Lecture 8: Policy Gradient I [2]

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CS234 Reinforcement Learning.

Winter 2018

- Additional reading: Sutton and Barto 2018 Chp. 13

[2]With many slides from or derived from David Silver and John Schulman and Pieter Abbeel

Last Time: We want RL Algorithms that Perform

- Optimization
- Delayed consequences
- Exploration
- Generalization
- And do it $\underbrace{\text{statistically}}_{}$ and computationally efficiently

high data
efficiency

Last Time: Generalization and Efficiency

- Can use structure and additional knowledge to help constrain and speed reinforcement learning

Class Structure

- Last time: Imitation Learning
- **This time: Policy Search**
- Next time: Policy Search Cont.

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1. Introduction
2. Policy Gradient
3. Score Function and Policy Gradient Theorem
4. Policy Gradient Algorithms and Reducing Variance

Policy-Based Reinforcement Learning

- In the last lecture we approximated the value or action-value function using parameters θ,

$$V_\theta(s) \approx V^\pi(s)$$

$$Q_\theta(s,a) \approx Q^\pi(s,a)$$

look up table

$s \rightarrow a$

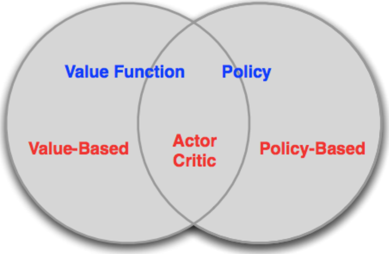
- A policy was generated directly from the value function
  - e.g. using ϵ-greedy
- In this lecture we will directly parametrize the policy

$$\pi_\theta(s,a) = \mathbb{P}[a|s,\theta]$$

- Goal is to find a policy π with the highest value function V^π
- We will focus again on model-free reinforcement learning

Value-Based and Policy-Based RL

- Value Based
  - Learnt Value Function
  - Implicit policy (e.g. ϵ-greedy)
- Policy Based
  - No Value Function
  - Learnt Policy
- Actor-Critic
  - Learnt Value Function
  - Learnt Policy



Advantages of Policy-Based RL

Advantages:

- Better convergence properties
- Effective in high-dimensional or continuous action spaces robotics
- Can learn stochastic policies

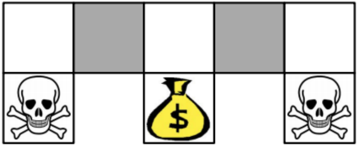
Disadvantages:

- Typically converge to a local rather than global optimum
- Evaluating a policy is typically data inefficient and high variance

Example: Rock-Paper-Scissors

- Two-player game of rock-paper-scissors
  - Scissors beats paper
  - Rock beats scissors
  - Paper beats rock
- Consider policies for iterated rock-paper-scissors
  - A deterministic policy is easily exploited
  - A uniform random policy is optimal (i.e. Nash equilibrium)

Example: Aliased Gridword (1)



- The agent cannot differentiate the grey states
- Consider features of the following form (for all N, E, S, W)

$$\phi(s, a) = \mathbb{1}(\text{wall to N}, a = \text{move E})$$

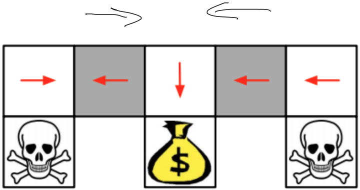
- Compare value-based RL, using an approximate value function

$$Q_\theta(s, a) = f(\phi(s, a), \theta)$$

- To policy-based RL, using a parametrised policy

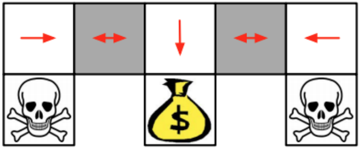
$$\pi_\theta(s, a) = g(\phi(s, a), \theta)$$

Example: Aliased Gridworld (2)



