Lecture 14: MCTS [2]

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CS234 Reinforcement Learning.

Winter 2018

[2] With many slides from or derived from David Silver

Class Structure

- Last time: Batch RL
- **This Time: MCTS**
- Next time: Human in the Loop RL

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Model-Based Reinforcement Learning

- *Last lecture*: learn policy directly from experience
- *Previous lectures*: learn value function directly from experience
- *This lecture*: learn model directly from experience
- and use planning to construct a value function or policy
- Integrate learning and planning into a single architecture

Model-Based and Model-Free RL

- Model-Free RL
  - No model
  - Learn value function (and/or policy) from experience

Model-Based and Model-Free RL

- Model-Free RL
  - No model
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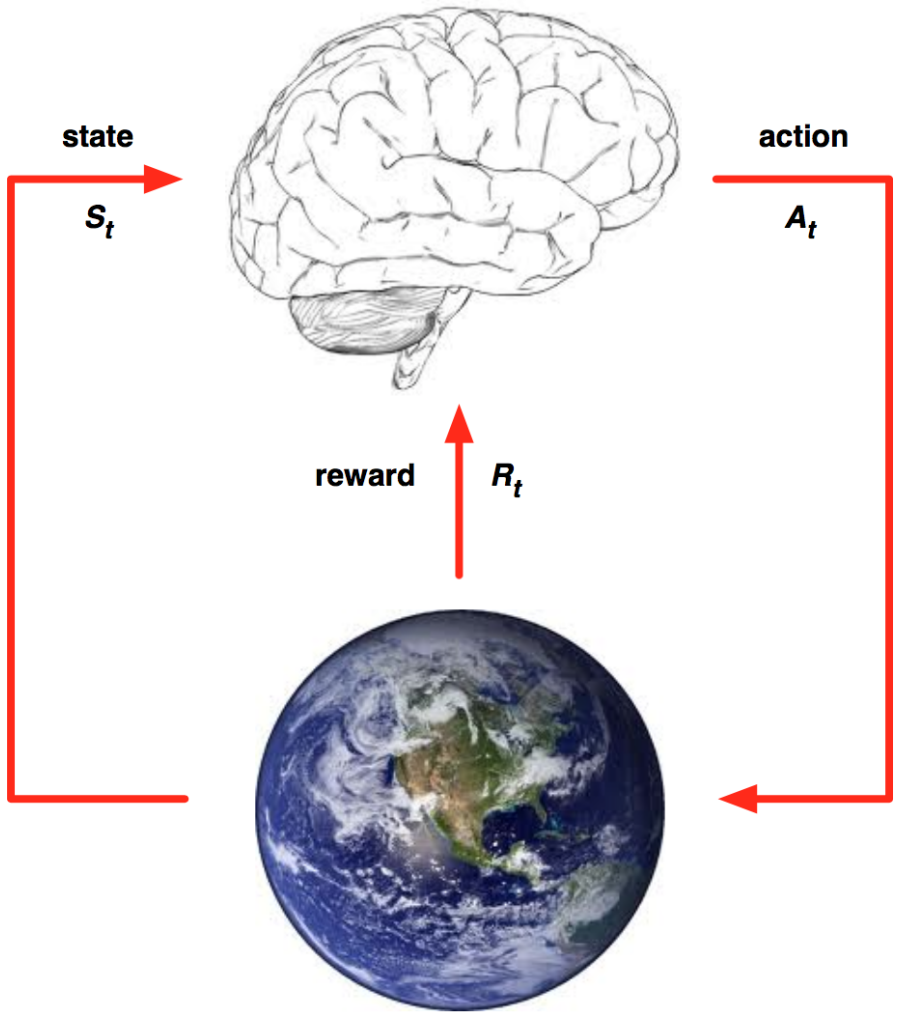
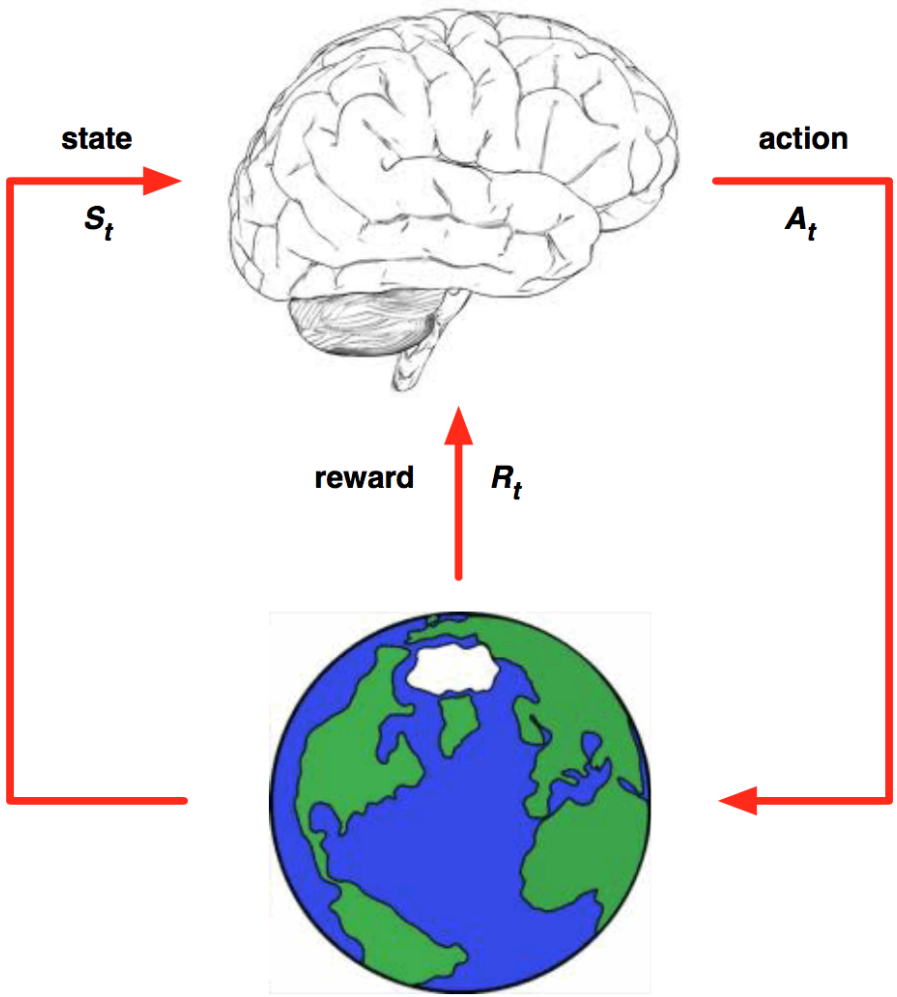
- Model-Based RL
  - Learn a model from experience
  - Plan value function (and/or policy) from model

