Natural Language Processing with Deep Learning

CS224N/Ling284

Christopher Manning

Lecture 5: Language Models and Recurrent Neural Networks

(oh, and finish neural dependency parsing ☺)

Lecture Plan

1. Neural dependency parsing (10 mins)
2. A bit more about neural networks (15 mins)
3. Language modeling + RNNs (55 mins)
   - A new NLP task: **Language Modeling**

     This is the most important concept in the class!

     motivates

   - A new family of neural networks: **Recurrent Neural Networks (RNNs)**

     Important and used in Ass4, but not the only way to build LMs

Reminders:

Now a hybrid class! In Nvidia Aud (remember 😷) or livecast or recordings.

You should have handed in Assignment 2 by today

In Assignment 3, out today, you build a neural dependency parser using PyTorch

1. Neural Dependency Parsing
Why might we gain from a neural dependency parser?

Categorical features are:

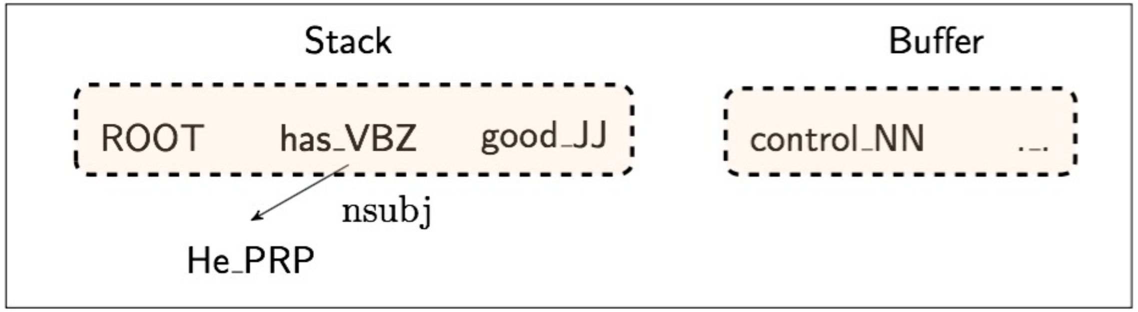
- Problem #1: sparse
- Problem #2: incomplete
- Problem #3: expensive to compute

More than 95% of parsing time is consumed by feature computation

$$s1.w = \text{good} \wedge s1.t = \text{JJ}$$
$$s2.w = \text{has} \wedge s2.t = \text{VBZ} \wedge s1.w = \text{good}$$
$$lc(s_2).t = \text{PRP} \wedge s_2.t = \text{VBZ} \wedge s_1.t = \text{JJ}$$
$$lc(s_2).w = \text{He} \wedge lc(s_2).l = \text{nsubj} \wedge s_2.w = \text{has}$$

Neural Approach:

learn a dense and compact feature representation



dense
dim = ~1000

0.1	0.9	-0.2	0.3	...	-0.1	-0.5

Neural Dependency Parser Model Architecture

Log loss (cross-entropy error) will be back-propagated to the embeddings

Softmax probabilities → { Shift , Left-Arc$_r$, Right-Arc$_r$ }

Output layer y

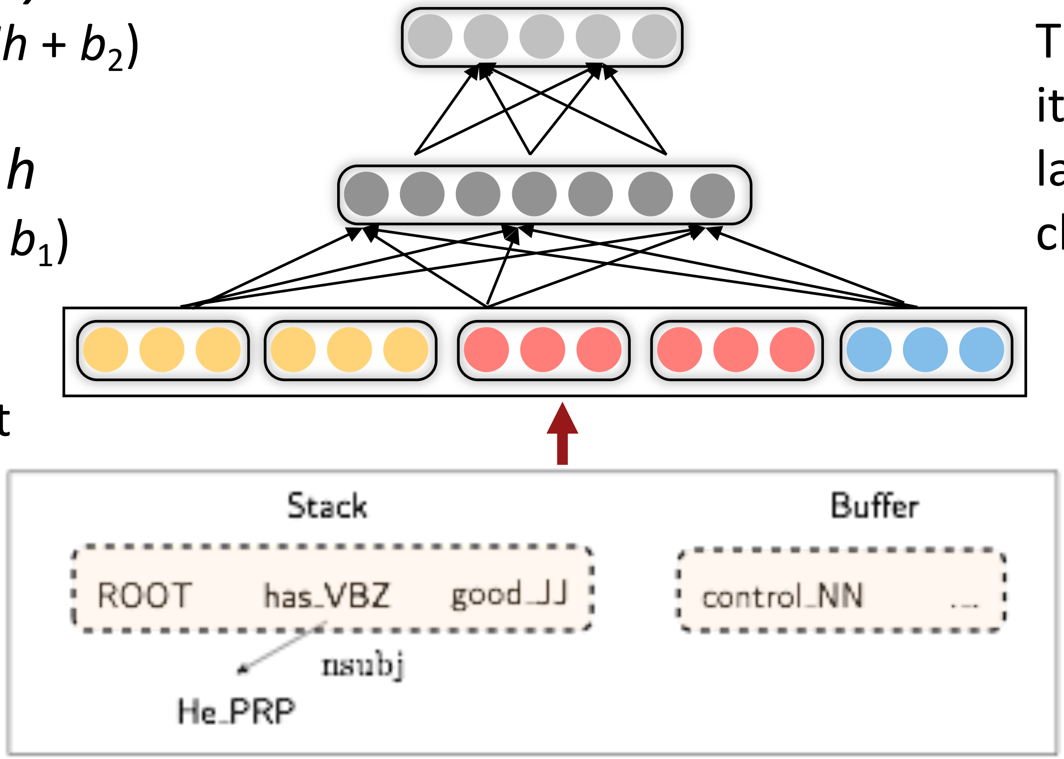
y = softmax($Uh + b_2$)

Hidden layer h

h = ReLU($Wx + b_1$)

Input layer x

lookup + concat



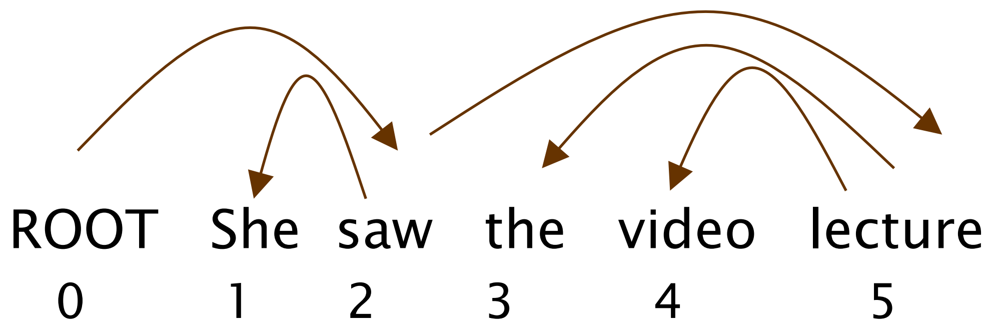
The hidden layer re-represents the input — it moves inputs around in an intermediate layer vector space—so it can be easily classified with a (linear) softmax

Wins:

Distributed representations!

Non-linear classifier!

1. Neural Dependency Parsing
Evaluation of Dependency Parsing: (labeled) dependency accuracy



$$\text{Acc} = \frac{\#\text{ correct deps}}{\#\text{ of deps}}$$

UAS = 4 / 5 = 80%

LAS = 2 / 5 = 40%

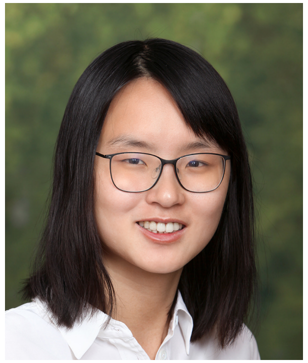
Gold

1	2	She	nsubj
2	0	saw	root
3	5	the	det
4	5	video	nn
5	2	lecture	obj

Parsed

1	2	She	nsubj
2	0	saw	root
3	4	the	det
4	5	video	nsubj
5	2	lecture	ccomp

A neural dependency parser [Chen and Manning 2014]



- Results on English parsing to Stanford Dependencies:
  - Unlabeled attachment score (UAS) = head
  - Labeled attachment score (LAS) = head and label
- Chen and Manning (2014) showed that neural networks can accurately determine the structure of sentences, supporting meaning interpretation. It was the first simple, successful neural dependency parser

Parser	UAS	LAS	sent. / s
MaltParser	89.8	87.2	469
MSTParser	91.4	88.1	10
TurboParser	**92.3**	89.6	8
C & M 2014	92.0	**89.7**	**654**

Further developments in transition-based neural dependency parsing

This work was further developed and improved by others, including in particular at Google

- Bigger, deeper networks with better tuned hyperparameters
- Beam search
- Global, conditional random field (CRF)-style inference over the decision sequence

Leading to SyntaxNet and the Parsey McParseFace model (2016):

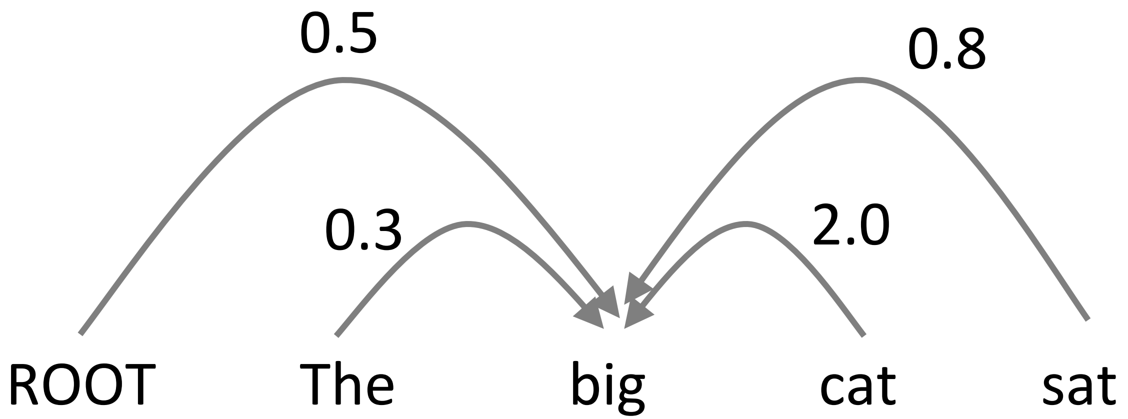
"The World's Most Accurate Parser" [in 2016]

https://research.googleblog.com/2016/05/announcing-syntaxnet-worlds-most.html

Method	UAS	LAS (PTB WSJ SD 3.3)
Chen & Manning 2014	92.0	89.7
Weiss et al. 2015	93.99	92.05
Andor et al. 2016	94.61	92.79

Graph-based dependency parsers

- Compute a score for every possible dependency (choice of head) for each word
  - Doing this well requires more than just knowing the two words
  - We need good "contextual" representations of each word token, which we will develop in the coming lectures
- Repeat the same process for each other word; find the best parse (MST algorithm)



e.g., picking the head for "big"

Graph-based dependency parsers

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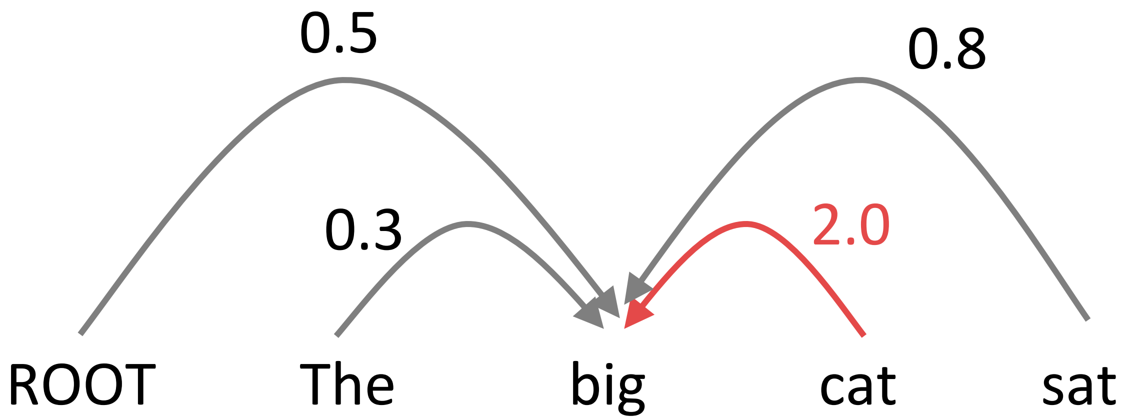

e.g., picking the head for “big”

A Neural graph-based dependency parser

[Dozat and Manning 2017; Dozat, Qi, and Manning 2017]

- This paper revived interest in graph-based dependency parsing in a neural world
  - Designed a biaffine scoring model for neural dependency parsing
    - Also crucially uses a neural sequence model, something we discuss next week
- Really great results!
  - **But slower than the simple neural transition-based parsers**
    - There are n^2 possible dependencies in a sentence of length n

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Chen & Manning 2014	92.0	89.7
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Andor et al. 2016	94.61	92.79
Dozat & Manning 2017	**95.74**	**94.08**

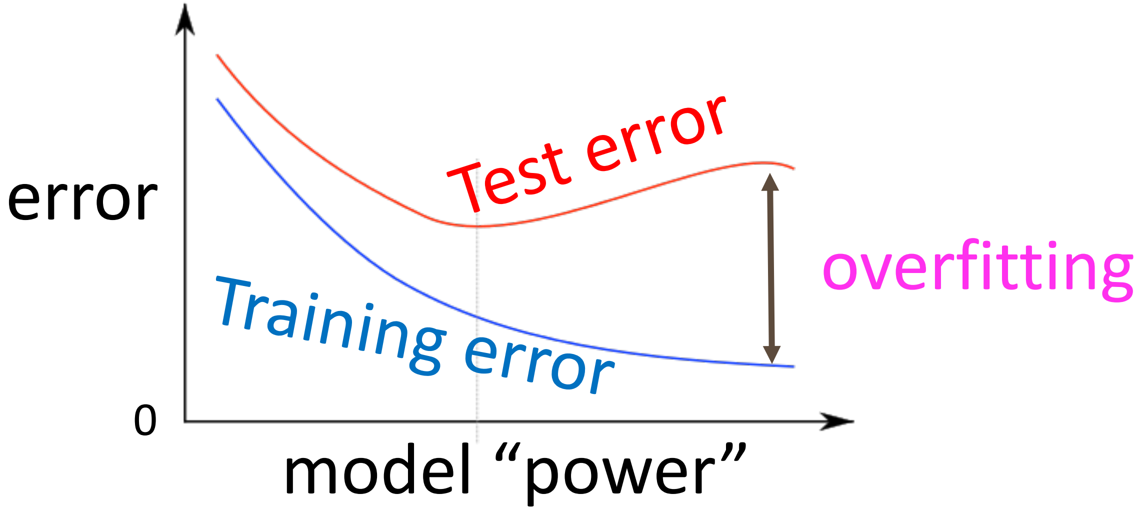
2. A bit more about neural networks

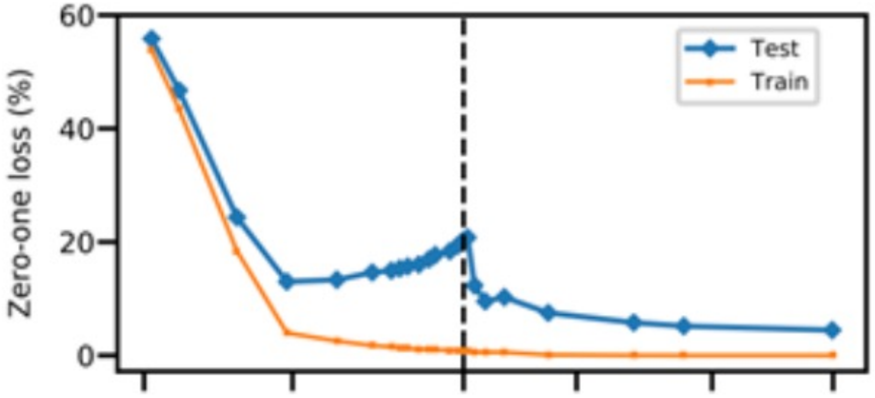
We have models with many parameters! Regularization!

