

Lecture 12: Batch RL

Emma Brunskill

CS234 Reinforcement Learning.

Winter 2018

Slides drawn from Philip Thomas with modifications

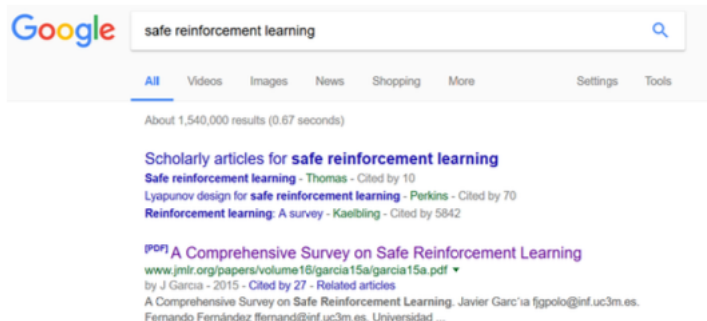
Class Structure

- Last time: Fast Reinforcement Learning / Exploration and Exploitation
- **This time: Batch RL**
- Next time: Monte Carlo Tree Search

Table of Contents

- 1 What makes an RL algorithm safe?
- 2 Notation
- 3 Create a safe batch reinforcement learning algorithm
 - Off-policy policy evaluation (OPE)
 - High-confidence off-policy policy evaluation (HCOPE)
 - Safe policy improvement (SPI)

What does it mean to for a reinforcement learning algorithm to be safe?



A Comprehensive Survey on Safe Reinforcement Learning

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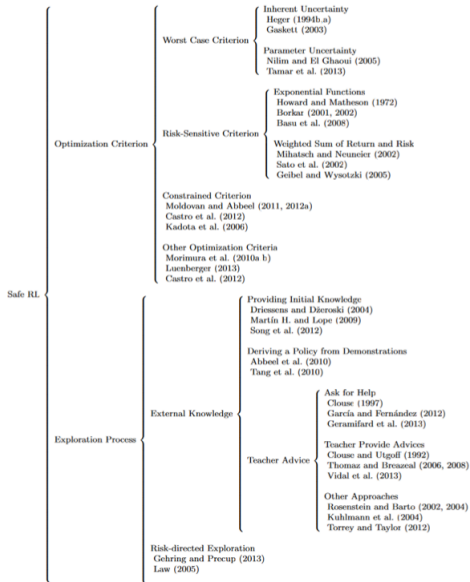
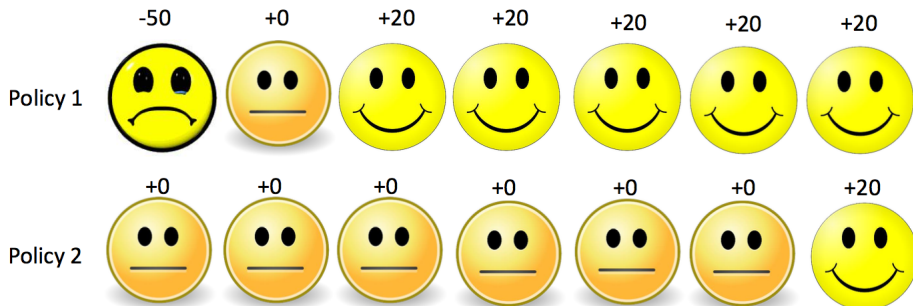


Table 1: Overview of the approaches for Safe Reinforcement Learning considered in this survey.

Changing the objective



Changing the objective

- Policy 1:
 - Reward = 0 with probability 0.999999
 - Reward = 10^9 with probability $1-0.999999$
 - Expected reward approximately 1000
- Policy 2:
 - Reward = 999 with probability 0.5
 - Reward = 1000 with probability 0.5
 - Expected reward 999.5

Safe and efficient off-policy reinforcement learning

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Another notion of safety (Munos et. al)

We start from the recent work of [Harutyunyan et al. \(2016\)](#), who show that naive off-policy policy evaluation, without correcting for the “off-policyness” of a trajectory, still converges to the desired Q^π value function provided the behavior μ and target π policies are not too far apart (the maximum allowed distance depends on the λ parameter). Their $Q^\pi(\lambda)$ algorithm learns from trajectories generated by μ simply by summing discounted off-policy corrected rewards at each time step. Unfortunately, the assumption that μ and π are close is restrictive, as well as difficult to uphold in the control case, where the target policy is greedy with respect to the current Q-function. **In that sense this algorithm is not *safe*: it does not handle the case of arbitrary “off-policyness”.**

Alternatively, the Tree-backup (TB(λ)) algorithm ([Precup et al., 2000](#)) tolerates arbitrary target/behavior discrepancies by scaling information (here called *traces*) from future temporal differences by the product of target policy probabilities. TB(λ) is not *efficient* in the “near on-policy” case (similar μ and π), though, as traces may be cut prematurely, blocking learning from full returns.

Reachability-Based Safe Learning with Gaussian Processes

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Shahab Kaynama

Jaime F. Fisac*
Melanie N. Zeilinger

Jeremy H. Gillula
Claire J. Tomlin

SAFE REINFORCEMENT LEARNING

A Dissertation Presented

by

PHILIP S. THOMAS

The Problem

- If you apply an existing method, do you have confidence that it will work?