- Under aliasing, an optimal deterministic policy will either
  - move W in both grey states (shown by red arrows)
  - move E in both grey states
- Either way, it can get stuck and never reach the money
- Value-based RL learns a near-deterministic policy
  - e.g. greedy or ϵ-greedy
- So it will traverse the corridor for a long time

Example: Aliased Gridworld (3)



- An optimal stochastic policy will randomly move E or W in grey states

$$\pi_\theta(\text{wall to N and W, move E}) = 0.5$$

$$\pi_\theta(\text{wall to N and W, move W}) = 0.5$$

- It will reach the goal state in a few steps with high probability
- Policy-based RL can learn the optimal stochastic policy

Policy Objective Functions

- Goal: given a policy $\pi_\theta(s,a)$ with parameters θ, find best θ
- But how do we measure the quality for a policy π_θ?
- In episodic environments we can use the start value of the policy

$$J_1(\theta) = V^{\pi_\theta}(s_1) = \mathbb{E}_{\pi_\theta}[v_1]$$

- In continuing environments we can use the average value

$$J_{avV}(\theta) = \sum_s \underbrace{d^{\pi_\theta}(s)}_{} V^{\pi_\theta}(s)$$

- where $d^{\pi_\theta}(s)$ is the stationary distribution of Markov chain for π_θ.
- Or the average reward per time-step

$$J_{avR}(\theta) = \sum_s d^{\pi_\theta}(s) \sum_a \pi_\theta(s,a) \mathcal{R}_s^a$$

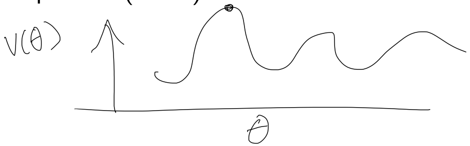
- For simplicity, today will mostly discuss the episodic case, but can easily extend to the continuing / infinite horizon case

Policy optimization

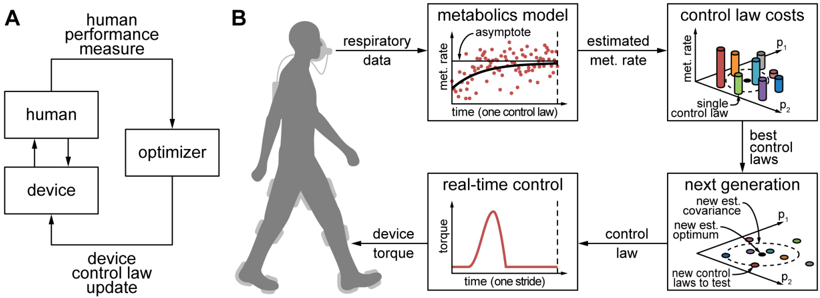
- Policy based reinforcement learning is an optimization problem
- Find policy parameters θ that maximize V^{π_θ}

Policy optimization

- Policy based reinforcement learning is an optimization problem
- Find policy parameters θ that maximize V^{π_θ}
- Can use gradient free optimization:
    - Hill climbing
    - Simplex / amoeba / Nelder Mead
    - Genetic algorithms
    - Cross-Entropy method (CEM)
    - Covariance Matrix Adaptation (CMA)



Recall Human-in-the-Loop Exoskeleton Optimization (Zhang et al. Science 2017)



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- Optimization was done using CMA-ES, variation of covariance matrix evaluation

Gradient Free Policy Optimization

- Can often work embarrassingly well: "discovered that evolution strategies (ES), an optimization technique that's been known for decades, rivals the performance of standard reinforcement learning (RL) techniques on modern RL benchmarks (e.g. Atari/MuJoCo)" (https://blog.openai.com/evolution-strategies/)

Gradient Free Policy Optimization

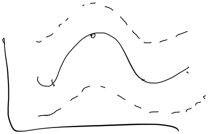
- Often a great simple baseline to try
- Benefits
  - Can work with any policy parameterizations, including non-differentiable
  - Frequently very easy to parallelize (computation vs data tradeoff)
- Limitations
  - Typically not very sample efficient because it ignores temporal structure

