Lecture 13: Fast Reinforcement Learning [1]

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CS234 Reinforcement Learning

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[1] With a few slides derived from David Silver

Refresh Your Knowledge Fast RL Part II

$Beta(c_1, c_2)$ $c_1 \simeq \#\text{pos} - 1$

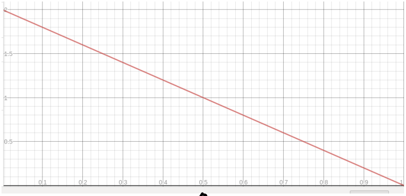
- The prior over arm 1 is Beta(1,2) (left) and arm 2 is a Beta(1,1) (right figure). Select all that are true. $c_2 \simeq \#\text{neg} - 1$
  1. Sample 3 params: 0.1,0.5,0.3. These are more likely to come from the Beta(1,2) distribution than Beta(1,1). T
  2. Sample 3 params: 0.2,0.5,0.8. These are more likely to come from the Beta(1,1) distribution than Beta(1,2). T
  3. It is impossible that the true Bernoulli parame is 0 if the prior is Beta(1,1). F
  4. Not sure

Bernoulli

"pos counts" "0 counts"

- The prior over arm 1 is Beta(1,2) (left) and arm 2 is a Beta(1,1) (right). The true parameters are arm 1 $\theta_1 = 0.4$ & arm 2 $\theta_2 = 0.6$. Thompson sampling = TS
  1. TS could sample $\theta = 0.5$ (arm 1) and $\theta = 0.55$ (arm 2). T
  2. For the sampled thetas (0.5,0.55) TS is optimistic with respect to the true arm parameters for all arms. F
  3. For the sampled thetas (0.5,0.55) TS will choose the true optimal arm for this round. T
  4. Not sure

Beta(1,2) $p(\theta)$



θ

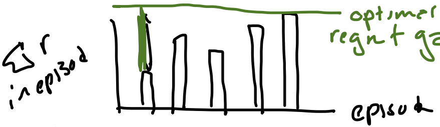
$p(\theta)$ Beta(1,1)

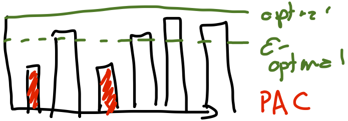
θ

Class Structure

- Last time: Fast Learning (Bayesian bandits to MDPs)
- **This time: Fast Learning III (MDPs)**
- Next time: Batch RL

Settings, Frameworks & Approaches





- Over these 3 lectures will consider 2 settings, multiple frameworks, and approaches
- Settings: Bandits (single decisions), MDPs
- Frameworks: evaluation criteria for formally assessing the quality of a RL algorithm. So far seen empirical evaluations, asymptotic convergence, regret, probably approximately correct
- Approaches: Classes of algorithms for achieving particular evaluation criteria in a certain set. So far for exploration seen: greedy, $\epsilon-$greedy, optimism, Thompson sampling, for multi-armed bandits

bandits → tabular MDP → deep RL

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Fast RL in Markov Decision Processes

- Very similar set of frameworks and approaches are relevant for fast learning in reinforcement learning
- Frameworks
  - Regret
  - Bayesian regret
  - Probably approximately correct (PAC)
- Approaches
  - Optimism under uncertainty
  - Probability matching / Thompson sampling
- Framework: Probably approximately correct

Fast RL in Markov Decision Processes

- Montezuma's revenge
- https://www.youtube.com/watch?v=ToSe_CUG0F4

Model-Based Interval Estimation with Exploration Bonus (MBIE-EB)

(Strehl and Littman, J of Computer & Sciences 2008)

tabular MDP $\quad r \in (0,1)$

Given ϵ, δ, m

$\beta = \frac{1}{1-\gamma}\sqrt{.5 \ln(2|S||A|m/\delta)}$

$n_{sas}(s,a,s') = 0 \quad s,a,s' \quad \cdot n_{sa}(0)$

$rc(s,a) = 0 \quad \tilde{Q}(s,a) = 1/(1-\gamma) \quad \forall s,a$

define initial dynamics model

$t = 0$, $s_t = s_{init}$

loop

$\quad a_t = \arg\max \tilde{Q}(s,a)$

$\quad$ observe reward r_t and s_{t+1}

$\quad n_{sas} + 1 \quad n_{sa} + 1$

$\quad rc = (rc \cdot (n_{sa} - 1) + r_t)/n_{sa}$ empirical reward

$\quad \hat{T}(s'|s,a) = n_{sas}(s,a,s') / n_{sa}(s,a)$

$\quad$ **while** not converged **do**

$$\tilde{Q}(s,a) = rc(s,a) + \gamma \sum_{s'} \hat{T}(s'|s,a) \max_{a'} \tilde{Q}(s',a') + \beta/\sqrt{n_{sa}(s,a)} \quad \forall s,a$$

