1/4 |
| b | 1/4 | 1/4 |

| | A | a |
|---|---|---|
| B | 0 | 0 |
| b | 0 | 0 |

A = 2/3

| | A | a |
|---|---|---|
| B | 1/3 | 1/6 |
| b | 1/3 | 1/6 |

| | A | a |
|---|---|---|
| B | $\lambda_A$ | |
| b | $\lambda_A$ | |

B = 2/3

| | A | a |
|---|---|---|
| B | 4/9 | 2/9 |
| b | 2/9 | 1/9 |

| | A | a |
|---|---|---|
| B | $\lambda_A+\lambda_B$ | $\lambda_B$ |
| b | $\lambda_A$ | |

## Feature Interaction

- If you want interaction terms, you have to add them:

Empirical

| | A | a |
|---|---|---|
| B | 1 | 1 |
| b | 1 | 0 |

| | A | a |
|---|---|---|
| B | | |
| b | | |

A = 2/3

| | A | a |
|---|---|---|
| B | 1/3 | 1/6 |
| b | 1/3 | 1/6 |

| | A | a |
|---|---|---|
| B | | |
| b | | |

B = 2/3

| | A | a |
|---|---|---|
| B | 4/9 | 2/9 |
| b | 2/9 | 1/9 |

| | A | a |
|---|---|---|
| B | | |
| b | | |

AB = 1/3

| | A | a |
|---|---|---|
| B | 1/3 | 1/3 |
| b | 1/3 | 0 |

- A disjunctive feature would also have done it (alone):

| | A | a |
|---|---|---|
| B | | |
| b | | |

| | A | a |
|---|---|---|
| B | 1/3 | 1/3 |
| b | 1/3 | 0 |

## Feature Interaction

- For loglinear/logistic regression models in statistics, it is standard to do a greedy stepwise search over the space of all possible interaction terms.
- This combinatorial space is exponential in size, but that's okay as most statistics models only have 4–8 features.
- In NLP, our models commonly use hundreds of thousands of features, so that's not okay.
- Commonly, interaction terms are added by hand based on linguistic intuitions.

## Example: NER Interaction

Previous-state and current-signature have interactions, e.g. P=PERS-C=Xx indicates C=PERS much more strongly than C=Xx and P=PERS independently.

This feature type allows the model to capture this interaction.

Local Context

| | Prev | Cur | Next |
|---|---|---|---|
| State | Other | ??? | ??? |
| Word | at | Grace | Road |
| Tag | IN | NNP | NNP |
| Sig | x | Xx | Xx |

Feature Weights

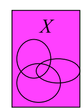
| Feature Type | Feature | PERS | LOC |
|---|---|---|---|
| Previous word | *at* | -0.73 | 0.94 |
| Current word | *Grace* | 0.03 | 0.00 |
| Beginning bigram | *<G* | 0.45 | -0.04 |
| Current POS tag | NNP | 0.47 | 0.45 |
| Prev and cur tags | IN NNP | -0.10 | 0.14 |
| Previous state | Other | -0.70 | -0.92 |
| Current signature | Xx | 0.80 | 0.46 |
| Prev state, cur sig | O-Xx | 0.68 | 0.37 |
| Prev-cur-next sig | x-Xx-Xx | -0.69 | 0.37 |
| P. state - p-cur sig | O-x-Xx | -0.20 | 0.82 |
| … | | | |
| Total: | | -0.58 | 2.68 |

## Classification

- What do these joint models of $P(X)$ have to do with conditional models $P(C|D)$?
- Think of the space $C \times D$ as a complex $X$.
  - $C$ is generally small (e.g., 2-100 topic classes)
  - $D$ is generally huge (e.g., number of documents)
- We can, in principle, build models over $P(C,D)$.
- This will involve calculating expectations of features (over $C \times D$):

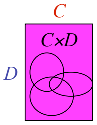
$$E(f_i) = \sum_{(c,d) \in (C,D)} P(c,d) f_i(c,d)$$

- Generally impractical: can't enumerate $d$ efficiently.




C

D




## Classification II

- $D$ may be huge or infinite, but only a few $d$ occur in our data.
- What if we add one feature for each $d$ and constrain its expectation to match our empirical data?

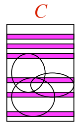
$$\forall (d) \in D \quad P(d) = \hat{P}(d)$$

- Now, most entries of $P(c,d)$ will be zero.
- We can therefore use the much easier sum:

$$E(f_i) = \sum_{(c,d) \in (C,D)} P(c,d) f_i(c,d)$$
$$= \sum_{(c,d) \in (C,D) \wedge \hat{P}(d) > 0} P(c,d) f_i(c,d)$$

C

D



## Classification III

- But if we've constrained the $D$ marginals

$$\forall (d) \in D \quad P(d) = \hat{P}(d)$$

then the only thing that can vary is the conditional distributions:

$$P(c,d) = P(c \mid d) P(d)$$
$$= P(c \mid d) \hat{P}(d)$$

- This is the connection between joint and conditional maxent / exponential models:
  - Conditional models can be thought of as joint models with marginal constraints.
- Maximizing joint likelihood and conditional likelihood of the data in this model are equivalent!