# Reinforcement Learning by the People and for the People: With a Focus on Lifelong / Meta / Transfer Learning

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CS234

Winter 2018

# Quiz Information

- Monday, in class
- See piazza for room information (Released by Friday)
- Cumulative (covers all material across class)
- Multiple choice quiz (for questions that are roughly on order of the level of difficulty, see examples at the end of this presentation. Focus on conceptual understanding rather than specific calculations, focus on the learning objectives in class (see listed on course webpage)

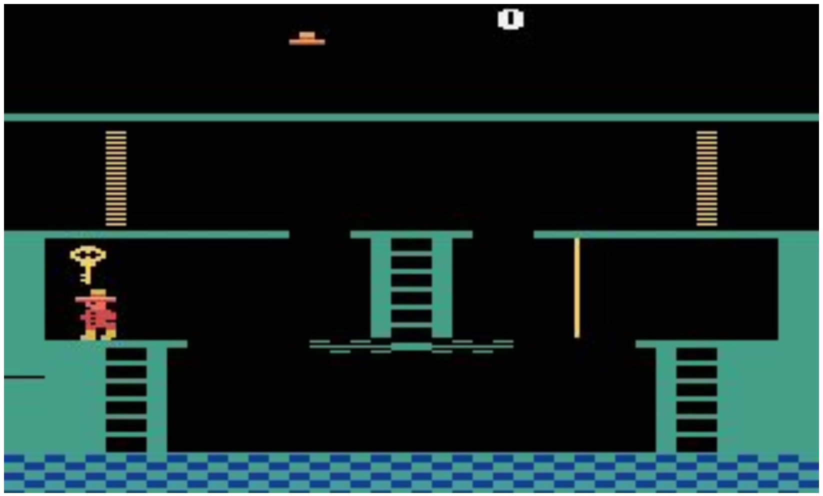
# Quiz Information

- Monday, in class
- See piazza for room information (Released by Friday)
- Cumulative (covers all material across class)
- Multiple choice quiz
- Individual + Team Component
  - First 45 minutes, individual component (4.5% of grade)
  - Rest of class: meet in small, pre-assigned groups, have to jointly decide on answers (0.5% of grade. Will be max of your group score and individual score. So group participation can only improve your grade!)
- Why? Another chance to reflect on your understanding, learn from others, and can improve your score
- SCPD students: see piazza information

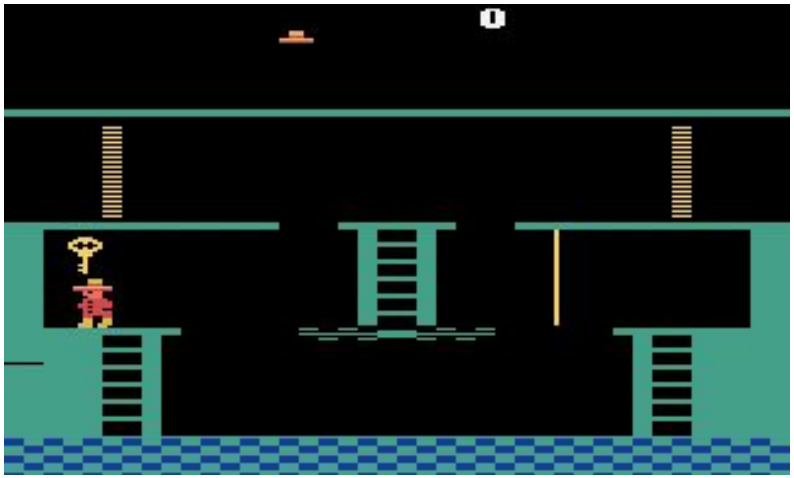
# Overview

- Last time: Monte Carlo Tree Search
- **This time: Human focused RL**
- Next time: Quiz

# Some Amazing Successes



# What About People?

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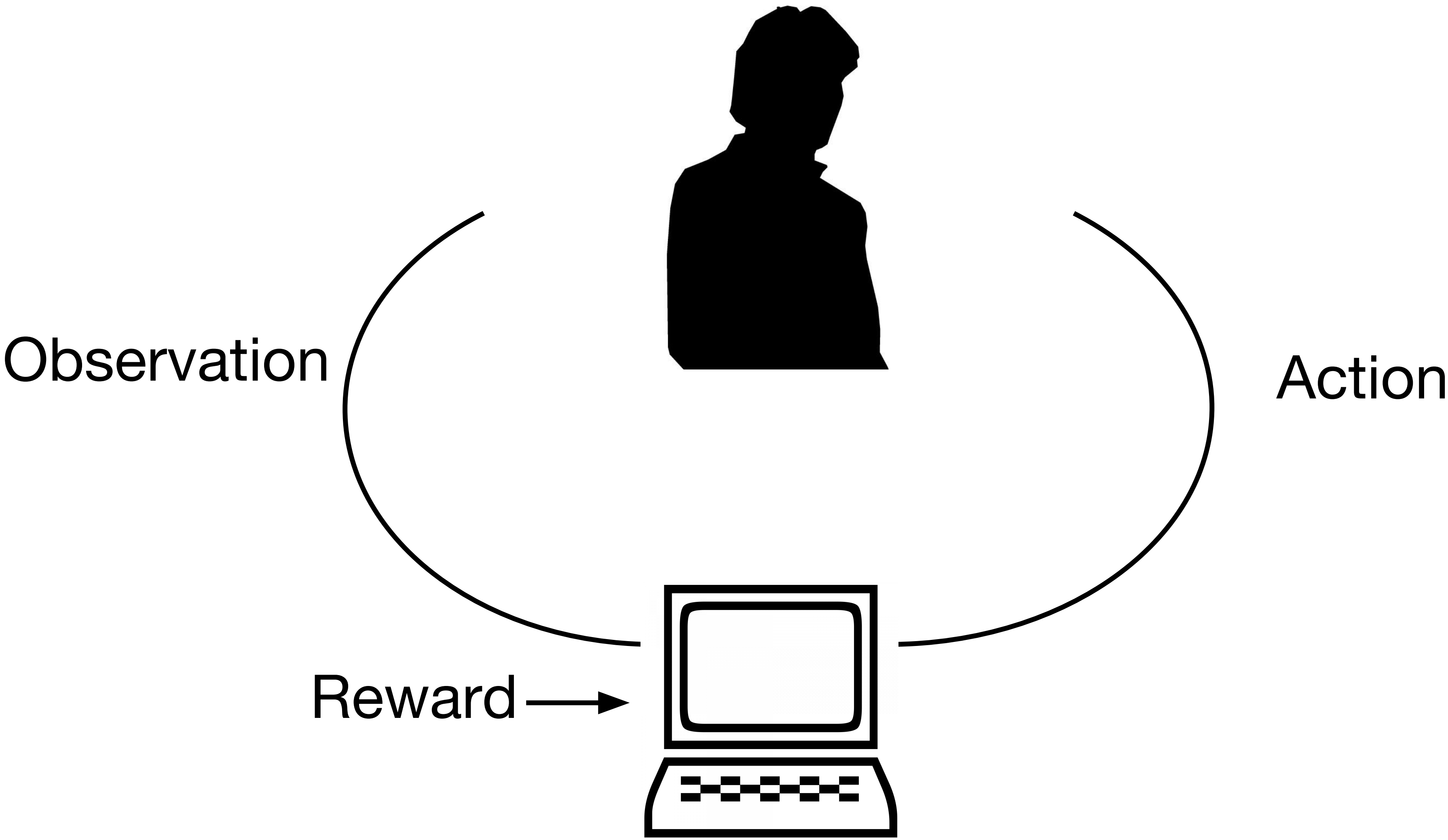
# Reinforcement Learning for the People and By the People




Policy: Map Observations → Actions
Goal: Choose actions to maximize expected rewards

# Today

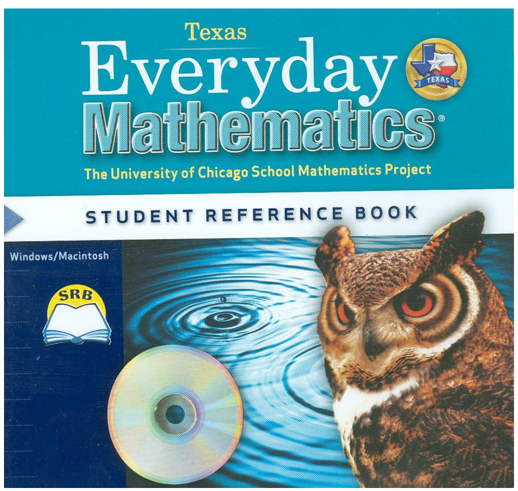
- Transfer learning / meta-learning / multi-task learning / lifelong learning for people focused domains
  - Small finite set of tasks
  - Large / continuous set of tasks

# Provably More Efficient Learners

- 1st (to our knowledge) Probably Approximately Correct (PAC) RL algorithm for discrete partially observable MDPs (Guo, Doroudi, Brunskill)
  - Polynomial sample complexity
- Near tight sample complexity bounds for finite horizon discrete MDP PAC RL (Dann and Brunskill, NIPS 2015)

# Limitations of Theoretical Bounds

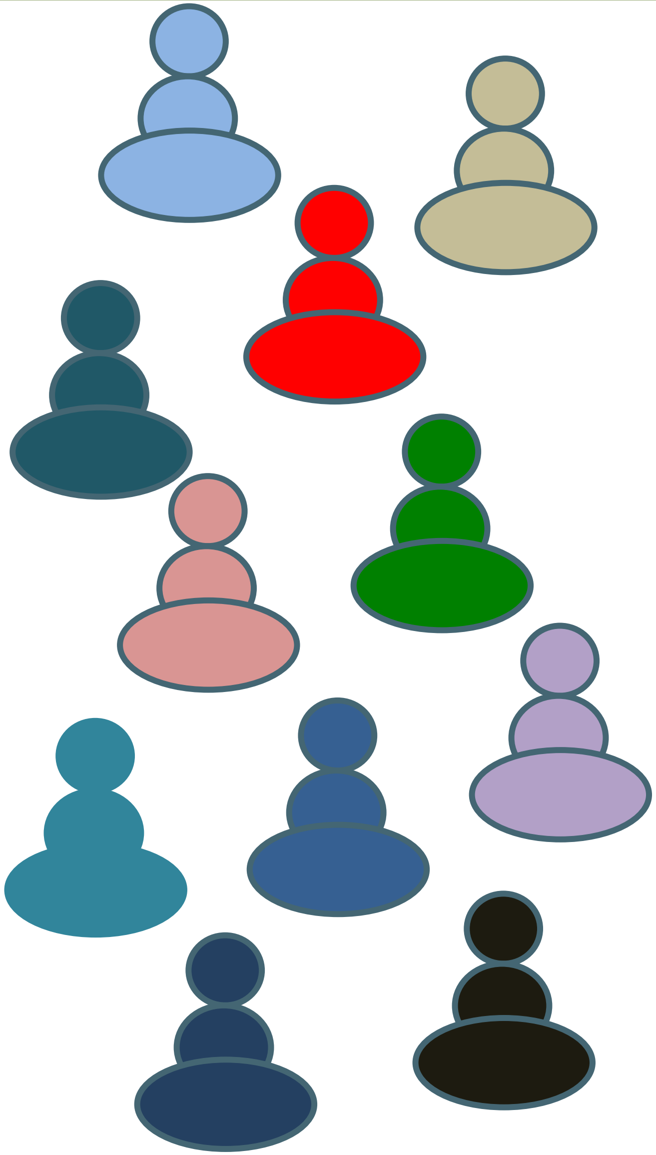
- Even our recent tighter bounds suggest need ~1000 samples per state—action pair
- And state—action space can be big!