Model-Free RL



Model-Based RL



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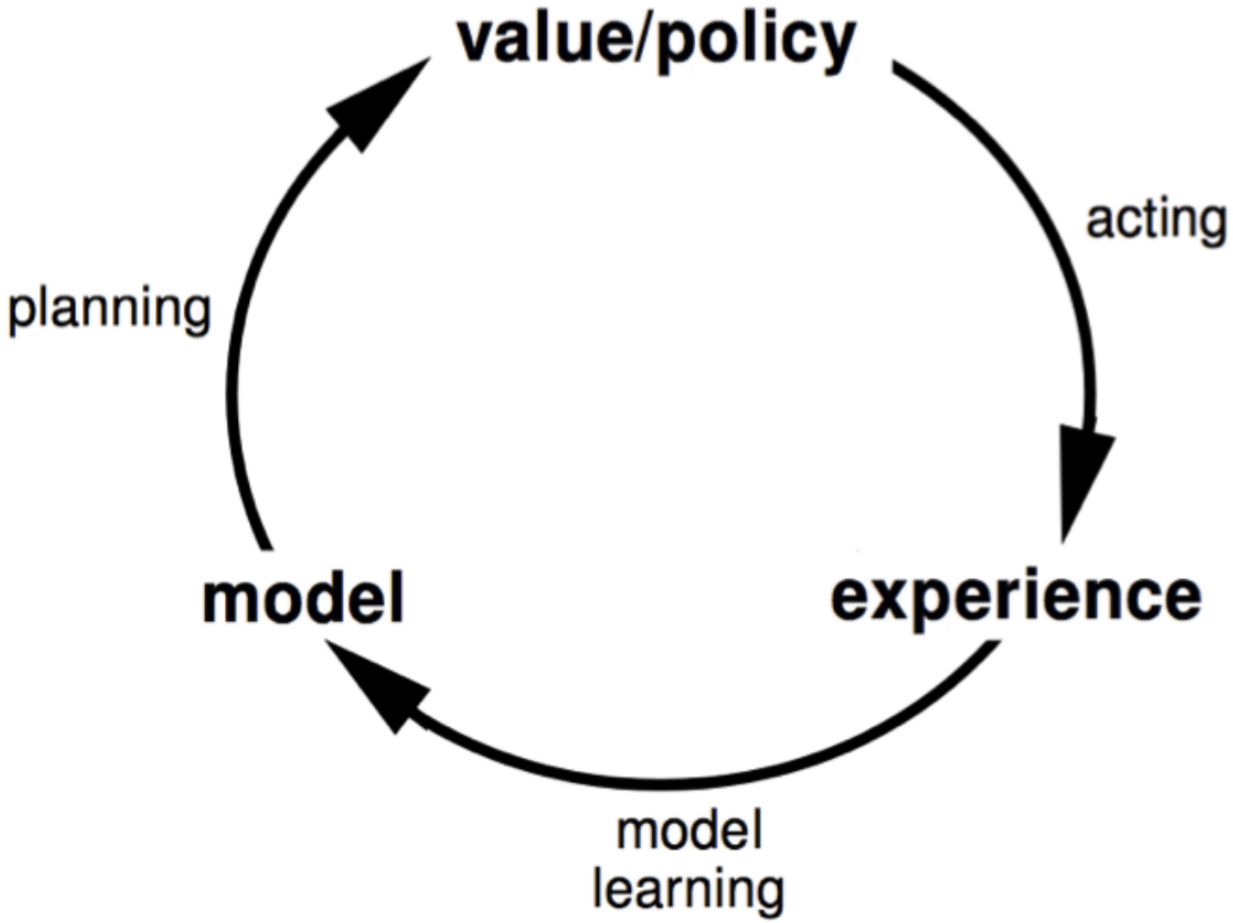
Model-Based RL



Advantages of Model-Based RL

- Advantages:
  - Can efficiently learn model by supervised learning methods
  - Can reason about model uncertainty
- Disadvantages
  - First learn a model, then construct a value function
    $\Rightarrow$ two sources of approximation error

What is a Model?

- A model $\mathcal{M}$ is a representation of an MDP $< \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} >$, parametrized by η
- We will assume state space $\mathcal{S}$ and action space $\mathcal{A}$ are known
- So a model $\mathcal{M} =< \mathcal{P}_\eta, \mathcal{R}_\eta >$ represents state transitions $\mathcal{P}_\eta \approx \mathcal{P}$ and rewards $\mathcal{R}_\eta \approx \mathcal{R}$

$$S_{t+1} \sim \mathcal{P}_\eta(S_{t+1} \mid S_t, A_t)$$

$$R_{t+1} = \mathcal{R}_\eta(R_{t+1} \mid S_t, A_t)$$

- Typically assume conditional independence between state transitions and rewards

$$\mathbb{P}[S_{t+1}, R_{t+1} \mid S_t, A_t] = \mathbb{P}[S_{t+1} \mid S_t, A_t]\mathbb{P}[R_{t+1} \mid S_t, A_t]$$

Model Learning

- Goal: estimate model $\mathcal{M}_\eta$ from experience $\{S_1, A_1, R_2, ..., S_T\}$
- This is a supervised learning problem

$$S_1, A_1 \rightarrow R_2, S_2$$
$$S_2 A_2 \rightarrow R_3, S_3$$
$$\vdots$$
$$S_{T-1}, A_{T-1} \rightarrow R_T, S_T$$

- Learning $s, a \rightarrow r$ is a regression problem
- Learning $s, a \rightarrow s'$ is a density estimation problem
- Pick loss function, e.g. mean-squared error, KL divergence, ...
- Find parameters η that minimize empirical loss

Examples of Models

- Table Lookup Model
- Linear Expectation Model
- Linear Gaussian Model
- Gaussian Process Model
- Deep Belief Network Model
- ...

Table Lookup Model

- Model is an explicit MDP, $\hat{\mathcal{P}}, \hat{\mathcal{R}}$
- Count visits $N(s, a)$ to each state action pair

$$\hat{\mathcal{P}}^a_{s,s'} = \frac{1}{N(s,a)} \sum_{t=1}^{T} \mathbb{1}(S_t, A_t, S_{t+1} = s, a, s')$$

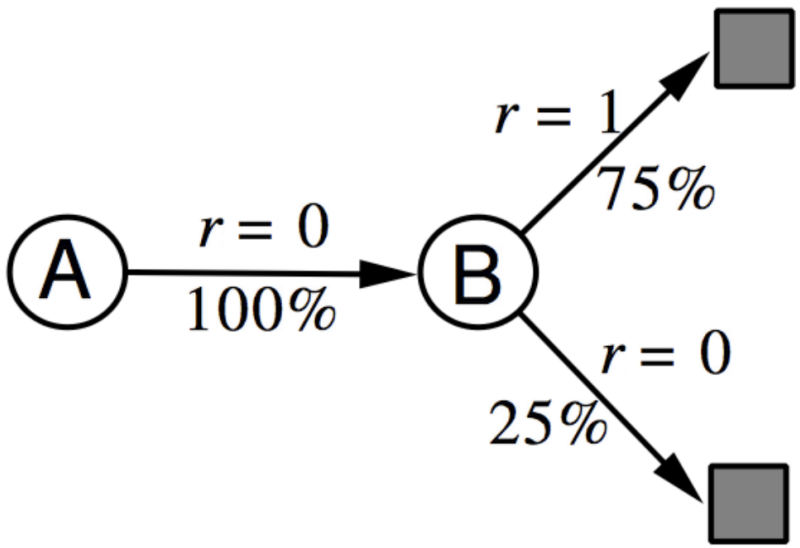
$$\hat{\mathcal{R}}^a_s = \frac{1}{N(s,a)} \sum_{t=1}^{T} \mathbb{1}(S_t, A_t = s, a)$$

- Alternatively
  - At each time-step t, record experience tuple $< S_t, A_t, R_{t+1}, S_{t+1} >$
  - To sample model, randomly pick tuple matching $< s, a, \cdot, \cdot >$

AB Example

- Two states A,B; no discounting; 8 episodes of experience

$$\begin{aligned}&\mathrm{A}, 0, \mathrm{B}, 0\\&\mathrm{B}, 1\\&\mathrm{B}, 1\\&\mathrm{B}, 1\\&\mathrm{B}, 1\\&\mathrm{B}, 1\\&\mathrm{B}, 1\\&\mathrm{B}, 0\end{aligned}$$



- We have constructed a table lookup model from the experience

Planning with a Model

- Given a model $\mathcal{M}_\eta =< \mathcal{P}_\eta, \mathcal{R}_\eta >$
- Solve the MDP $< \mathcal{S}, \mathcal{A}, \mathcal{P}_\eta, \mathcal{R}_\eta >$
- Using favourite planning algorithm
  - Value iteration
  - Policy iteration
  - Tree search
  - ...