- A full loss function includes **regularization** over all parameters θ, e.g., L2 regularization:

$$J(\theta) = \frac{1}{N}\sum_{i=1}^{N} -\log\left(\frac{e^{f_{y_i}}}{\sum_{c=1}^{C} e^{f_c}}\right) + \lambda \sum_k \theta_k^2$$

- Classic view: Regularization works to prevent overfitting when we have a lot of features (or later a very powerful/deep model, etc.)
- Now: Regularization produces models that generalize well when we have a "big" model
  - We do not care that our models overfit on the training data, even though they are **hugely** overfit





Dropout (Srivastava, Hinton, Krizhevsky, Sutskever, & Salakhutdinov 2012/JMLR 2014)

Preventing Feature Co-adaptation = Good Regularization Method! Use it widely!

- Training time: at each instance of evaluation (in online SGD-training), randomly set 50% of the inputs to each neuron to 0
- Test time: halve the model weights (now twice as many)
- (Except usually only drop first layer inputs a little (~15%) or not at all)
- This prevents feature co-adaptation: A feature cannot only be useful in the presence of particular other features
- In a single layer: A kind of middle-ground between Naïve Bayes (where all feature weights are set independently) and logistic regression models (where weights are set in the context of all others)
- Can be thought of as a form of model bagging (i.e., like an ensemble model)
- Nowadays usually thought of as strong, feature-dependent regularizer [Wager, Wang, & Liang 2013]

“Vectorization”

- E.g., looping over word vectors versus concatenating them all into one large matrix and then multiplying the softmax weights with that matrix:

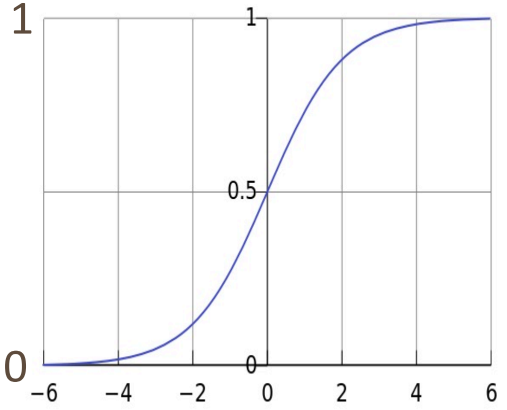
```
from numpy import random
N = 500 # number of windows to classify
d = 300 # dimensionality of each window
C = 5 # number of classes
W = random.rand(C,d)
wordvectors_list = [random.rand(d,1) for i in range(N)]
wordvectors_one_matrix = random.rand(d,N)

%timeit [W.dot(wordvectors_list[i]) for i in range(N)]
%timeit W.dot(wordvectors_one_matrix)
```

- 1000 loops, best of 3: **639 µs** per loop
  10000 loops, best of 3: **53.8 µs** per loop ← Now using a single a C x N matrix
- Matrices are awesome!!! Always try to use vectors and matrices rather than for loops!
- The speed gain goes from 1 to 2 orders of magnitude with GPUs!

Non-linearities, old and new

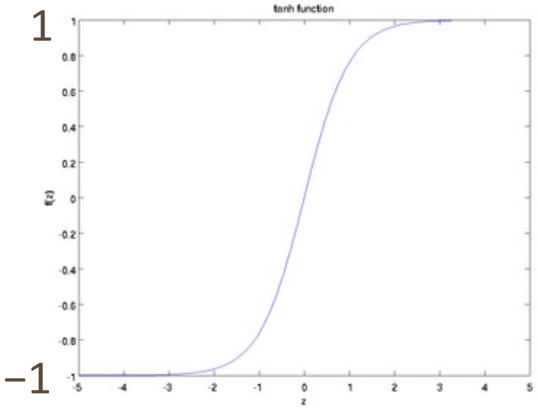
logistic ("sigmoid")

$$f(z) = \frac{1}{1+\exp(-z)}.$$



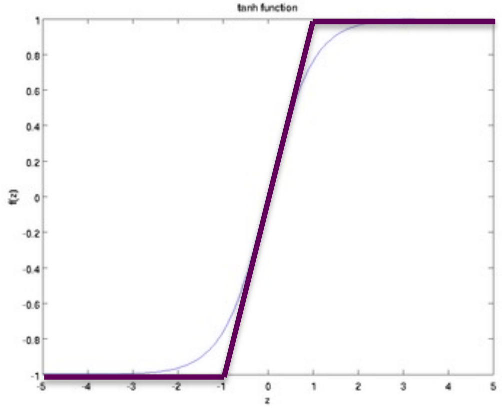
tanh

$$f(z) = \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}},$$



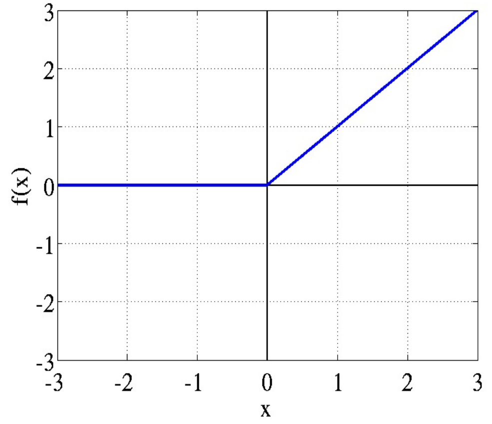
hard tanh

$$\text{HardTanh}(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 <= x <= 1 \\ 1 & \text{if } x > 1 \end{cases}$$



ReLU (Rectified Linear Unit)

$$\text{ReLU}(z) = \max(z, 0)$$



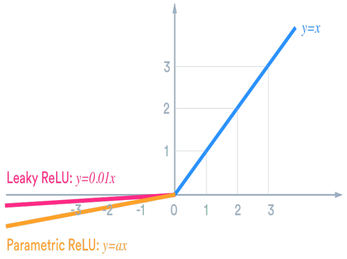
tanh is just a rescaled and shifted sigmoid (2 × as steep, [−1,1]):

$$\tanh(z) = 2\text{logistic}(2z) - 1$$

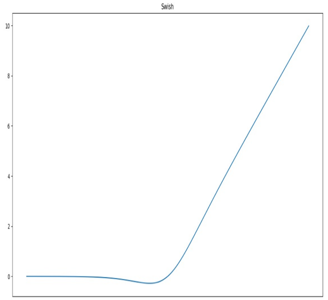
Both logistic and tanh are still used in various places (e.g., to get a probability), but are no longer the defaults for making deep networks

For building a deep network, the first thing you should try is ReLU — it trains quickly and performs well due to good gradient backflow

Leaky ReLU / Parametric ReLU



Swish [Ramachandran, Zoph & Le 2017]



Parameter Initialization

- You normally must initialize weights to small random values (i.e., not zero matrices!)
  - To avoid symmetries that prevent learning/specialization
- Initialize hidden layer biases to 0 and output (or reconstruction) biases to optimal value if weights were 0 (e.g., mean target or inverse sigmoid of mean target)
- Initialize **all other weights** ~ Uniform(–*r*, *r*), with *r* chosen so numbers get neither too big or too small [later the need for this is removed with use of layer normalization]
- Xavier initialization has variance inversely proportional to fan-in n_{in} (previous layer size) and fan-out n_{out} (next layer size):

$$\mathrm{Var}(W_i) = \frac{2}{n_{\mathrm{in}} + n_{\mathrm{out}}}$$

Optimizers

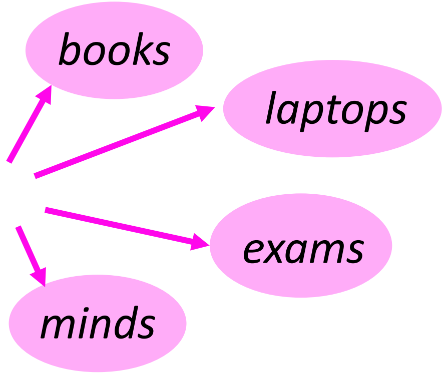
- Usually, plain SGD will work just fine!
  - However, getting good results will often require hand-tuning the learning rate
    - E.g., start it higher and halve it every *k* epochs (passes through full data, **shuffled** or sampled)
- For more complex nets and situations, or just to avoid worry, you often do better with one of a family of more sophisticated "adaptive" optimizers that scale the adjustment to individual parameters by an accumulated gradient.
  - These models give differential per-parameter learning rates
    - Adagrad
    - RMSprop
    - Adam ← A fairly good, safe place to begin in many cases
    - AdamW
    - SparseAdam
    - …
  - Can just start them with an initial learning rate, around 0.001

3. Language Modeling + RNNs

Language Modeling

- **Language Modeling** is the task of predicting what word comes next

the students opened their ______



- More formally: given a sequence of words $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(t)}$, compute the probability distribution of the next word $\boldsymbol{x}^{(t+1)}$:

$$P(\boldsymbol{x}^{(t+1)} \mid \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$$

where $\boldsymbol{x}^{(t+1)}$ can be any word in the vocabulary $V = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_{|V|}\}$

- A system that does this is called a **Language Model**

Language Modeling

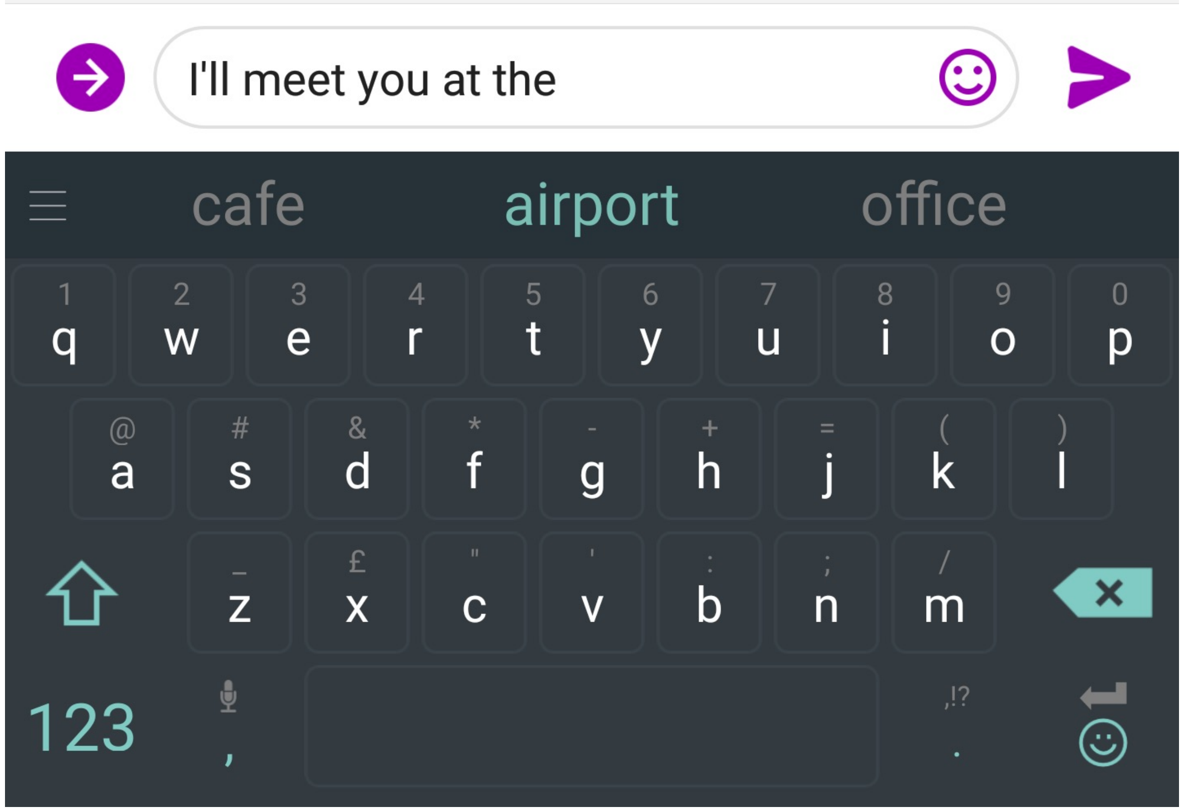
- You can also think of a Language Model as a system that assigns a probability to a piece of text

- For example, if we have some text $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$, then the probability of this text (according to the Language Model) is:

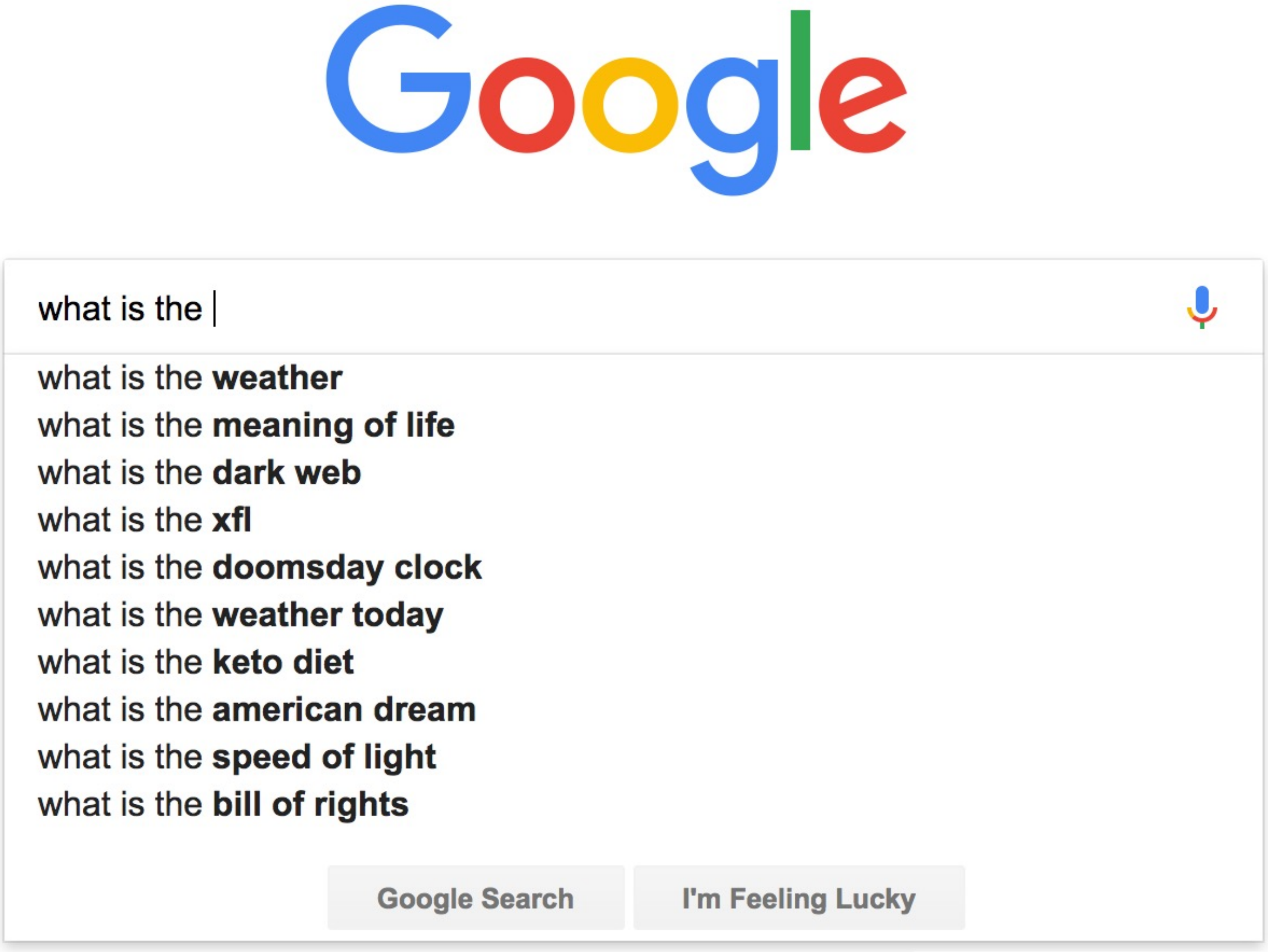
$$
\begin{aligned}
P(\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}) &= P(\boldsymbol{x}^{(1)}) \times P(\boldsymbol{x}^{(2)} | \, \boldsymbol{x}^{(1)}) \times \cdots \times P(\boldsymbol{x}^{(T)} | \, \boldsymbol{x}^{(T-1)}, \ldots, \boldsymbol{x}^{(1)}) \\
&= \prod_{t=1}^{T} \underbrace{P(\boldsymbol{x}^{(t)} | \, \boldsymbol{x}^{(t-1)}, \ldots, \boldsymbol{x}^{(1)})}
\end{aligned}
$$

This is what our LM provides

You use Language Models every day!



You use Language Models every day!



n-gram Language Models

the students opened their ______

- **Question**: How to learn a Language Model?
- **Answer** (pre- Deep Learning): learn an *n*-gram Language Model!

- **Definition:** An *n*-gram is a chunk of *n* consecutive words.
  - unigrams: “the”, “students”, “opened”, ”their”
  - bigrams: “the students”, “students opened”, “opened their”
  - trigrams: “the students opened”, “students opened their”
  - four-grams: “the students opened their”

- **Idea:** Collect statistics about how frequent different n-grams are and use these to predict next word.

n-gram Language Models

- First we make a Markov assumption: $x^{(t+1)}$ depends only on the preceding *n*-1 words

n-1 words

$$P(\boldsymbol{x}^{(t+1)}|\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(1)}) = P(\boldsymbol{x}^{(t+1)}|\underbrace{\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)}}_{n-1 \text{ words}})$$ (assumption)

prob of a n-gram

prob of a (n-1)-gram

$$= \frac{P(\boldsymbol{x}^{(t+1)},\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}{P(\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}$$ (definition of conditional prob)

- **Question:** How do we get these *n*-gram and (*n*-1)-gram probabilities?
- **Answer:** By counting them in some large corpus of text!

$$\approx \frac{\text{count}(\boldsymbol{x}^{(t+1)},\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}{\text{count}(\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}$$ (statistical approximation)

n-gram Language Models: Example

Suppose we are learning a 4-gram Language Model.

~~as the proctor started the clock, the~~ *students opened their* ______

discard

condition on this

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\text{count(students opened their } \boldsymbol{w})}{\text{count(students opened their)}}$$

For example, suppose that in the corpus:

- "students opened their" occurred 1000 times
- "students opened their books" occurred 400 times
  - → P(books | students opened their) = 0.4
- "students opened their exams" occurred 100 times
  - → P(exams | students opened their) = 0.1

Should we have discarded the "proctor" context?

Sparsity Problems with n-gram Language Models

Sparsity Problem 1

Problem: What if *"students opened their w"* never occurred in data? Then w has probability 0!

(Partial) Solution: Add small δ to the count for every $w \in V$. This is called *smoothing*.

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\text{count}(\text{students opened their } \boldsymbol{w})}{\text{count}(\text{students opened their})}$$

Sparsity Problem 2

Problem: What if *"students opened their"* never occurred in data? Then we can't calculate probability for *any* w!

(Partial) Solution: Just condition on *"opened their"* instead. This is called *backoff*.

Note: Increasing *n* makes sparsity problems *worse.* Typically, we can't have *n* bigger than 5.

Storage Problems with n-gram Language Models

Storage: Need to store count for all *n*-grams you saw in the corpus.

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\text{count}(\text{students opened their } \boldsymbol{w})}{\text{count}(\text{students opened their})}$$

Increasing *n* or increasing corpus increases model size!

n-gram Language Models in practice

- You can build a simple trigram Language Model over a 1.7 million word corpus (Reuters) in a few seconds on your laptop*

Business and financial news

today the _______

get probability distribution

```
company  0.153
bank     0.153
price    0.077
italian  0.039
emirate  0.039
      …
```

Sparsity problem: not much granularity in the probability distribution

Otherwise, seems reasonable!

*** Try for yourself:** https://nlpforhackers.io/language-models/

Generating text with a n-gram Language Model

You can also use a Language Model to generate text

today the ________

condition on this

get probability distribution

company	0.153
bank	0.153
price	0.077
italian	0.039
emirate	0.039
…	

sample

Generating text with a n-gram Language Model

You can also use a Language Model to generate text

today the price ________

condition on this

get probability distribution

of	0.308
for	0.050
it	0.046
to	0.046
is	0.031
…	

sample

Generating text with a n-gram Language Model

You can also use a Language Model to generate text

today the price of ________

condition on this

get probability distribution

the	0.072
18	0.043
oil	0.043
its	0.036
gold	0.018
…	

sample

Generating text with a n-gram Language Model

You can also use a Language Model to generate text

today the price of gold per ton , while production of shoe lasts and shoe industry , the bank intervened just after it considered and rejected an imf demand to rebuild depleted european stocks , sept 30 end primary 76 cts a share .

Surprisingly grammatical!

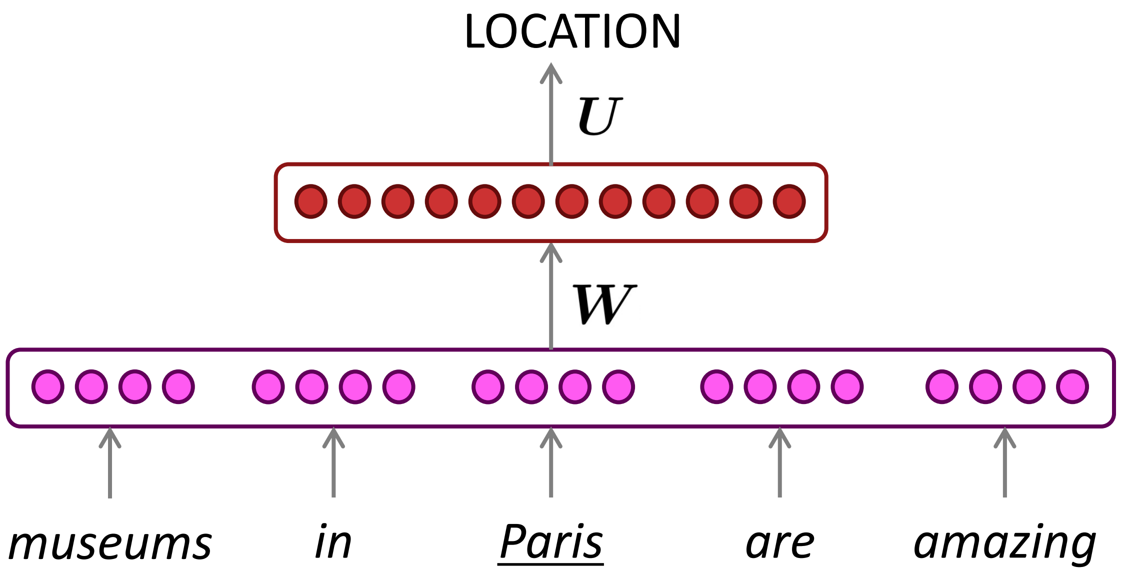
…but **incoherent.** We need to consider more than three words at a time if we want to model language well.

But increasing *n* worsens sparsity problem, and increases model size…

How to build a *neural* Language Model?

- Recall the Language Modeling task:
  - Input: sequence of words $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(t)}$
  - Output: prob dist of the next word $P(\boldsymbol{x}^{(t+1)} | \ \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$

- How about a window-based neural model?
  - We saw this applied to Named Entity Recognition in Lecture 3:



A fixed-window neural Language Model

~~*as the proctor started the clock*~~ *the students opened their* ______

discard

fixed window

A fixed-window neural Language Model

output distribution

$$\hat{\boldsymbol{y}} = \text{softmax}(\boldsymbol{U}\boldsymbol{h} + \boldsymbol{b}_2) \in \mathbb{R}^{|V|}$$

hidden layer

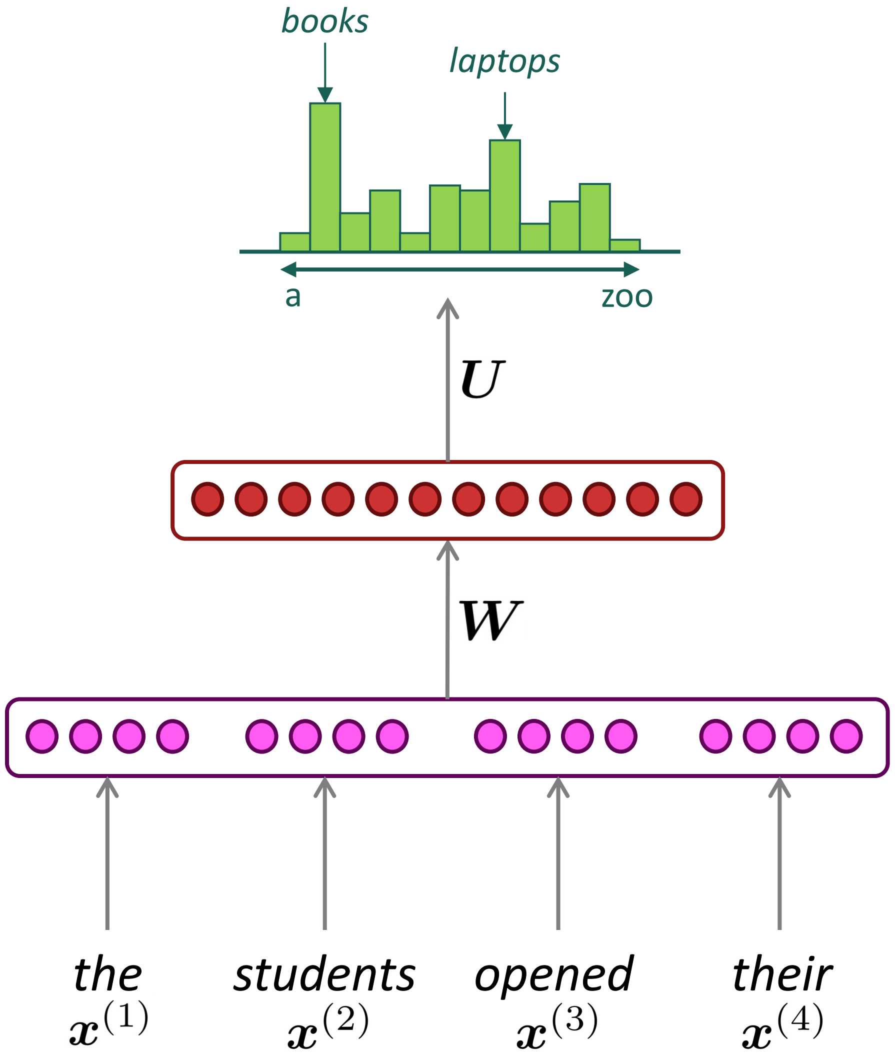
$$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{e} + \boldsymbol{b}_1)$$

concatenated word embeddings

$$\boldsymbol{e} = [\boldsymbol{e}^{(1)}; \boldsymbol{e}^{(2)}; \boldsymbol{e}^{(3)}; \boldsymbol{e}^{(4)}]$$

words / one-hot vectors

$$\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \boldsymbol{x}^{(3)}, \boldsymbol{x}^{(4)}$$



A fixed-window neural Language Model

Approximately: Y. Bengio, et al. (2000/2003): A Neural Probabilistic Language Model

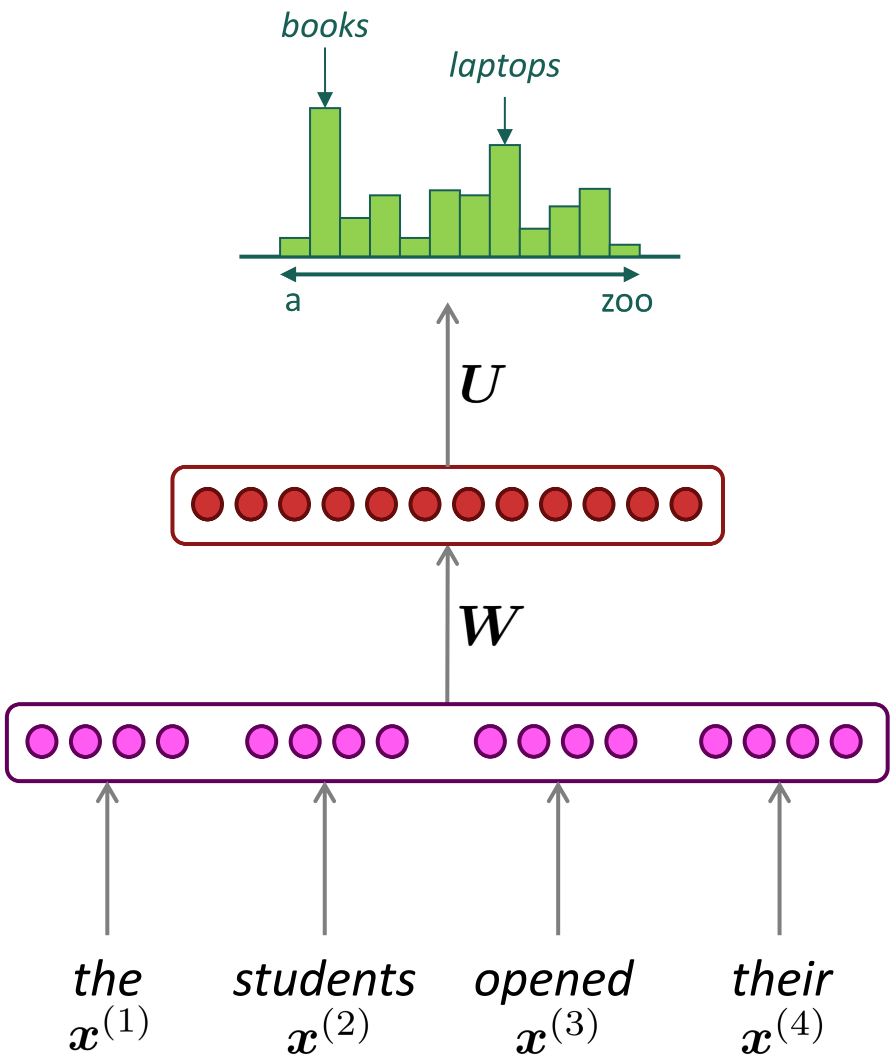
Improvements over *n*-gram LM:

- No sparsity problem
- Don't need to store all observed *n*-grams

Remaining **problems**:

- Fixed window is too small
- Enlarging window enlarges W
- Window can never be large enough!
- $x^{(1)}$ and $x^{(2)}$ are multiplied by completely different weights in W. No symmetry in how the inputs are processed.