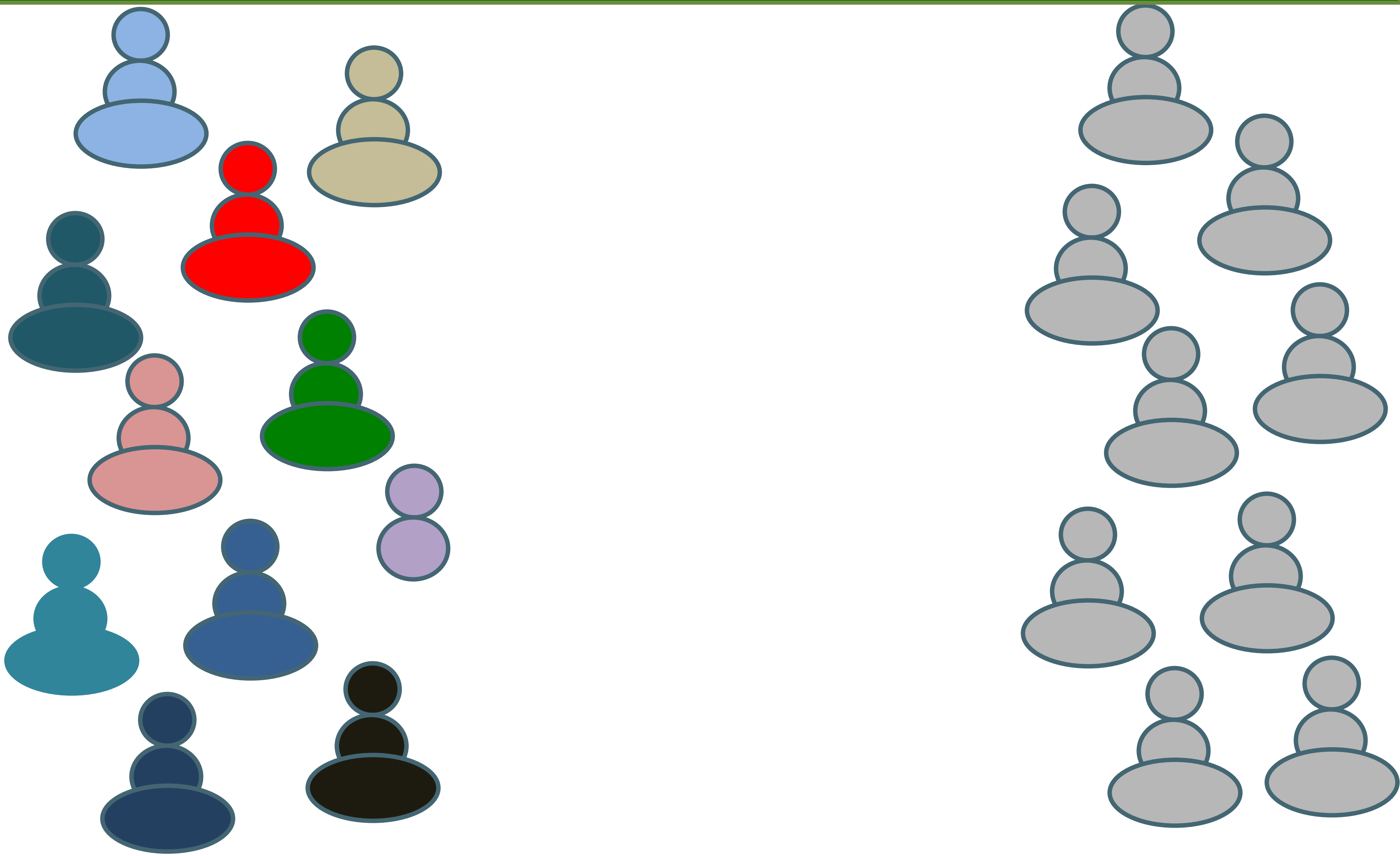

**$2^{100}$**

Possible knowledge states

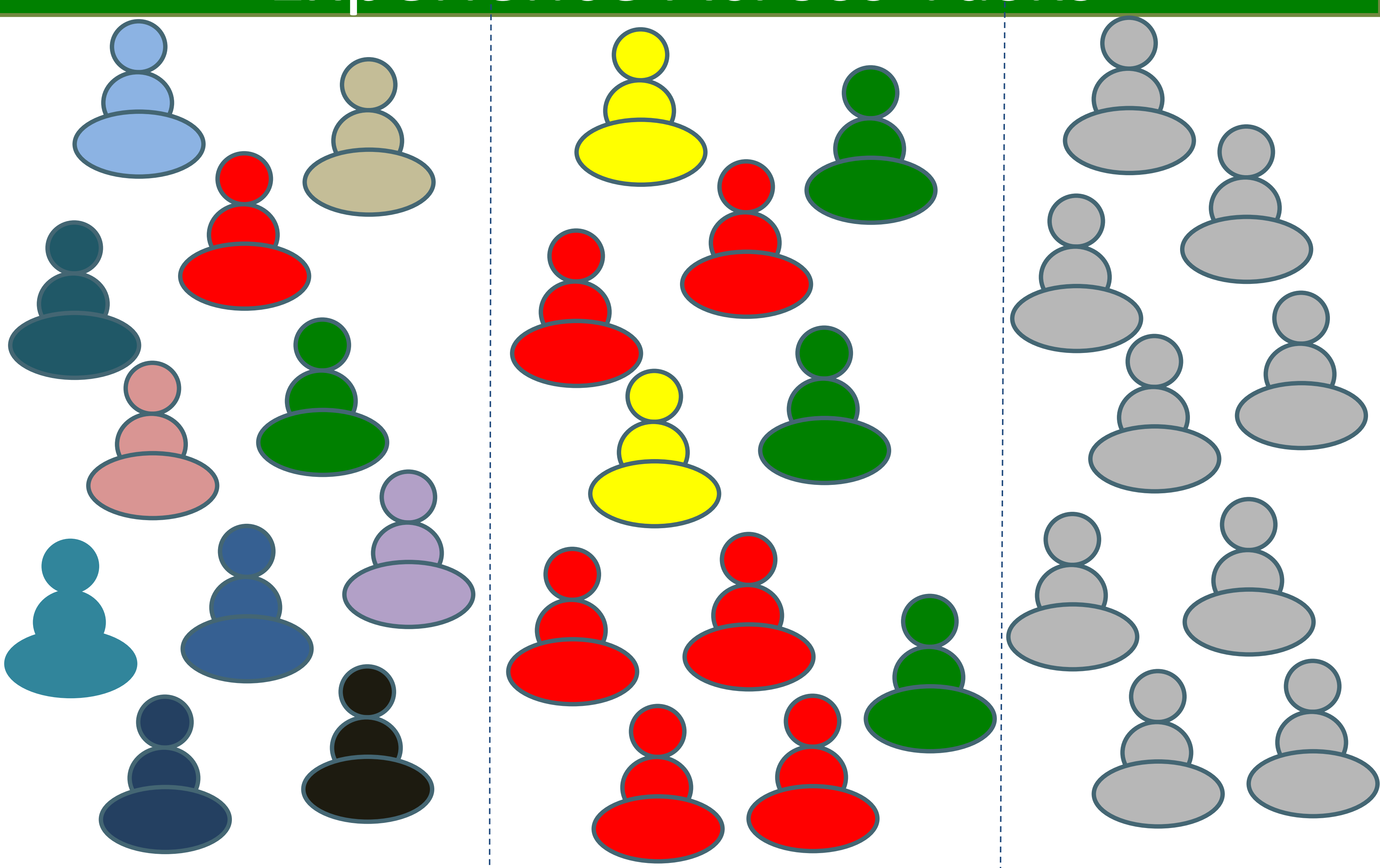
# Types of Tasks: All Different



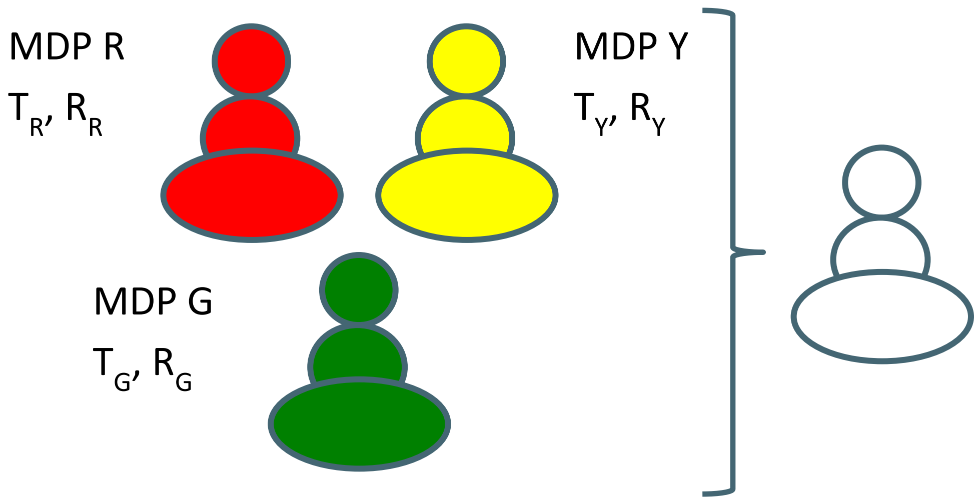
# Types of Tasks: All the Same -- Can Share Experience!
# Transfer / Lifelong Learning



# Finite Set of Tasks: Can Also Share Experience Across Tasks



# 1st: If Know New Task is 1 of M Tasks, Can That Speed Learning?

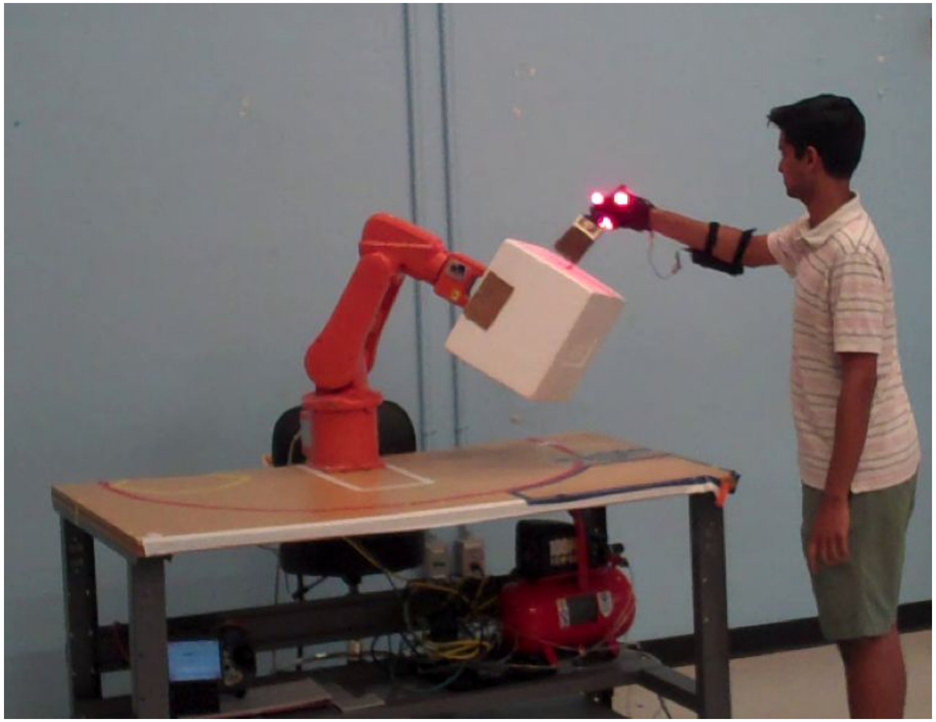



if told you identity of task
could just run $\pi^*$ for
that MDP

# Approach 1: Simple Policy Class: Small Finite Set of Models or Policies

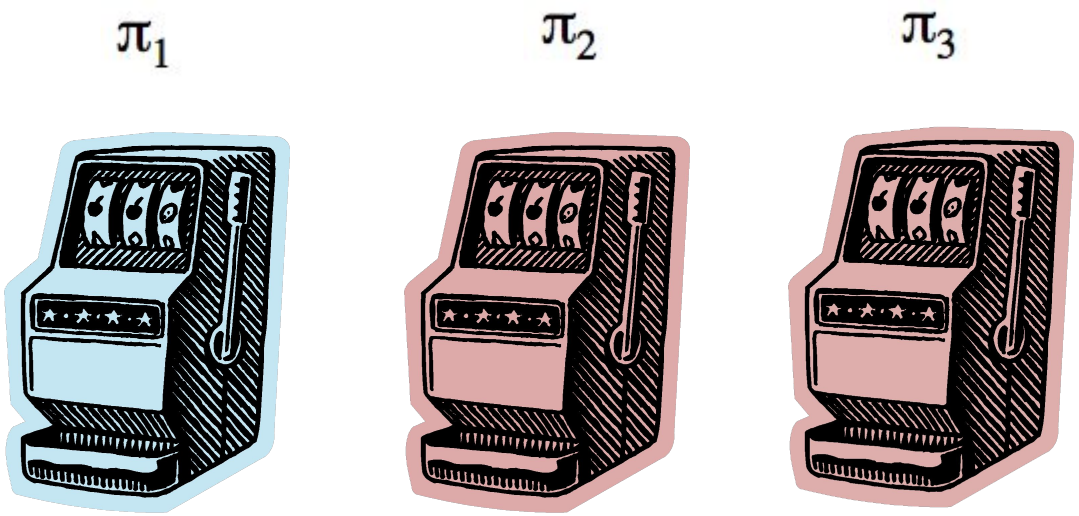
- If set is small, finding a good policy is much easier

Preference Modeling



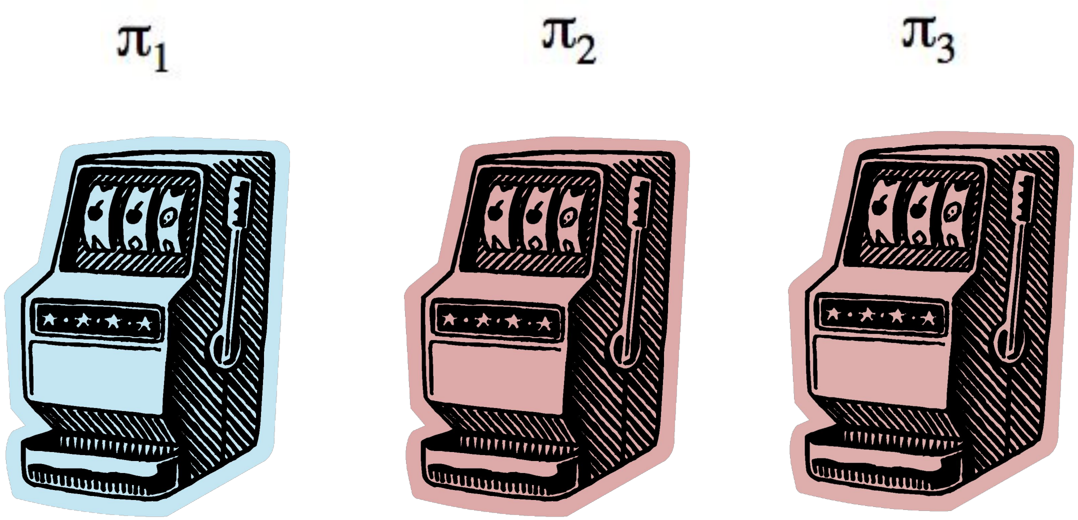
Nikolaidis et al. HRI 2015

# RL with Policy Advice




Azar, Lazaric, Brunskill, ECML 2013

# RL with Policy Advice

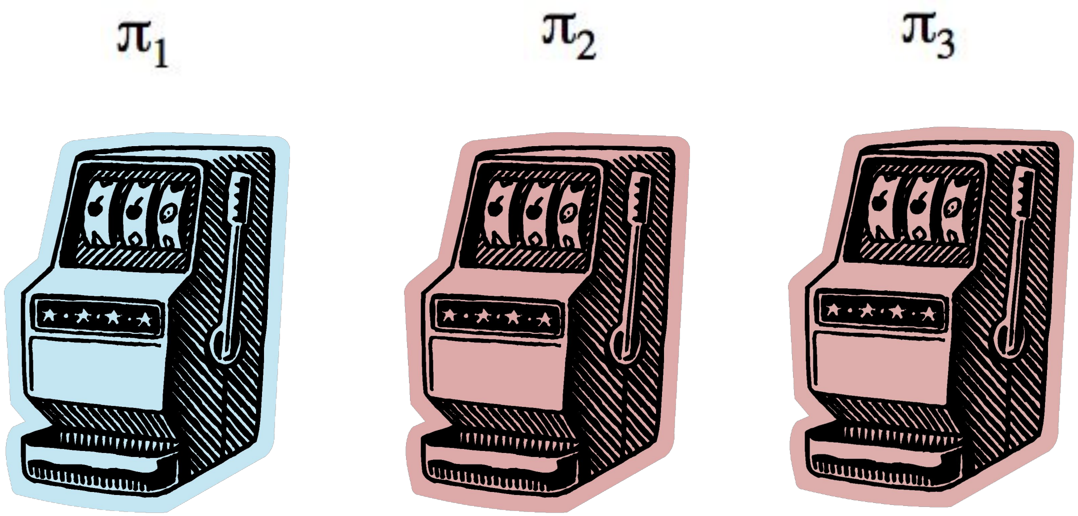



- Treat as a multi-armed bandit problem!
- Pulling an arm now corresponds to executing one of M policies
- What is the bandit reward?
  - Normally reward of arm
  - Here arms are policies
  - If in episodic setting, reward is just sum of rewards in an episode
  - In infinite horizon problem what is reward?

like estimate $V^{\pi_i}$ in current domain

Azar, Lazaric, Brunskill, ECML 2013

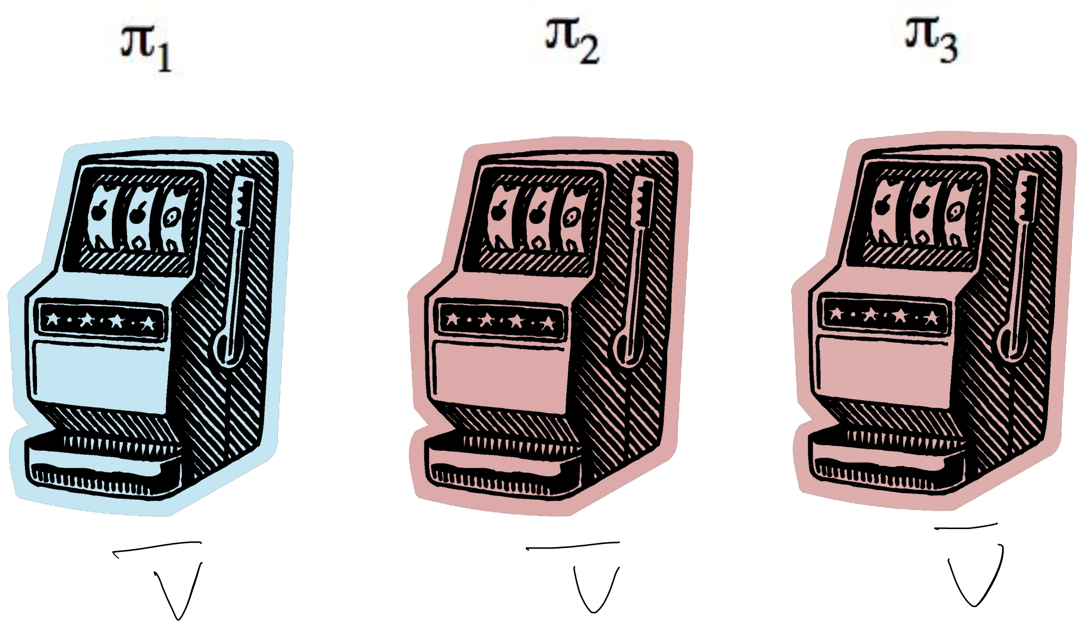
# RL with Policy Advice




- Treat as a multi-armed bandit problem!
- Pulling an arm now corresponds to executing one of M policies
- Have to figure out how many steps to execute a policy to get an estimate of its return
- Requires some mild assumptions on mixing and reachability

Azar, Lazaric, Brunskill, ECML 2013

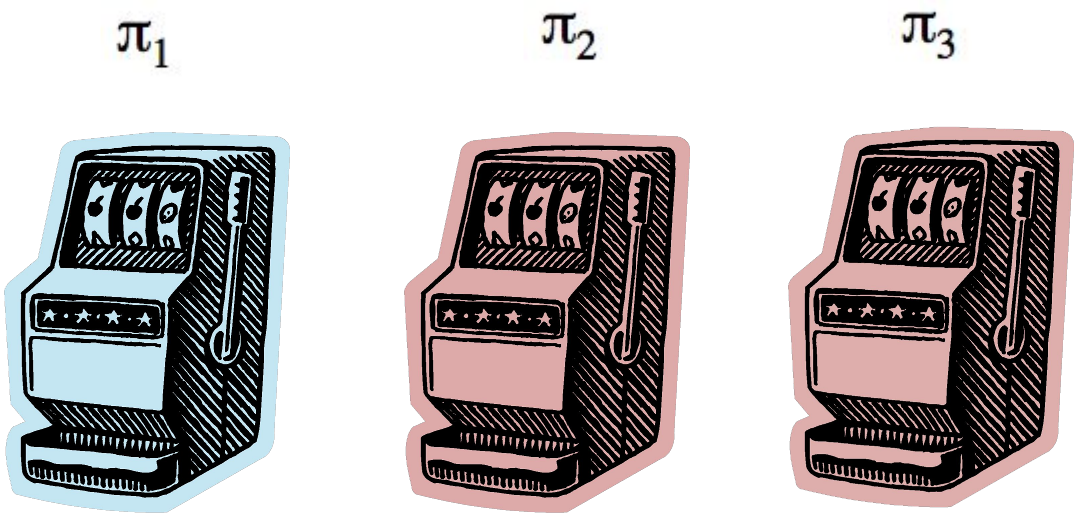
# Which Policy to Pull?