Sample-Based Planning

- A simple but powerful approach to planning
- Use the model only to generate samples
- Sample experience from model

$$S_{t+1} \sim \mathcal{P}_\eta(S_{t+1} \mid S_t, A_t)$$

$$R_{t+1} = \mathcal{R}_\eta(R_{t+1} \mid S_t, A_t)$$

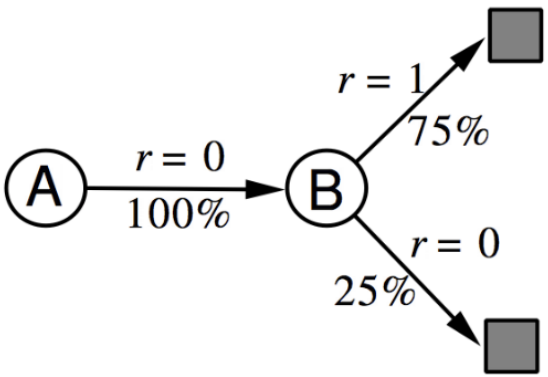
- Apply model-free RL to samples, e.g.:
  - Monte-Carlo control
  - Sarsa
  - Q-learning
- Sample-based planning methods are often more efficient

Back to the AB Example

- Construct a table-lookup model from real experience
- Apply model-free RL to sampled experience

Real experience

A, 0, B, 0
B, 1
B, 1
B, 1
B, 1
B, 1
B, 1
B, 0



Sampled experience

B, 1
B, 0
B, 1
A, 0 B, 1
B, 1
A, 0 B, 1
B, 1
B, 0

- e.g. Monte-Carlo learning: $V(A) = 1$, $V(B) = 0.75$

Planning with an Inaccurate Model

- Given an imperfect model $< \mathcal{P}_\eta, \mathcal{R}_\eta > \neq < \mathcal{P}, \mathcal{R} >$
- Performance of model-based RL is limited to optimal policy for approximate MDP $< \mathcal{S}, \mathcal{A}, \mathcal{P}_\eta, \mathcal{R}_\eta >$
- i.e. Model-based RL is only as good as the estimated model
- When the model is inaccurate, planning process will compute a sub-optimal policy
- Solution 1: when model is wrong, use model-free RL
- Solution 2: reason explicitly about model uncertainty

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Real and Simulated Experience

- We consider two sources of experience
- Real experience Sampled from environment (true MDP)

$$S' \sim \mathcal{P}^a_{s,s'}$$

$$R = \mathcal{R}^a_s$$

- Simulated experience Sampled from model (approximate MDP)

$$S' \sim \mathcal{P}_\eta(S' \mid S, A)$$

$$R = \mathcal{R}_\eta(R \mid S, A)$$

Integrating Learning and Planning

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Integrating Learning and Planning

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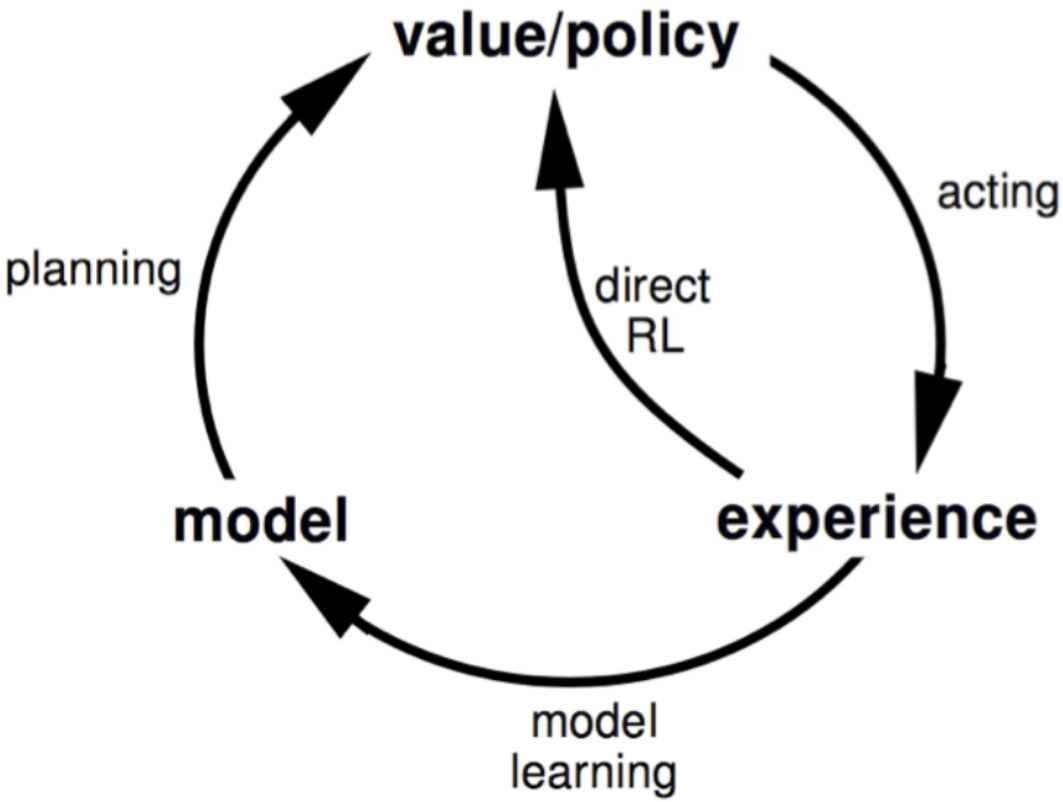
- Dyna
  - Learn a model from real experience
  - Learn and plan value function (and/or policy) from real and simulated experience

Dyna Architecture



Dyna-Q Algorithm

Initialize $Q(s,a)$ and $Model(s,a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$

Do forever:

(a) $S \leftarrow$ current (nonterminal) state

(b) $A \leftarrow \varepsilon$-greedy(S,Q)

(c) Execute action A; observe resultant reward, R, and state, S'

(d) $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$

(e) $Model(S,A) \leftarrow R, S'$ (assuming deterministic environment)

(f) Repeat n times:

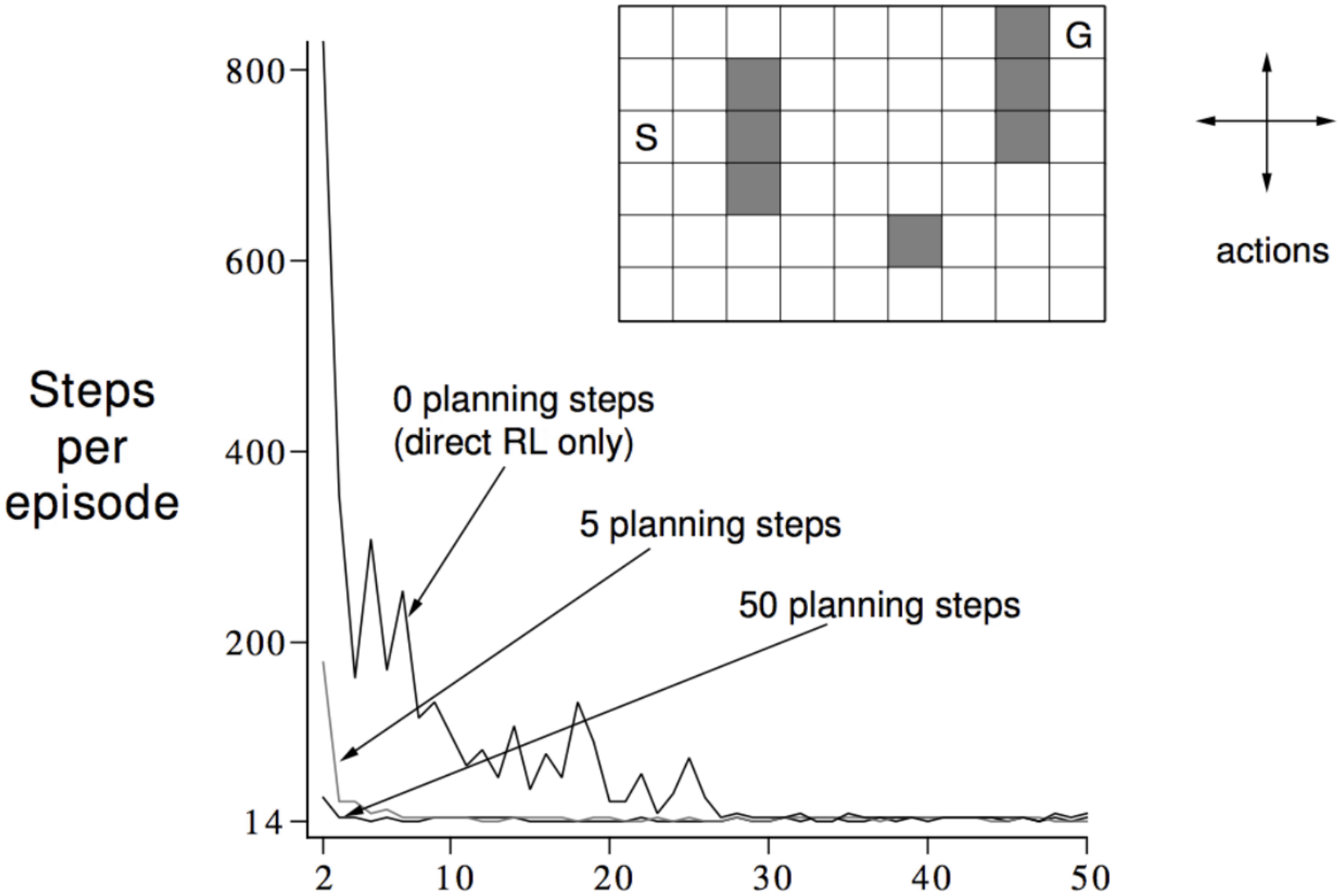
$S \leftarrow$ random previously observed state