$\quad$ **end while**

end loop

Model-Based Interval Estimation with Exploration Bonus (MBIE-EB)

(Strehl and Littman, J of Computer & Sciences 2008)

```
Given ϵ, δ, m
β = 1/(1−γ) √(0.5 ln(2|S||A|m/δ))
n_sas(s, a, s′) = 0, ∀s ∈ S, a ∈ A, s′ ∈ S
rc(s, a) = 0, n_sa(s, a) = 0, Q̃(s, a) = 1/(1 − γ), ∀ s ∈ S, a ∈ A
t = 0, s_t = s_init
loop
    while not converged do
    end while
end loop
```

Model-Based Interval Estimation with Exploration Bonus (MBIE-EB)

(Strehl and Littman, J of Computer & Sciences 2008)

Given ϵ, δ, m
$\beta = \frac{1}{1-\gamma}\sqrt{0.5 \ln(2|S||A|m/\delta)}$
$n_{sas}(s,a,s') = 0, \ \forall s \in S, \ a \in A, \ s' \in S$
$rc(s,a) = 0, \ n_{sa}(s,a) = 0, \ \tilde{Q}(s,a) = 1/(1-\gamma), \ \forall \ s \in S, \ a \in A$
$t = 0, \ s_t = s_{init}$
loop
  $a_t = \arg\max_{a \in \mathcal{A}} \tilde{Q}(s_t, a)$
  Observe reward r_t and state s_{t+1}
  $n_{sa}(s_t,a_t) = n_{sa}(s_t,a_t) + 1, \ n_{sas}(s_t,a_t,s_{t+1}) = n_{sas}(s_t,a_t,s_{t+1}) + 1$
  $rc(s_t,a_t) = \frac{rc(s_t,a_t)(n_{sa}(s_t,a_t)-1)+r_t}{n_{sa}(s_t,a_t)}$
  $\hat{R}(s_t,a_t) = rc(s_t,a_t)$ and $\hat{T}(s'|s_t,a_t) = \frac{n_{sas}(s_t,a_t,s')}{n_{sa}(s_t,a_t)}, \ \forall s' \in S$
  while not converged **do**
    $\tilde{Q}(s,a) = \hat{R}(s,a) + \gamma \sum_{s'} \hat{T}(s'|s,a) \max_{a'} \tilde{Q}(s',a) + \frac{\beta}{\sqrt{n_{sa}(s,a)}}, \ \forall \ s \in S, \ a \in A$
  end while
end loop

update each s, a at most m times

Framework: PAC for MDPs

- For a given ϵ and δ, A RL algorithm $\mathcal{A}$ is PAC if on all but N steps, the action selected by algorithm $\mathcal{A}$ on time step t, a_t, is ϵ-close to the optimal action, where N is a polynomial function of $(|S|, |A|, \gamma, \epsilon, \delta)$
- Is this true for all algorithms?

No

with prob at least $1-\delta$

MBIE-EB is a PAC RL Algorithm

$\epsilon = .1 \quad \gamma = .9$

Theorem 2. *Suppose that* ϵ *and* δ *are two real numbers between* 0 *and* 1 *and* $M = \langle S, A, T, \mathcal{R}, \gamma \rangle$ *is any MDP. There exists an input* $m = m(\frac{1}{\epsilon}, \frac{1}{\delta})$, *satisfying* $m(\frac{1}{\epsilon}, \frac{1}{\delta}) = O(\frac{|S|}{\epsilon^2(1-\gamma)^4} + \frac{1}{\epsilon^2(1-\gamma)^4} \ln \frac{|S||A|}{\epsilon(1-\gamma)\delta})$, *and* $\beta = (1/(1-\gamma))\sqrt{\ln(2|S||A|m/\delta)/2}$ *such that if MBIE-EB is executed on MDP* M, *then the following holds. Let* $\mathcal{A}_t$ *denote MBIE-EB's policy at time* t *and* s_t *denote the state at time* t. *With probability at least* $1-\delta$, $V_M^{\mathcal{A}_t}(s_t) \geqslant V_M^*(s_t) - \epsilon$ *is true for all but* $\underbrace{O(\frac{|S||A|}{\epsilon^3(1-\gamma)^6}(|S| + \ln \frac{|S||A|}{\epsilon(1-\gamma)\delta}) \ln \frac{1}{\delta} \ln \frac{1}{\epsilon(1-\gamma)})}_{10^{10}}$ *timesteps* t.