Policy optimization

- Policy based reinforcement learning is an optimization problem
- Find policy parameters θ that maximize V^{π_θ}
- Can use gradient free optimization:
- Greater efficiency often possible using gradient
  - Gradient descent
  - Conjugate gradient
  - Quasi-newton
- We focus on gradient descent, many extensions possible
- And on methods that exploit sequential structure

Bayesian optimiz

GPs



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Policy Gradient

- Define $V(\theta) = V^{\pi_\theta}$ to make explicit the dependence of the value on the policy parameters
- Assume episodic MDPs (easy to extend to related objectives, like average reward)

Policy Gradient

- Define $V(\theta) = V^{\pi_\theta}$ to make explicit the dependence of the value on the policy parameters
- Assume episodic MDPs
- Policy gradient algorithms search for a *local* maximum in $V(\theta)$ by ascending the gradient of the policy, w.r.t parameters θ

$$\nabla\theta = \alpha \nabla_\theta V(\theta)$$

- Where $\nabla_\theta V(\theta)$ is the policy gradient

$$\underbrace{\nabla_\theta V(\theta)}_{\text{only need to estim up to a proportion}} = \begin{pmatrix} \frac{\delta V(\theta)}{\delta \theta_1} \\ \vdots \\ \frac{\delta V(\theta)}{\delta \theta_n} \end{pmatrix}$$

policy has parameters $\theta_1 \ldots \theta_n$

- and α is a step-size parameter

Computing Gradients by Finite Differences

- To evaluate policy gradient of $\pi_\theta(s, a)$
- For each dimension $k \in [1, n]$
  - Estimate kth partial derivative of objective function w.r.t. θ
  - By perturbing θ by small amount ϵ in kth dimension

$$\frac{\delta V(\theta)}{\delta \theta_k} \approx \frac{V(\theta + \epsilon u_k) - V(\theta)}{\epsilon}$$

where u_k is a unit vector with 1 in kth component, 0 elsewhere.

Computing Gradients by Finite Differences

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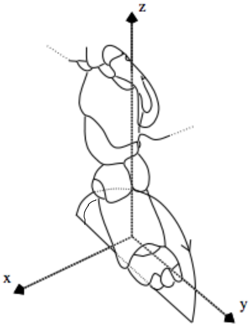
where u_k is a unit vector with 1 in kth component, 0 elsewhere.

- Uses n evaluations to compute policy gradient in n dimensions
- Simple, noisy, inefficient - but sometimes effective
- Works for arbitrary policies, even if policy is not differentiable

Training AIBO to Walk by Finite Difference Policy Gradient[27]



- Goal: learn a fast AIBO walk (useful for Robocup)
- Adapt these parameters by finite difference policy gradient
- Evaluate performance of policy by field traversal time

[27] Kohl and Stone. Policy gradient reinforcement learning for fast quadrupedal locomotion. ICRA 2004. http://www.cs.utexas.edu/ ai-lab/pubs/icra04.pdf

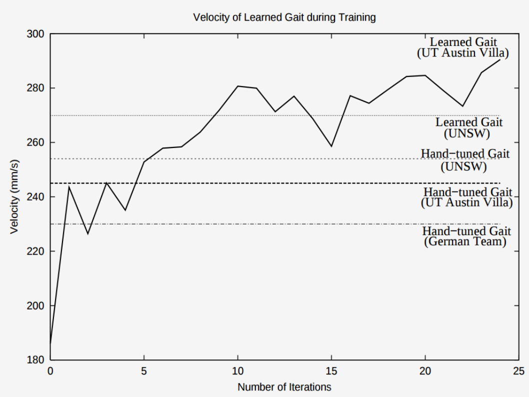
AIBO Policy Parameterization

- AIBO walk policy is open-loop policy
- No state, choosing set of action parameters that define an ellipse
- Specified by 12 continuous parameters (elliptical loci)
    - The front locus (3 parameters: height, x-pos., y-pos.)
    - The rear locus (3 parameters)
    - Locus length
    - Locus skew multiplier in the x-y plane (for turning)
    - The height of the front of the body
    - The height of the rear of the body
    - The time each foot takes to move through its locus
    - The fraction of time each foot spends on the ground
- New policies: for each parameter, randomly add (ϵ, 0, or $-\epsilon$)