$A \leftarrow$ random action previously taken in S

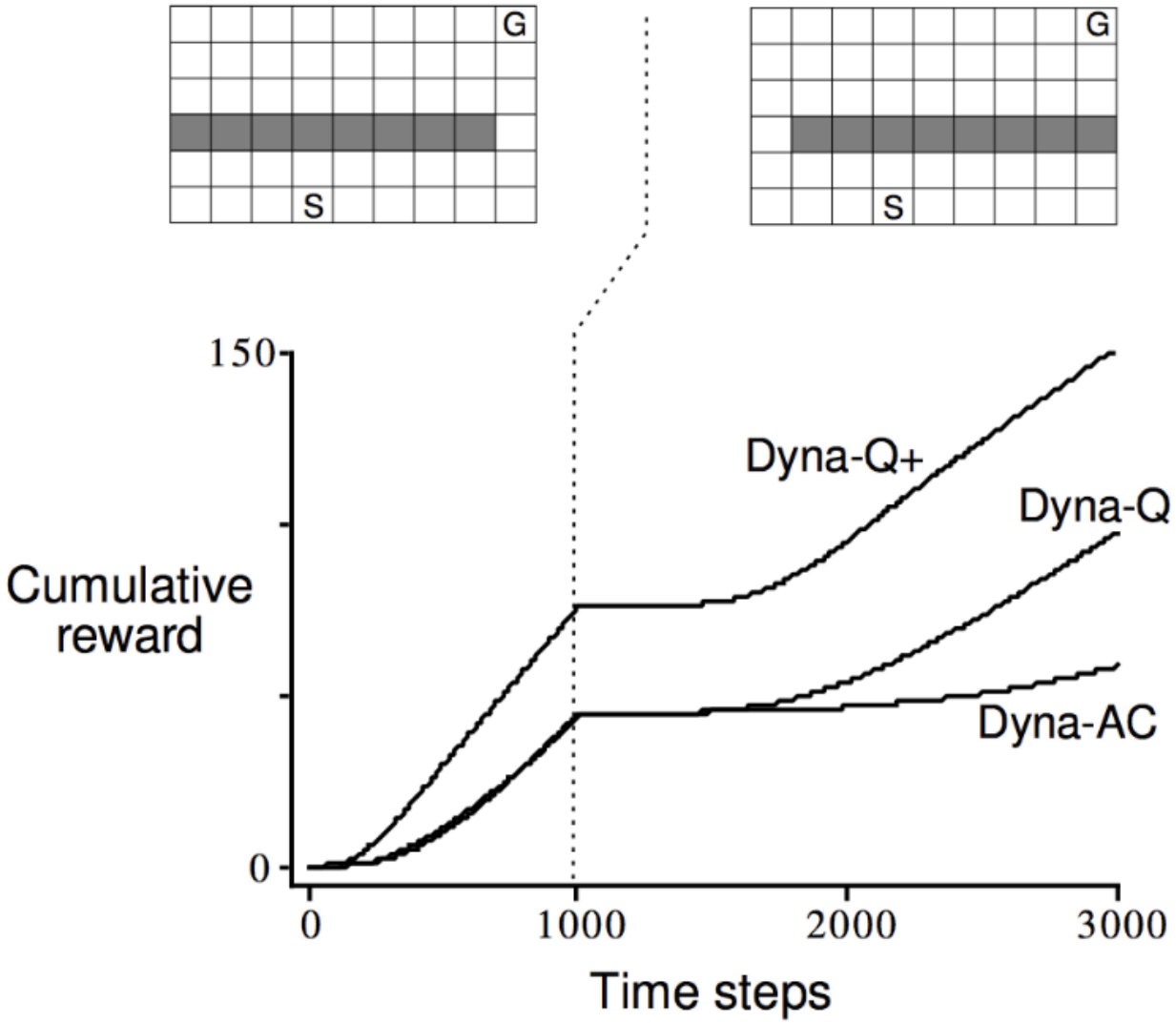
$R, S' \leftarrow Model(S,A)$

$Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$

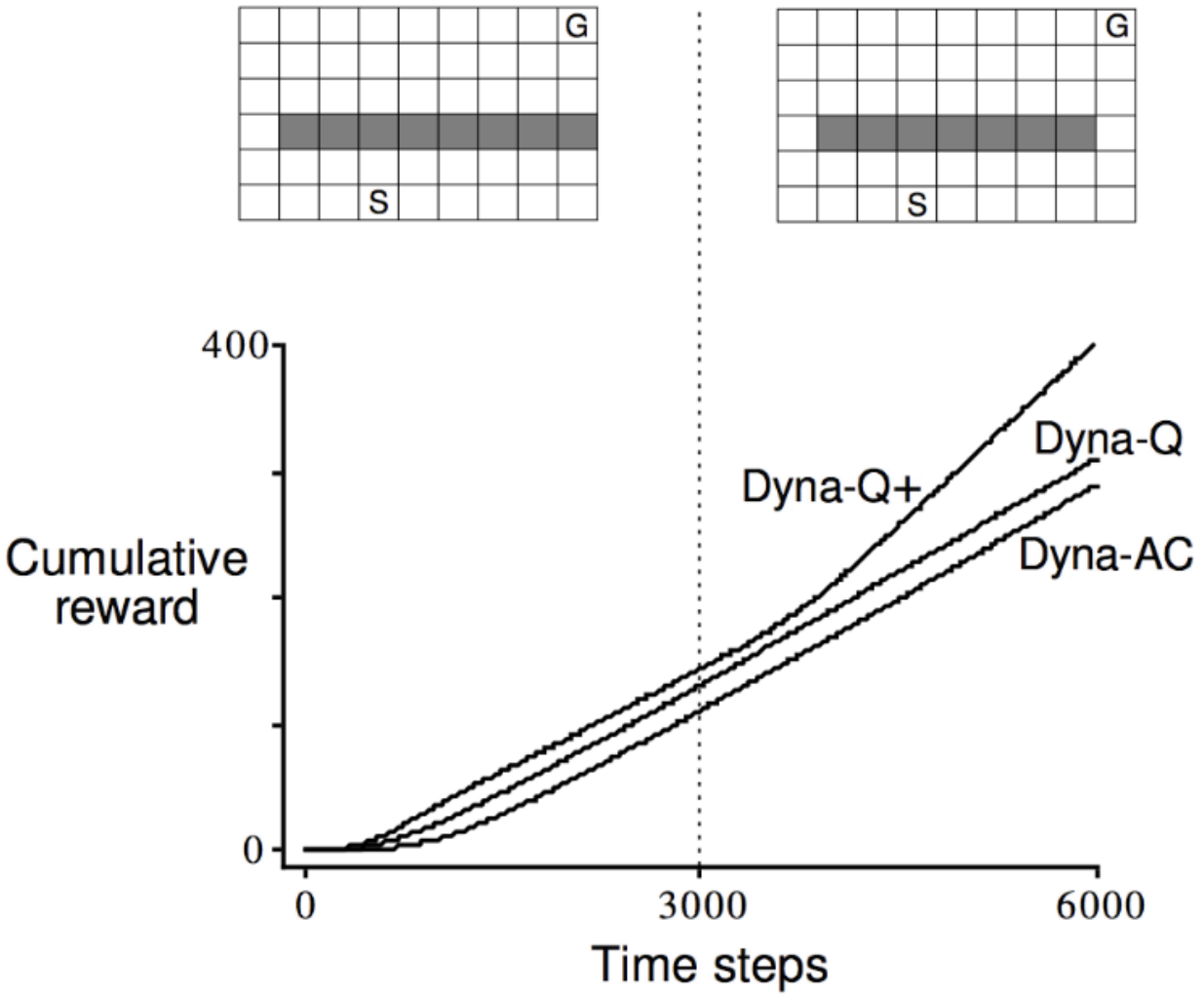
Dyna-Q on a Simple Maze



Dyna-Q with an Inaccurate Model



Dyna-Q with an Inaccurate Model (2)