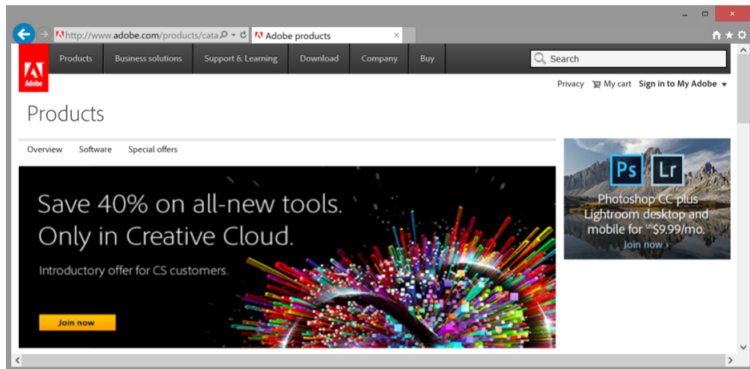
Reinforcement learning success



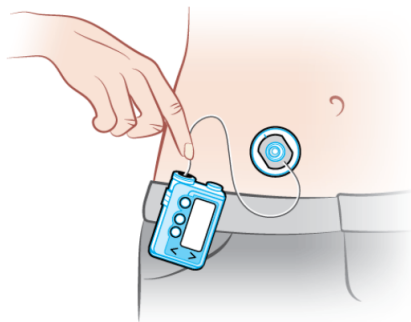
A property of many real applications

- Deploying "bad" policies can be costly or dangerous

Deploying bad policies can be costly



Deploying bad policies can be dangerous



What property should a safe batch reinforcement learning algorithm have?

- Given past experience from current policy/policies, produce a new policy
 - “Guarantee that with probability at least $1 - \delta$, will not change your policy to one that is worse than the current policy.”
 - You get to choose δ
 - Guarantee not contingent on the tuning of any hyperparameters

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- Policy π : $\pi(a) = P(a_t = a \mid s_t = s)$
- History: $H = (s_1, a_1, r_1, s_2, a_2, r_2, \dots, s_L, a_L, r_L)$
- Historical data: $D = \{H_1, H_2, \dots, H_n\}$
- Historical data from behavior policy, π_b
- Objective:

$$V^\pi = \mathbb{E}[\sum_{t=1}^L \gamma^t R_t \mid \pi]$$

Safe batch reinforcement learning algorithm

- Reinforcement learning algorithm, $\mathcal{A}$
- Historical data, D , which is a random variable
- Policy produced by the algorithm, $\mathcal{A}(D)$, which is a random variable
- a safe batch reinforcement learning algorithm, $\mathcal{A}$, satisfies:

$$\Pr(V^{\mathcal{A}(D)} \geq V^{\pi_b} \geq 1 - \delta)$$

or, in general

$$\Pr(V^{\mathcal{A}(D)} \geq V_{min}) \geq 1 - \delta$$

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Create a safe batch reinforcement learning algorithm

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 - Use a concentration inequality to convert the n independent and unbiased estimates of V^{π_e} into a $1 - \delta$ confidence lower bound on V^{π_e}
- Safe policy improvement (SPI)
 - Use HCOPE method to create a safe batch reinforcement learning algorithm, a

Off-policy policy evaluation (OPE)



Importance Sampling (Reminder)

$$IS(D) = \frac{1}{n} \sum_{i=1}^n \left(\prod_{t=1}^L \frac{\pi_e(a_t | s_t)}{\pi_b(a_t | s_t)} \right) \left(\sum_{t=1}^L \gamma^t R_t^i \right)$$

$$\mathbb{E}[IS(D)] = V^{\pi_e}$$

Create a safe batch reinforcement learning algorithm

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High-confidence off-policy policy evaluation (HCOPE)



Hoeffding's inequality

- Let $X_1, \dots, X_n$ be n independent identically distributed random variables such that $X_i \in [0, b]$
- Then with probability at least $1 - \delta$:

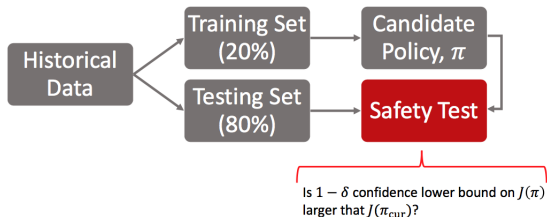
$$\mathbb{E}[X_i] \geq \frac{1}{n} \sum_{i=1}^n X_i - b \sqrt{\frac{\ln(1/\delta)}{2n}},$$

where $X_i = \frac{1}{n} \sum_{i=1}^n (w_i \sum_{t=1}^L \gamma^t R_t^i)$ in our case.

Safe policy improvement (SPI)



Safe policy improvement (SPI)



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WON'T WORK!