AIBO Policy Experiments

- "All of the policy evaluations took place on actual robots... only human intervention required during an experiment involved replacing discharged batteries ... about once an hour."
- Ran on 3 Aibos at once
- Evaluated 15 policies per iteration.
- Each policy evaluated 3 times (to reduce noise) and averaged
- Each iteration took 7.5 minutes
- Used $\eta = 2$ (learning rate for their finite difference approach)

Training AIBO to Walk by Finite Difference Policy Gradient Results



- Authors discuss that performance is likely impacted by: initial starting policy parameters, ϵ (how much policies are perturbed), η (how much to change policy), as well as policy parameterization

AIBO Walk Policies

- https://www.cs.utexas.edu/Ãustin Villa/?p=research/learned_walk

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Computing the gradient analytically

- We now compute the policy gradient *analytically*
- Assume policy π_θ is differentiable whenever it is non-zero
- and we know the gradient $\nabla_\theta \pi_\theta(s, a)$

Likelihood Ratio Policies

- Denote a state-action trajectory as $\tau = (s_0, a_0, r_0, ..., s_{T-1}, a_{T-1}, r_{T-1}, s_T)$
- Use $R(\tau) = \sum_{t=0}^{T} R(s_t, a_t)$ to be the sum of rewards for a trajectory τ

Likelihood Ratio Policies

- Denote a state-action trajectory as $\tau = (s_0, a_0, r_0, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T)$
- Use $R(\tau) = \sum_{t=0}^{T} R(s_t, a_t)$ to be the sum of rewards for a trajectory τ
- Policy value is

$$V(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{T} R(s_t, a_t); \pi_\theta\right] = \sum_{\tau} P(\tau; \theta) R(\tau), \tag{1}$$

- where $P(\tau; \theta)$ is used to denote the probability over trajectories when executing policy $\pi(\theta)$
- In this new notation, our goal is to find the policy parameters θ:

$$\arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau; \theta) R(\tau). \tag{2}$$

Likelihood Ratio Policy Gradient

- Goal is to find the policy parameters θ:

$$\arg\max_\theta V(\theta) = \arg\max_\theta \sum_\tau P(\tau;\theta)R(\tau). \quad (3)$$

- Take the gradient with respect to θ:

$$\begin{aligned}
\nabla_\theta V(\theta) &= \nabla_\theta \sum_\tau P(\tau;\theta)R(\tau) \\
&= \sum_\tau \nabla_\theta P(\tau;\theta)R(\tau) \\
&= \sum_\tau \frac{P(\tau;\theta)}{P(\tau;\theta)} \nabla_\theta P(\tau;\theta) R(\tau) \\
&= \sum_\tau R(\tau) P(\tau;\theta) \underbrace{\frac{\nabla_\theta P(\tau;\theta)}{P(\tau;\theta)}}_{\text{likelihood ratio}} \\
&= \sum_\tau R(\tau) P(\tau;\theta) \nabla_\theta \log P(\tau;\theta)
\end{aligned}$$

Likelihood Ratio Policy Gradient

- Goal is to find the policy parameters θ:

$$\arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau;\theta)R(\tau). \tag{4}$$

- Take the gradient with respect to θ:

$$\begin{aligned}
\nabla_\theta V(\theta) &= \nabla_\theta \sum_{\tau} P(\tau;\theta)R(\tau) \\
&= \sum_{\tau} \nabla_\theta P(\tau;\theta)R(\tau) \\
&= \sum_{\tau} \frac{P(\tau;\theta)}{P(\tau;\theta)} \nabla_\theta P(\tau;\theta)R(\tau) \\
&= \sum_{\tau} P(\tau;\theta)R(\tau) \underbrace{\frac{\nabla_\theta P(\tau;\theta)}{P(\tau;\theta)}}_{\text{likelihood ratio}} \\
&= \sum_{\tau} P(\tau;\theta)R(\tau)\nabla_\theta \log P(\tau;\theta)
\end{aligned}$$