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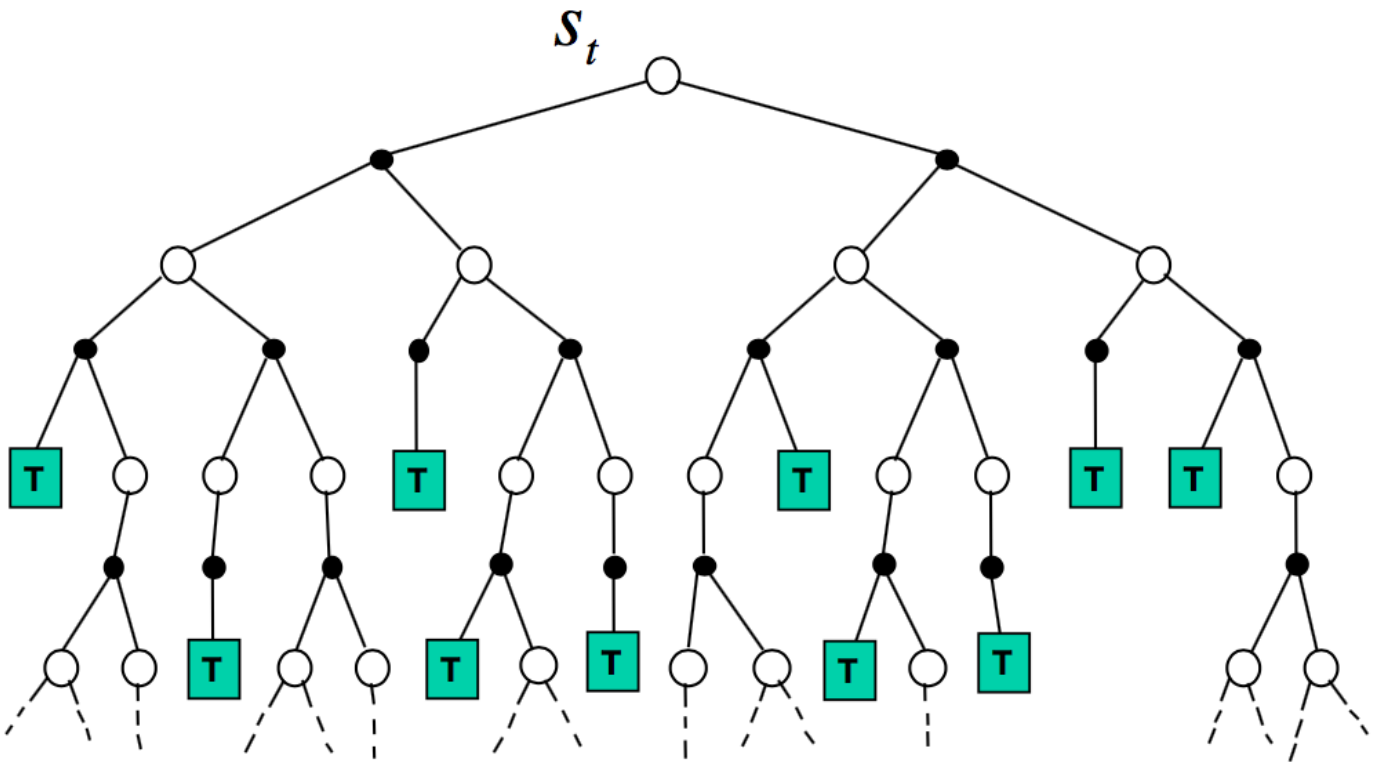
Forward Search

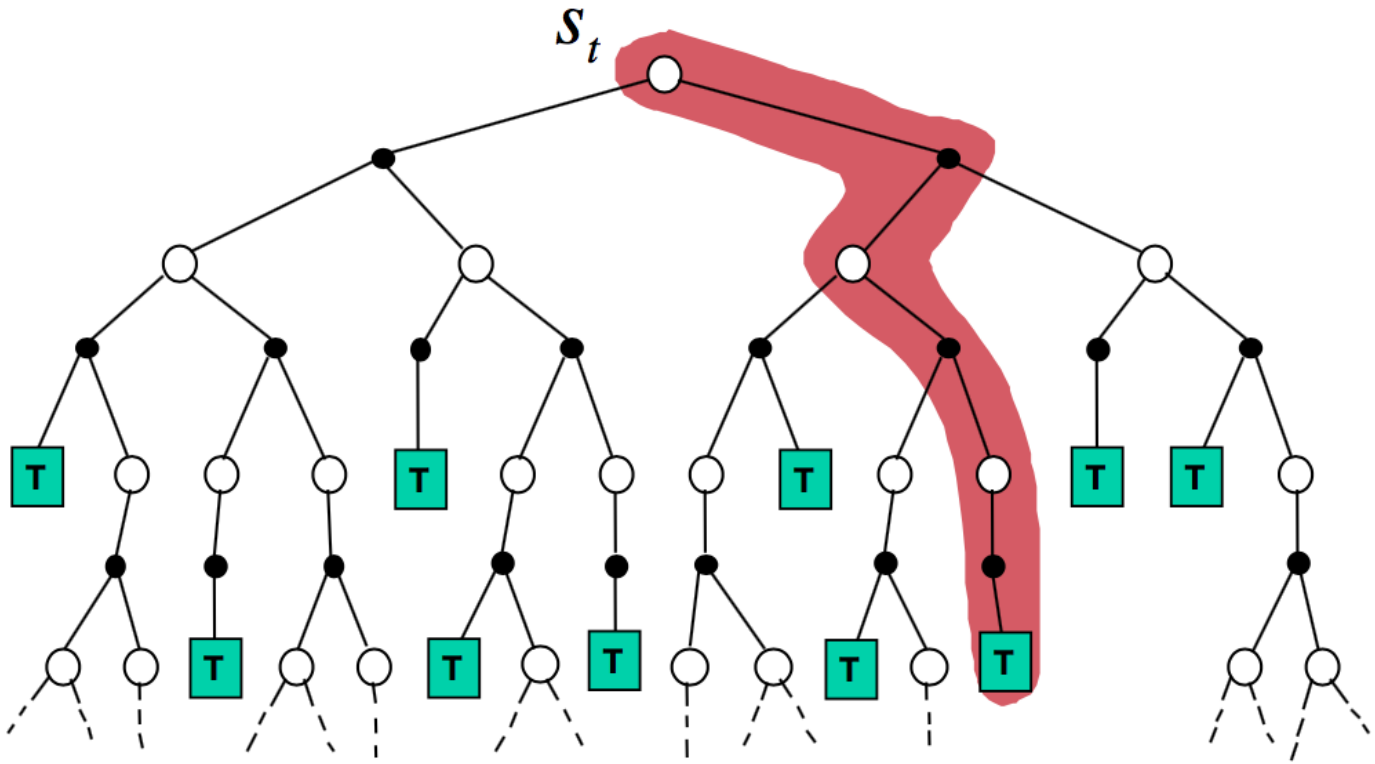
- Forward search algorithms select the best action by lookahead
- They build a search tree with the current state st at the root
- Using a model of the MDP to look ahead



- No need to solve whole MDP, just sub-MDP starting from now

Simulation-Based Search

- Forward search paradigm using sample-based planning
- Simulate episodes of experience from now with the model
- Apply model-free RL to simulated episodes



Simulation-Based Search (2)

- Simulate episodes of experience from now with the model

$$\{S_t^k, A_t^k, R_{t+1}^k, ..., S_T^k\}_{k=1}^K \sim \mathcal{M}_v$$

- Apply model-free RL to simulated episodes
  - Monte-Carlo control $\rightarrow$ Monte-Carlo search
  - Sarsa $\rightarrow$ TD search