Off-policy policy evaluation (revisited)

- Importance sampling (IS):

$$IS(D) = \frac{1}{n} \sum_{i=1}^n \left(\prod_{t=1}^L \frac{\pi_e(a_t | s_t)}{\pi_b(a_t | s_t)} \right) \left(\sum_{t=1}^L \gamma^t R_t^i \right)$$

- Per-decision importance sampling (PDIS)

$$PSID(D) = \sum_{t=1}^L \gamma^t \frac{1}{n} \sum_{i=1}^n \left(\prod_{\tau=1}^t \frac{\pi_e(a_\tau | s_\tau)}{\pi_b(a_\tau | s_\tau)} \right) R_t^i$$

Off-policy policy evaluation (revisited)

- Importance sampling (IS):

$$IS(D) = \frac{1}{n} \sum_{i=1}^n w_i \left(\sum_{t=1}^L \gamma^t R_t^i \right)$$

- Weighted importance sampling (WIS)

$$WIS(D) = \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i \left(\sum_{t=1}^L \gamma^t R_t^i \right)$$

Off-policy policy evaluation (revisited)

- Weighted importance sampling (WIS)

$$WIS(D) = \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i \left(\sum_{t=1}^L \gamma^t R_t^i \right)$$

- NOT unbiased. When $n = 1$, $\mathbb{E}[WIS] = J(\pi_b)$
- Strongly consistent estimator of V^{π_e}
 - i.e. $\Pr(\lim_{n \rightarrow \infty} WIS(D) = V^{\pi_e}) = 1$
 - If
 - Finite horizon
 - One behavior policy, or bounded rewards

Off-policy policy evaluation (revisited)

- Weighted per-decision importance sampling
 - Also called consistent weighted per-decision importance sampling
 - A fun exercise!

Control variates

- Given: X
- Estimate: $\mu = \mathbb{E}[X]$
- $\hat{\mu} = X$
- Unbiased: $\mathbb{E}[\hat{\mu}] = \mathbb{E}[X] = \mu$
- Variance: $\text{Var}(\hat{\mu}) = \text{Var}(X)$

Control variates

- Given: $X, Y, \mathbb{E}[Y]$
- Estimate: $\mu = \mathbb{E}[X]$
- $\hat{\mu} = X - Y + \mathbb{E}[Y]$
- Unbiased:
$$\mathbb{E}[\hat{\mu}] = \mathbb{E}[X - Y + \mathbb{E}[Y]] = \mathbb{E}[X] - \mathbb{E}[Y] + \mathbb{E}[Y] = \mathbb{E}[X] = \mu$$
- Variance:

$$\begin{aligned}\text{Var}(\hat{\mu}) &= \text{Var}(X - Y + \mathbb{E}[Y]) = \text{Var}(X - Y) \\ &= \text{Var}(X) + \text{Var}(Y) - 2\text{Cov}(X, Y)\end{aligned}$$

- Lower variance if $2\text{Cov}(X, Y) > \text{Var}(Y)$
- We call Y a control variate
- We saw this idea before: baseline term in policy gradient estimation

Off-policy policy evaluation (revisited)

- Idea: add a control variate to importance sampling estimators
 - X is the importance sampling estimator
 - Y is a control variate build from an approximate model of the MDP
 - $\mathbb{E}[Y] = 0$ in this case
 - $PDIS_{CV}(D) = PDIS(D) - CV(D)$
- Called the doubly robust estimator (Jiang and Li, 2015)
 - Robust to (1) poor approximate model, and (2) error in estimates of π_b
 - If the model is poor, the estimates are still unbiased
 - If the sampling policy is unknown, but the model is good, MSE will still be low
 - $DR(D) = PDIS_{CV}(D)$
- Non-recursive and weighted forms, as well as control variate view provided by Thomas and Brunskill (2016)

Off-policy policy evaluation (revisited)

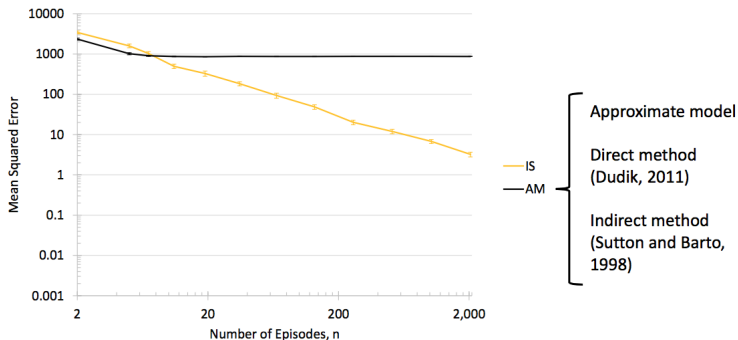
$$DR(\pi_e \mid D) = \frac{1}{n} \sum_{i=1}^n \sum_{t=0}^{\infty} \gamma^t w_t^i (R_t^i - \hat{q}^{\pi_e}(S_t^i, A_t^i)) + \gamma^t \rho_{t-1}^i \hat{v}^{\pi_e}(S_t^i),$$

where $w_t^i = \prod_{\tau=1}^t \frac{\pi_e(a_\tau \mid s_\tau)}{\pi_b(a_\tau \mid s_\tau)}$