- Keep upper bound on avg. reward per policy
- Just like upper confidence bound algorithm in earlier lectures
- Use to optimistically select policy

Azar, Lazaric, Brunskill, ECML 2013

# RL with Policy Advice




- Regret bounds indp of S-A space, sqrt(# policies)

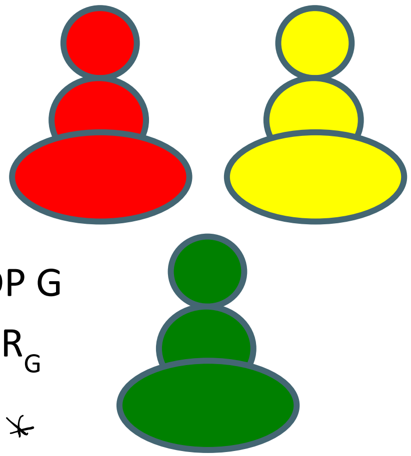
Azar, Lazaric, Brunskill, ECML 2013

# What if Have M Models Instead of M Policies?

MDP R

$T_R$, $R_R$

$V^*_R$ $\pi^*_R$



MDP Y

$T_Y$, $R_Y$

$V^*_Y$

$\pi^*_Y$

MDP G

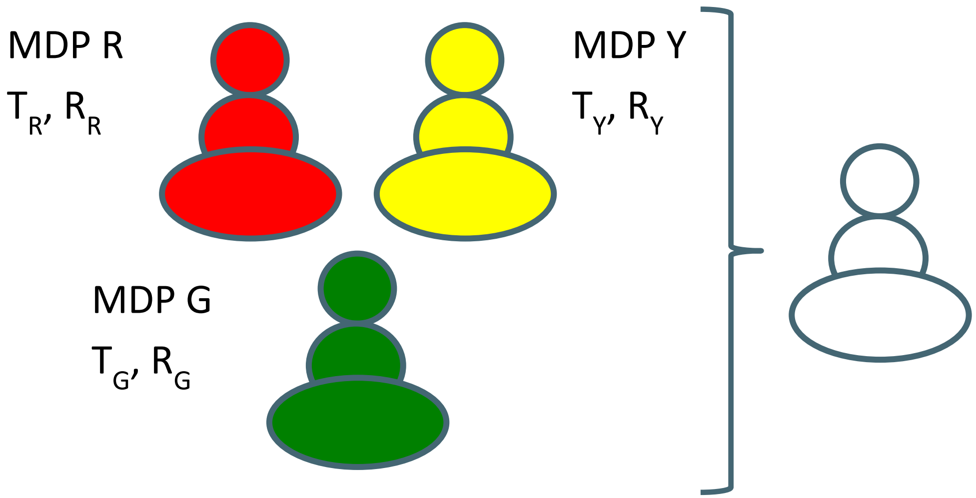
$T_G$, $R_G$

$V^*_G$

$\pi^*_G$

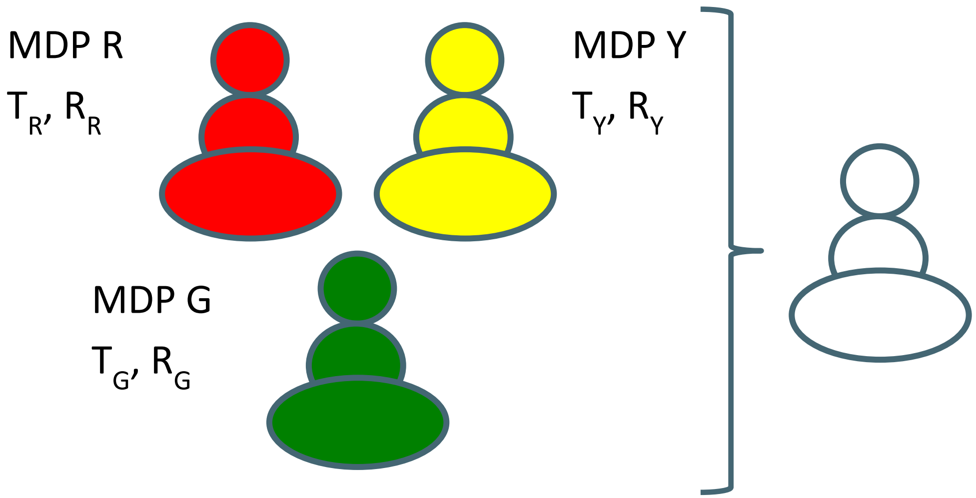
Brunskill & Li, UAI 2013

# What if Have M Models Instead of M Policies? New MDP 1 of M Models




Brunskill & Li, UAI 2013

# New MDP 1 of M Models But Don't Know Which

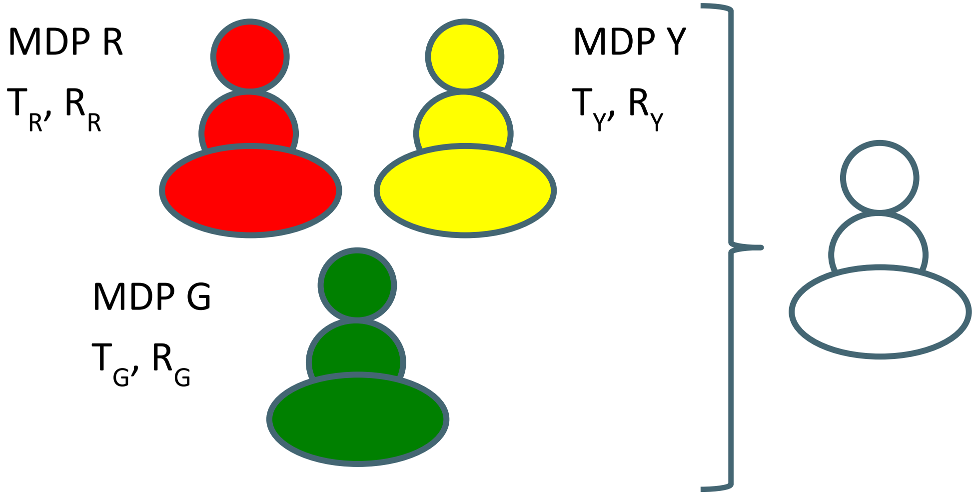



Act in it for H steps

$<s_1, a_1, r_1, s_2, a_2, r_2, s_3, a_3, \ldots s_H>$

Brunskill & Li, UAI 2013

# Learning as Classification

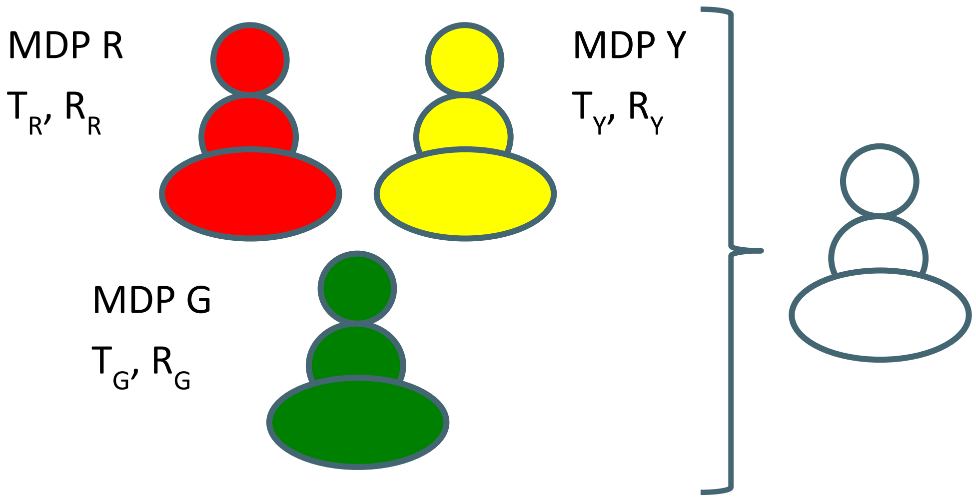



Act in it for H steps
$<s_1,a_1,r_1,s_2,a_2,r_2,s_3,a_3,\ldots s_H>$

- If knew identify of new MDP, would know optimal policy
- Try to identify which MDP the new task is

Brunskill & Li, UAI 2013

# Learning as Classification

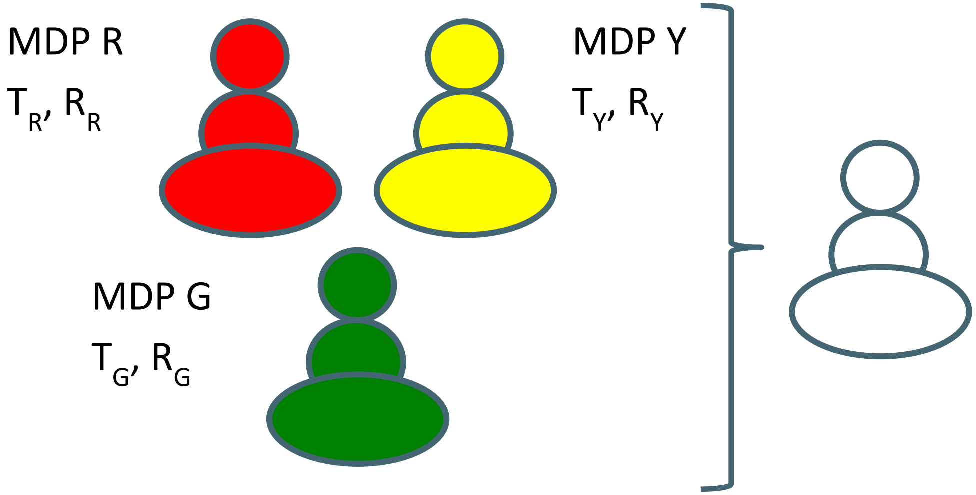



Act in it for H steps

$<s_1,a_1,r_1,s_2,a_2,r_2,s_3,a_3,\ldots s_H>$

- Maintain set of MDPs that the new task could be
  - Initially this is the full set of MDPs

# Learning as Classification

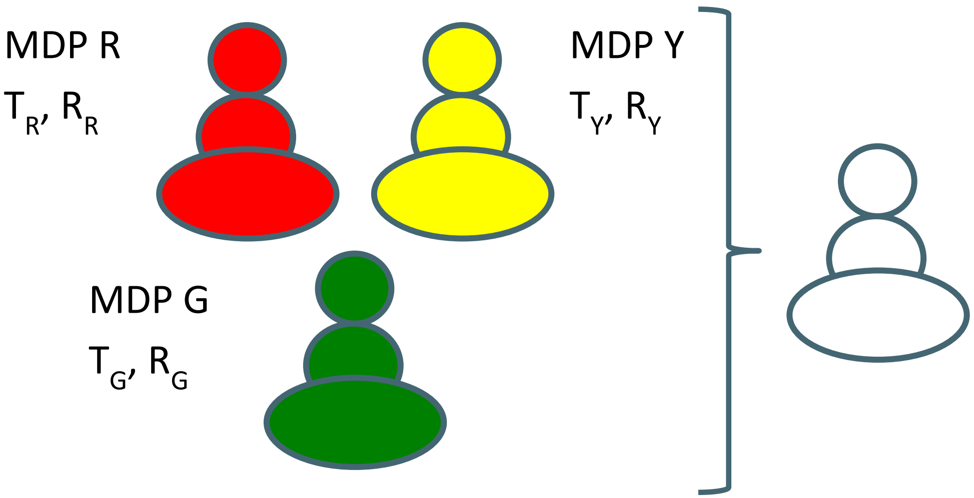



Act in it for H steps
$<s_1,a_1,r_1,s_2,a_2,r_2,s_3,a_3,\ldots s_H>$

- Maintain set of MDPs that the new task could be
  - Initially this is the full set of MDPs
- Track L2 error of model predictions of observed transitions (s,a,r,s') in current task
- Eliminate MDP *i* from the set if error is too large-- very unlikely current task is MDP *i*
- Use to identify current task as 1 of M tasks

Brunskill & Li, UAI 2013

# Directed Classification




Act in it for H steps

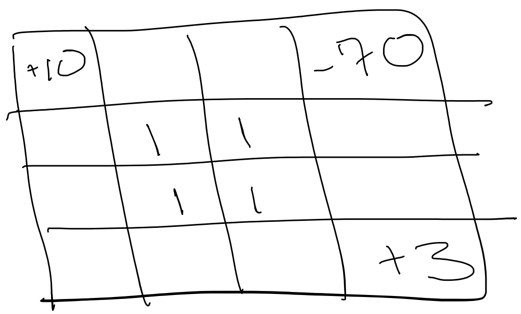
$<s_1,a_1,r_1,s_2,a_2,r_2,s_3,a_3,\ldots s_H>$

- Can strategically gather data to identify task
- Prioritize visiting (s,a) pairs where the possible MDPs disagree in their models