Simple Monte-Carlo Search

- Given a model $\mathcal{M}_v$ and a simulation policy π
- For each action $a \in \mathcal{A}$
  - Simulate K episodes from current (real) state s_t

$$\{s_t, a, R^k_{t+1}, ..., S^k_T\}^K_{k=1} \sim \mathcal{M}_v, \pi$$

  - Evaluate actions by mean return (Monte-Carlo evaluation)

$$Q(s_t, a) = \frac{1}{K} \sum_{k=1}^{K} G_t \xrightarrow{P} q_\pi(s_t, a) \tag{1}$$

- Select current (real) action with maximum value

$$a_t = \underset{a \in A}{\operatorname{argmin}}\, Q(s_t, a)$$

Recall Expectimax Tree

- If have a MDP model $\mathcal{M}_v$
- Can compute optimal $q(s, a)$ values for current state by constructing an expectimax tree

- Limitations: Size of tree scales as

Monte-Carlo Tree Search (MCTS)

- Given a model $\mathcal{M}_v$
- Build a search tree rooted at the current state s_t
- Samples actions and next states
- Iteratively construct and update tree by performing K simulation episodes starting from the root state
- After search is finished, select current (real) action with maximum value in search tree

$$a_t = \underset{a \in A}{\operatorname{argmin}}\, Q(s_t, a)$$

Monte-Carlo Tree Search

- Simulating an episode involves two phases (in-tree, out-of-tree)
  - Tree policy: pick actions to maximize $Q(S, A)$
  - Roll out policy: e.g. pick actions randomly, or another policy
- To evaluate the value of a tree node i at state action pair (s, a), average over all rewards received from that node onwards across simulated episodes in which this tree node was reached

$$Q(i) = \frac{1}{N(i)} \sum_{k=1}^{K} \sum_{u=t}^{T} \mathbb{1}(i \in epi.k) G_k(i) \xrightarrow{P} q(s, a) \tag{2}$$

- Under mild conditions, converges to the optimal search tree, $Q(S, A) \rightarrow q^*(S, A)$

Upper Confidence Tree (UCT) Search

- How to select what action to take during a simulated episode?
- UCT: borrow idea from bandit literature and treat each node where can select actions as a multi-armed bandit (MAB) problem
- Maintain an upper confidence bound over reward of each arm

$$Q(s,a,i) = \frac{1}{N(s,a,i)} \sum_{k=1}^{K} \sum_{u=t}^{T} \mathbb{1}(i \in epi.k) G_k(s,a,i) + c\sqrt{\frac{ln n(s)}{n(s,a)}} \quad (3)$$

- For simplicity can treat each node as a separate MAB
- For simulated episode k at node i, select action/arm with highest upper bound to simulate and expand (or evaluate) in the tree

$$a_{ik} = \arg\max Q(s,a,i) \quad (4)$$

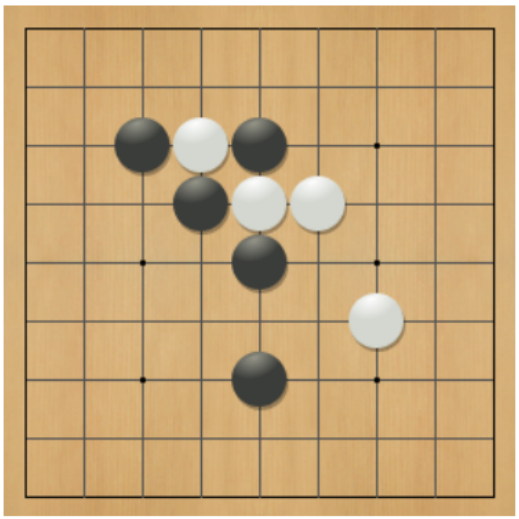
- This implies that the policy used to simulate episodes with (and expand/update the tree) can change across each episode

Case Study: the Game of Go

- The ancient oriental game of Go is 2500 years old
- Considered to be the hardest classic board game
- Considered a grand challenge task for AI (John McCarthy)
- Traditional game-tree search has failed in Go
- Check your understanding: does playing Go involve learning to make decisions in a world where dynamics and reward model are unknown?

Rules of Go

- Usually played on 19x19, also 13x13 or 9x9 board
- Simple rules, complex strategy
- Black and white place down stones alternately
- Surrounded stones are captured and removed
- The player with more territory wins the game



Position Evaluation in Go

- How good is a position s
- Reward function (undiscounted):

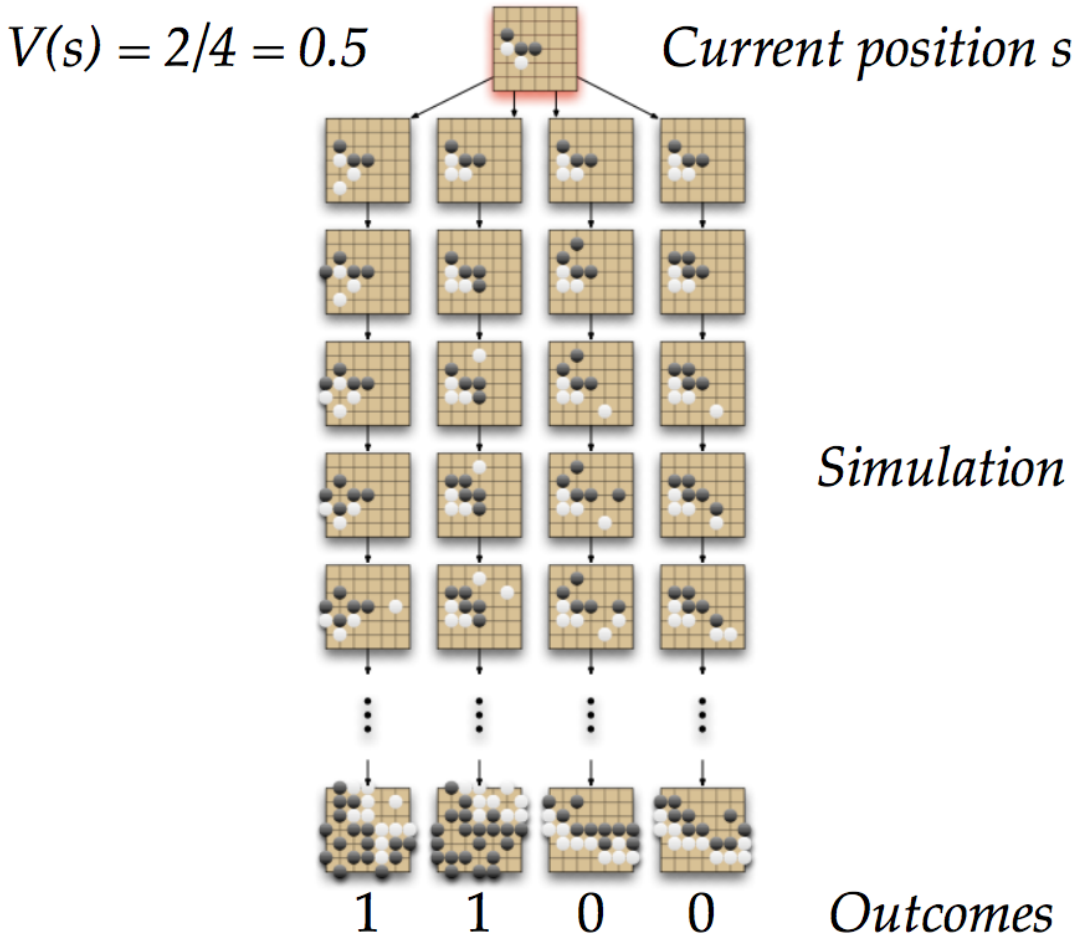
$$R_t = 0 \text{ for all non-terminal steps } t < T$$

$$R_T = \begin{cases} 1, & \text{if Black wins.} \\ 0, & \text{if White wins.} \end{cases} \tag{5}$$