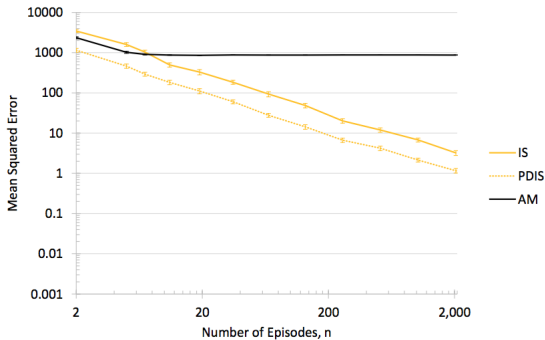
- Recall: we want the control variate Y to cancel with X :

$$R - q(S, A) + \gamma v(S')$$

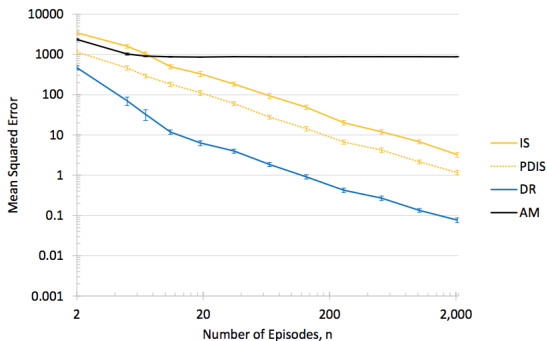
Empirical Results (Gridworld)



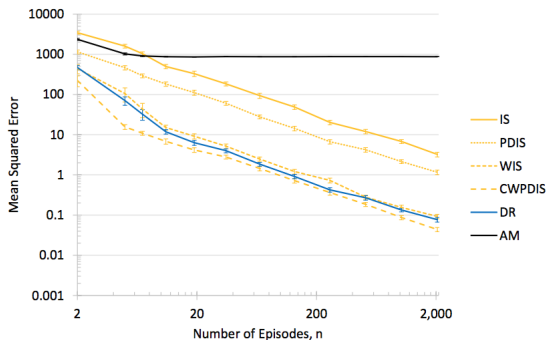
Empirical Results (Gridworld)



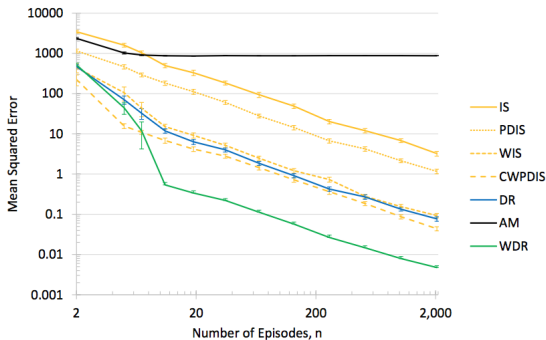
Empirical Results (Gridworld)



Empirical Results (Gridworld)



Empirical Results (Gridworld)

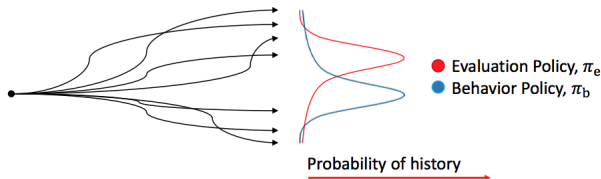


Off-policy policy evaluation (revisited): Blending

- Importance sampling is unbiased but high variance
- Model based estimate is biased but low variance
- Doubly robust is one way to combine the two
- Can also trade between importance sampling and model based estimate within a trajectory
- MAGIC estimator (Thomas and Brunskill 2016)
- Can be particularly useful when part of the world is non-Markovian in the given model, and other parts of the world are Markov

Off-policy policy evaluation (revisited)

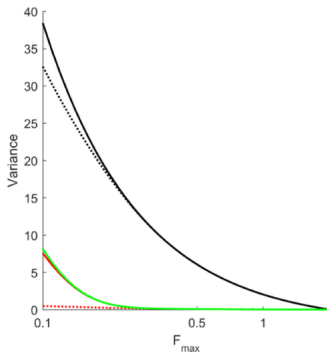
- What if $\text{supp}(\pi_e \subset \text{supp}(\pi_b))$
- There is a state-action pair, (s, a) , such that $\pi_e(a | s) = 0$, but $\pi_b(a | s) \neq 0$.
- If we see a history where (s, a) occurs, what weight should we give it?
- $IS(D) = \frac{1}{n} \sum_{i=1}^n \left(\prod_{t=1}^L \frac{\pi_e(a_t | s_t)}{\pi_b(a_t | s_t)} \right) \left(\sum_{t=1}^L \gamma^t R_t^i \right)$



Off-policy policy evaluation (revisited)

- What if there are zero samples ($n = 0$)?
 - The importance sampling estimate is undefined
- What if no samples are in $\text{supp}(\pi_e)$ (or $\text{supp}(p)$ in general)?
 - Importance sampling says: the estimate is zero
 - Alternate approach: undefined
- Importance sampling estimator is unbiased if $n > 0$
- Alternate approach will be unbiased given that at least one sample is in the support of p
- Alternate approach detailed in Importance Sampling with Unequal Support (Thomas and Brunskill, AAAI 2017)

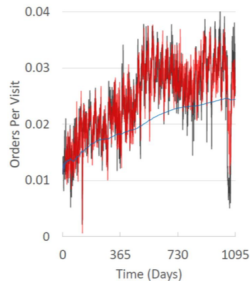
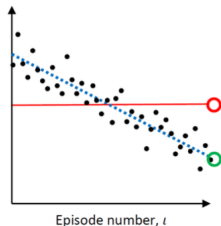
Off-policy policy evaluation (revisited)