A Sufficient Set of Conditions to Make a RL Algorithm PAC

- Strehl, A. L., Li, L., & Littman, M. L. (2006). Incremental model-based learners with formal learning-time guarantees. In Proceedings of the Twenty-Second Conference on Uncertainty in Artificial Intelligence (pp. 485-493)

A Sufficient Set of Conditions to Make a RL Algorithm PAC

1. optimism $Q_t(s,a) \geq Q^*(s,a) - \epsilon \quad \forall s, a, t.$

$Q_t = 1/(1-\gamma)$

2. accuracy

$V_t(s) - V_{\mathcal{M}}^{\pi_t}(s) \leq \epsilon$

MDP related to true MDP & MDP defined in MBIE-EB

"known set"

s, a known if sampled enough that have a good enough estimate of its reward & dynamics

3) Bounded learning complexity

$+\Big[$ - total # of updates to Q
- # times we visit an unknown (s,a) pair

$\leq C_1(\epsilon, \delta)$

A RL alg that is executed on any MDP will follow an $O(\epsilon)$-optimal policy on all but ~~a~~ N steps

$$N = O\left(\frac{C_1(\epsilon,\delta)}{\epsilon(1-\gamma^2)} \ln\frac{1}{\delta} \ln\frac{1}{\epsilon(1-\gamma)}\right)$$

w/prob $\geq 1-2\delta$

needs to be poly in S, A, etc. for RL alg to be PAC

How Does MBIE-EB Fulfill these Conditions?

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Refresher: Bayesian Bandits

- **Bayesian bandits** exploit prior knowledge of rewards, $p[\mathcal{R}]$
- They compute posterior distribution of rewards $p[\mathcal{R} \mid h_t]$, where $h_t = (a_1, r_1, \dots, a_{t-1}, r_{t-1})$
- Use posterior to guide exploration
  - Upper confidence bounds (Bayesian UCB)
  - Probability matching (Thompson Sampling)
- Better performance if prior knowledge is accurate

$p(\theta|r) \propto p(r|\theta)p(\theta)$

- Consider a bandit problem where the reward of an arm is a binary outcome $\{0, 1\}$ sampled from a Bernoulli with parameter θ
  - E.g. Advertisement click through rate, patient treatment succeeds/fails, ...

exponential families

- The Beta distribution $Beta(\alpha, \beta)$ is conjugate for the Bernoulli distribution

analytically

$$p(\theta|\alpha, \beta) = \theta^{\alpha-1}(1-\theta)^{\beta-1}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

where $\Gamma(x)$ is the Gamma function.

- Assume the prior over θ is a $Beta(\alpha, \beta)$ as above
- Then after observed a reward $r \in \{0, 1\}$ then updated posterior over θ is $Beta(r + \alpha, 1 - r + \beta)$

Thompson Sampling for Bandits

Initialize prior over each arm a, $p(\mathcal{R}_a)$
loop
  For each arm a **sample** a reward distribution $\mathcal{R}_a$ from posterior
  Compute action-value function $Q(a) = \mathbb{E}[\mathcal{R}_a]$
  $a_t = \arg\max_{a \in \mathcal{A}} Q(a)$
  Observe reward r
  Update posterior $p(\mathcal{R}_a | r)$ using Bayes law
end loop

Bayesian Model-Based RL

tabular to start

- Maintain posterior distribution over **MDP** models
- Estimate both transition and rewards, $p[\mathcal{P}, \mathcal{R} \mid h_t]$, where $h_t = (s_1, a_1, r_1, \ldots, s_t)$ is the history
- Use posterior to guide exploration
  - Upper confidence bounds (Bayesian UCB)
  - Probability matching (Thompson sampling)

Thompson Sampling: Model-Based RL

multinomial
$p(s' \mid s, a)$

1 prior per (s,a)
tuple for rewards
1 for dyn. per (s,a)

gridword

s_1	s_2
s_3	s_4

- Thompson sampling implements probability matching

Dirichlet
conj.
for
multinomials

$$\begin{aligned}\pi(s,a \mid h_t) &= \mathbb{P}[Q(s,a) \geq Q(s,a'), \forall a' \neq a \mid h_t] \\ &= \mathbb{E}_{\mathcal{P},\mathcal{R}\mid h_t}\left[\mathbb{1}(a = \arg\max_{a\in\mathcal{A}} Q(s,a))\right]\end{aligned}$$