Likelihood Ratio Policy Gradient

- Goal is to find the policy parameters θ:

$$\arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau;\theta)R(\tau). \tag{5}$$

- Take the gradient with respect to θ:

$$\nabla_\theta V(\theta) = \sum_{\tau} P(\tau;\theta)R(\tau)\nabla_\theta \log P(\tau;\theta)$$

- Approximate with empirical estimate for m sample paths under policy π_θ:

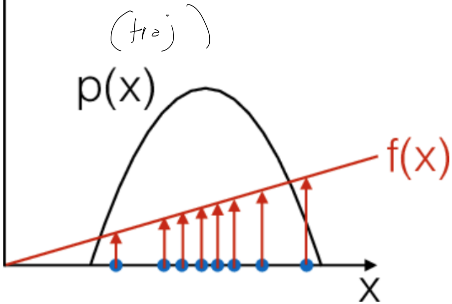
$$\nabla_\theta V(\theta) \approx \hat{g} = (1/m)\sum_{i=1}^{m} \underbrace{R(\tau^{(i)})}\nabla_\theta \underbrace{\log P(\tau^{(i)};\theta)}$$

Score Function Gradient Estimator: Intuition

- Consider generic form of $R(\tau^{(i)})\nabla_\theta \log P(\tau^{(i)};\theta)$:
  $\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$
- $f(x)$ measures how good the sample x is.
- Moving in the direction $\hat{g}_i$ pushes up the logprob of the sample, in proportion to how good it is
- *Valid even if $f(x)$ is discontinuous, and unknown, or sample space (containing x) is a discrete set*

Score Function Gradient Estimator: Intuition

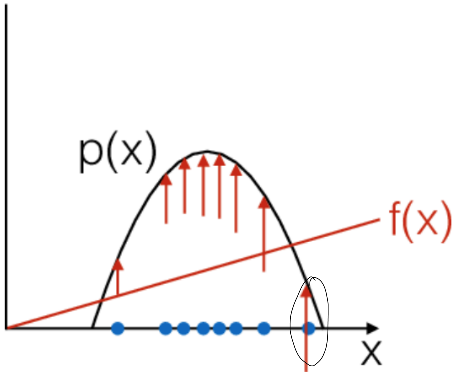
$$\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$$



(in our case R)

Score Function Gradient Estimator: Intuition

$$\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$$



Decomposing the Trajectories Into States and Actions

- Approximate with empirical estimate for m sample paths under policy π_θ:

$$\nabla_\theta V(\theta) \approx \hat{g} = (1/m)\sum_{i=1}^{m} R(\tau^{(i)})\nabla_\theta \log P(\tau^{(i)}; \theta)$$

reward
fct

unknown

$$\nabla_\theta \log P(\tau^{(i)};\theta) = \nabla_\theta \log \Big[\underbrace{\mu(s_0)}_{\text{start state}} \cdot \prod_{i=0}^{T-1} \pi_\theta(a_i|s_i)\, P(s_{i+1}|a_i, s_i) \Big]$$

$$= \underbrace{\nabla_\theta \Big[\log \mu(s_0)}_{\Rightarrow 0} + \sum_{i=0}^{T-1} \log \pi_\theta(a_i|s_i) + \underbrace{\sum_{i=0}^{T-1} \log P(s_{i+1}|a_i, s_i)}_{\Rightarrow 0 \text{ because indep of } \theta} \Big]$$

$$= \sum_{i=0}^{T-1} \nabla_\theta \log \pi_\theta(a_i|s_i)$$

Decomposing the Trajectories Into States and Actions

- Approximate with empirical estimate for m sample paths under policy π_θ:

$$\nabla_\theta V(\theta) \approx \hat{g} = (1/m)\sum_{i=1}^{m} R(\tau^{(i)})\nabla_\theta \log P(\tau^{(i)})$$

$$\begin{aligned}
\nabla_\theta \log P(\tau^{(i);\theta}) &= \nabla_\theta \log \Bigg[\underbrace{\mu(s_0)}_{\text{Initial state distrib.}} \prod_{t=0}^{T-1} \underbrace{\pi_\theta(a_t|s_t)}_{\text{policy}} \underbrace{P(s_{t+1}|s_t,a_t)}_{\text{dynamics model}} \Bigg] \\
&= \nabla_\theta \left[\log \mu(s_0) + \sum_{t=0}^{T-1} \log \pi_\theta(a_t|s_t) + \log P(s_{t+1}|s_t,a_t) \right] \\
&= \sum_{t=0}^{T-1} \underbrace{\nabla_\theta \log \pi_\theta(a_t|s_t)}_{\text{no dynamics model required!}}
\end{aligned}$$

Score Function

- Define score function as $\underbrace{\nabla_\theta \log \pi_\theta(s, a)}$

Likelihood Ratio / Score Function Policy Gradient

- Putting this together
- Goal is to find the policy parameters θ:

$$\arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau;\theta)R(\tau). \tag{6}$$

- Approximate with empirical estimate for m sample paths under policy π_θ using score function:

$$\begin{aligned}
\nabla_\theta V(\theta) \quad &\approx \quad \hat{g} = (1/m)\sum_{i=1}^{m} R(\tau^{(i)}) \nabla_\theta \log P(\tau^{(i)};\theta) \\
&= \quad (1/m)\sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)}|s_t^{(i)})
\end{aligned}$$

Do not need to know dynamics model

Policy Gradient Theorem

- The policy gradient theorem generalizes the likelihood ratio approach

Theorem

*For any differentiable policy $\pi_\theta(s, a)$,
for any of the policy objective function $J = J_1$, (episodic reward), J_{avR} (average reward per time step), or $\frac{1}{1-\gamma} J_{avV}$ (average value),
the policy gradient is*

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s, a) Q^{\pi_\theta}(s, a)]$$

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Likelihood Ratio / Score Function Policy Gradient

- $$\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} \underline{R(\tau^{(i)})} \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)}|s_t^{(i)})$$

- Unbiased but very noisy
- Fixes that can make it practical
  - Temporal structure
  - Baseline
- Next time will discuss some additional tricks

Policy Gradient: Use Temporal Structure

- Previously:

decomposing reward

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau \left[\left(\sum_{t=0}^{T-1} r_t \right) \left(\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \right) \right]$$

- We can repeat the same argument to derive the gradient estimator for a single reward term $r_{t'}$.

$$\nabla_\theta \mathbb{E}[r_{t'}] = \mathbb{E} \left[r_{t'} \sum_{t=0}^{t'} \underline{\nabla_\theta \log \pi_\theta(a_t|s_t)} \right]$$

- Summing this formula over t, we obtain

$V(\theta) =$

$$\nabla_\theta \mathbb{E}[R] = \mathbb{E} \left[\sum_{t'=0}^{T-1} r_{t'} \sum_{t=0}^{t'} \nabla_\theta \log \pi_\theta(a_t|s_t) \right]$$

$$= \mathbb{E} \left[\sum_{t=0}^{T-1} \underline{\nabla_\theta \log \pi_\theta(a_t, s_t)} \sum_{t'=t}^{T-1} r_{t'} \right]$$

Policy Gradient: Use Temporal Structure

- Recall for a particular trajectory $\tau^{(i)}$, $\sum_{t'=t}^{T-1} r_{t'}^{(i)}$ is the return $G_t^{(i)}$

$$\nabla_\theta \mathbb{E}[R] \approx (1/m) \sum_{i=1}^{m} \sum_{t=0}^{T-1} \underbrace{\nabla_\theta \log \pi_\theta(a_t, s_t)} G_t^{(i)}$$