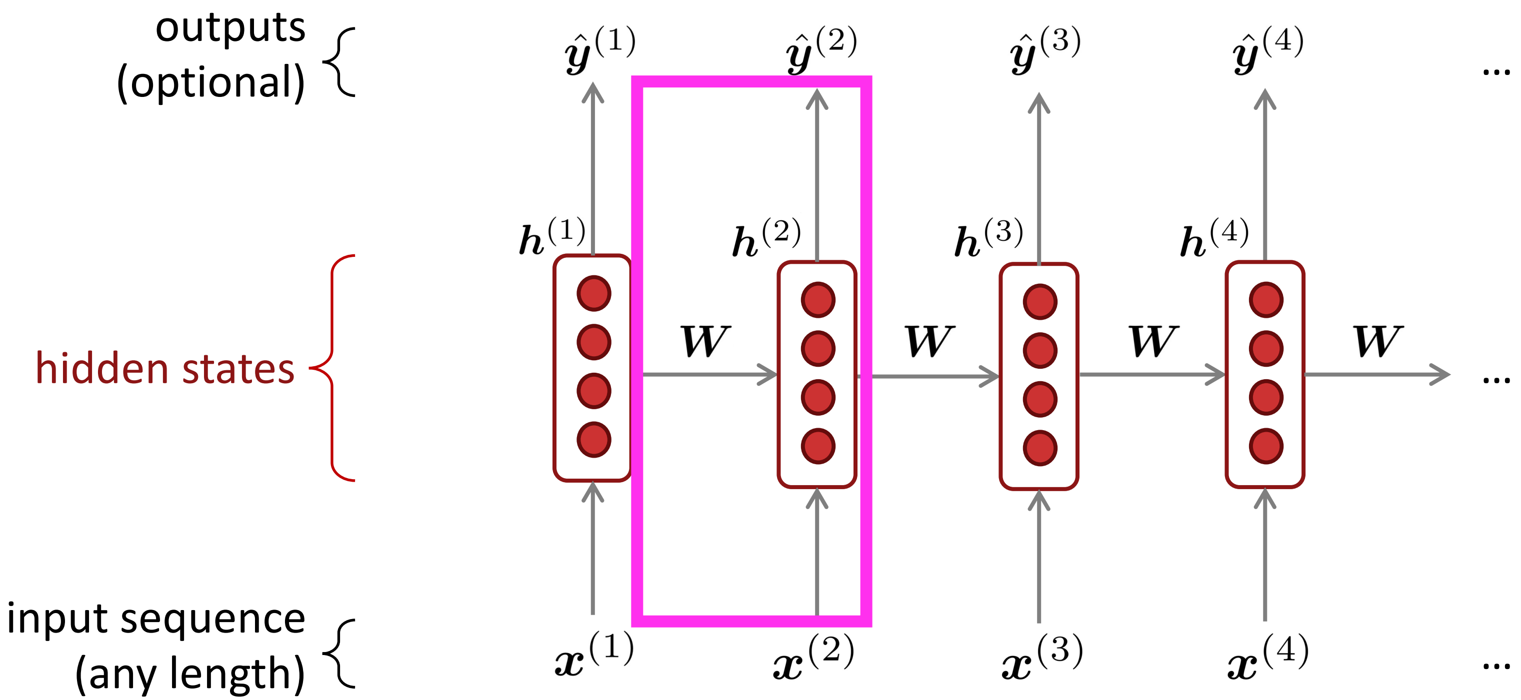
We need a neural architecture that can process *any length input*



Recurrent Neural Networks (RNN)

A family of neural architectures

Core idea: Apply the same weights W *repeatedly*



A Simple RNN Language Model

$$\hat{\boldsymbol{y}}^{(4)} = P(\boldsymbol{x}^{(5)}|\text{the students opened their})$$

output distribution

$$\hat{\boldsymbol{y}}^{(t)} = \text{softmax}\left(\boldsymbol{U}\boldsymbol{h}^{(t)} + \boldsymbol{b}_2\right) \in \mathbb{R}^{|V|}$$

hidden states

$$\boldsymbol{h}^{(t)} = \sigma\left(\boldsymbol{W}_h\boldsymbol{h}^{(t-1)} + \boldsymbol{W}_e\boldsymbol{e}^{(t)} + \boldsymbol{b}_1\right)$$

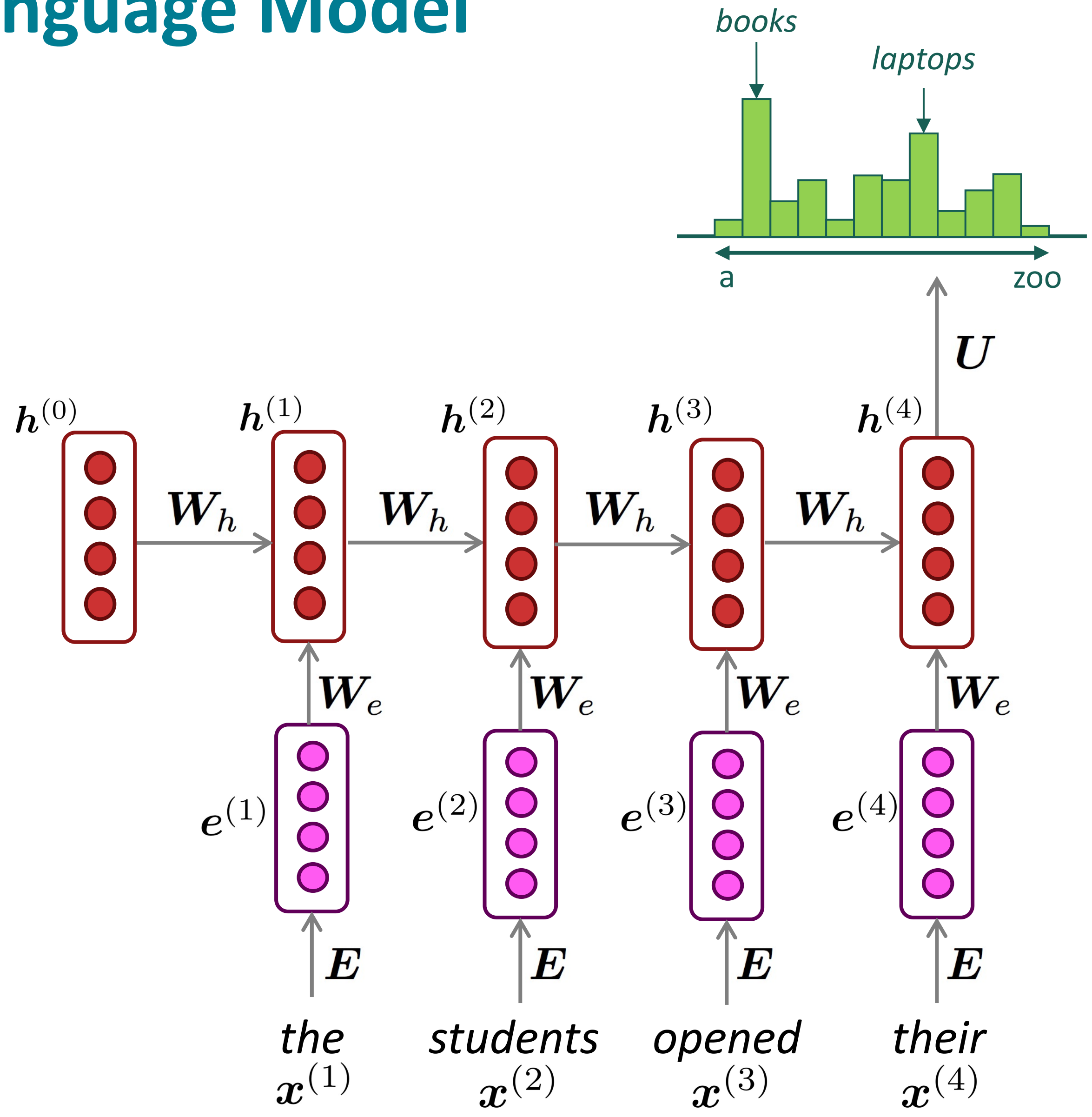
$\boldsymbol{h}^{(0)}$ is the initial hidden state

word embeddings

$$\boldsymbol{e}^{(t)} = \boldsymbol{E}\boldsymbol{x}^{(t)}$$

words / one-hot vectors

$\boldsymbol{x}^{(t)} \in \mathbb{R}^{|V|}$



Note: *this input sequence could be much longer now!*

RNN Language Models

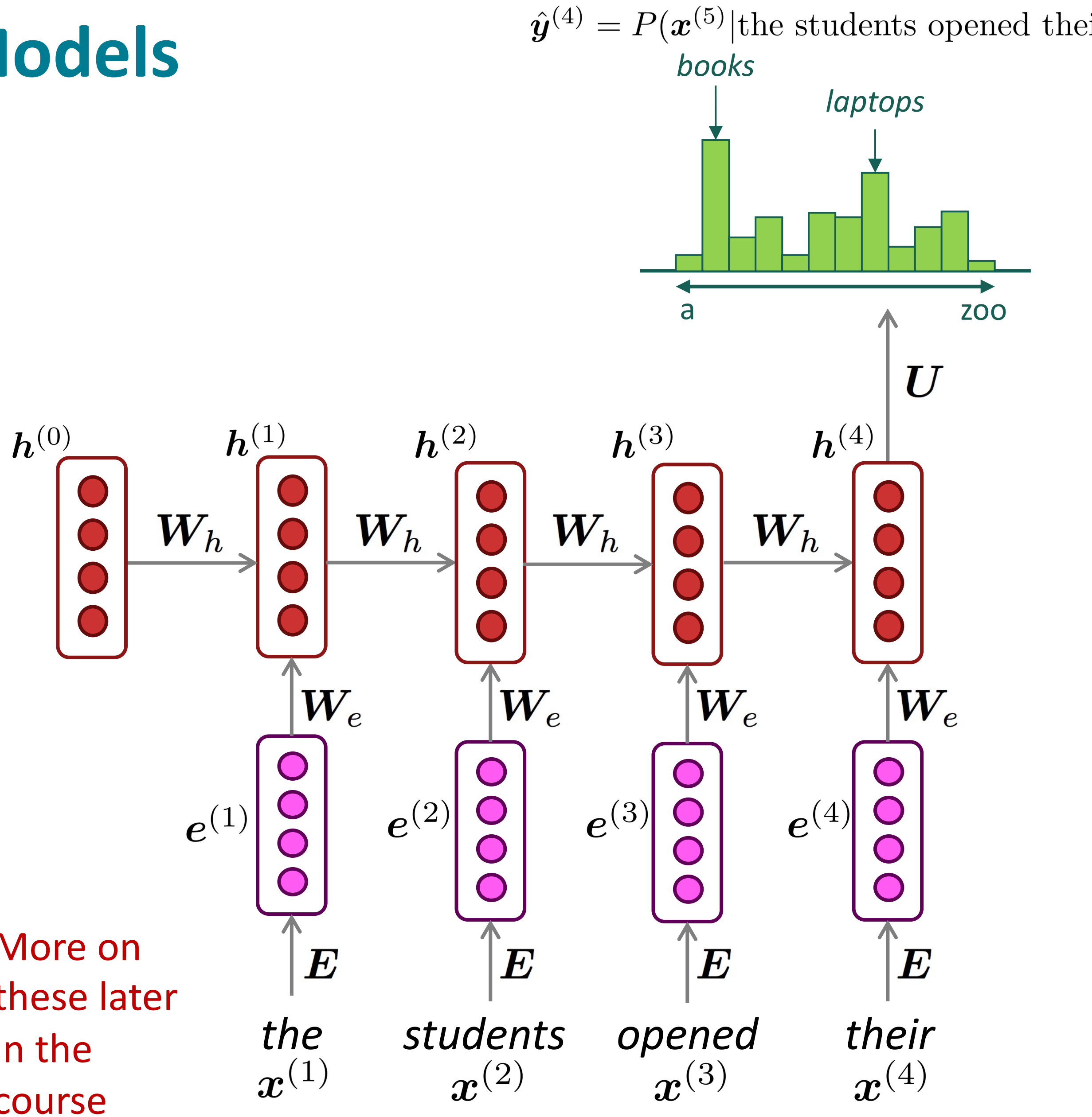
RNN **Advantages**:

- Can process any length input
- Computation for step *t* can (in theory) use information from many steps back
- Model size doesn't increase for longer input context
- Same weights applied on every timestep, so there is symmetry in how inputs are processed.