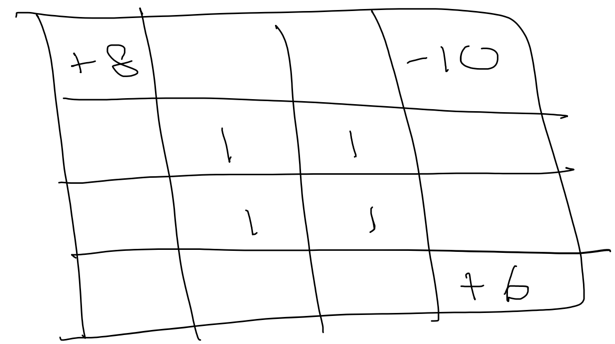
Brunskill & Li, UAI 2013

# Grid World Example: Directed Exploration

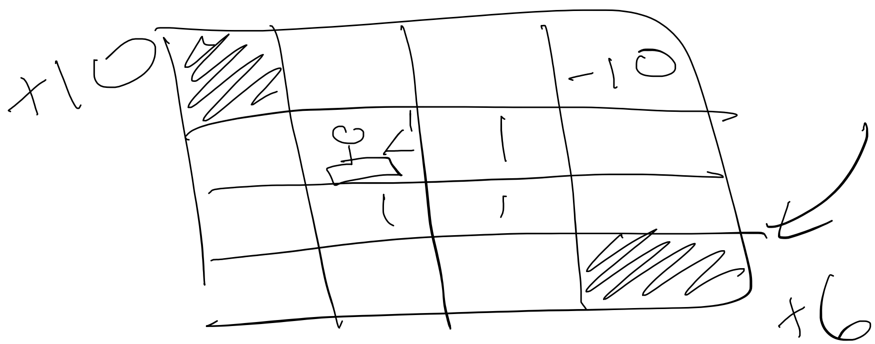
MDP G




MDP R

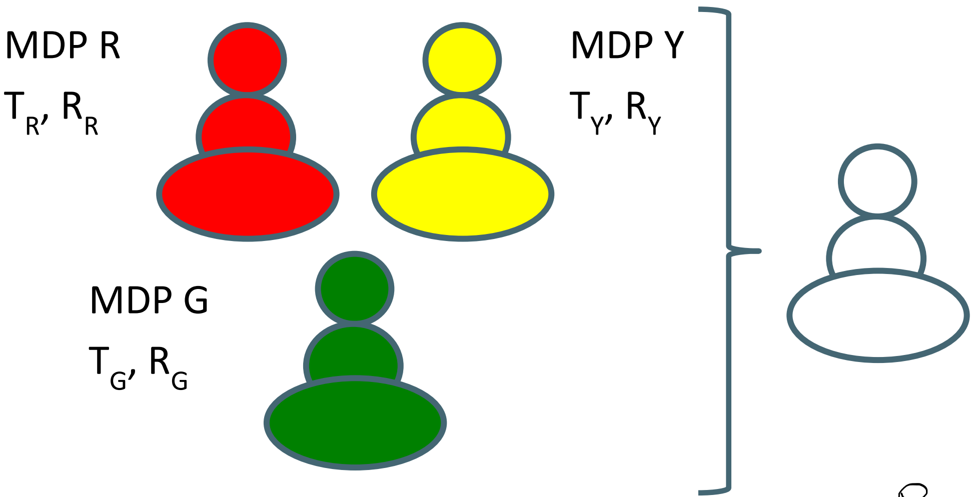



grid world




places MDPs disagree

# Intuition: Why This Speeds Learning




Act in it for H steps

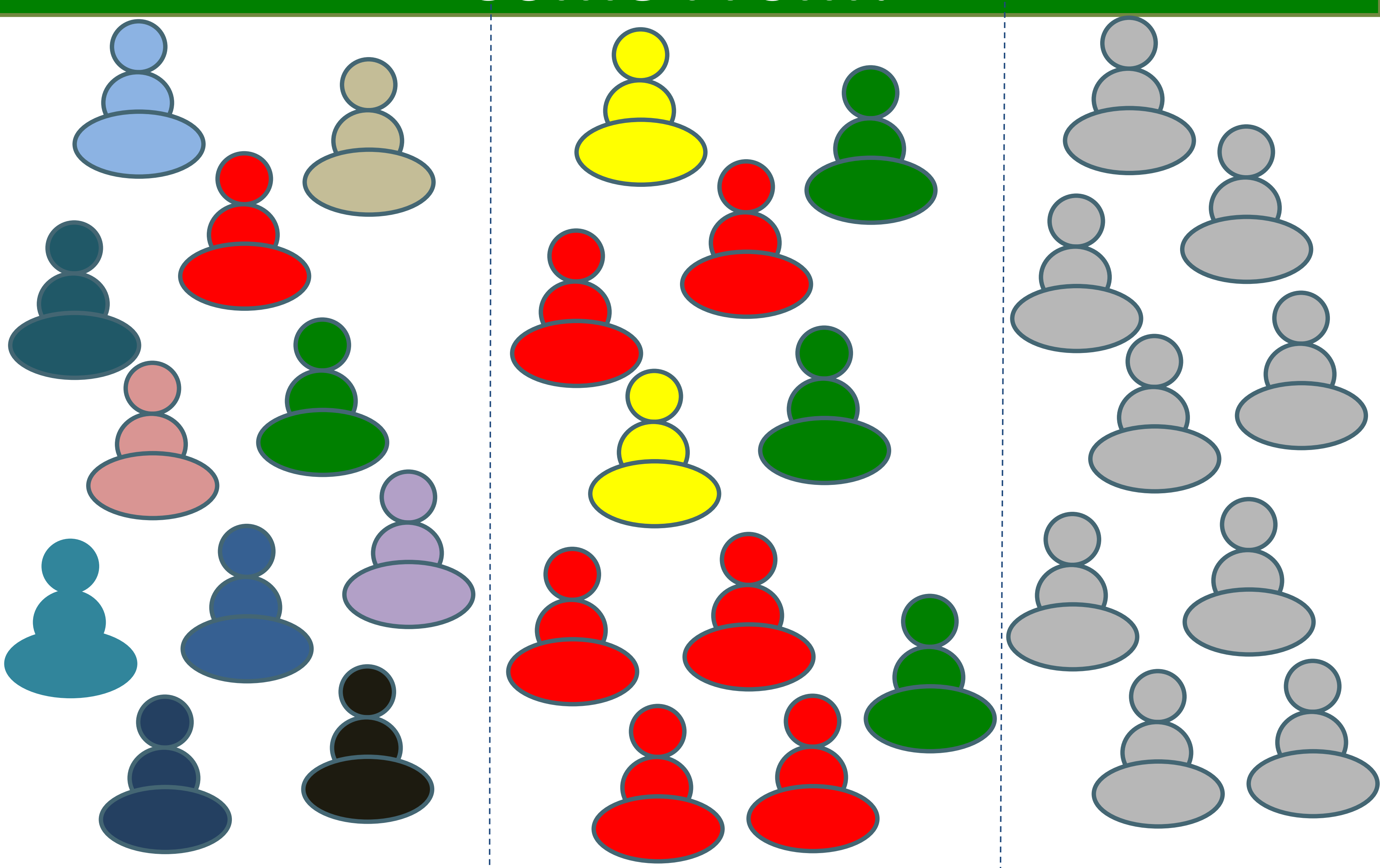
$<s_1,a_1,r_1,s_2,a_2,r_2,s_3,a_3,\ldots s_H>$

R | T  $R(s,a)$  $T(s'|s,a)$

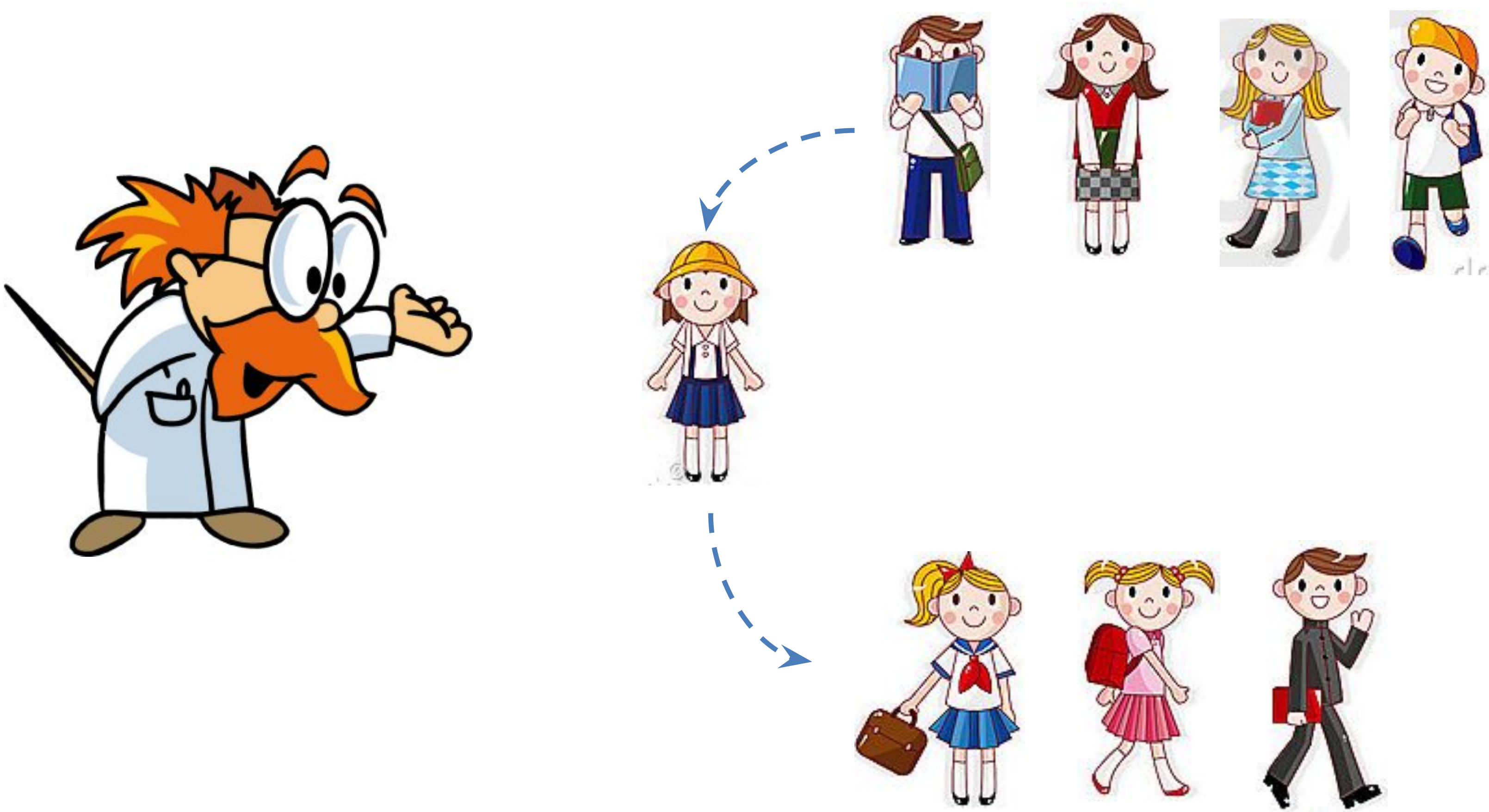
- If MDPs agree (have same model parameters) for most (s,a) pairs, only a few (s,a) pairs need to visit
  - To classify task
  - To learn parameters (all others are known)
- If MDPs differ in most (s,a) pairs, easy to classify task

Brunskill & Li, UAI 2013

# But Where Do These Clustered Tasks Come From?



# Personalization & Transfer Learning for Sequential Decision Making Tasks



Possible to guarantee learning speed increases across tasks? data

# Why is Transfer Learning Hard?

- What should we transfer?
  - Models?
  - Value functions?
  - Policies?

# Why is Transfer Learning Hard?

- What should we transfer?
  - Models?
  - Value functions?
  - Policies?
- The dangers of negative transfer
  - What if prior tasks are unrelated to current task, or worse, misleading
  - **Check your understanding**: Can we ever guarantee that we can avoid negative transfer without additional assumptions? (Why or why not?) NO

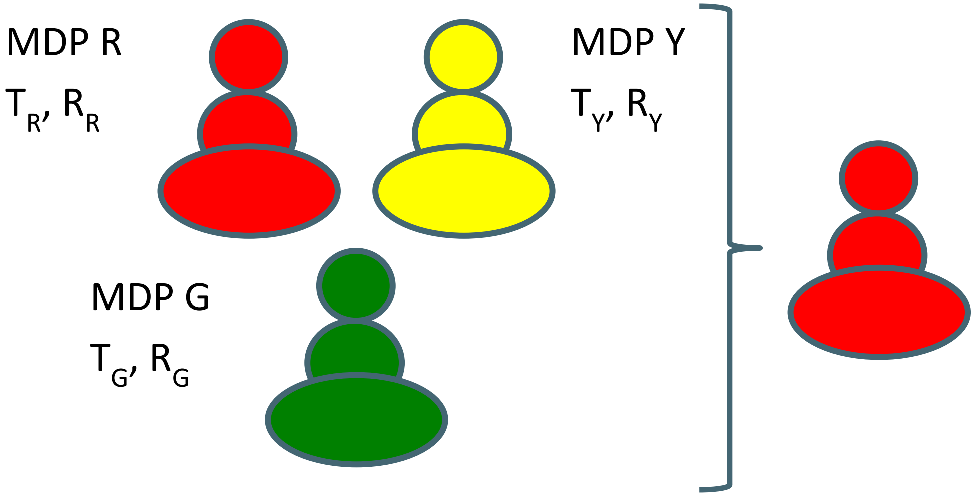
# Formalizing Learning Speed in Decision Making Tasks

Sample complexity:

number of actions may choose whose value is potentially far from optimal action's value

Can sample complexity get smaller by leveraging prior tasks?

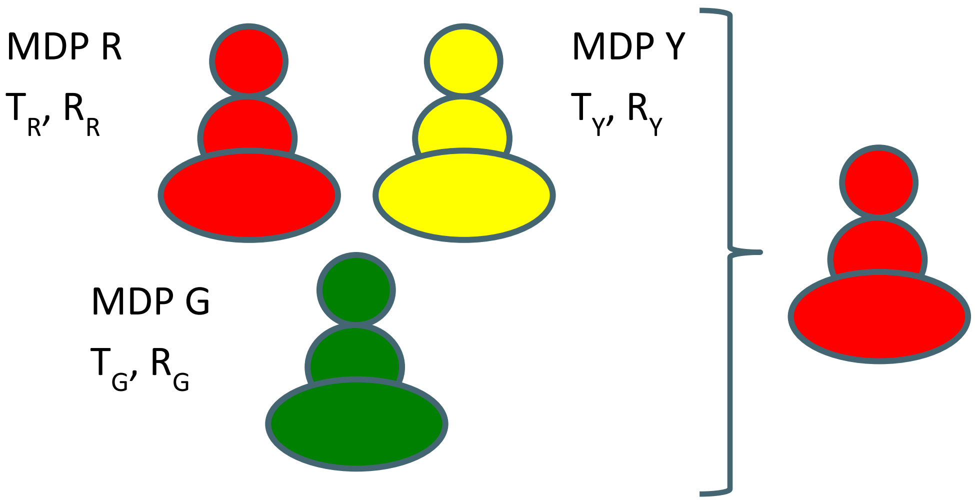
# Example: Multitask Learning Across Finite Set of Markov Decision Processes




Sample a task from finite set of MDPs

Brunskill & Li, UAI 2013

# Example: Multitask Learning Across Finite Set of Markov Decision Processes

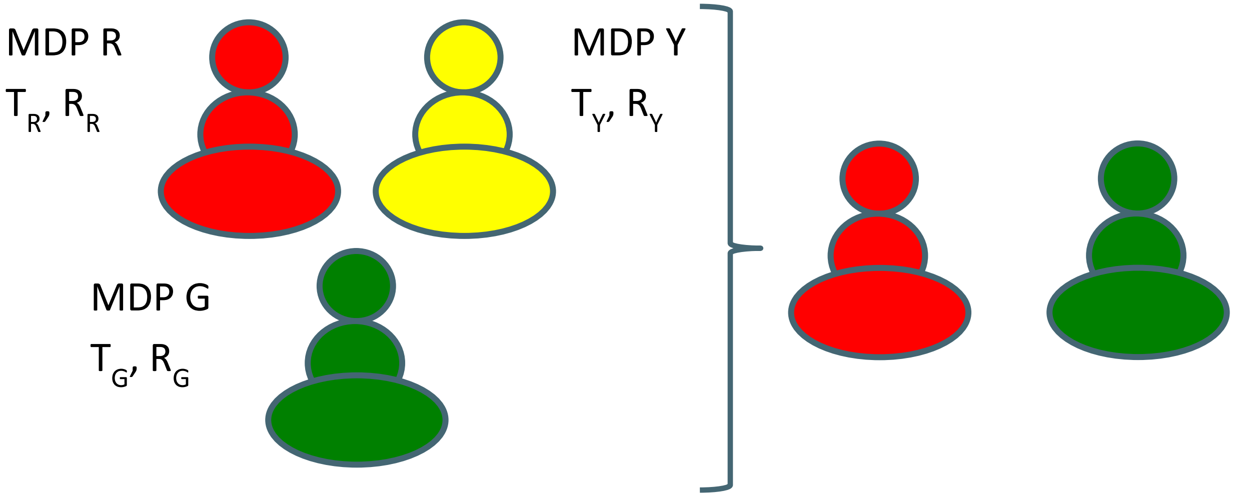



Act in it for H steps

$<s_1, a_1, r_1, s_2, a_2, r_2, s_3, a_3, \ldots s_H>$

Brunskill & Li, UAI 2013

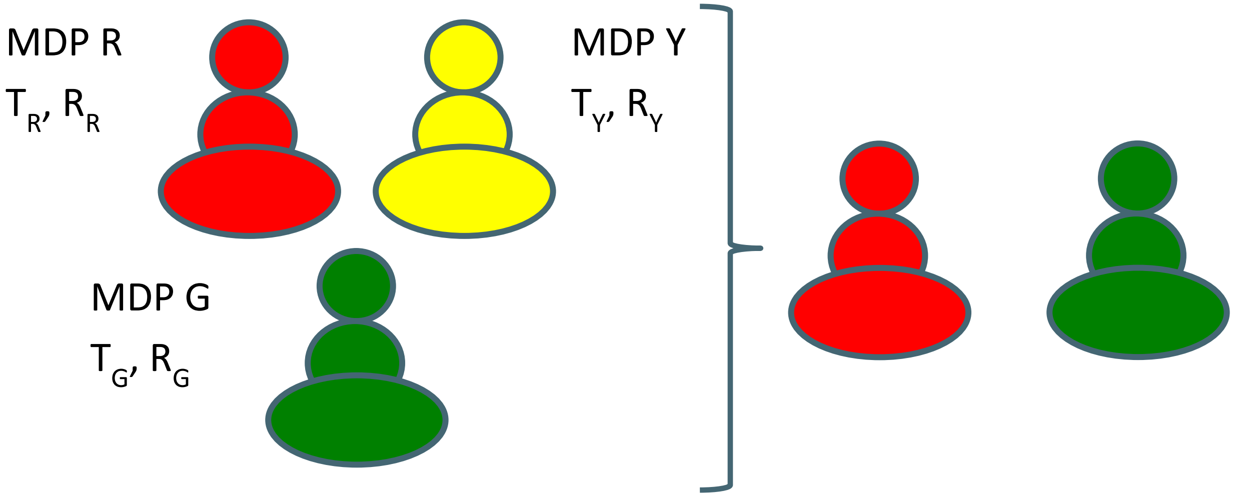
# Example: Multitask Learning Across Finite Set of Markov Decision Processes