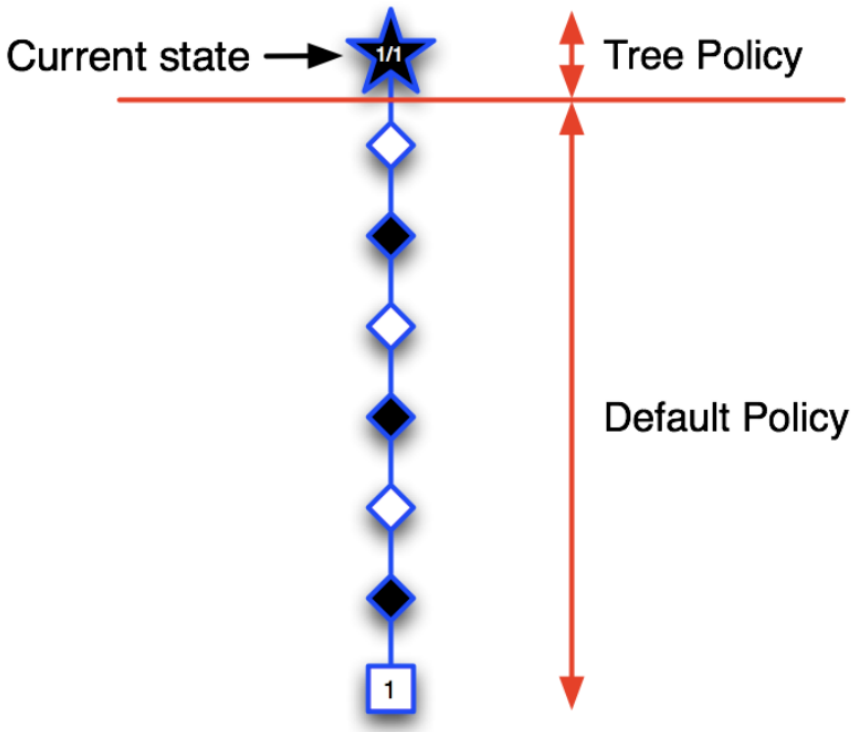
- Policy $\pi =< \pi_B, \pi_W >$ selects moves for both players
- Value function (how good is position s):

$$v_\pi(s) = \mathbb{E}_\pi[R_T \mid S = s] = \mathbb{P}[\text{Black wins} \mid S = s]$$
$$v^*(s) = \max_{\pi_B} \min_{\pi_W} v_\pi(s)$$

Monte-Carlo Evaluation in Go

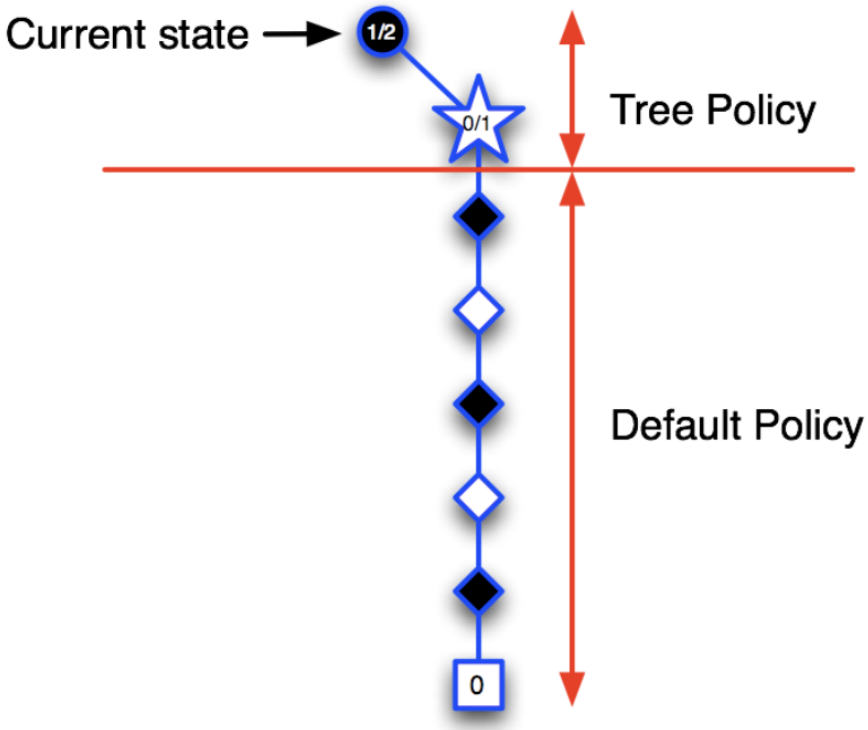


Applying Monte-Carlo Tree Search (1)

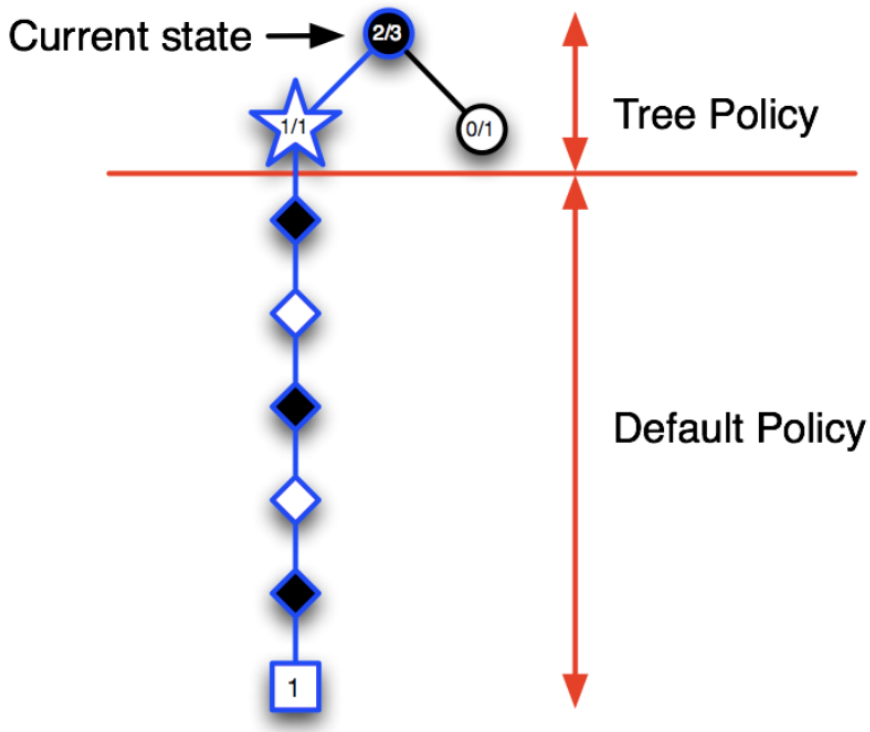


- Go is a 2 player game so tree is a minimax tree instead of expectimax
- White minimizes future reward and Black maximizes future reward when computing action to simulate

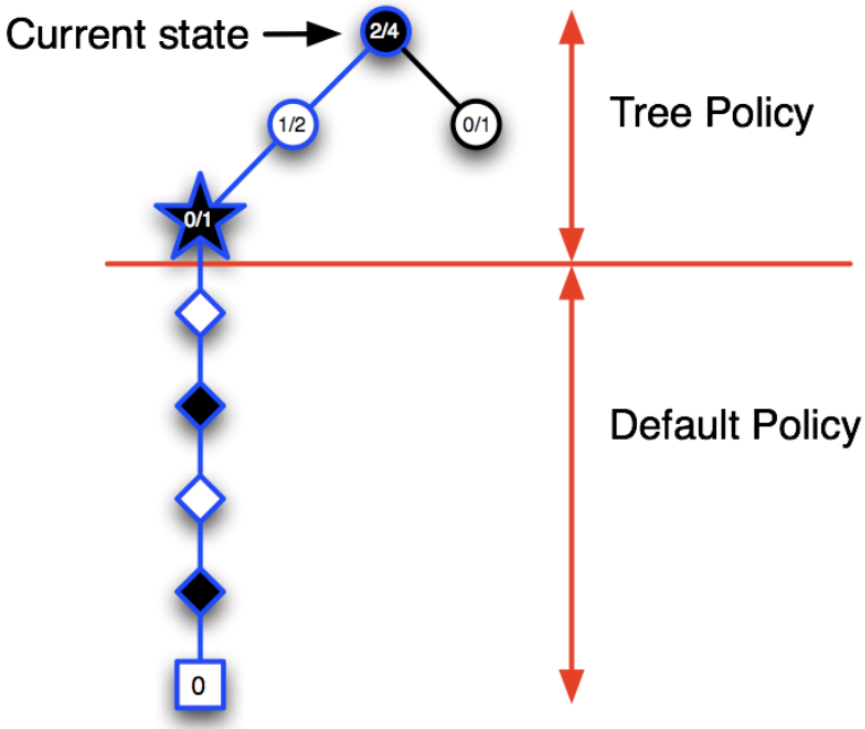
Applying Monte-Carlo Tree Search (2)