RNN **Disadvantages**:

- Recurrent computation is slow
- In practice, difficult to access information from many steps back

More on these later in the course



Training an RNN Language Model

- Get a big corpus of text which is a sequence of words $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$
- Feed into RNN-LM; compute output distribution $\hat{\boldsymbol{y}}^{(t)}$ for *every step t*.
  - i.e. predict probability dist of *every word*, given words so far

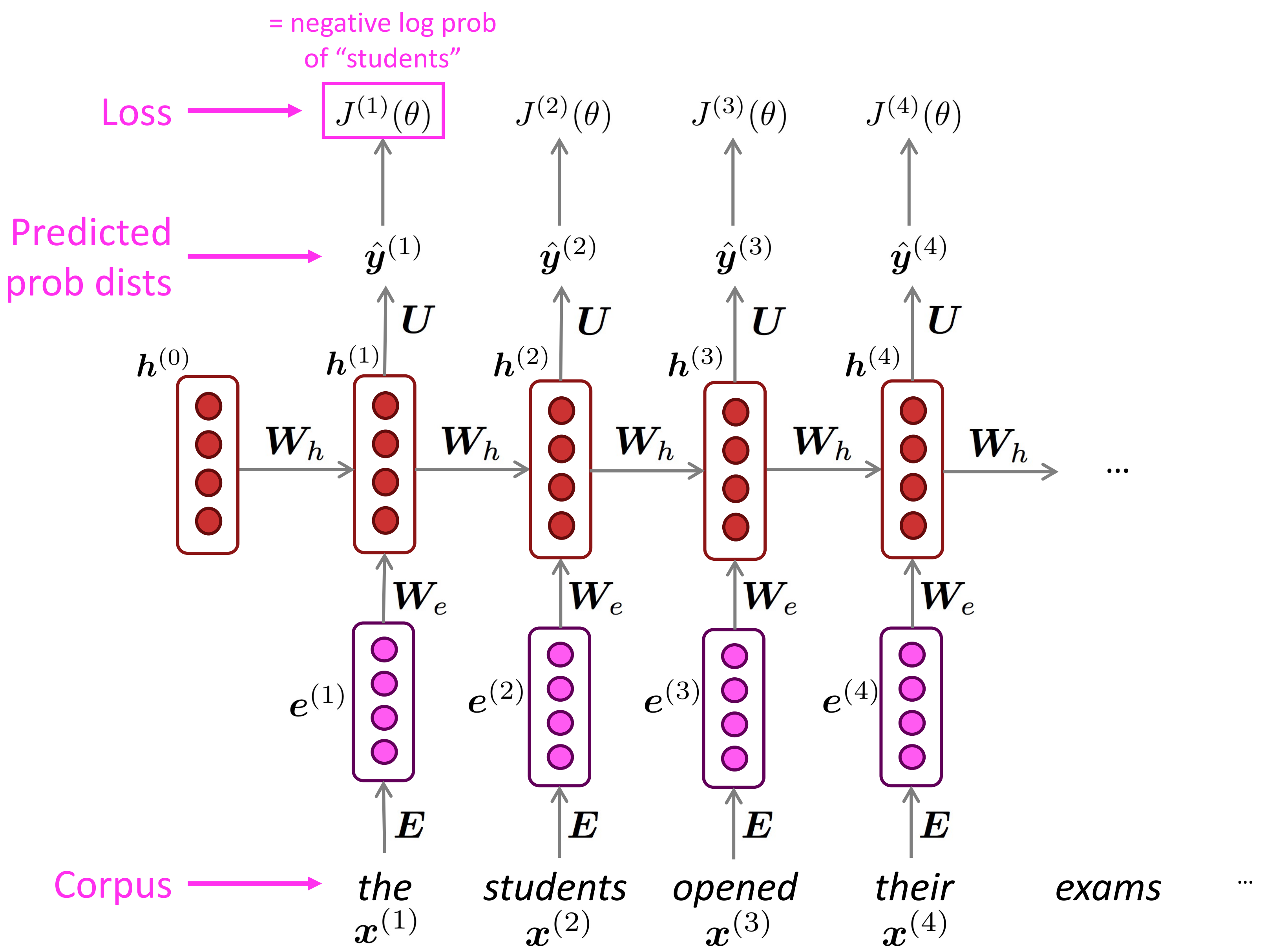
- Loss function on step *t* is cross-entropy between predicted probability distribution $\hat{\boldsymbol{y}}^{(t)}$, and the true next word $\boldsymbol{y}^{(t)}$ (one-hot for $\boldsymbol{x}^{(t+1)}$):

$$J^{(t)}(\theta) = CE(\boldsymbol{y}^{(t)}, \hat{\boldsymbol{y}}^{(t)}) = -\sum_{w \in V} \boldsymbol{y}_w^{(t)} \log \hat{\boldsymbol{y}}_w^{(t)} = -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}_{t+1}}^{(t)}$$

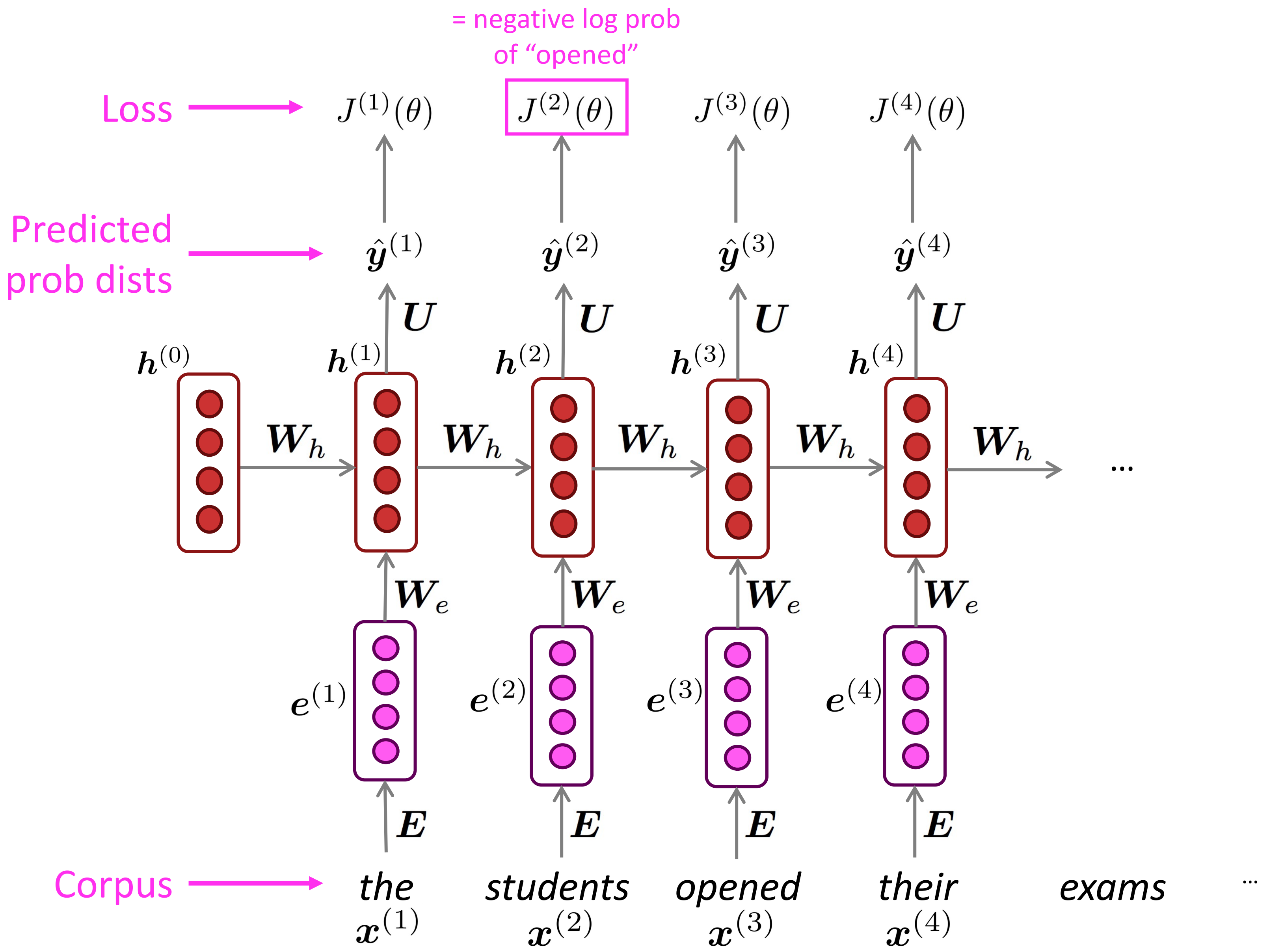
- Average this to get overall loss for entire training set:

$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J^{(t)}(\theta) = \frac{1}{T} \sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}_{t+1}}^{(t)}$$