Again sample a MDP…

Brunskill & Li, UAI 2013

# Example: Multitask Learning Across Finite Set of Markov Decision Processes

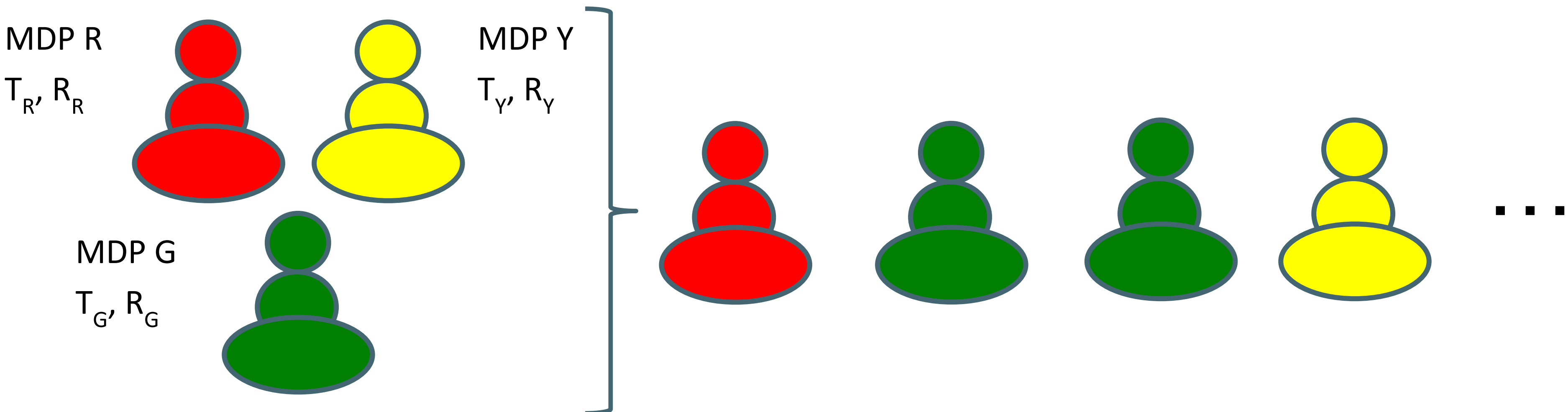



Act in it for H steps

$\langle s_1, a_1, r_1, s_2, a_2, r_2, s_3, a_3, \ldots s_H \rangle$

Brunskill & Li, UAI 2013

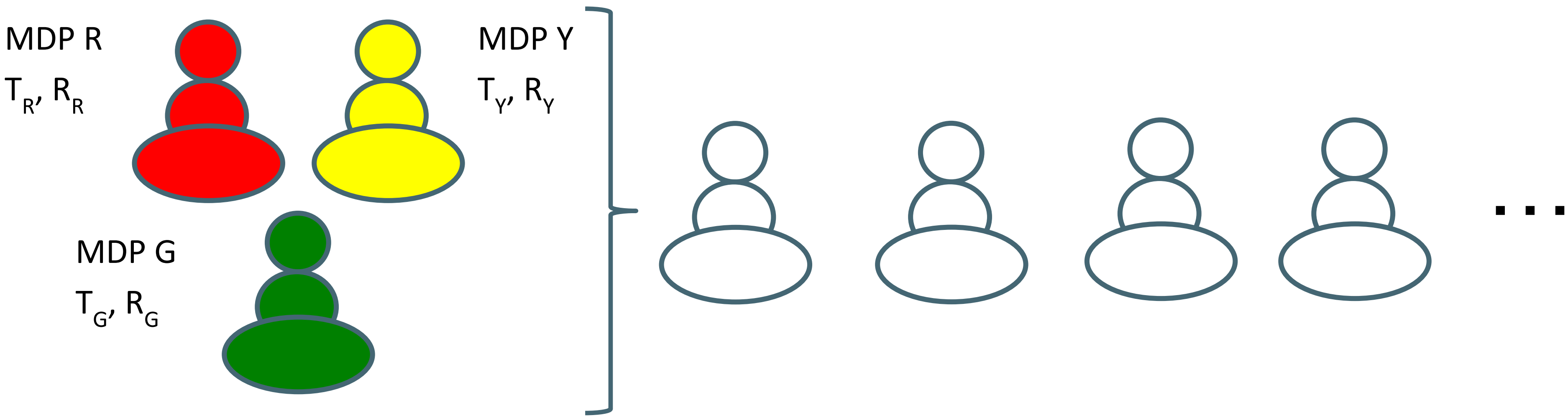
# Example: Multitask Learning Across Finite Set of Markov Decision Processes




Series of tasks
Act in each task for H steps

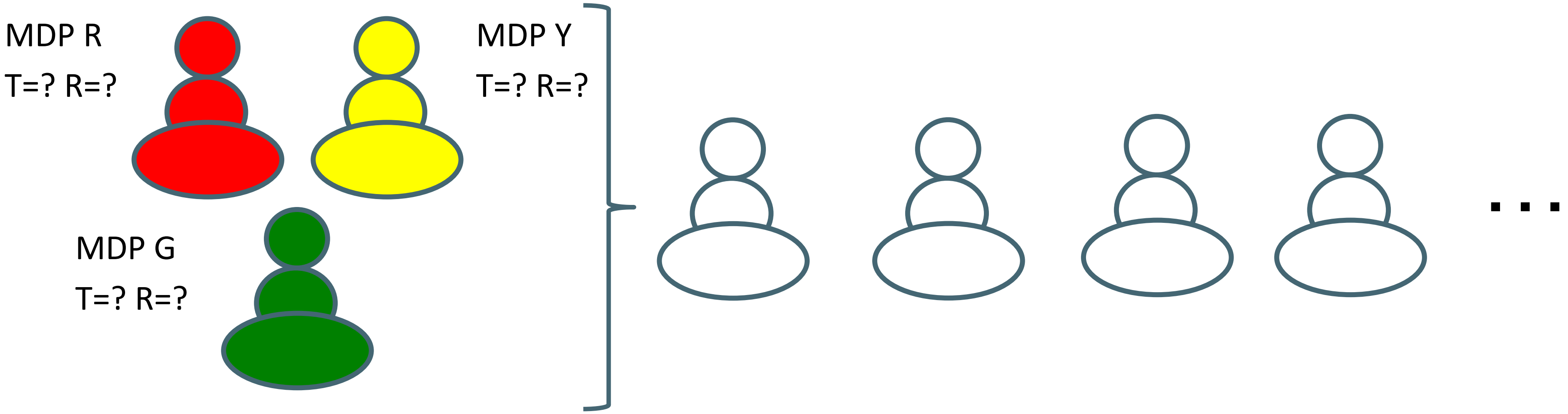
Brunskill & Li, UAI 2013

# Example: Multitask Learning Across Finite Set of Markov Decision Processes




Brunskill & Li, UAI 2013

# Example: Multitask Learning Across Finite Set of Markov Decision Processes




Brunskill & Li, UAI 2013

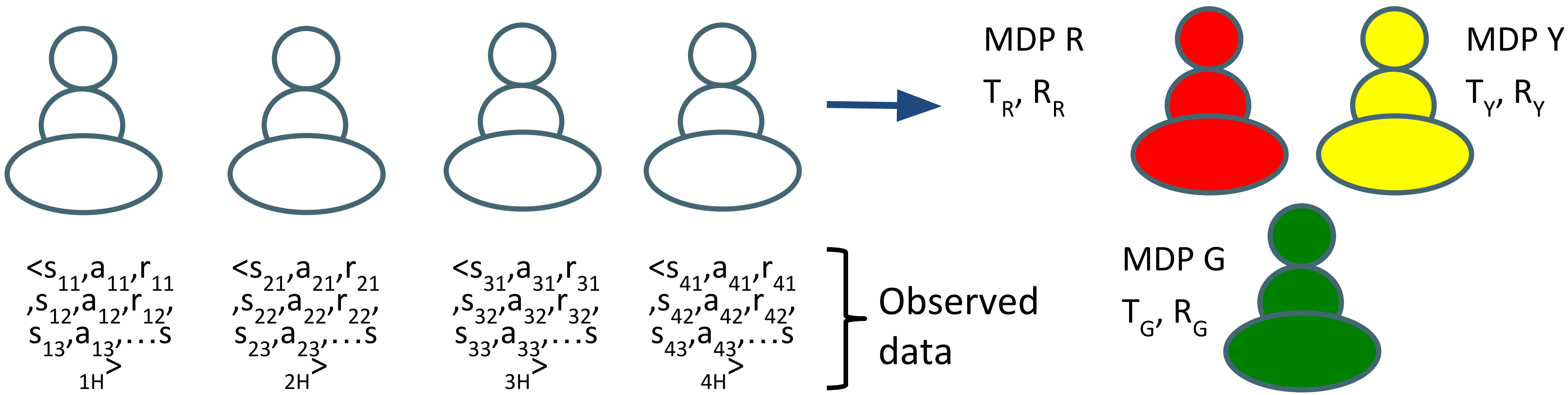
# 2 Key Challenges in Multi-task / Lifelong Learning Across Decision Making Tasks

1. How to summarize past experience in old tasks?
2. How to use prior experience to accelerate learning / improve performance in new tasks? (if have 1 of M tasks/policies)

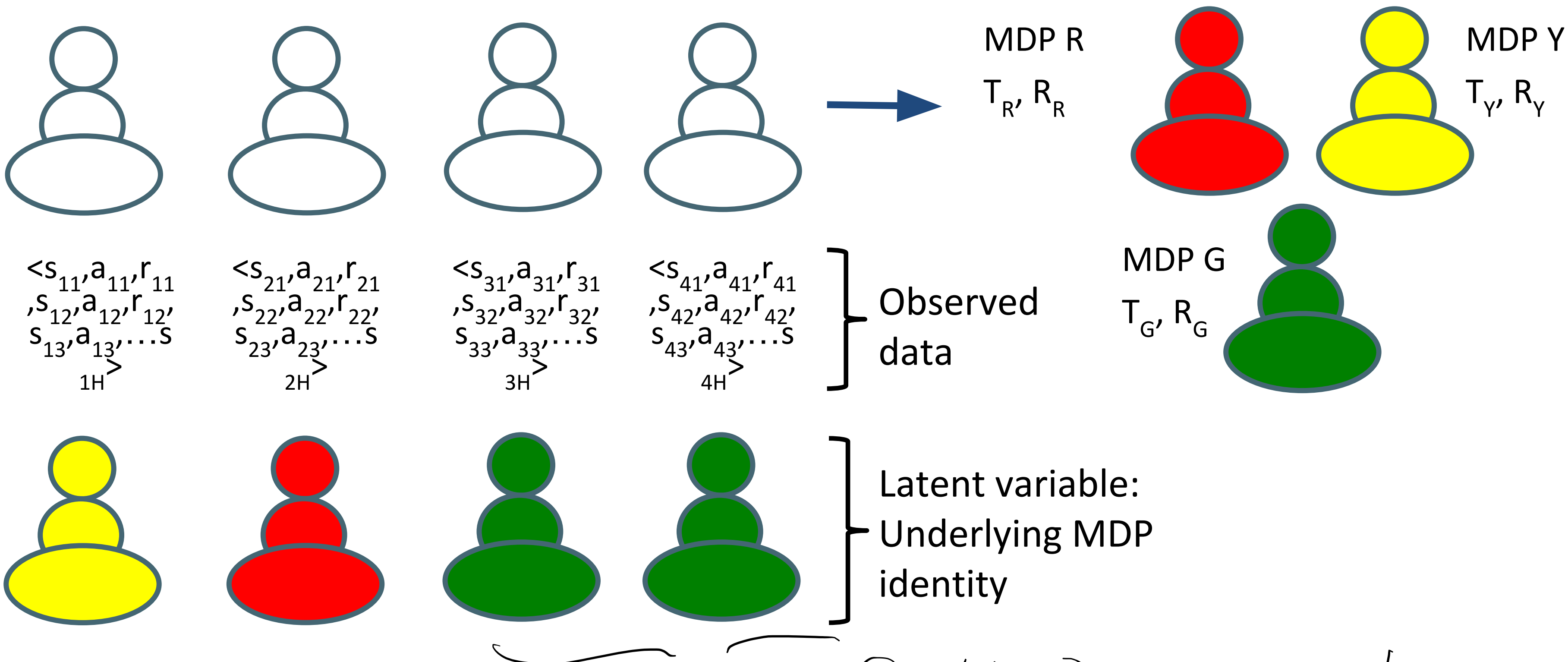
# Summarizing Past Task Experience

- Assume a finite (potentially large) set of sequential decision making tasks
- Learn models of tasks from data

# Latent Variable Modeling



# Latent Variable Modeling




→ MLE of MDP parameters

# Latent Variable Modeling Background

- Formally hard problem
- Expectation Maximization has weak theoretical guarantees
- Recent finite sample bounds on learned parameter estimates

# Separability for Latent Variable Modeling

Assume for any 2 finite state—action MDPs $M_i$ & $M_j$, there exists at least one state—action pair such that

$$\|\underbrace{\theta_i(\cdot|s,a)}_{} - \underbrace{\theta_j(\cdot|s,a)}_{\text{Vector of transition \& reward parameters for (s,a) for MDP } M_j}\| > \Gamma$$

$\exists s, a$

$| r_1(s,a) - r_2(s,a) | > \epsilon$

MDP

Note: to guarantee ε-optimal performance, very small differences in models are irrelevant. *Implies above property always holds in discrete MDPs for some Γ = f(ε)*

Brunskill & Li, UAI 2013

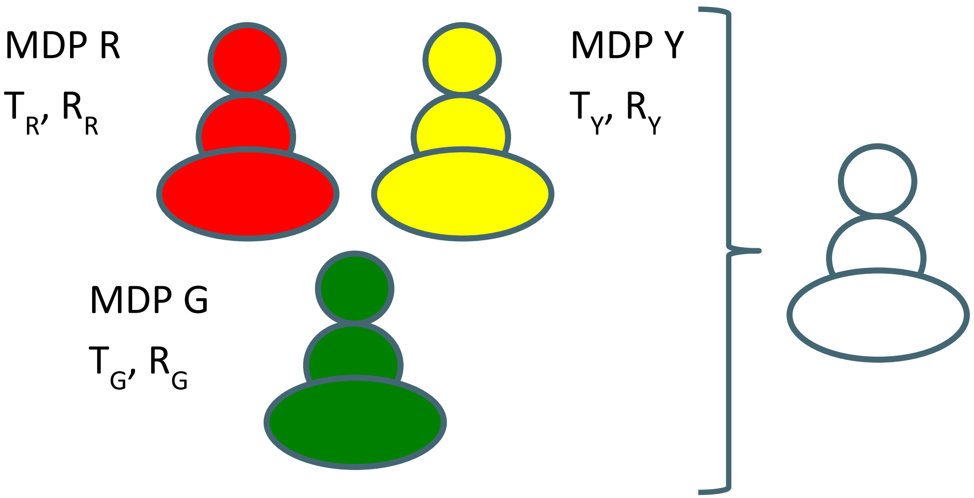
# Implications of Separability for Learning & Representing Task Knowledge