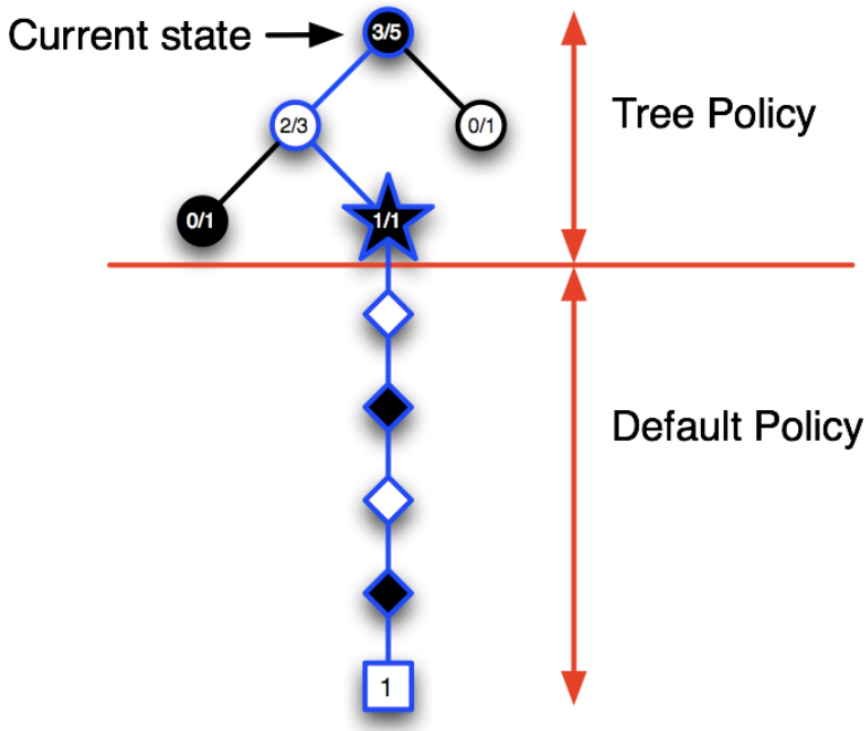
Applying Monte-Carlo Tree Search (3)



Applying Monte-Carlo Tree Search (4)



Applying Monte-Carlo Tree Search (5)



Advantages of MC Tree Search

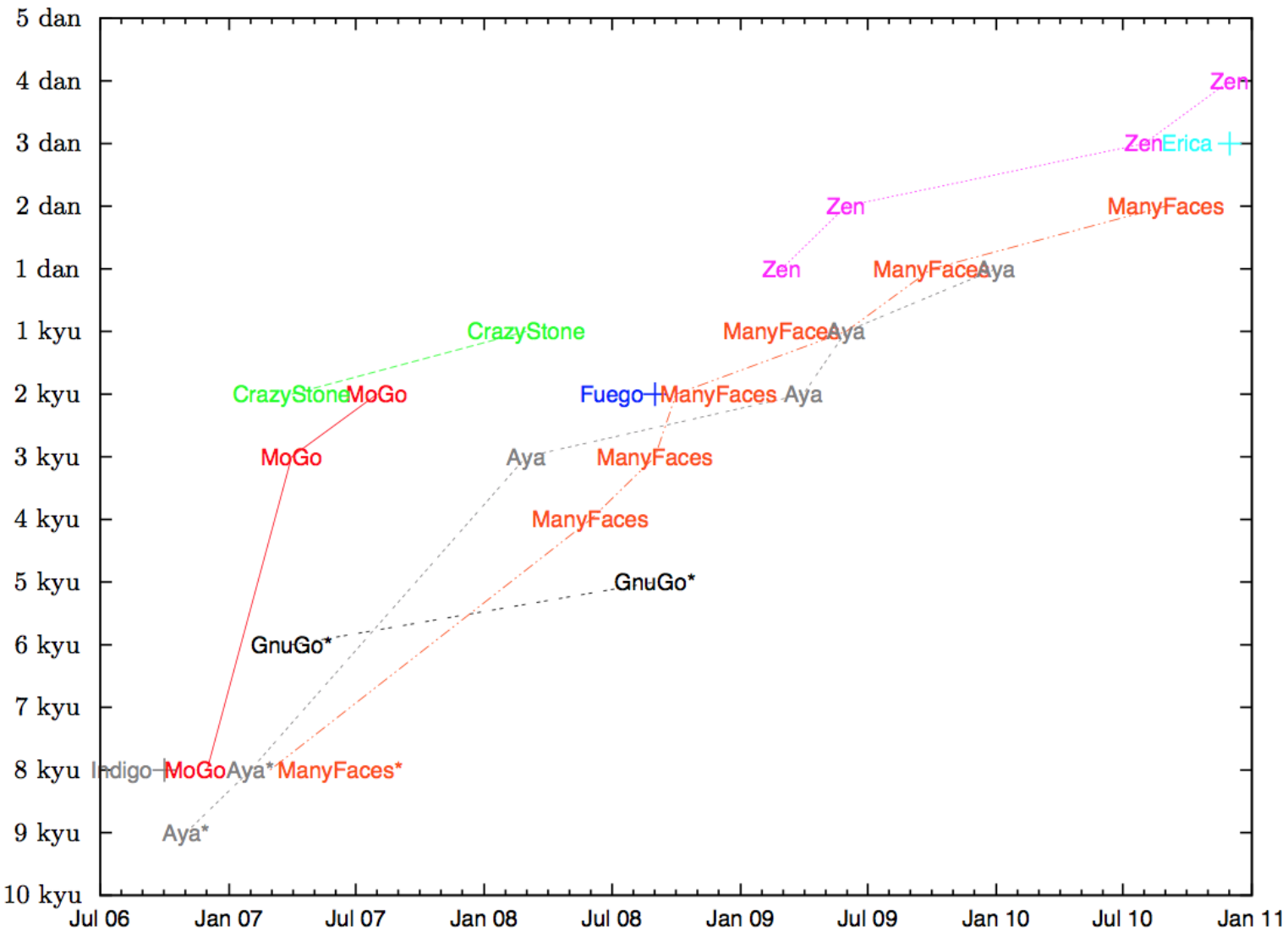
- Highly selective best-first search
- Evaluates states dynamically (unlike e.g. DP)
- Uses sampling to break curse of dimensionality
- Works for "black-box" models (only requires samples)
- Computationally efficient, anytime, parallelisable

In more depth: Upper Confidence Tree (UCT) Search

- UCT: borrow idea from bandit literature and treat each tree node where can select actions as a multi-armed bandit (MAB) problem
- Maintain an upper confidence bound over reward of each arm and select the best arm
- Check your understanding: Why is this slightly strange? Hint: why were upper confidence bounds a good idea for exploration/ exploitation? Is there an exploration/ exploitation problem during simulated episodes?[52]

[52] Relates to metalevel reasoning (for an example related to Go see "Selecting Computations: Theory and Applications", Hay, Russell, Tolpin and Shimony 2012)

MCTS and Early Go Results



MCTS Variants

- UCT and vanilla MCTS are just the beginning
- Potential extensions / alterations?

MCTS Variants

- UCT and vanilla MCTS are just the beginning
- Potential extensions / alterations?
  - Use a better rollout policy (have a policy network? Learned from expert data or from data gathered in the real world)
  - Learn a value function (can be used in combination with simulated trajectories to get a state-action estimate, can be used to bias initial actions considered, can be used to avoid having to rollout to the full episode length, ...)
  - Many other possibilities

MCTS and AlphaGo / AlphaZero ...

- MCTS was a critical advance for defeating Go
- Several new versions including AlphaGo Zero and AlphaZero which have even more impressive performance
- AlphaZero has also been applied to other games now including Chess

Class Structure

- Last time: Batch RL
- **This Time: MCTS**
- Next time: Human in the Loop RL