Monte-Carlo Policy Gradient (REINFORCE)

- Leverages likelihood ratio / score function and temporal structure

$$\Delta\theta_t = \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) G_t \tag{7}$$

REINFORCE:

Initialize policy parameters θ arbitrarily

for each episode $\{s_1, a_1, r_2, \cdots, s_{T-1}, a_{T-1}, r_T\} \sim \pi_\theta$ **do**

  for $t = 1$ to $T-1$ **do**

    $\theta \leftarrow \theta + \alpha \nabla_\theta \underbrace{\log \pi_\theta(s_t, a_t)}_{} G_t$

  endfor

endfor

return θ

Differentiable Policy Classes

- Many choices of differentiable policy classes including:
  - Softmax
  - Gaussian
  - Neural networks

Softmax Policy

- Weight actions using linear combination of features $\phi(s,a)^T\theta$
- Probability of action is proportional to exponentiated weight

$$\pi_\theta(s,a) = e^{\phi(s,a)^T\theta} / (\sum_a e^{\phi(s,a)^T\theta}) \tag{8}$$

discrete action spaces

- The score function is

$$\nabla_\theta \log \pi_\theta(s,a) = \phi(s,a) - \mathbb{E}\pi_\theta[\phi(s,\cdot)]$$

Gaussian Policy

- In continuous action spaces, a Gaussian policy is natural
- Mean is a linear combination of state features $\mu(s) = \phi(s)^T\theta$
- Variance may be fixed σ^2, or can also parametrised
- Policy is Gaussian $a \sim \mathcal{N}(\mu(s), \sigma^2)$
- The score function is

$$\nabla_\theta \log \pi_\theta(s, a) = \frac{(a - \mu(s))\phi(s)}{\sigma^2}$$

Likelihood Ratio / Score Function Policy Gradient

- $$\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$
- Unbiased but very noisy
- Fixes that can make it practical
  - Temporal structure
  - **Baseline**
- Next time will discuss some additional tricks

Policy Gradient: Introduce Baseline

- Reduce variance by introducing a *baseline* $b(s)$

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t, \theta) \left(\sum_{t'=t}^{T-1} r_{t'} - b(s_t) \right) \right]$$

- For any choice of $b(s)$, gradient estimator is unbiased.
- Near optimal choice is expected return, $b(s_t) \approx \mathbb{E}[r_t + r_{t+1} + \cdots + r_{T-1}]$
- Interpretation: increase logprob of action a_t proportionally to how much returns $\sum_{t'=t}^{T-1} r_{t'}$ are better than expected

Baseline $b(s)$ Does Not Introduce Bias–Derivation

$$\mathbb{E}_\tau[\nabla_\theta \log \pi(a_t|s_t,\theta) b(s_t)]$$
$$= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[\mathbb{E}_{s_{(t+1):T},a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t,\theta) b(s_t)]\right]$$

Baseline $b(s)$ Does Not Introduce Bias–Derivation

$$
\begin{aligned}
&\mathbb{E}_\tau[\nabla_\theta \log \pi(a_t|s_t,\theta) b(s_t)] \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[\mathbb{E}_{s_{(t+1):T},a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t,\theta) b(s_t)]\right] \text{ (break up expectation)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\mathbb{E}_{s_{(t+1):T},a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t,\theta)]\right] \text{ (pull baseline term out)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}[b(s_t)\mathbb{E}_{a_t}[\nabla_\theta \log \pi(a_t|s_t,\theta)]] \text{ (remove irrelevant variables)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\sum_a \pi_\theta(a_t|s_t)\frac{\nabla_\theta \pi(a_t|s_t,\theta)}{\pi_\theta(a_t|s_t)}\right] \text{ (likelihood ratio)} \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\sum_a \nabla_\theta \pi(a_t|s_t,\theta)\right] \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[b(s_t)\nabla_\theta \sum_a \pi(a_t|s_t,\theta)\right] \\
&= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}[b(s_t)\nabla_\theta 1] \\
&= \mathbb{E}_{s_0:t,a_0:(t-1)}[b(s_t)\cdot 0] = 0
\end{aligned}
$$