- Assume can visit any part of the decision making task an unbounded number of times
- If time horizon per task sufficiently long, can learn $O(\Gamma)$-accurate task parameters with high probability

$\rightarrow$ Can correctly cluster tasks

Brunskill & Li, UAI 2013

# Recall: Using Task Models to Accelerate Learning in New Task*




Act in it for H steps
$<s_1,a_1,r_1,s_2,a_2,r_2,s_3,a_3,\ldots s_H>$

- Track L2 error of model predictions of observed transitions (s,a,r,s') in current task
- Use to identify current task as 1 of M tasks

Brunskill & Li, UAI 2013

# Sample Complexity Substantially Improved

**Theorem 1** *Given any* $\epsilon$ *and* $\delta$, *run Algorithm 1 for* $T$ *tasks, each for* $H = O\left(DSA(\max(\frac{1}{\Gamma^2}\log\frac{T}{\delta}, SD^2)\right)$ *steps. Then, the algorithm will select an* $\epsilon$*-optimal policy on all but at most* $\tilde{O}\left(\frac{\zeta V_{\max}}{\epsilon(1-\gamma)}\right)$ *steps, with probability at least* $1-\delta$, *where*

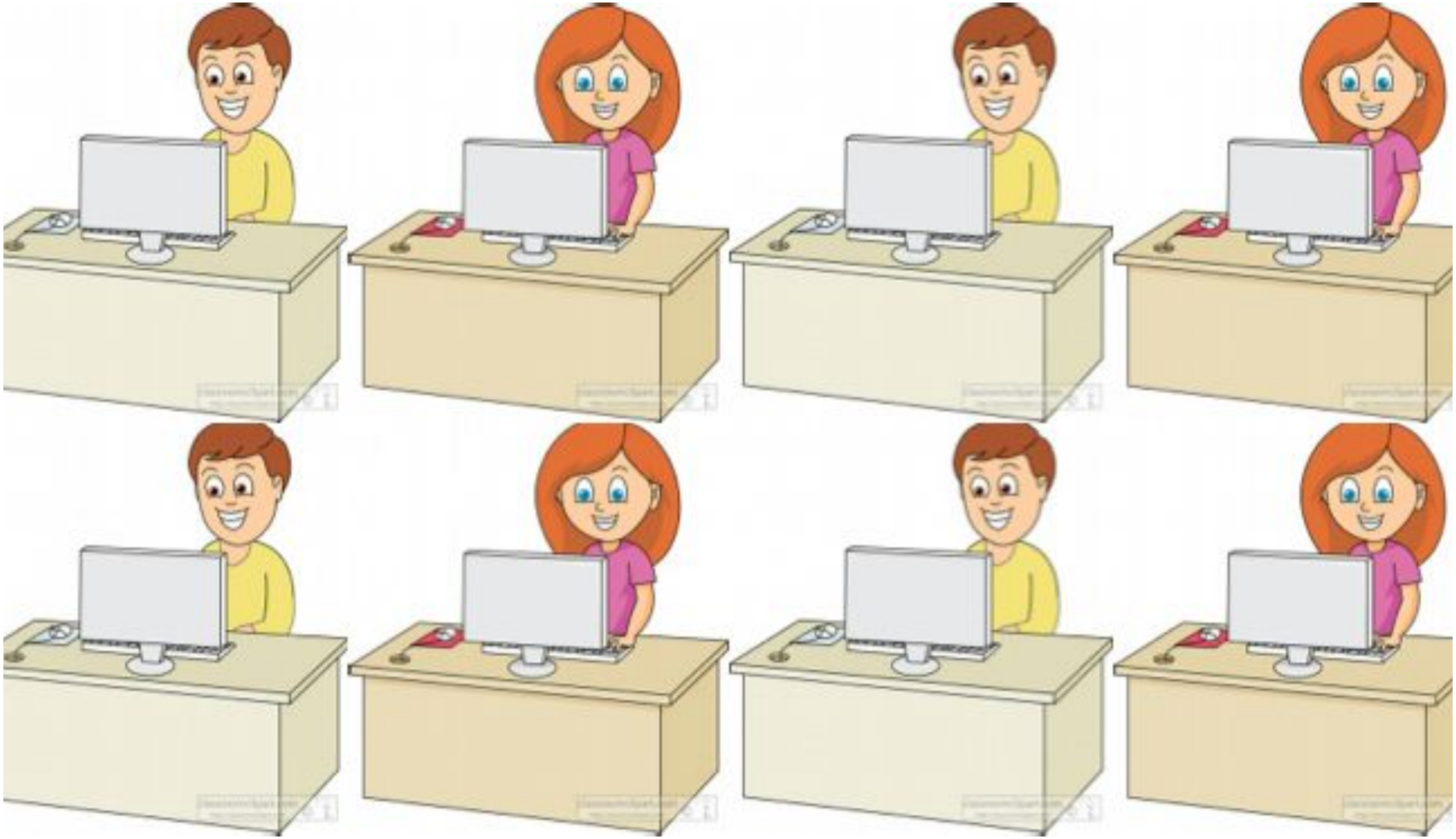
$$\zeta = O\left(T_1\zeta_s + \bar{C}\zeta_s + (T-T_1)\frac{\bar{C}D}{\Gamma^2}\right),$$

*and* $\zeta_s = \tilde{O}\left(\frac{NSAV_{\max}^2}{\epsilon^2(1-\gamma)^2}\right)$, *with probability at least* $1-\delta$.

- 1st result, to our knowledge, that *multi-task learning can provably speed learning* in later sequential decision making tasks

Brunskill & Li, UAI 2013 & in prep

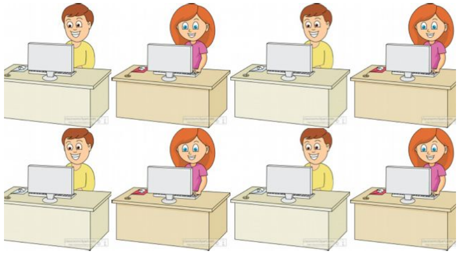
# Concurrent RL



Or all customers using Amazon, or patients, or robot farm...

# Concurrent but Independent

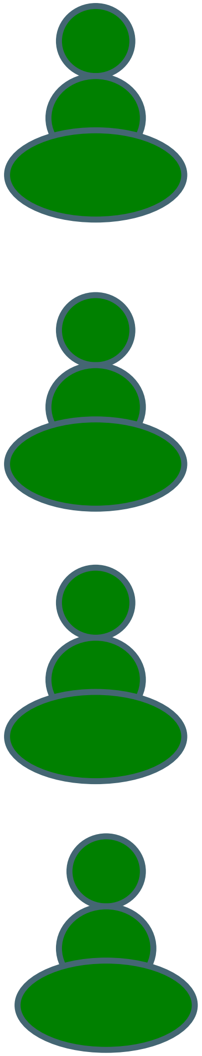
# Concurrent but Independent



- Very little prior work on concurrent RL
- Except encouraging empirical paper that might be very useful for customers (Silver et al. 2013)

# Concurrent RL in Same MDP

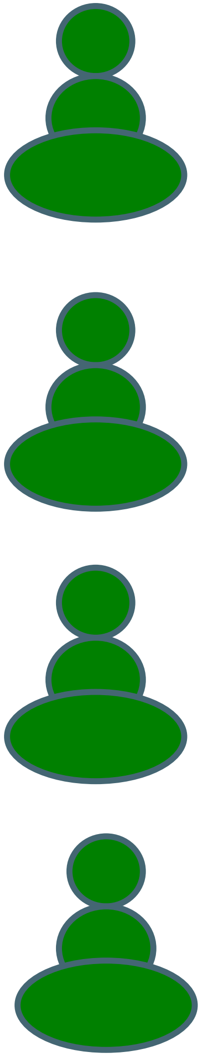
- N copies of same task
- Best possible improvement in how long takes to learn a good policy?



Guo and Brunskill, AAAI 2015

# Concurrent RL in Same MDP

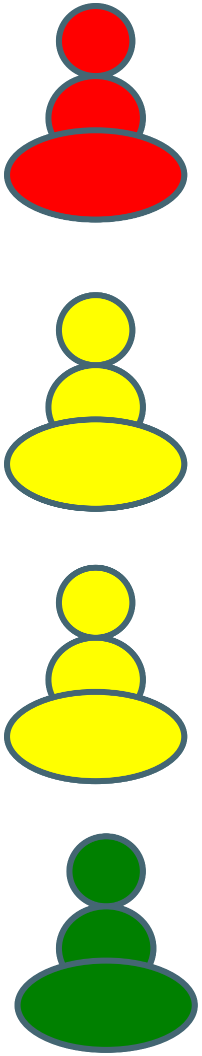
- N copies of same task
- Best possible improvement in how long takes to learn a good policy?
- Linear improvement
- Proved this for sample complexity (within minor restrictions)
- Interesting:
  - Needed no explicit coordination
  - Algorithm: concurrent MBIE



Guo and Brunskill, AAAI 2015

# Concurrent RL in Finite Set of MDPs

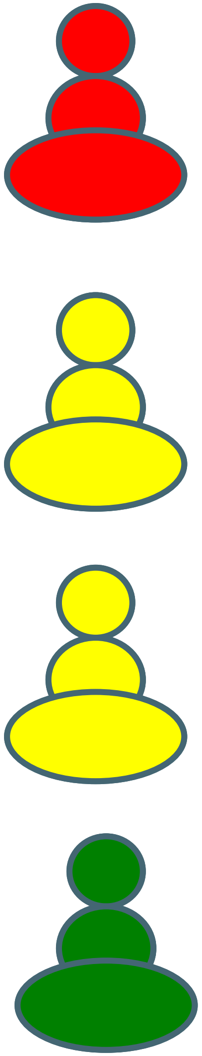
- Task identity unknown
- Just know there is a finite set
- Latent variable modeling!
- Assume separability again



Guo and Brunskill, AAAI 2015

# Concurrent RL in Finite Set of MDPs

- For **t=1:T steps**
  - Explore state-action space in each MDP
- Cluster tasks
- Run concurrent MBIE in each cluster for all future time steps

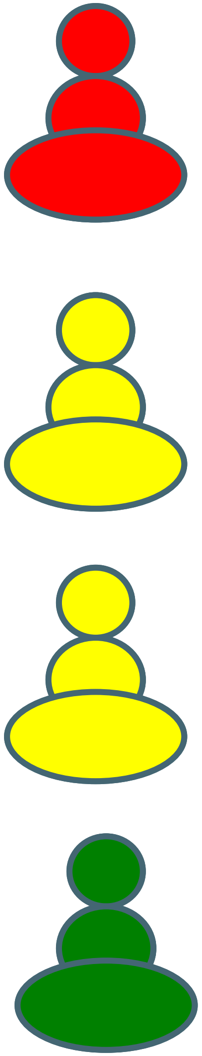


Guo and Brunskill, AAAI 2015

# Can Be Much Faster!

- If samples to cluster << samples to learn optimal policy

  ≈ Linear speedup*



*Sample complexity over not sharing data

Guo and Brunskill, AAAI 2015

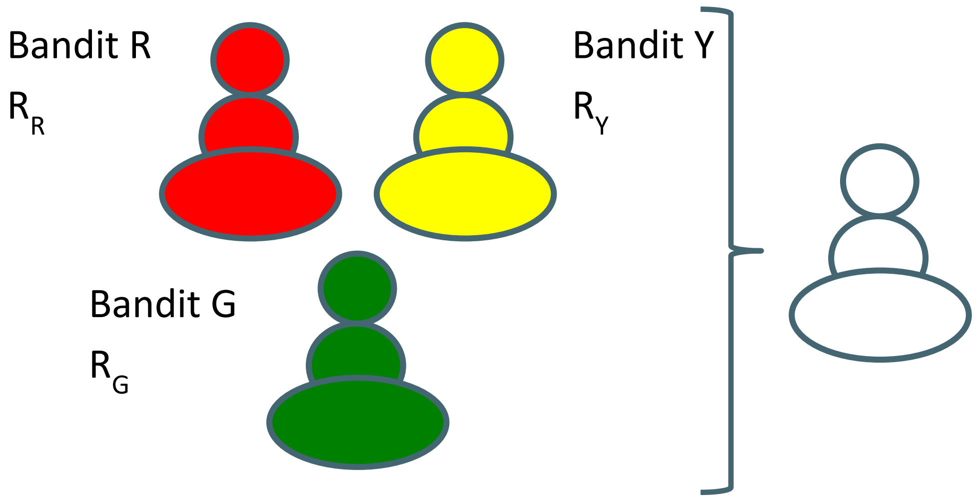
# 2 Key Challenges in Multi-task / Lifelong Learning Across Decision Making Tasks

1. How to summarize past experience in old tasks? Latent variable modeling
   - Separability assumption
   - **Alternate assumptions?**
2. How to use prior experience to accelerate learning / improve performance in new tasks?

# Method of Moments for Latent Variable Modeling

- Required # of interaction steps per task is very short
- Need to be able to visit all relevant state/actions during that time

# Regret Bounds for Multitask Learning across Latent Bandits




Act in it for H steps

$\langle a_1, r_1,, a_2, r_2,, a_3, \ldots s_H \rangle$

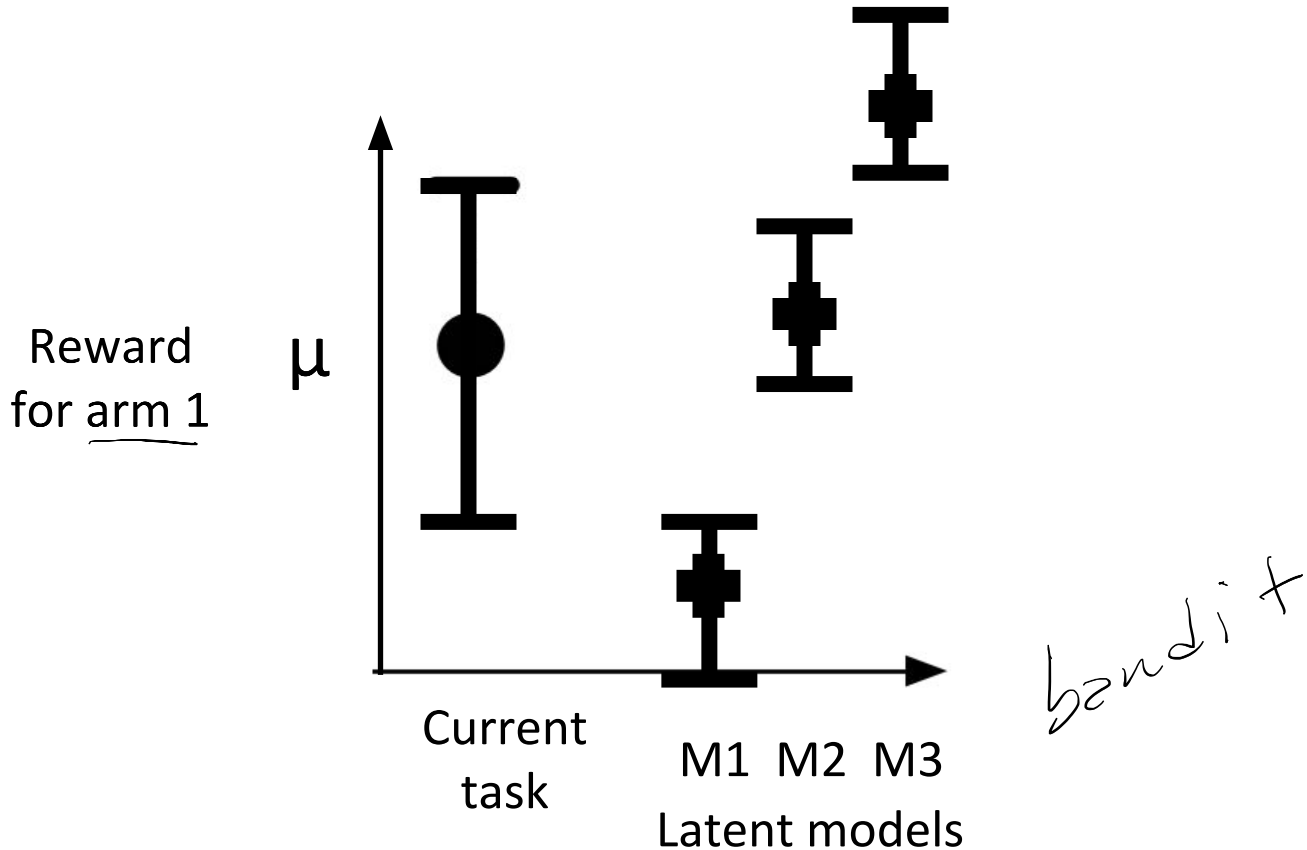
Azar, Lazaric & Brunskill, NIPS 2013

# Method of Moments to Learn Multitask Latent Bandit Parameters

- Used robust tensor power method (Anandkumar et al. 2014)
- Yields confidence bounds over latent bandit parameters