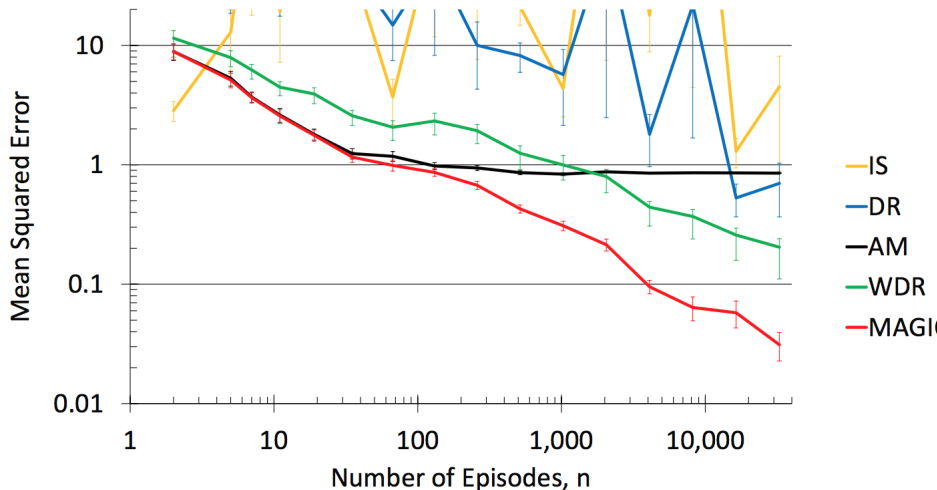
— IS, IS (conditional), — US, US (conditional) — MSE(US)

Off-policy policy evaluation (revisited)

- Thomas et. al. Predictive Off-Policy Policy Evaluation for Nonstationary Decision Problems, with Applications to Digital Marketing (AAAI 2017)



Off-policy policy evaluation (revisited)



Create a safe batch reinforcement learning algorithm

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High-confidence off-policy policy evaluation (revisited)

- Consider using IS + Hoeffding's inequality for HCOPE on mountain car

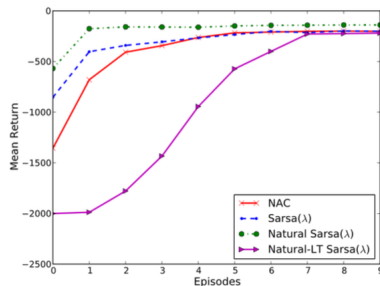
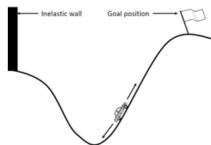
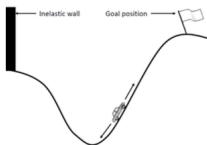


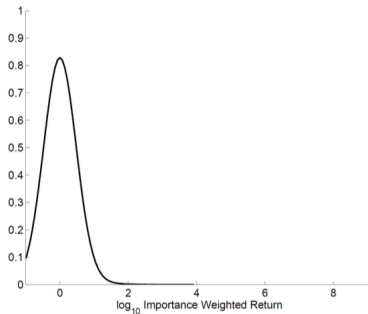
Figure 3: Mountain Car (Sarsa(λ))
Natural Temporal Difference Learning, Dabney and Thomas, 2014

High-confidence off-policy policy evaluation (revisited)

- Using 100,000 trajectories
- Evaluation policy's true performance is $0.19 \in [0, 1]$
- We get a 95% confidence lower bound of: -5,8310,000



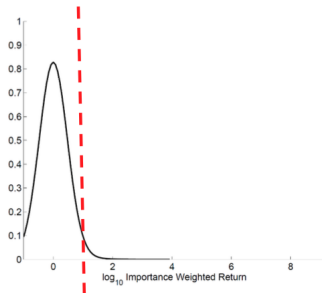
What went wrong



$$w_i = \prod_{t=1}^L \frac{\pi_e(a_t | s_t)}{\pi_b(a_t | s_t)}$$

High-confidence off-policy policy evaluation (revisited)

- Removing the upper tail only decreases the expected value.



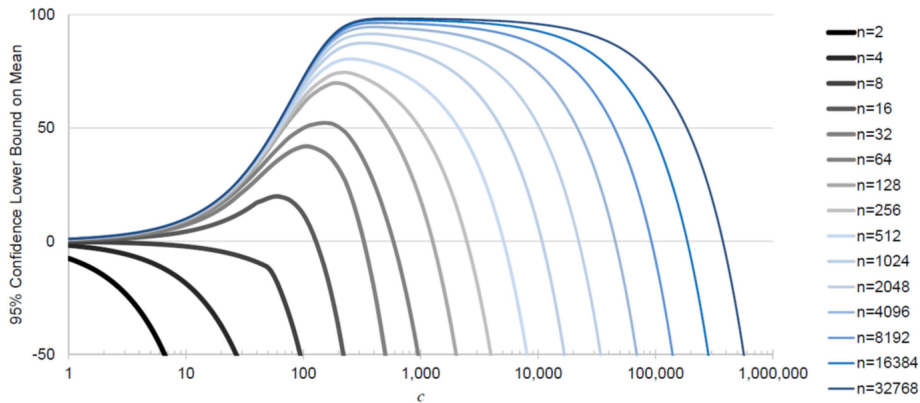
High-confidence off-policy policy evaluation (revisited)