- Use Bayes law to compute posterior distribution $p[\mathcal{P}, \mathcal{R} \mid h_t]$
- **Sample** an MDP $\mathcal{P}, \mathcal{R}$ from posterior
- Solve MDP using favorite planning algorithm to get $Q^*(s,a)$
- Select optimal action for sample MDP, $a_t = \arg\max_{a\in\mathcal{A}} Q^*(s_t, a)$

✓ for sampled MDP

rewards Bernoulli

$\forall s, a \quad p(r_{sa} = \theta) = \text{Beta} \ldots$

Thompson Sampling for MDPs

Initialize prior over the dynamics and reward models for each (s, a), $p(\mathcal{R}_{as})$, $p(\mathcal{T}(s'|s, a))$ ex. Beta Dirichlet

Initialize state s_0

loop

  Sample a MDP $\mathcal{M}$: for each (s, a) pair, sample a dynamics model $\mathcal{T}(s'|s, a)$ and reward model $\mathcal{R}(s, a)$

  Compute $Q^*_{\mathcal{M}}$, optimal value for MDP $\mathcal{M}$

  $a_t = \arg\max_{a \in \mathcal{A}} Q^*_{\mathcal{M}}(s_t, a)$

  Observe reward r_t and next state s_{t+1}

  Update posterior $p(\mathcal{R}_{a_t s_t}|r_t)$, $p(\mathcal{T}(s'|s_t, a_t)|s_{t+1})$ using Bayes rule

  $t = t + 1$

end loop

Check Your Understanding: Fast RL III

- Strategic exploration in MDPs (select all):
  F 1. Doesn't really matter because the distribution of data is independent of the policy followed
  T 2. Can involve using optimism with respect to both the possible dynamics and reward models in order to compute an optimistic Q function
  F 3. Is known as PAC if the number of time steps on which a less than near optimal decision is made is guaranteed to be less than an ~~exponential~~ function of the problem domain parameters (state space cardinality, etc). polynomial to be true
  4. Not sure
- In Thompson sampling for MDPs:
  F 1. TS samples the reward model parameters and could use the empirical average for the dynamics model parameters and obtain the same performance generally false
  T 2. Must perform MDP planning everytime the posterior is updated for all sh[illegible]
  F 3. Has the same computational cost each step as Q-learning
  4. Not sure

Resampling in Coordinated Exploration

- Concurrent PAC RL. Guo and Brunskill. AAAI 2015
- Coordinated Exploration in Concurrent Reinforcement Learning. Dimakopoulou and Van Roy. ICML 2018
- https://www.youtube.com/watch?v=xjGK-wm0PkI&feature=youtu.be

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Generalization and Strategic Exploration

- Active area of ongoing research: combine generalization & strategic exploration
- Many approaches are grounded by principles outlined here
  - Optimism under uncertainty
  - Thompson sampling

Generalization and Optimism

- Recall MBIE-EB algorithm for finite state and action domains
- What needs to be modified for continuous / extremely large state and/or action spaces?

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    $n_{sa}(s_t,a_t) = n_{sa}(s_t,a_t) + 1,\ n_{sas}(s_t,a_t,s_{t+1}) = n_{sas}(s_t,a_t,s_{t+1}) + 1$

    $rc(s_t,a_t) = \frac{rc(s_t,a_t)(n_{sa}(s_t,a_t)-1)+r_t}{n_{sa}(s_t,a_t)}$

    $\hat{R}(s_t,a_t) = rc(s_t,a_t)$ and $\hat{T}(s'|s_t,a_t) = \frac{n_{sas}(s_t,a_t,s')}{n_{sa}(s_t,a_t)},\ \forall s' \in S$

    while not converged **do**

        $\tilde{Q}(s,a) = \hat{R}(s,a) + \gamma \sum_{s'} \hat{T}(s'|s,a) \max_{a'} \tilde{Q}(s',a) + \frac{\beta}{\sqrt{n_{sa}(s,a)}},\ \forall\ s \in S,\ a \in A$

    end while

end loop

Generalization and Optimism

- Recall MBIE-EB algorithm for finite state and action domains
- What needs to be modified for continuous / extremely large state and/or action spaces?
- Estimating uncertainty
  - Counts of (s,a) and (s,a,s') tuples are not useful if we expect only to encounter any state once
- Computing a policy
  - Model-based planning will fail
- So far, model-free approaches have generally had more success than model-based approaches for extremely large domains
  - Building good transition models to predict pixels is challenging