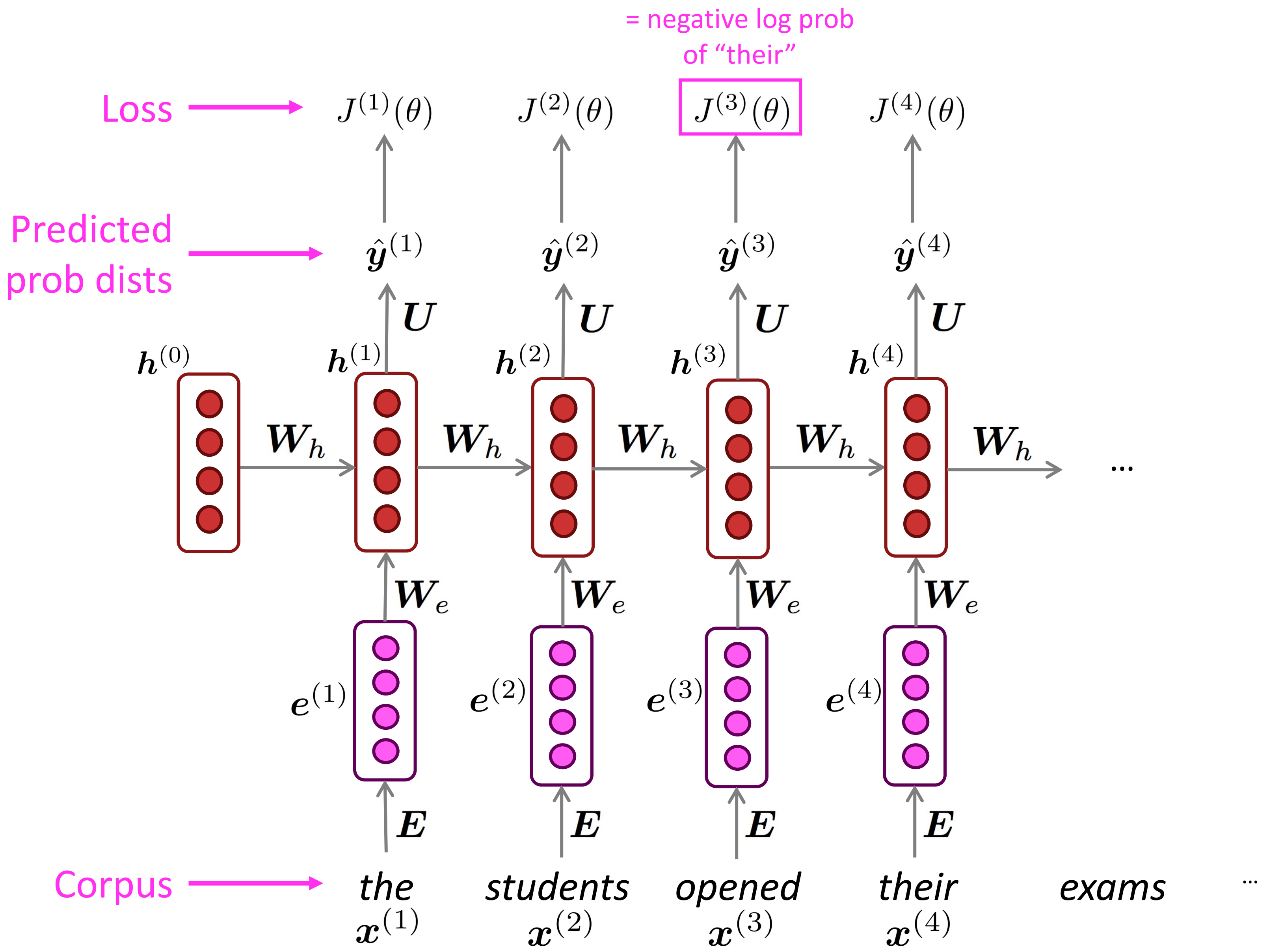
Training an RNN Language Model



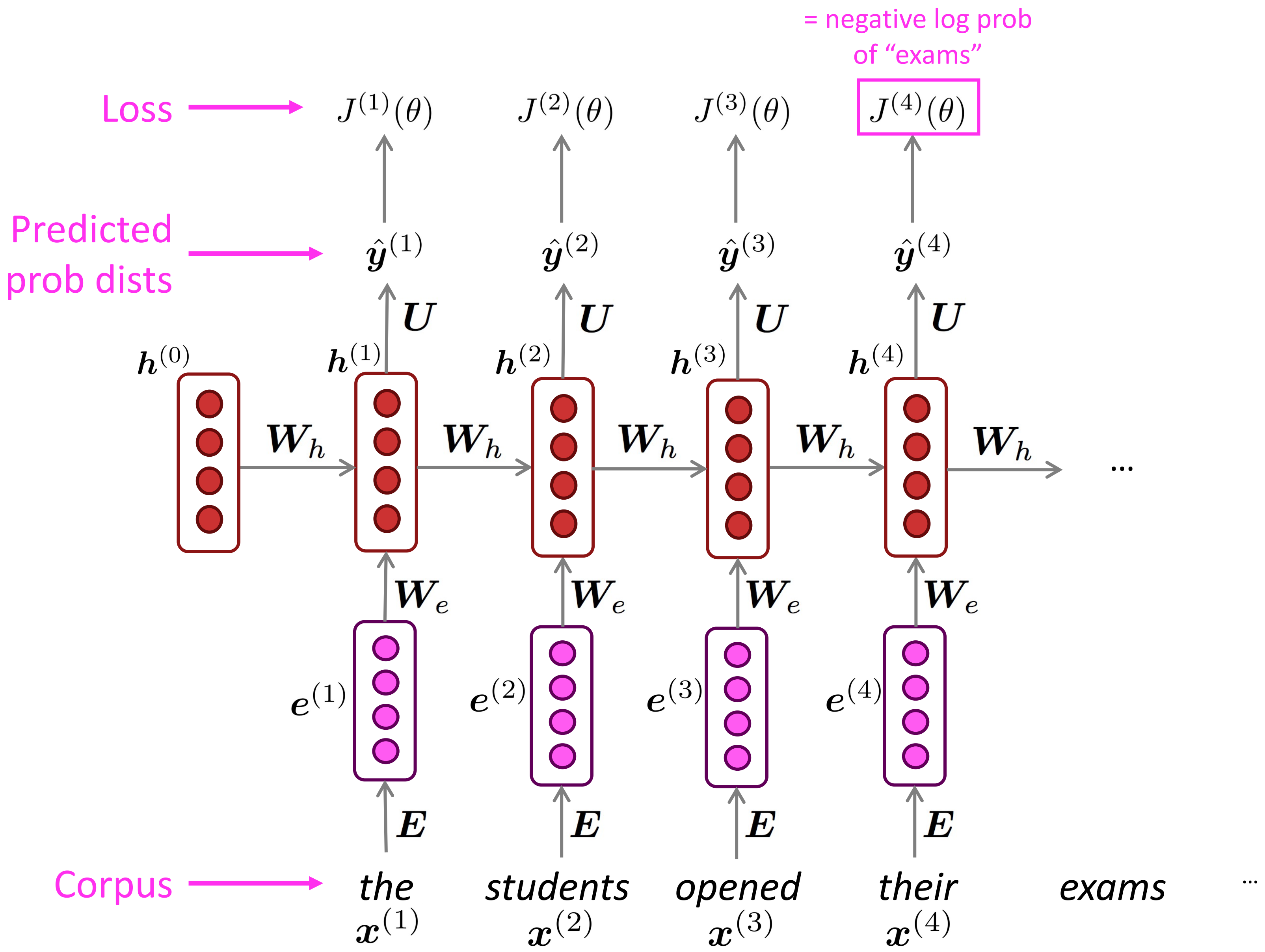
Training an RNN Language Model



Training an RNN Language Model

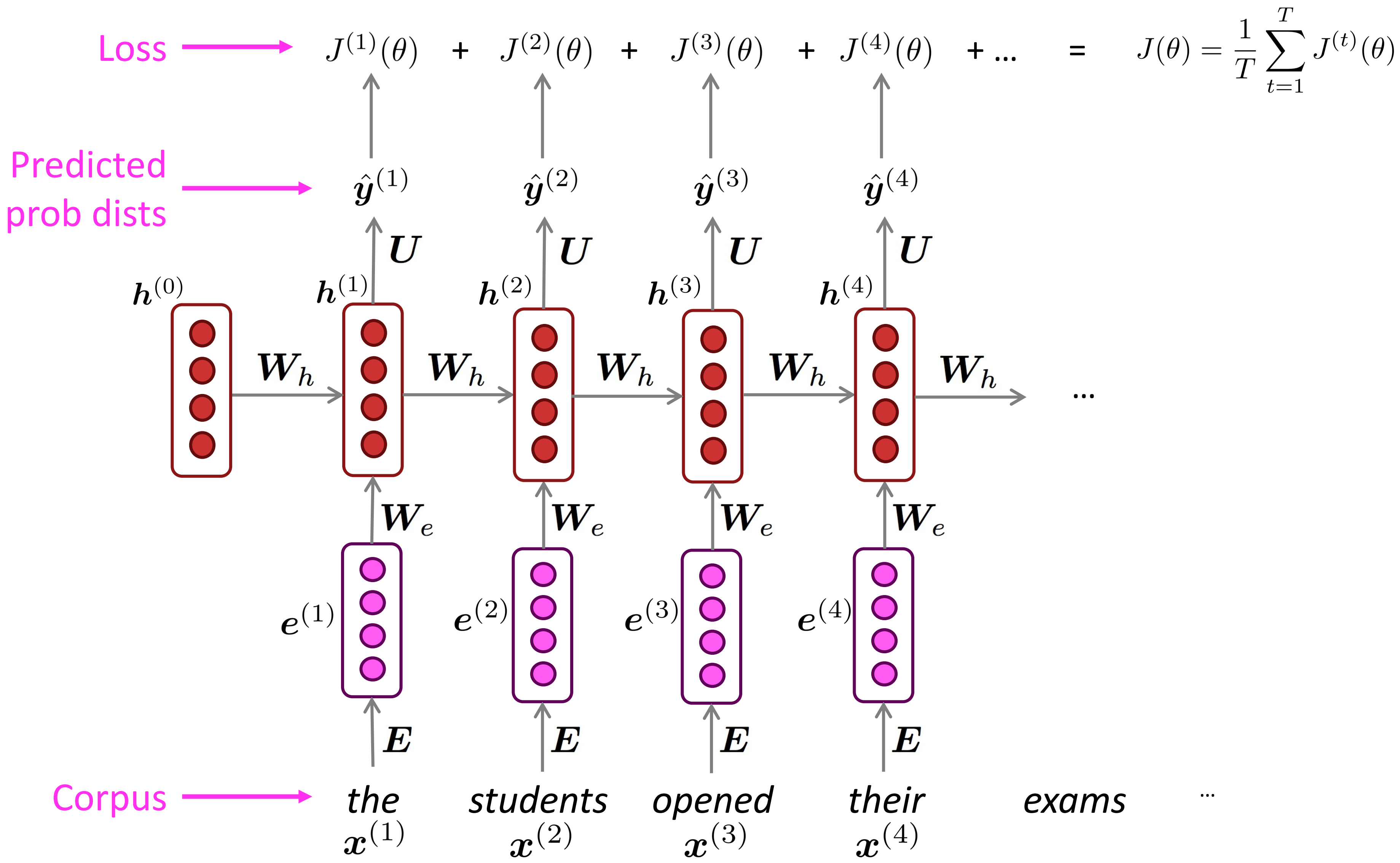


Training an RNN Language Model



Training an RNN Language Model

"Teacher forcing"



Training a RNN Language Model

- However: Computing loss and gradients across entire corpus $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$ is too expensive!

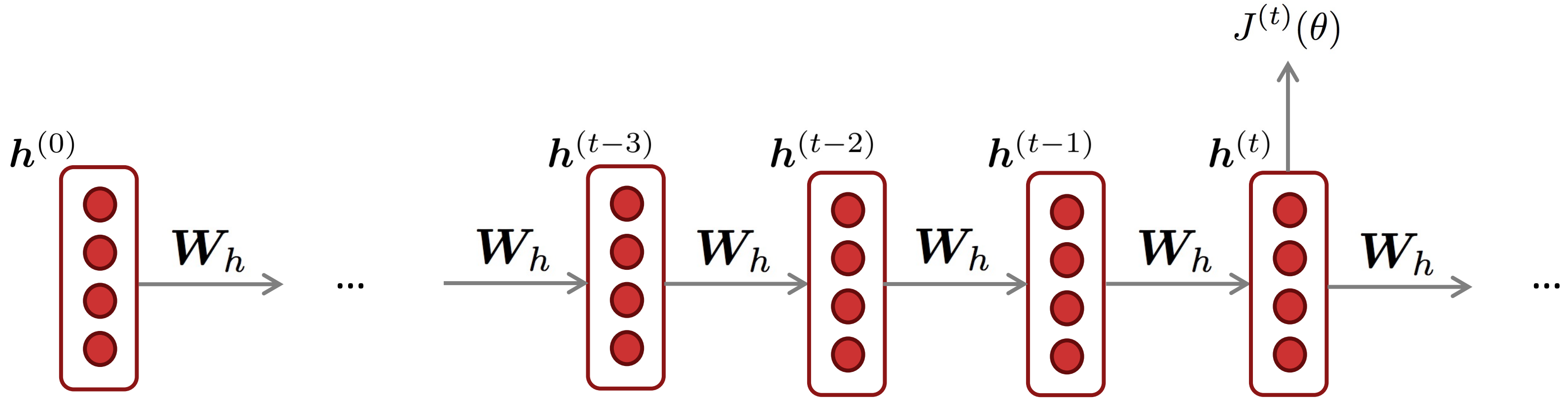
$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J^{(t)}(\theta)$$

- In practice, consider $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$ as a sentence (or a document)

- Recall: Stochastic Gradient Descent allows us to compute loss and gradients for small chunk of data, and update.

- Compute loss $J(\theta)$ for a sentence (actually, a batch of sentences), compute gradients and update weights. Repeat.

Backpropagation for RNNs



Question: What's the derivative of $J^{(t)}(\theta)$ w.r.t. the repeated weight matrix $\boldsymbol{W}_h$?

Answer: $$\frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} = \sum_{i=1}^{t} \left. \frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} \right|_{(i)}$$

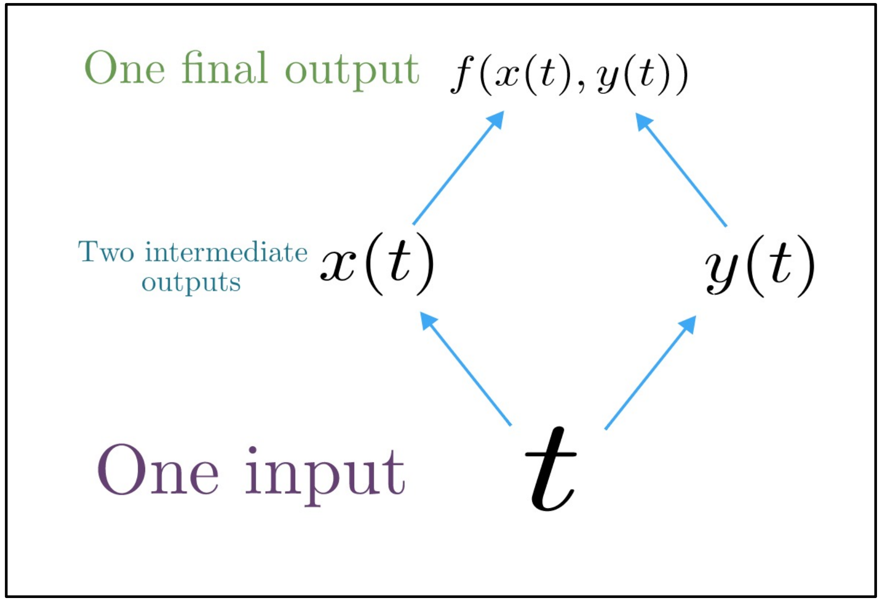
"The gradient w.r.t. a repeated weight is the sum of the gradient w.r.t. each time it appears"

Why?

Multivariable Chain Rule

- Given a multivariable function $f(x, y)$, and two single variable functions $x(t)$ and $y(t)$, here's what the multivariable chain rule says:

$$\underbrace{\frac{d}{dt} f(x(t), y(t))}_{\text{Derivative of composition function}} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$



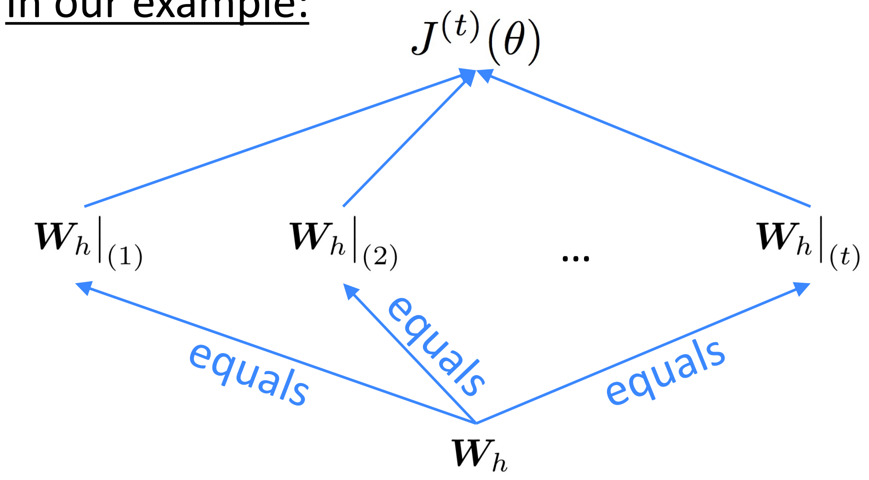
Source:
https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/differentiating-vector-valued-functions/a/multivariable-chain-rule-simple-version

Backpropagation for RNNs: Proof sketch

- Given a multivariable function $f(x, y)$, and two single variable functions $x(t)$ and $y(t)$, here's what the multivariable chain rule says:

$$\underbrace{\frac{d}{dt} f(x(t), y(t))}_{\text{Derivative of composition function}} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

In our example:



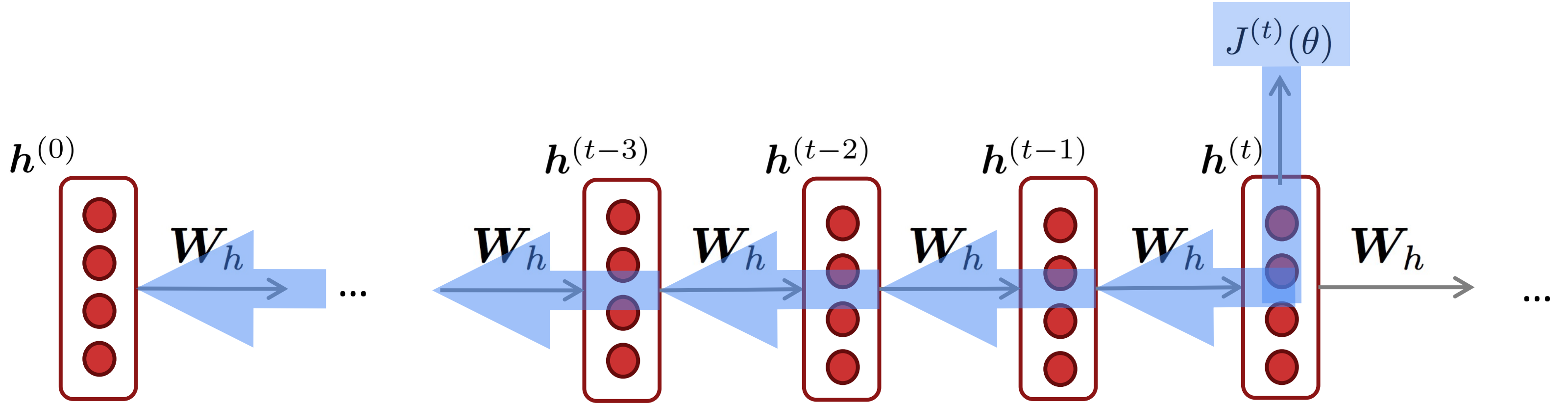
Apply the multivariable chain rule:

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{W}_h} = \sum_{i=1}^{t} \left.\frac{\partial J^{(t)}}{\partial \boldsymbol{W}_h}\right|_{(i)} \underbrace{\frac{\partial \boldsymbol{W}_h|_{(i)}}{\partial \boldsymbol{W}_h}}_{= 1}$$

$$= \sum_{i=1}^{t} \left.\frac{\partial J^{(t)}}{\partial \boldsymbol{W}_h}\right|_{(i)}$$

Source:
https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/differentiating-vector-valued-functions/a/multivariable-chain-rule-simple-version

Backpropagation for RNNs



$$\frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} = \sum_{i=1}^{t} \left. \frac{\partial J^{(t)}}{\partial \boldsymbol{W_h}} \right|_{(i)}$$