- Thomas et. al, High confidence off-policy evaluation, AAAI 2015

Theorem 1. Let $X_1, \dots, X_n$ be n independent real-valued random variables such that for each $i \in \{1, \dots, n\}$, we have $\mathbb{P}[0 \leq X_i] = 1$, $\mathbb{E}[X_i] \leq \mu$, and some threshold value $c_i > 0$. Let $\delta > 0$ and $Y_i := \min\{X_i, c_i\}$. Then with probability at least $1 - \delta$, we have

$$\mu \geq \underbrace{\left(\sum_{i=1}^n \frac{1}{c_i}\right)^{-1} \sum_{i=1}^n \frac{Y_i}{c_i}}_{\text{empirical mean}} - \underbrace{\left(\sum_{i=1}^n \frac{1}{c_i}\right)^{-1} \frac{7n \ln(2/\delta)}{3(n-1)}}_{\text{term that goes to zero as } 1/n \text{ as } n \rightarrow \infty} - \underbrace{\left(\sum_{i=1}^n \frac{1}{c_i}\right)^{-1} \sqrt{\frac{\ln(2/\delta)}{n-1} \sum_{i,j=1}^n \left(\frac{Y_i}{c_i} - \frac{Y_j}{c_j}\right)^2}}_{\text{term that goes to zero as } 1/\sqrt{n} \text{ as } n \rightarrow \infty}. \quad (3)$$

High-confidence off-policy policy evaluation (revisited)



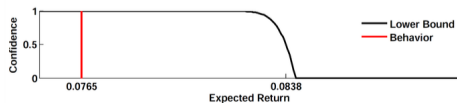
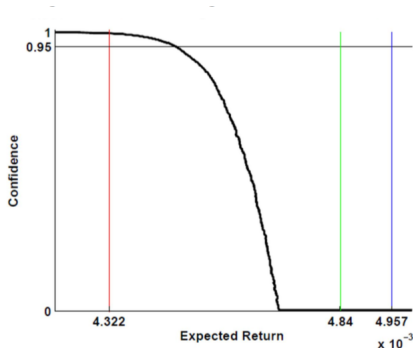
High-confidence off-policy policy evaluation (revisited)

- Use 20% of the data to optimize c
- Use 80% to compute lower bound with optimized c
- Mountain car results:

	CUT	Chernoff-Hoeffding	Maurer	Anderson	Bubeck et al.
95% Confidence lower bound on the mean	0.145	-5,831,000	-129,703	0.055	-.046

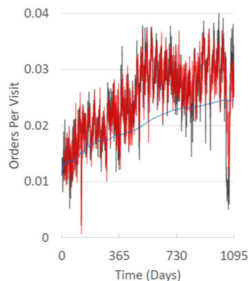
High-confidence off-policy policy evaluation (revisited)

Digital marketing:



High-confidence off-policy policy evaluation (revisited)

Cognitive dissonance:



$$\mathbb{E}[X_i] \geq \frac{1}{n} \sum_{i=1}^n X_i - b \sqrt{\frac{\ln(1/\delta)}{2n}}$$

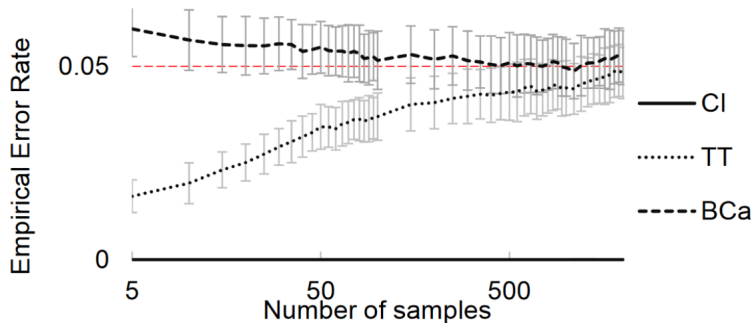
High-confidence off-policy policy evaluation (revisited)

- Student's t-test
 - Assumes that $IS(D)$ is normally distributed
 - By the central limit theorem, it (is as $n \rightarrow \infty$)

$$\Pr \left(\mathbb{E} \left[\frac{1}{n} \sum_{i=1}^n X_i \right] \geq \frac{1}{n} \sum_{i=1}^n X_i \right) = \frac{\sqrt{\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2}}{\sqrt{n}} t_{1-\delta, n-1} \\ \geq 1 - \delta$$

- Efron's Bootstrap methods (e.g., BCa)
 - Also, without importance sampling: Hanna, Stone, and Niekum, AAMAS 2017

High-confidence off-policy policy evaluation (revisited)



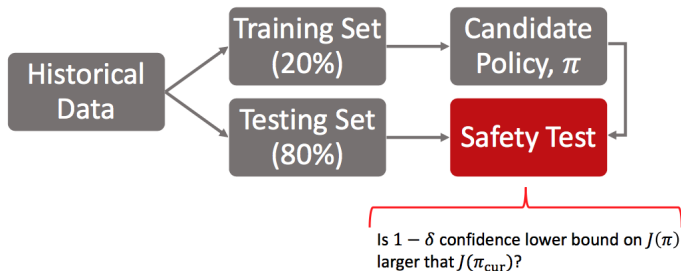
P. S. Thomas. Safe reinforcement learning (PhD Thesis, 2015)