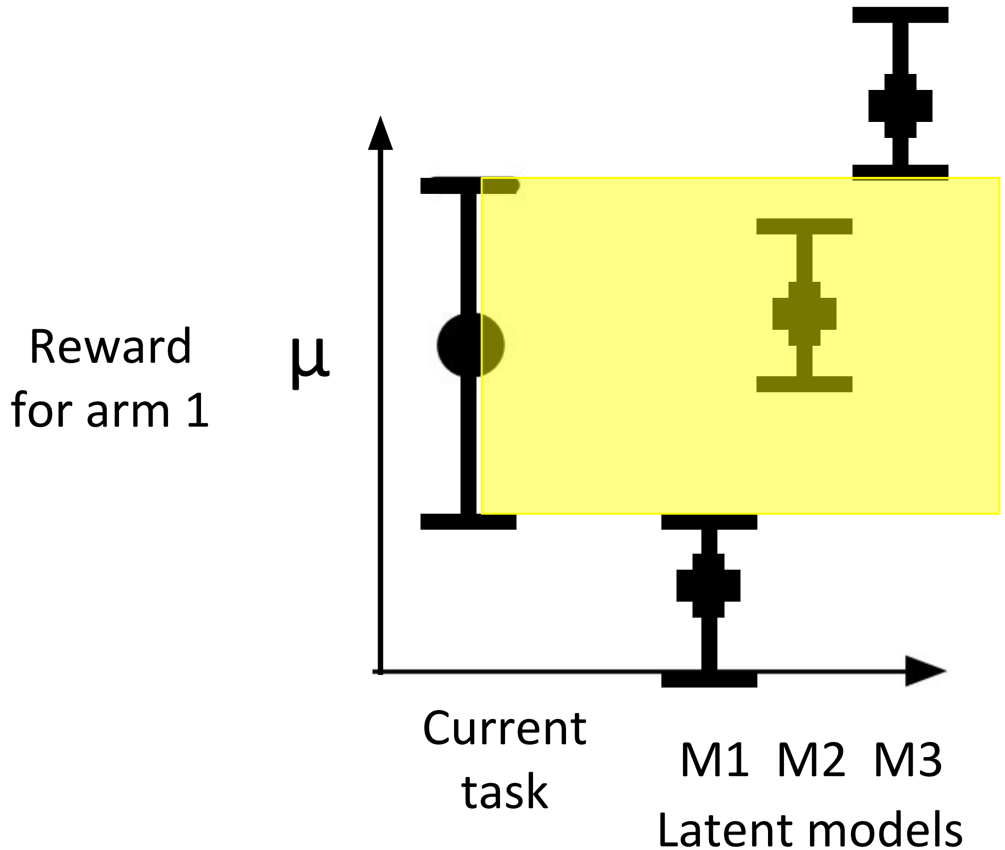
Azar, Lazaric & Brunskill, NIPS 2013

# Using Prior Information to Speed Learning in Latent Bandits



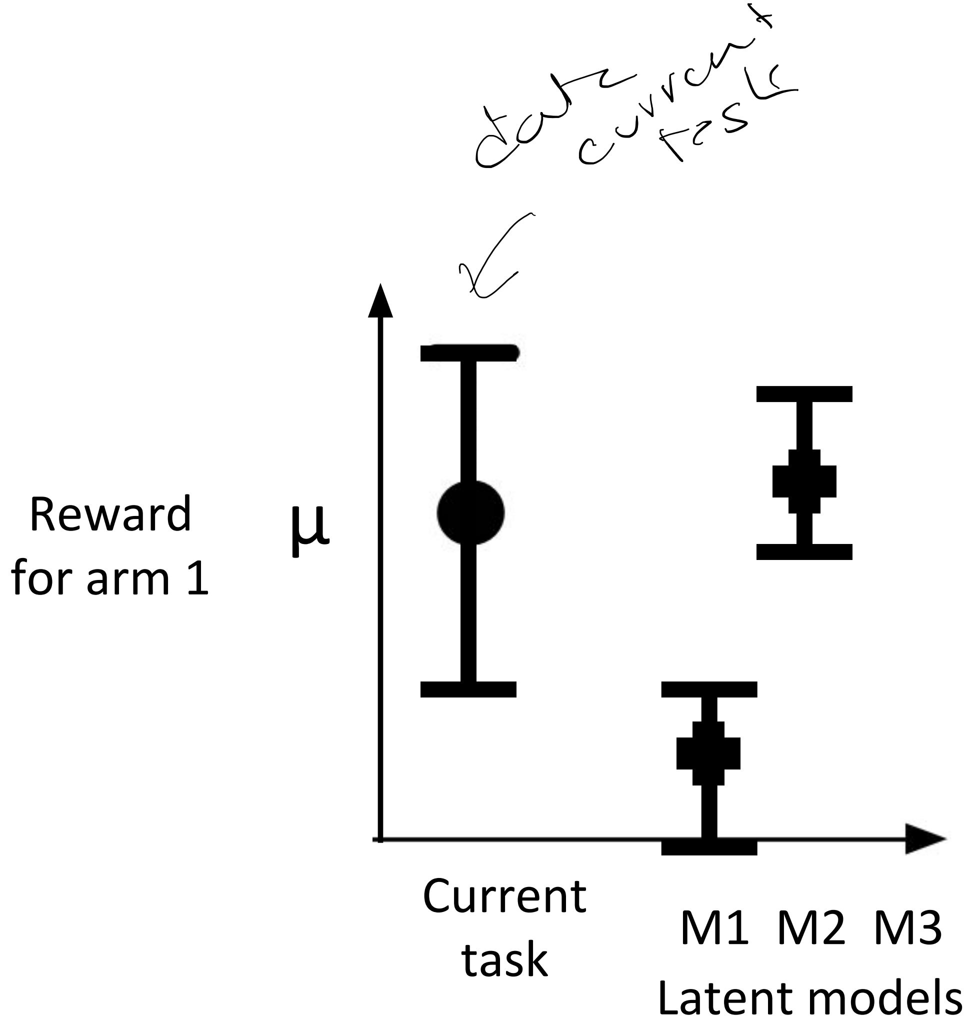

Azar, Lazaric & Brunskill, NIPS 2013

# Active Set is Models Compatible with Current Task's Data



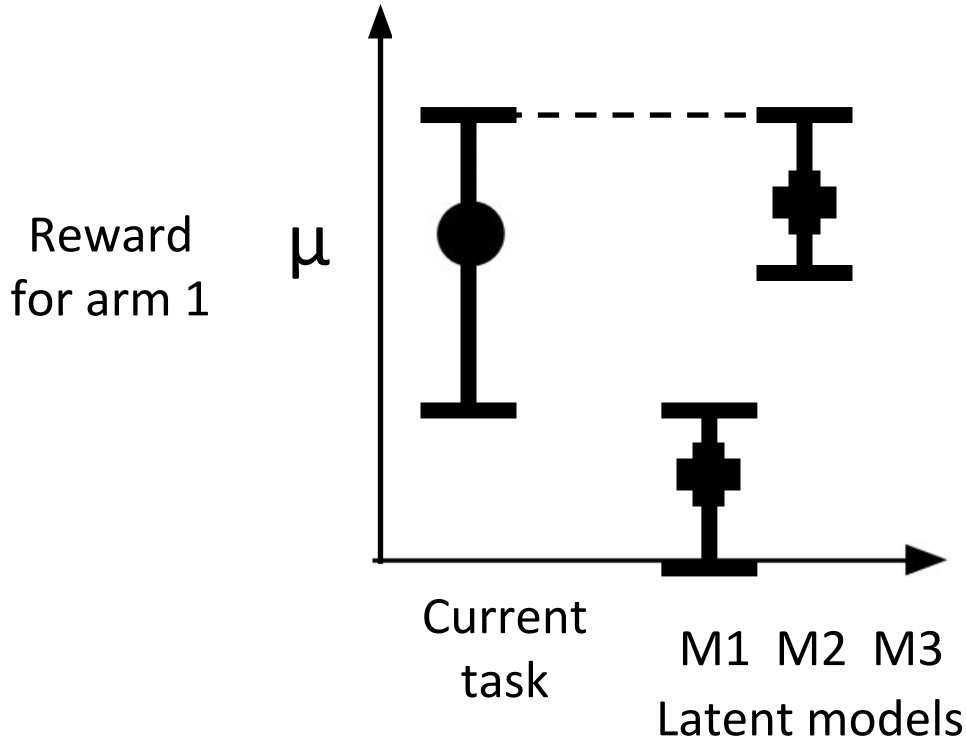

Azar, Lazaric & Brunskill, NIPS 2013

# Active Set is Models Compatible with Current Task's Data




Azar, Lazaric & Brunskill, NIPS 2013

# Upper Bound is Now Upper Bound of Active Set




Azar, Lazaric & Brunskill, NIPS 2013

# Regret of Transfer Upper Confidence Bound for Multitask Bandits

Theorem. If tUCB is run over J tasks of n steps, where each task is drawn from a set of models Θ, then with probability at least 1 – δ, its cumulative regret is

$$\mathcal{R}_J \leq JK + \sum_{j=1}^{J} \sum_{i \in \mathcal{A}_1^j} \min\left\{ \frac{2\log\left(2mKn^2/\delta\right)}{\Delta_i(\bar{\theta}^j)^2}, \frac{\log\left(2mKn^2/\delta\right)}{2\min_{\theta \in \Theta_{i,+}^j(\bar{\theta}^j)} \widehat{\Gamma}_i^j(\theta; \bar{\theta}^j)^2} \right\} \Delta_i(\bar{\theta}^j)$$

$$+ \sum_{j=1}^{J} \sum_{i \in \mathcal{A}_2^j} \frac{2\log\left(2mKn^2/\delta\right)}{\Delta_i(\bar{\theta}^j)},$$

where K = # arms, $\mathcal{A}_1^j$ = the set of best arms of models that can be discarded during task j, $\mathcal{A}_2^j$ = the set of best arms of models that cannot be discarded during task j

& m = # of models

Azar, Lazaric & Brunskill, NIPS 2013

# Converges to Regret as if Knew Models!

Theorem. If tUCB is run over J tasks of n steps, where each task is drawn from a set of models Θ, then with probability at least 1 – δ, its cumulative regret is

$$\mathcal{R}_J \leq JK + \sum_{j=1}^{J} \sum_{i \in \mathcal{A}_1^j} \min\left\{ \frac{2\log\left(2mKn^2/\delta\right)}{\Delta_i(\bar{\theta}^j)^2}, \frac{\log\left(2mKn^2/\delta\right)}{2\min_{\theta \in \Theta_{i,+}^j(\bar{\theta}^j)} \widehat{\Gamma}_i^j(\theta; \bar{\theta}^j)^2} \right\} \Delta_i(\bar{\theta}^j)$$

where K = # arms, $\mathcal{A}_1^j$ = the set of best arms of models that can be discarded d…ıg task j,

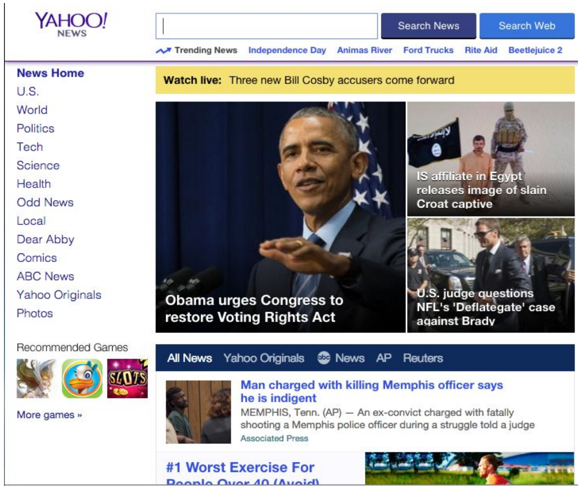
& m = # of models

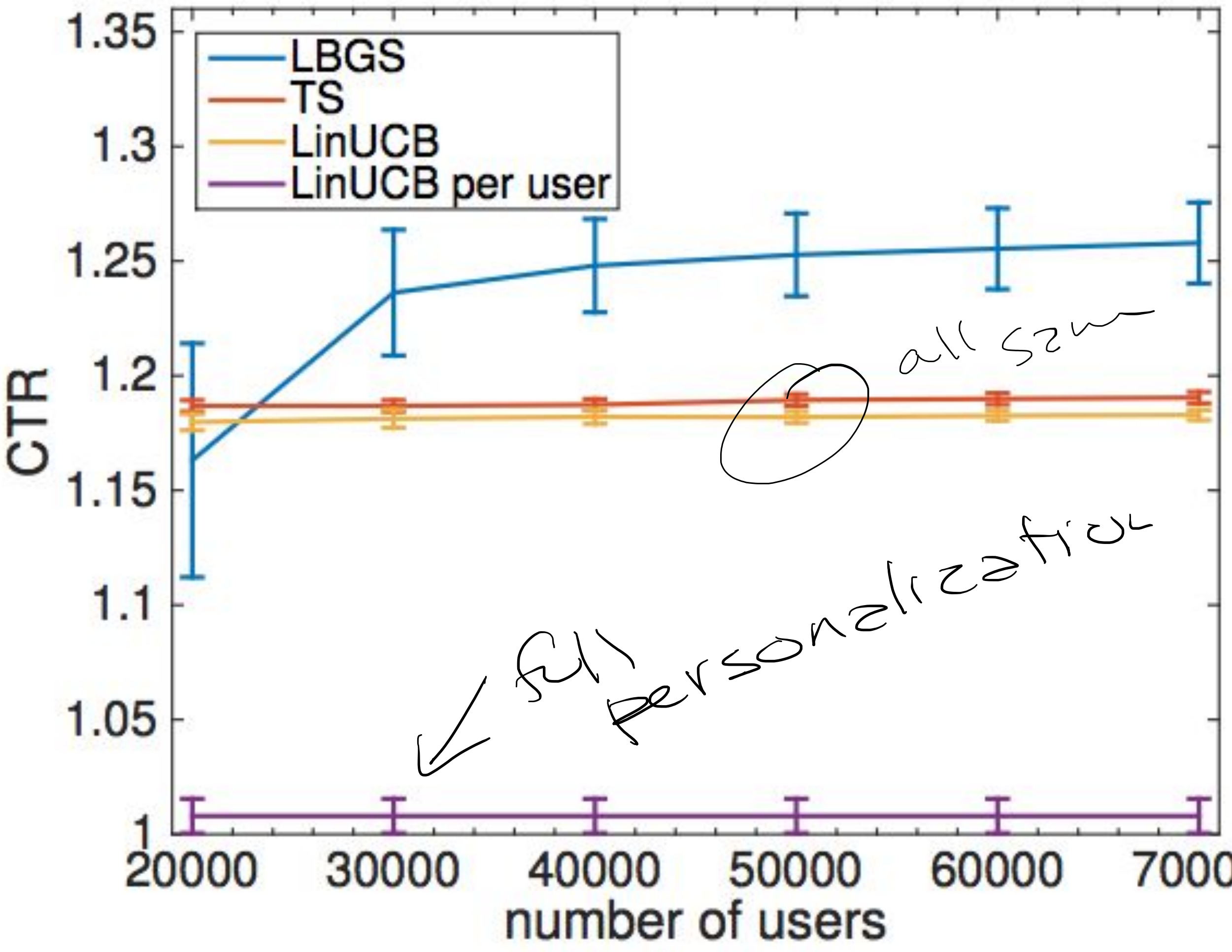
Azar, Lazaric & Brunskill, NIPS 2013

# Multitask Learning & Partial Personalization: Additional Work

- Separability assumptions
  - Concurrent RL (Guo & B., AAAI 2015)
  - Multi-task RL options learning (Li & B. ICML 2014)
  - Continuous-state multi-task RL (Liu, Guo & B. AAMAS 2016 16)
- Method of moments
  - Contextual latent bandits