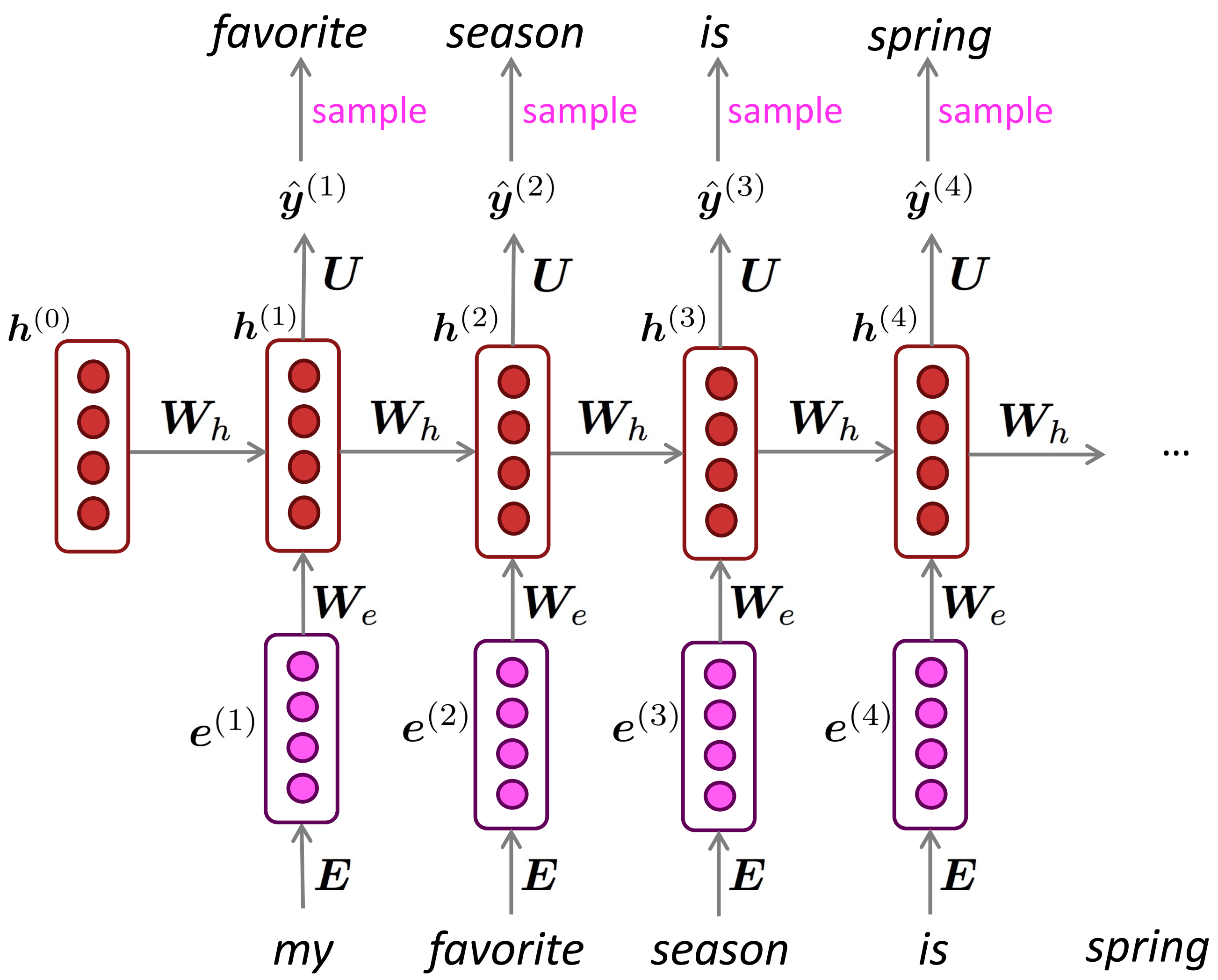
Question: How do we calculate this?

Answer: Backpropagate over timesteps *i*=*t*,…,0, summing gradients as you go. This algorithm is called **"backpropagation through time"** [Werbos, P.G., 1988, *Neural Networks* **1**, and others]

In practice, often "truncated" after ~20 timesteps for training efficiency reasons

Generating text with a RNN Language Model

Just like a n-gram Language Model, you can use a RNN Language Model to generate text by repeated sampling. Sampled output becomes next step's input.



Generating text with an RNN Language Model

Let's have some fun!

- You can train an RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on Obama speeches:

The United States will step up to the cost of a new challenges of the American people that will share the fact that we created the problem. They were attacked and so that they have to say that all the task of the final days of war that I will not be able to get this done.

Source: https://medium.com/@samim/obama-rnn-machine-generated-political-speeches-c8abd18a2ea0

Generating text with an RNN Language Model

Let's have some fun!

- You can train an RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on *Harry Potter*:

"Sorry," Harry shouted, panicking—"I'll leave those brooms in London, are they?"

"No idea," said Nearly Headless Nick, casting low close by Cedric, carrying the last bit of treacle Charms, from Harry's shoulder, and to answer him the common room perched upon it, four arms held a shining knob from when the spider hadn't felt it seemed. He reached the teams too.

Source: https://medium.com/deep-writing/harry-potter-written-by-artificial-intelligence-8a9431803da6

Generating text with an RNN Language Model

Let’s have some fun!

- You can train an RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on recipes:

```
     Title: CHOCOLATE RANCH BARBECUE
Categories: Game, Casseroles, Cookies, Cookies
     Yield: 6 Servings

     2 tb Parmesan cheese -- chopped
     1 c  Coconut milk
     3    Eggs, beaten

 Place each pasta over layers of lumps. Shape mixture into the moderate oven and simmer
 until firm. Serve hot in bodied fresh, mustard, orange and cheese.

 Combine the cheese and salt together the dough in a large skillet; add the ingredients
 and stir in the chocolate and pepper.
```

Source: https://gist.github.com/nylki/1efbaa36635956d35bcc

Generating text with a RNN Language Model

Let’s have some fun!

- You can train a RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on paint color names:

Ghasty Pink 231 137 165	Sand Dan 201 172 143
Power Gray 151 124 112	Grade Bat 48 94 83
Navel Tan 199 173 140	Light Of Blast 175 150 147
Bock Coe White 221 215 236	Grass Bat 176 99 108
Horble Gray 178 181 196	Sindis Poop 204 205 194
Homestar Brown 133 104 85	Dope 219 209 179
Snader Brown 144 106 74	Testing 156 101 106
Golder Craam 237 217 177	Stoner Blue 152 165 159
Hurky White 232 223 215	Burble Simp 226 181 132
Burf Pink 223 173 179	Stanky Bean 197 162 171
Rose Hork 230 215 198	Turdly 190 164 116

This is an example of a character-level RNN-LM (predicts what character comes next)

Source: http://aiweirdness.com/post/160776374467/new-paint-colors-invented-by-neural-network

Evaluating Language Models

- The standard evaluation metric for Language Models is perplexity.

$$\text{perplexity} = \prod_{t=1}^{T} \left(\frac{1}{P_{\text{LM}}(\boldsymbol{x}^{(t+1)} | \, \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})} \right)^{1/T}$$

Normalized by number of words

Inverse probability of corpus, according to Language Model

- This is equal to the exponential of the cross-entropy loss $J(\theta)$:

$$= \prod_{t=1}^{T} \left(\frac{1}{\hat{\boldsymbol{y}}^{(t)}_{\boldsymbol{x}_{t+1}}} \right)^{1/T} = \exp\left(\frac{1}{T} \sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}^{(t)}_{\boldsymbol{x}_{t+1}} \right) = \exp(J(\theta))$$

Lower perplexity is better!

RNNs have greatly improved perplexity

n-gram model →

Increasingly complex RNNs

Model	Perplexity
Interpolated Kneser-Ney 5-gram (Chelba et al., 2013)	67.6
RNN-1024 + MaxEnt 9-gram (Chelba et al., 2013)	51.3
RNN-2048 + BlackOut sampling (Ji et al., 2015)	68.3
Sparse Non-negative Matrix factorization (Shazeer et al., 2015)	52.9
LSTM-2048 (Jozefowicz et al., 2016)	43.7
2-layer LSTM-8192 (Jozefowicz et al., 2016)	30
Ours small (LSTM-2048)	43.9
Ours large (2-layer LSTM-2048)	39.8

Perplexity improves (lower is better)

Source: https://research.fb.com/building-an-efficient-neural-language-model-over-a-billion-words/

Why should we care about Language Modeling?

- Language Modeling is a benchmark task that helps us measure our progress on understanding language
- Language Modeling is a subcomponent of many NLP tasks, especially those involving generating text or estimating the probability of text:
    - Predictive typing
    - Speech recognition
    - Handwriting recognition
    - Spelling/grammar correction
    - Authorship identification
    - Machine translation
    - Summarization
    - Dialogue
    - etc.
- Language Modeling has been extended to cover everything else in NLP: GPT-3 is an LM!

Recap

- **Language Model**: A system that predicts the next word

- **Recurrent Neural Network**: A family of neural networks that:
  - Take sequential input of any length
  - Apply the same weights on each step
  - Can optionally produce output on each step

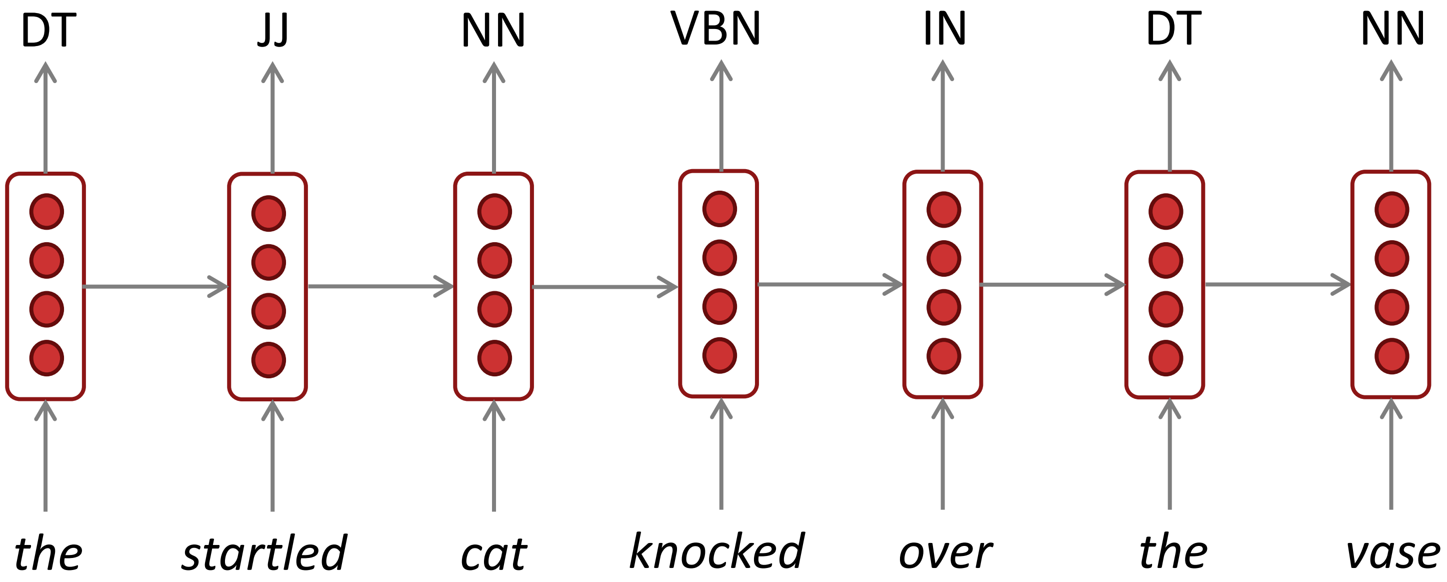
- Recurrent Neural Network $\neq$ Language Model

- We've shown that RNNs are a great way to build a LM.

- But RNNs are useful for much more!