Recall: Value Function Approximation with Control

- For Q-learning use a TD target $r + \gamma \max_{a'} \hat{Q}(s', a'; \boldsymbol{w})$ which leverages the max of the current function approximation value

$$\Delta \boldsymbol{w} = \alpha(r(s) + \gamma \max_{a'} \hat{Q}(s', a'; \boldsymbol{w}) - \hat{Q}(s, a; \boldsymbol{w}))\nabla_{\boldsymbol{w}} \hat{Q}(s, a; \boldsymbol{w})$$

Recall: Value Function Approximation with Control

- For Q-learning use a TD target $r + \gamma \max_{a'} \hat{Q}(s', a'; \boldsymbol{w})$ which leverages the max of the current function approximation value

$$\Delta \boldsymbol{w} = \alpha(r(s) + r_{bonus}(s, a) + \gamma \max_{a'} \hat{Q}(s', a'; \boldsymbol{w}) - \hat{Q}(s, a; \boldsymbol{w})) \nabla_{\boldsymbol{w}} \hat{Q}(s, a; \boldsymbol{w})$$

Recall: Value Function Approximation with Control

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$$\Delta \boldsymbol{w} = \alpha(r(s) + r_{bonus}(s, a) + \gamma \max_{a'} \hat{Q}(s', a'; \boldsymbol{w}) - \hat{Q}(s, a; \boldsymbol{w})) \nabla_{\boldsymbol{w}} \hat{Q}(s, a; \boldsymbol{w})$$

- $r_{bonus}(s, a)$ should reflect uncertainty about future reward from (s, a)
- Approaches for deep RL that make an estimate of visits / density of visits include: Bellemare et al. NIPS 2016; Ostrovski et al. ICML 2017; Tang et al. NIPS 2017
- Note: bonus terms are computed at time of visit. During episodic replay can become outdated.

Benefits of Strategic Exploration: Montezuma's revenge

1st 2 rooms

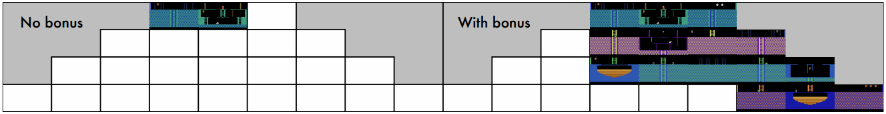

Figure 3: "Known world" of a DQN agent trained for 50 million frames with (**right**) and without (**left**) count-based exploration bonuses, in MONTEZUMA'S REVENGE.

Figure: Bellemare et al. "Unifying Count-Based Exploration and Intrinsic Motivation"

- Enormously better than standard DQN with ϵ-greedy approach

Generalization and Strategic Exploration: Thompson Sampling

- Leveraging Bayesian perspective has also inspired some approaches
- One approach: Thompson sampling over representation & parameters (Mandel, Liu, Brunskill, Popovic IJCAI 2016)

Generalization and Strategic Exploration: Thompson Sampling

- Leveraging Bayesian perspective has also inspired some approaches
- One approach: Thompson sampling over representation & parameters (Mandel, Liu, Brunskill, Popovic IJCAI 2016)
- For scaling up to very large domains, again useful to consider model-free approaches
- Non-trivial: would like to be able to sample from a posterior over possible Q^*
- Bootstrapped DQN (Osband et al. NIPS 2016)
  - Train C DQN agents using bootstrapped samples
  - When acting, choose action with highest Q value over any of the C agents
  - Some performance gain, not as effective as reward bonus approaches

Generalization and Strategic Exploration: Thompson Sampling

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- One approach: Thompson sampling over representation & parameters (Mandel, Liu, Brunskill, Popovic IJCAI 2016)
- For scaling up to very large domains, again useful to consider model-free approaches
- Non-trivial: would like to be able to sample from a posterior over possible Q^*
- Bootstrapped DQN (Osband et al. NIPS 2016)
- Efficient Exploration through Bayesian Deep Q-Networks (Azizzadenesheli, B, Anandkumar, NeurIPS workshop 2017)
  - Use deep neural network
  - On last layer use Bayesian linear regression
  - Be optimistic with respect to the resulting posterior
  - Very simple, empirically much better than just doing linear regression on last layer or bootstrapped DQN, not as good as reward bonuses in some cases

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Summary: What You Are Expected to Know

- Define the tension of exploration and exploitation in RL and why this does not arise in supervised or unsupervised learning
- Be able to define and compare different criteria for "good" performance (empirical, convergence, asymptotic, regret, PAC)
- Be able to map algorithms discussed in detail in class to the performance criteria they satisfy

Class Structure

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