Create a safe batch reinforcement learning algorithm

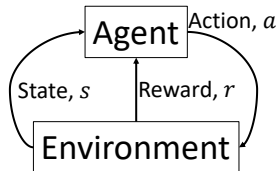
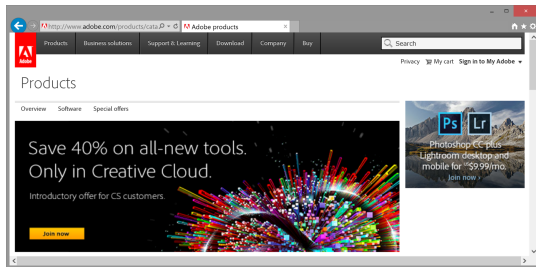
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Safe policy improvement (revisited)

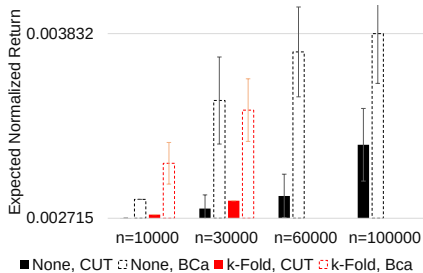
Thomas et. al, ICML 2015



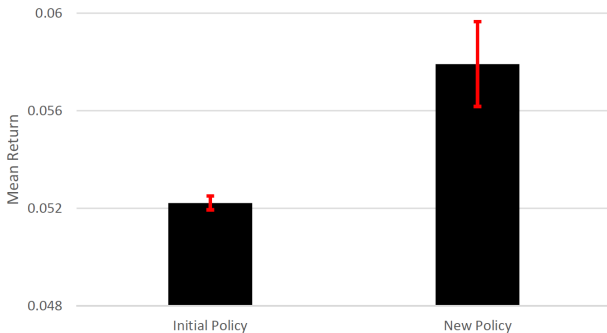
Empirical Results: Digital Marketing



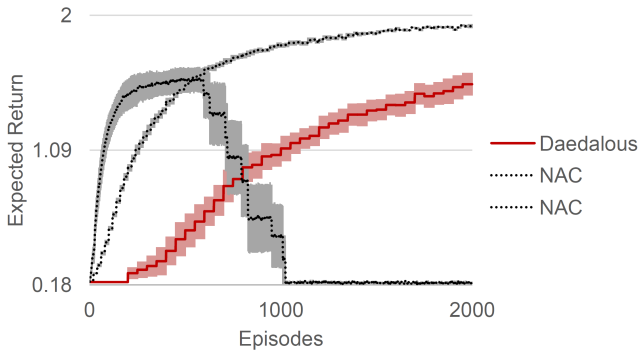
Empirical Results: Digital Marketing



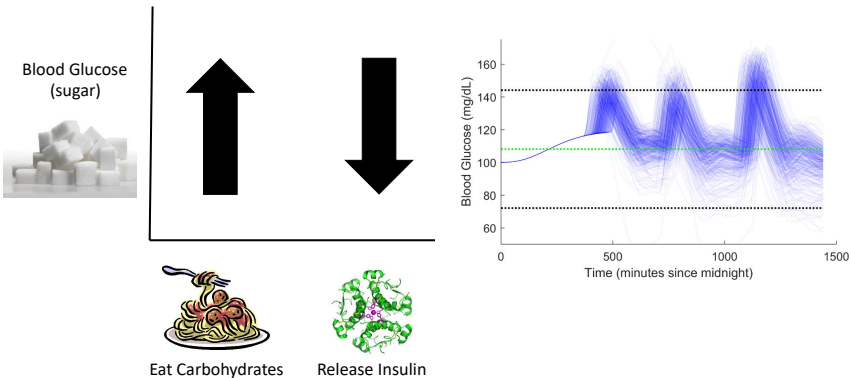
Empirical Results: Digital Marketing



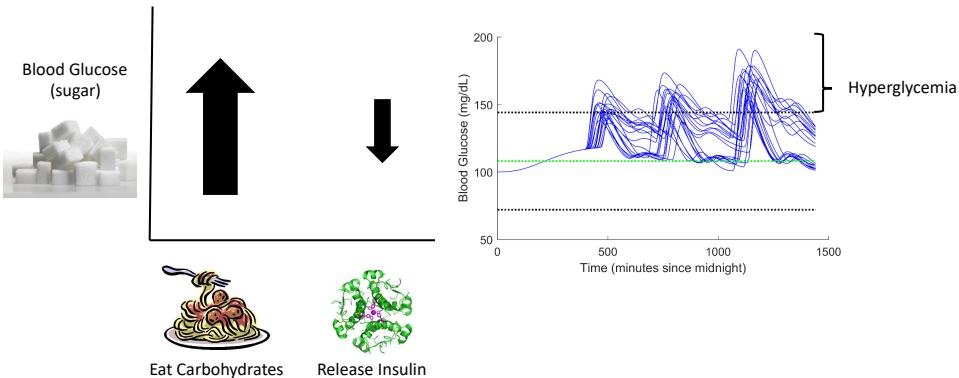
Empirical Results: Digital Marketing



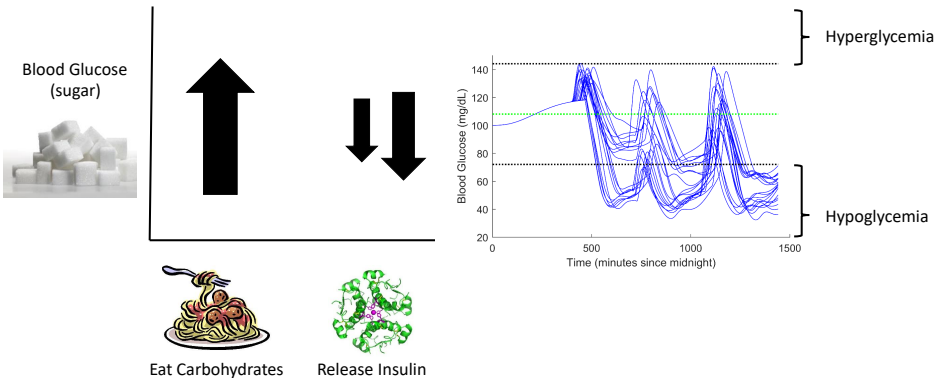
Example Results : Diabetes Treatment



Example Results : Diabetes Treatment



Example Results : Diabetes Treatment



Example Results : Diabetes Treatment

$$\text{injection} = \frac{\text{blood glucose} - \text{target blood glucose}}{CF} + \frac{\text{meal size}}{CR}$$

January							February							March						
S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S
1	2	3	4	5	6	7				1	2	3	4				1	2	3	4
8	9	10	11	12	13	14	5	6	7	8	9	10	11	5	6	7	8	9	10	11
15	16	17	18	19	20	21	12	13	14	15	16	17	18	12	13	14	15	16	17	18
22	23	24	25	26	27	28	19	20	21	22	23	24	25	19	20	21	22	23	24	25
29	30	31					26	27	28					26	27	28	29	30	31	

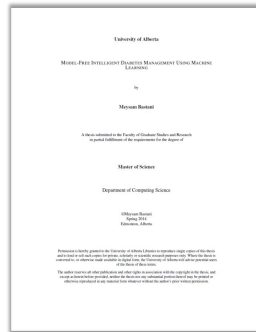
April							May							June							
S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	
						1				1	2	3	4	5	6				1	2	3
2	3	4	5	6	7	8	7	8	9	10	11	12	13	4	5	6	7	8	9	10	
9	10	11	12	13	14	15	14	15	16	17	18	19	20	11	12	13	14	15	16	17	
16	17	18	19	20	21	22	21	22	23	24	25	26	27	18	19	20	21	22	23	24	
23	24	25	26	27	28	29	28	29	30	31				25	26	27	28	29	30		