# Offline Evaluation of Online Latent Contextual Bandit for News Personalization



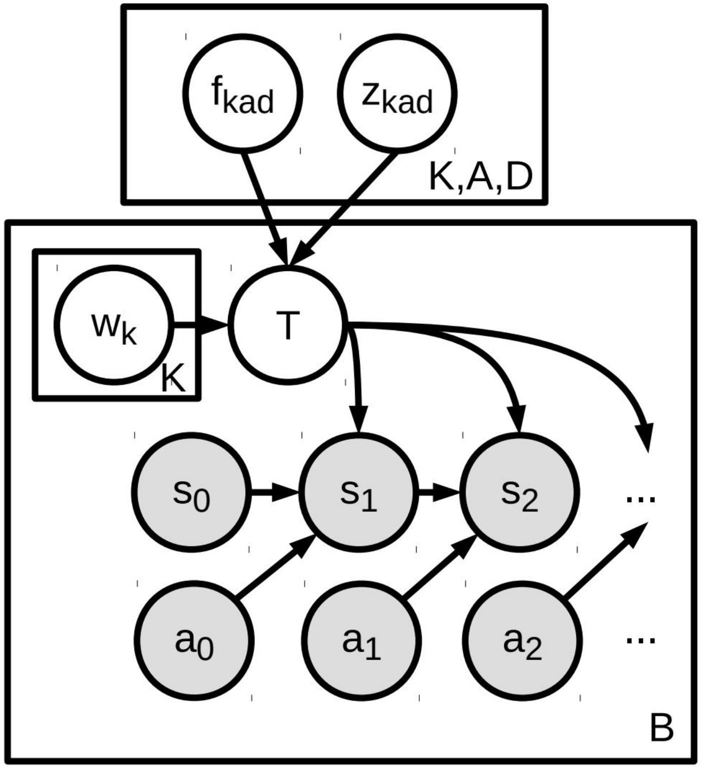




Zhou and Brunskill IJCAI 2016

# From Finite Set of Groups to Continuous Similarity: Hidden Parameter MDPs

- Allow for smooth linear parameterization of dynamics model

$$
\begin{aligned}
(s'_d - s_d) &\sim \textstyle\sum_k^K z_{kad} w_{kb} f_{kad}(s) + \epsilon \\
\epsilon &\sim N(0, \sigma^2_{nad}),
\end{aligned}
$$

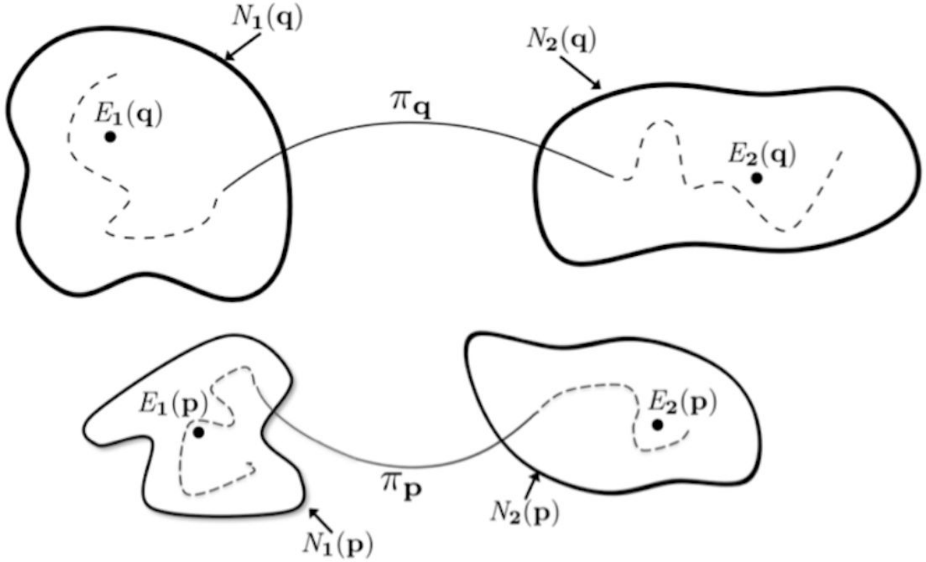


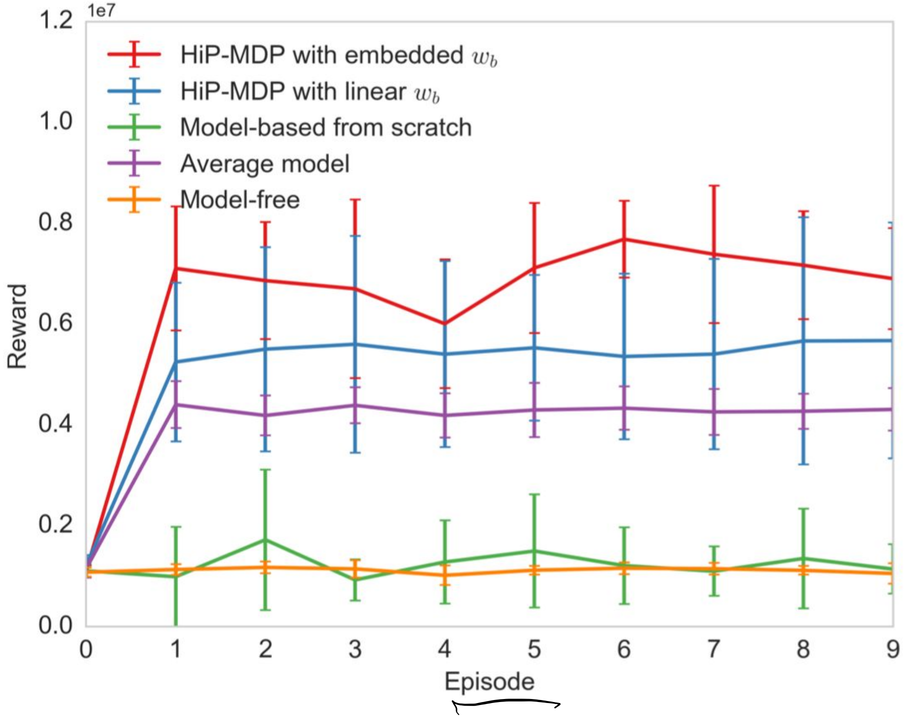


Doshi-Velez, F., & Konidaris, G. (2016, July). Hidden parameter Markov decision processes: A semiparametric regression approach for discovering latent task parametrizations. In *IJCAI: proceedings of the conference*(Vol. 2016, p. 1432).

# Hidden Parameter MDPs ++

- Use Bayesian Neural Nets for dynamics
- Benefits for HIV Treatment simulation
- Each episode new patient

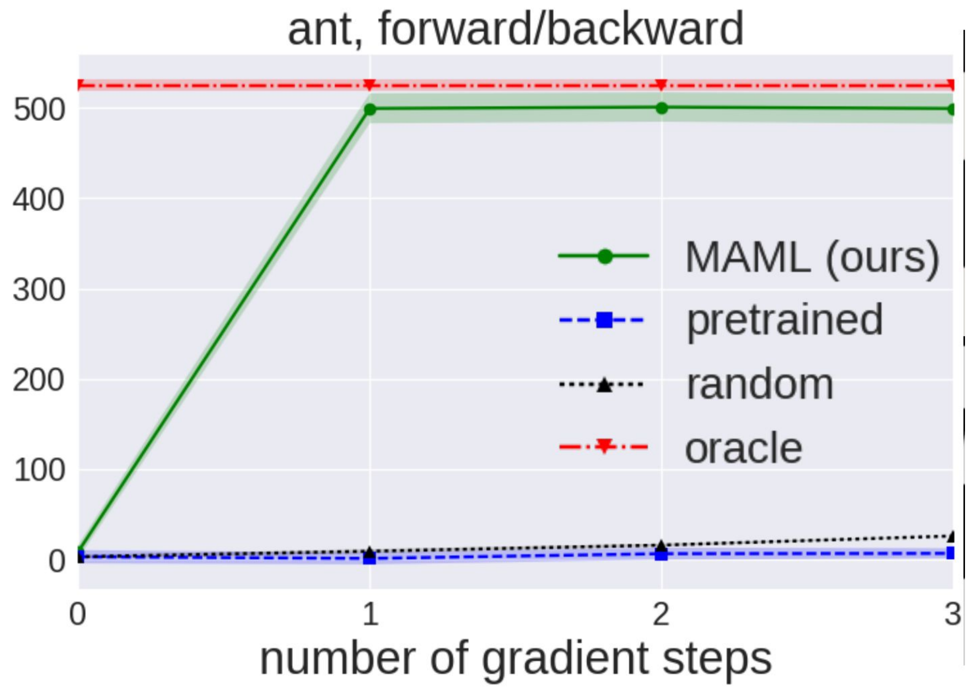



TW Killian, G Konidaris, F Doshi-Velez. Robust and Efficient Transfer Learning with Hidden Parameter Markov Decision Processes. NIPS 2017.

# Model-Agnostic Meta-Learning for Fast Adaptation of Deep Networks

- Transfer / meta learning is useful broadly in tasks involving people
- Deep reinforcement learning to find good shared representation (Finn, Abbeel, Levine ICML 2017)
- Fast transfer by encouraging shared representation learning across tasks



# Open Issues

- What if the domains have different state or action spaces?
- When do we need new models or policies?
  - How do we identify when not to transfer?

# Notes

- A number of the algorithms & results above combined ideas from multiple parts of the class
  - Sample efficient learning
  - Batch reinforcement learning
  - Generalization
- Many important additional challenges, in particular for human focused RL
  - What is the reward?
  - Moving beyond expectation / safe RL
  - Trustworthy and interpretable RL

# What You Should Know From Today

- List the terms used to describe sharing knowledge as learn across tasks ( transfer / lifelong / meta learning)
- Define negative transfer
- Be able to give at least one example application where transfer learning could be useful

# Summary

Next time: quiz

2 sided page of notes allowed

# Problems in Related Classes (Of Similar Difficulty Level)

Q. Thinking about Reinforcement Learning (select which ones are true):

(a) The maximization of the future cumulative reward allows to Reinforcement Learning to perform global decisions with local information

(b) Q-learning is a temporal difference RL method that does not need a model of the task to learn the action value function

(c) Reinforcement Learning only can be applied to problems with a finite number of states

(d) In Markov Decision Problems (MDP) the future actions from a state depend on the previous states

Q. Thinking about reinforcement learning which one (only 1) of the following statements is true:

(a) Estimation using Dynamic Programming is less computational costly than using Temporal Difference Learning

(b) Estimating using Montecarlo methods has the advantage that it is not needed to have absorbent states in the problem

(c) Temporal Difference learning allows on-line learning and Montecarlo methods need complete

training sequences for estimation

(d) Dynamic Programming and Montecarlo methods only work if we know the transitions probabilities for the actions and the reward function

# Problems in Related Classes (Of Similar Difficulty Level)

Q. In RL the discount factor (select all that are true)

A. Is specified in the interval $-1,0$

B. Is important for convergence

C. Adjusts the balance between immediate and delayed rewards

https://www.uio.no/studier/emner/matnat/ifi/INF3490/h14/inf3490-exam-2014.pdf

# Problems in Related Classes (Of Similar Difficulty Level)

Pacman is the model of rationality and seeks to maximize his expected utility, but that doesn't mean he never plays games.

**(a)** [3 pts] **Q-Learning to Play under a Conspiracy.** Pacman does tabular Q-learning (where every state-action pair has its own Q-value) to figure out how to play a game against the adversarial ghosts. As he likes to explore, Pacman always plays a random action. After enough time has passed, every state-action pair is visited infinitely often. The learning rate decreases as needed. For any game state $s$, the value $\max_a Q(s,a)$ for the learned $Q(s,a)$ is equal to (for complete search trees)

- ◯ The minimax value where Pacman maximizes and ghosts minimize.
- ◯ The expectimax value where Pacman maximizes and ghosts act uniformly at random.
- ◯ The expectimax value where Pacman plays uniformly at random and ghosts minimize.
- ◯ The expectimax value where both Pacman and ghosts play uniformly at random.
- ◯ None of the above.

**(b)** [3 pts] **Feature-based Q-Learning the Game under a Conspiracy.** Pacman now runs feature-based Q-learning. The Q-values are equal to the evaluation function $\sum_{i=1}^{n} w_i f_i(s,a)$ for weights $w$ and features $f$. The number of features is much less than the number of states. As he likes to explore, Pacman always plays a random action. After enough time has passed, every state-action pair is visited infinitely often. The learning rate decreases as needed. The value $\max_a Q(s,a)$ for the learned $Q(s,a)$ is equal to (for complete search trees)

- ◯ The minimax value where Pacman maximizes and ghosts minimize and the same evaluation function is used at the leaves.
- ◯ The expectimax value where Pacman maximizes and ghosts act uniformly at random and the same evaluation function is used at the leaves.
- ◯ The expectimax value where Pacman plays uniformly at random and ghosts minimize and the same evaluation function is used at the leaves.
- ◯ The expectimax value where both Pacman and ghosts play uniformly at random and the same evaluation function is used at the leaves.
- ◯ None of the above.

https://s3-us-west-2.amazonaws.com/cs188websitecontent/exams/fa13_midterm1.pdf

# Problems in Related Classes (Of Similar Difficulty Level)

**(c)** [2 pts] **A Costly Game.** Pacman is now stuck playing a new game with only costs and no payoff. Instead of maximizing expected utility $V(s)$, he has to minimize expected costs $J(s)$. In place of a reward function, there is a cost function $C(s, a, s')$ for transitions from $s$ to $s'$ by action $a$. We denote the discount factor by $\gamma \in (0, 1)$. $J^*(s)$ is the expected cost incurred by the optimal policy. Which one of the following equations is satisfied by $J^*$?

- ◯ $J^*(s) = \min_a \sum_{s'} [C(s, a, s') + \gamma \max_{a'} T(s, a', s') * J^*(s')]$
- ◯ $J^*(s) = \min_{s'} \sum_a T(s, a, s')[C(s, a, s') + \gamma * J^*(s')]$
- ◯ $J^*(s) = \min_a \sum_{s'} T(s, a, s')[C(s, a, s') + \gamma * \max_{s'} J^*(s')]$
- ◯ $J^*(s) = \min_{s'} \sum_a T(s, a, s')[C(s, a, s') + \gamma * \max_{s'} J^*(s')]$
- ◯ $J^*(s) = \min_a \sum_{s'} T(s, a, s')[C(s, a, s') + \gamma * J^*(s')]$
- ◯ $J^*(s) = \min_{s'} \sum_a [C(s, a, s') + \gamma * J^*(s')]$

**(d)** [2 pts] **It's a conspiracy again!** The ghosts have rigged the costly game so that once Pacman takes an action they can pick the outcome from all states $s' \in S'(s, a)$, the set of all $s'$ with non-zero probability according to $T(s, a, s')$. Choose the correct Bellman-style equation for Pacman against the adversarial ghosts.

- ◯ $J^*(s) = \min_a \max_{s'} T(s, a, s')[C(s, a, s') + \gamma * J^*(s')]$
- ◯ $J^*(s) = \min_{s'} \sum_a T(s, a, s')[\max_{s'} C(s, a, s') + \gamma * J^*(s')]$
- ◯ $J^*(s) = \min_a \min_{s'} [C(s, a, s') + \gamma * \max_{s'} J^*(s')]$
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- ◯ $J^*(s) = \min_{s'} \sum_a T(s, a, s')[\max_{s'} C(s, a, s') + \gamma * \max_{s'} J^*(s')]$
- ◯ $J^*(s) = \min_a \min_{s'} T(s, a, s')[C(s, a, s') + \gamma * J^*(s')]$



https://s3-us-west-2.amazonaws.com/cs188websitecontent/exams/fa13_midterm1.pdf