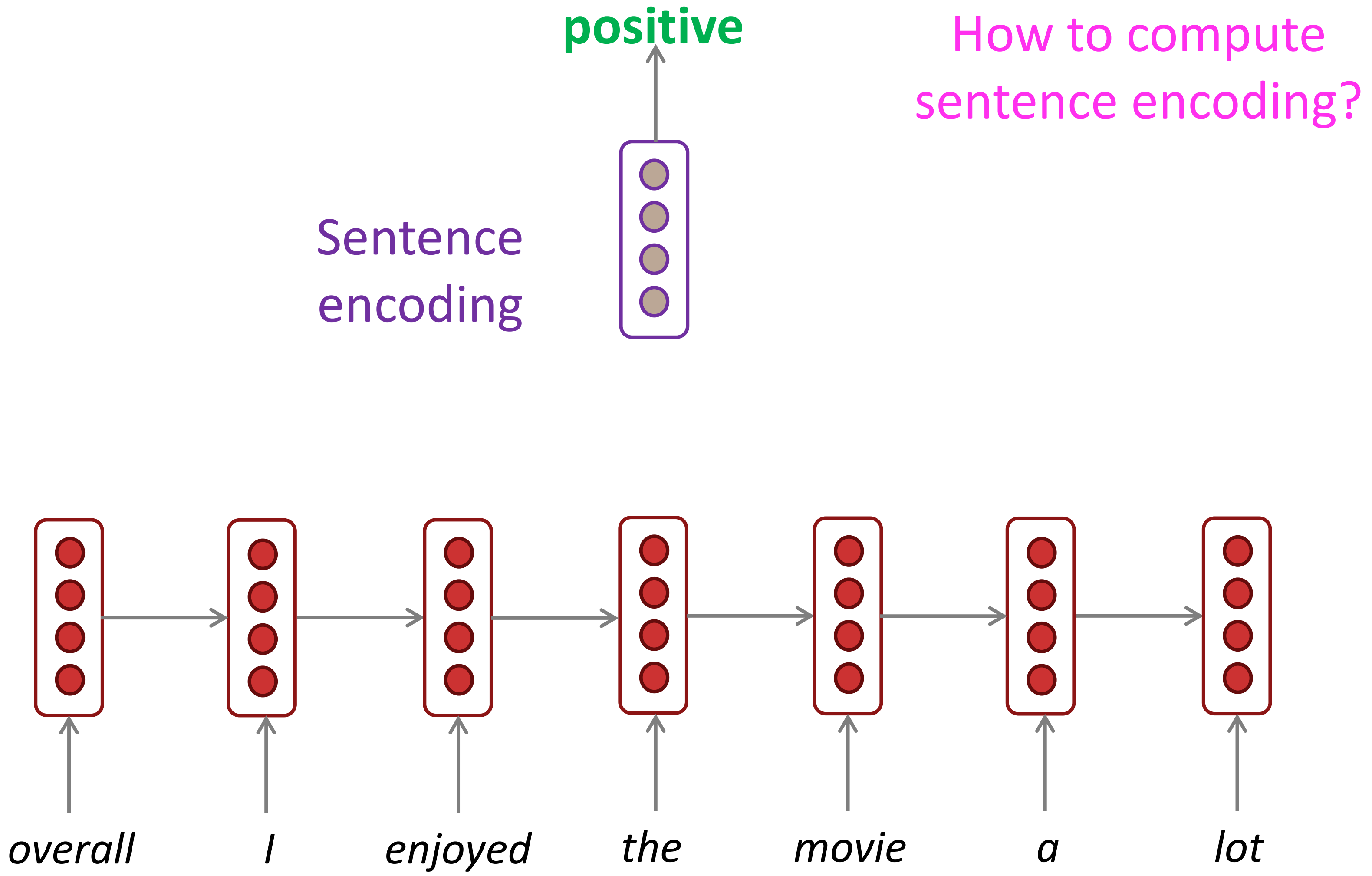
RNNs can be used for tagging

e.g., **part-of-speech tagging**, named entity recognition



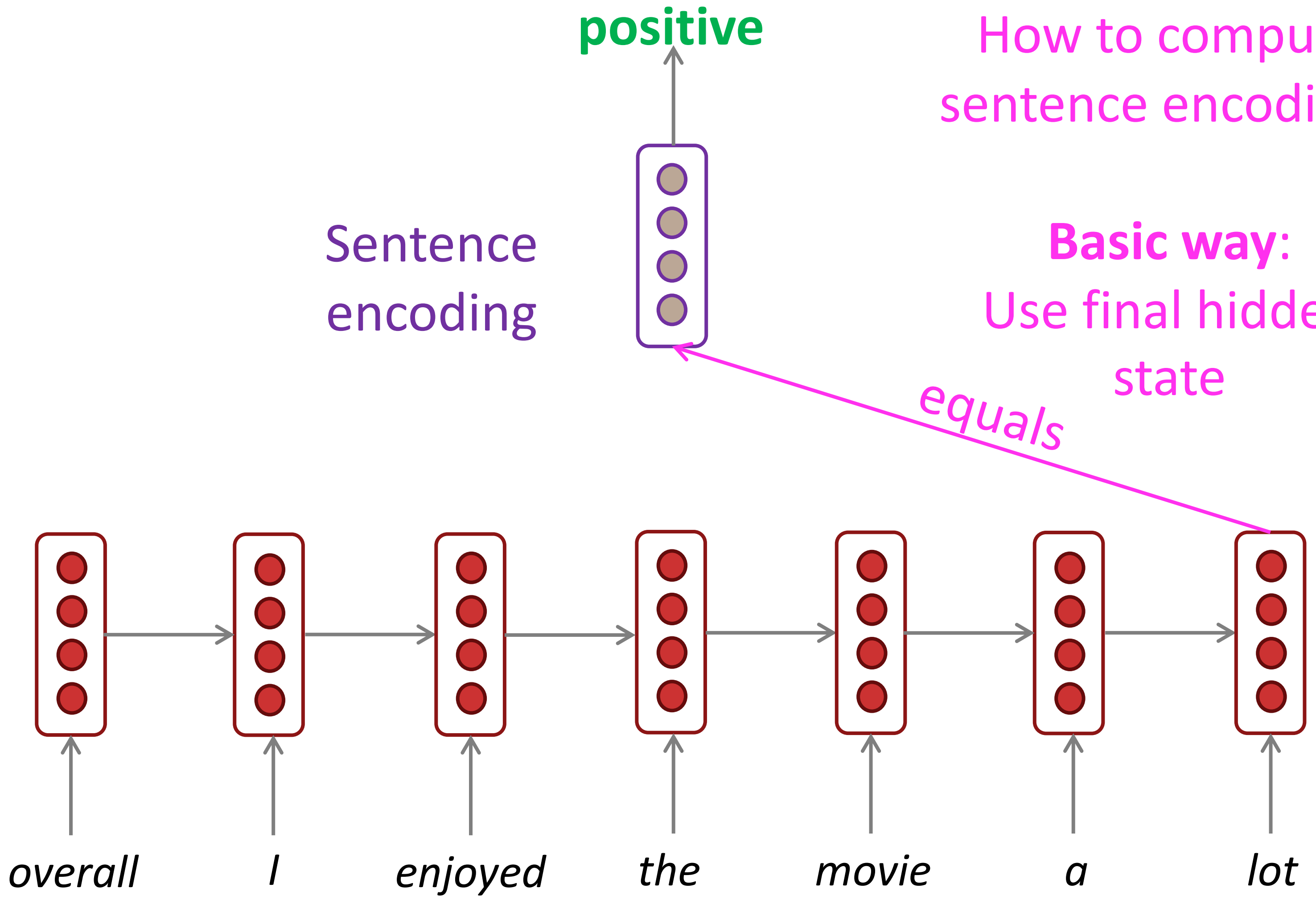
RNNs can be used for sentence classification

e.g., **sentiment classification**



RNNs can be used for sentence classification

e.g., **sentiment classification**

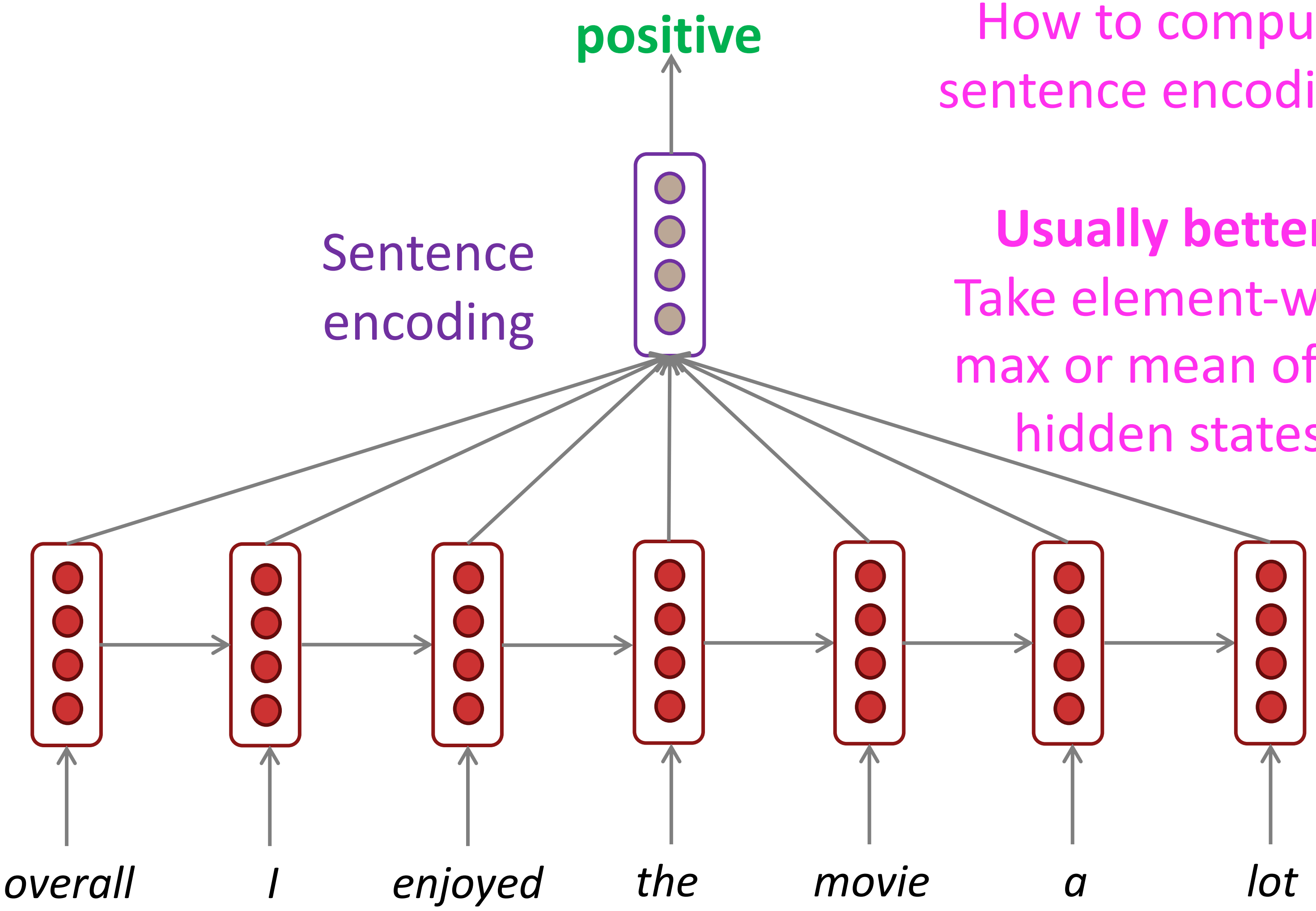


How to compute sentence encoding?

Basic way:
Use final hidden state

RNNs can be used for sentence classification

e.g., **sentiment classification**



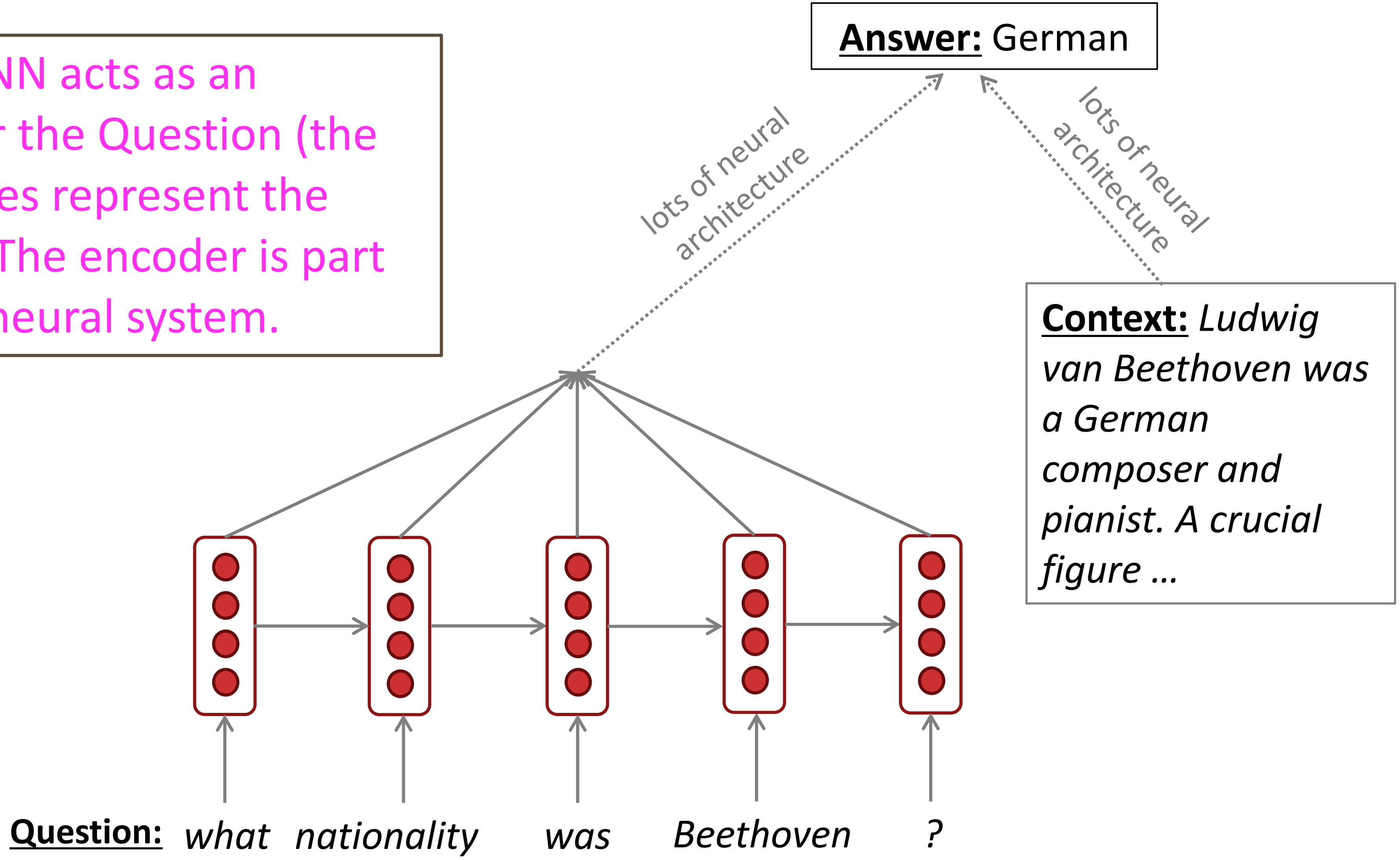
How to compute sentence encoding?

Usually better: Take element-wise max or mean of all hidden states

RNNs can be used as an encoder module

e.g., **question answering**, machine translation, *many other tasks!*

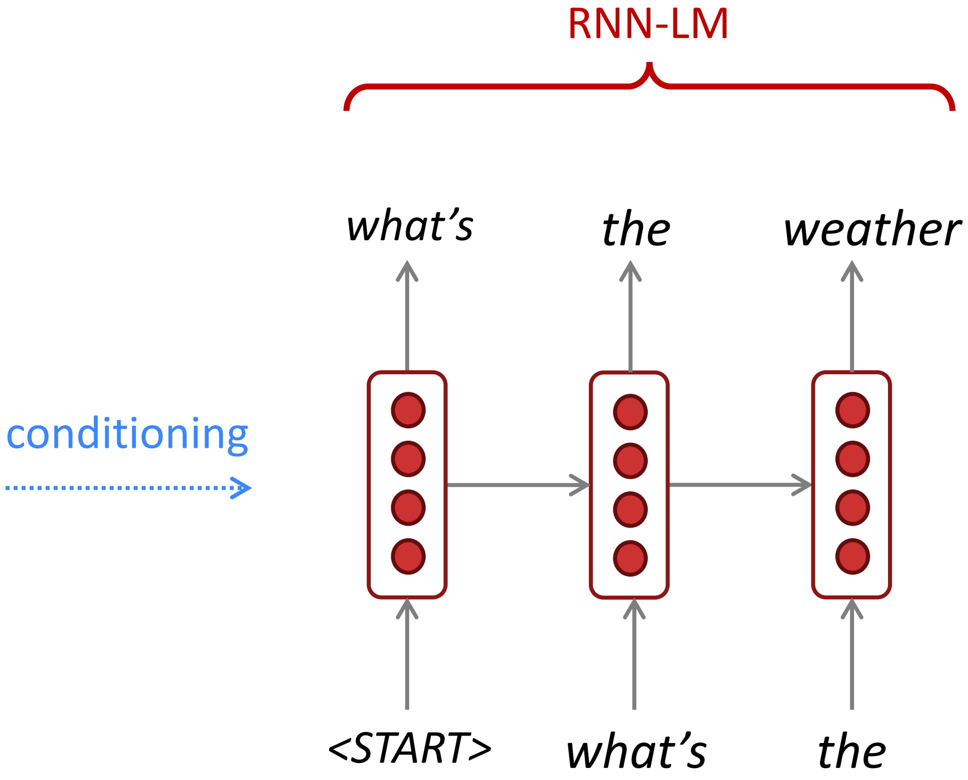
Here the RNN acts as an **encoder** for the Question (the hidden states represent the Question). The encoder is part of a larger neural system.



RNN-LMs can be used to generate text

e.g., **speech recognition**, machine translation, summarization

Input (audio)



This is an example of a *conditional language model*.

We'll see Machine Translation in much more detail later.

Terminology and a look forward

The RNN described in this lecture = **simple**/vanilla/**Elman** RNN

Next lecture: You will learn about other RNN flavors

like GRU  and LSTM  and multi-layer RNNs 

By the end of the course: You will understand phrases like
"stacked bidirectional LSTM with residual connections and self-attention"