30																					

Example Results : Diabetes Treatment

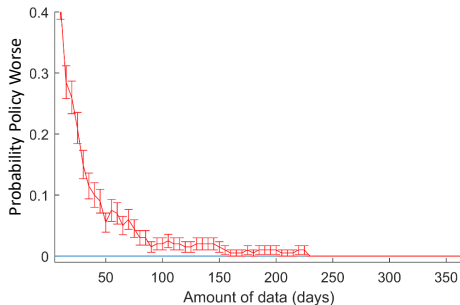
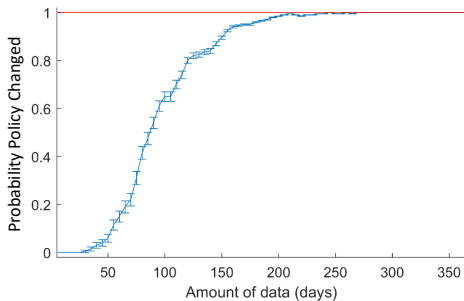


Intelligent Diabetes Management

T1DMS
Type 1 Diabetes Metabolic Simulator



Example Results : Diabetes Treatment



Other Relevant Work

- How to deal with long horizons? (Guo, Thomas, Brunskill NIPS 2017)
- How to deal with importance sampling being “unfair”? (Doroudi, Thomas and Brunskill, best paper UAI 2017)
- What to do when the behavior policy is not known?
- What to do when the behavior policy is deterministic?
- What to do when care about doing safe exploration?
- What to do when care about performance on a single trajectory
- For last two, see great work by Marco Pavone’s group, Pieter Abbeel’s group, Shie Mannor’s group and Claire Tomlin’s group, amongst others

Off Policy Policy Evaluation and Selection

- Very important topic: healthcare, education, marketing, ...
- Insights are relevant to on policy learning
- Big focus of my lab
- A number of others on campus also working in this area (e.g. Stefan Wager, Susan Athey...)
- Very interesting area at the intersection of causality and control

What You Should Know: Off Policy Policy Evaluation and Selection

- Be able to define and apply importance sampling for off policy policy evaluation
- Define some limitations of IS (variance)
- List a couple alternatives (weighted IS, doubly robust)
- Define why we might want safe reinforcement learning
- Define the scope of the guarantees implied by safe policy improvement as defined in this lecture

Class Structure

- Last time: Exploration and Exploitation
- **This time: Batch RL**
- Next time: Monte Carlo Tree Search