"Vanilla" Policy Gradient Algorithm

Initialize policy parameter θ, baseline b
for iteration$=1, 2, \cdots$ **do**
  Collect a set of trajectories by executing the current policy
  At each timestep in each trajectory, compute
    the *return* $R_t = \sum_{t'=t}^{T-1} r_{t'}$, and
    the *advantage estimate* $\hat{A}_t = R_t - b(s_t)$.
  Re-fit the baseline, by minimizing $||b(s_t) - R_t||^2$,
    summed over all trajectories and timesteps.
  Update the policy, using a policy gradient estimate $\hat{g}$,
    which is a sum of terms $\nabla_\theta \log \pi(a_t|s_t, \theta)\hat{A}_t$.
    (Plug $\hat{g}$ into SGD or ADAM)
endfor

Practical Implementation with Autodiff

- Usual formula $\sum_t \nabla_\theta \log \pi(a_t|s_t;\theta)\hat{A}_t$ is inifficient–want to batch data
- Define "surrogate" function using data from current batch

$$L(\theta) = \sum_t \log \pi(a_t|s_t;\theta)\hat{A}_t$$

- Then policy gradient estimator $\hat{g} = \nabla_\theta L(\theta)$
- Can also include value function fit error

$$L(\theta) = \sum_t \left(\log \pi(z_t|s_t;\theta)\hat{A}_t - ||V(s_t) - \hat{R}_t||^2\right)$$

Value Functions

- Recall Q-function / state-action-value function:

$$Q^{\pi,\gamma}(s,a) = \mathbb{E}_\pi \left[r_0 + \gamma r_1 + \gamma^2 r_2 \cdots | s_0 = s, a_0 = a \right]$$

- State-value function can serve as a great baseline

$$\begin{aligned} V^{\pi,\gamma}(s) &= \mathbb{E}_\pi \left[r_0 + \gamma r_1 + \gamma^2 r_2 \cdots | s_0 = s \right] \\ &= \mathbb{E}_{a\sim\pi}[Q^{\pi,\gamma}(s,a)] \end{aligned}$$

- Advantage function: Combining Q with baseline V

$$A^{\pi,\gamma}(s,a) = Q^{\pi,\gamma}(s,a) - V^{\pi,\gamma}(s)$$

N-step estimators

- Can also consider blending between TD and MC estimators for the target to substitute for the true state-action value function.

$$\hat{R}_t^{(1)} = r_t + \gamma V(s_{t+1})$$
$$\hat{R}_t^{(2)} = r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2}) \qquad \cdots$$
$$\hat{R}_t^{(\text{inf})} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+1} + \cdots$$

- If subtract baselines from the above, get advantage estimators

$$\hat{A}_t^{(1)} = r_t + \gamma V(s_{t+1}) - V(s_t)$$
$$\hat{A}_t^{(2)} = r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2}) - V(s_t)$$
$$\hat{A}_t^{(\text{inf})} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+1} + \cdots - V(s_t)$$

- $\hat{A}_t^{(a)}$ has low variance & high bias. $\hat{A}_t^{(\infty)}$ high variance but low bias. (Why? Like which model-free policy estimation techniques?)
- Using intermediate k (say, 20) can give an intermediate amount of bias and variance.

Application: Robot Locomotion

Learning to Walk in 20 Minutes

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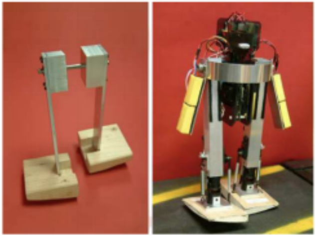

Class Structure

- Last time: Imitation Learning
- **This time: Policy Search**
- Next time: Policy Search Cont.