

Lecture 14: Batch RL

Emma Brunskill

CS234 Reinforcement Learning.

Winter 2020

Slides drawn from Philip Thomas with modifications

*Note: we only went carefully through slides before slide 34. The remaining slides are kept for those interested but will not be material required for the quiz. See the last slide for a summary of what you should know

Refresh Your Understanding: Fast RL III

Select all that are true:

- ✓ • Thompson sampling for MDPs the posterior over the dynamics can be updated after each transition
- ✓ • When using a Beta prior for a Bernoulli reward parameter for an (s,a) pair, the posterior after N samples of that pair time steps can be the same as after $N+2$ samples $R \in \{0,1\}$ $1/(1-\gamma)$
- ✓ • The optimism bonuses discussed for MBIE-EB depend on the maximum reward but not on the maximum value function $R_{max}/(1-\gamma)$
- ✓ • In class we discussed adding a bonus term to the policy gradient update for a (s,a,r,s') tuple using Q-learning with function approximation. Adding this bonus term will ensure all Q estimates used to make decisions online using DQN are optimistic with respect to Q^*
- Not sure density approx

Class Structure

- Last time: Fast Reinforcement Learning
- **This time: Batch RL**
- Next time: Guest Lecture

A Scientific Experiment

A Group

$$\frac{3}{6} + \frac{2}{8} = \frac{18}{24} = \frac{3}{4}$$

2 Finally, reduce the sum to lowest terms:

$$\frac{2}{10} + \frac{3}{4} = \frac{19}{20} = \frac{19}{20}$$



1 Compare these fractions using the cross-multiplication strategy.

$$\frac{4}{5} \quad \frac{9}{10}$$

4 x 10 = 40 9 x 5 = 45

40 < 45

Avg Score: 95

A Scientific Experiment

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Avg Score: 92

What Should We Do For a New Student?

10P new group

sample size

distrib of scores

A Group

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Involves Counterfactual Reasoning

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Involves Generalization

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Batch Reinforcement Learning

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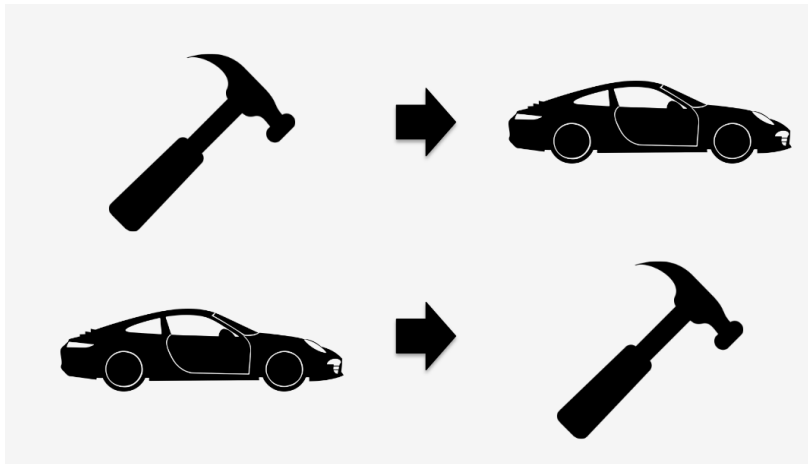
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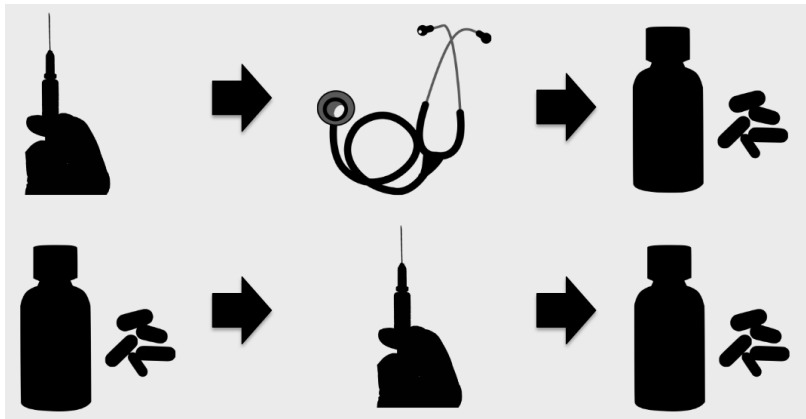
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???

Batch RL



Batch RL



The Problem

- If you apply an existing method, do you have confidence that it will work?

A property of many real applications

- Deploying "bad" policies can be costly or dangerous

What property should a safe batch reinforcement learning algorithm have?

TRPO
policy gradient
methods

- Given past experience from current policy/policies, produce a new policy
 - “Guarantee that with probability at least $1 - \delta$, will not change your policy to one that is worse than the current policy.”
 - You get to choose δ
 - Guarantee not contingent on the tuning of any hyperparameters

Table of Contents

1 Notation

2 Create a safe batch reinforcement learning algorithm

- Off-policy policy evaluation (OPE)
- Safe policy improvement (SPI)

- Policy π : $\pi(a \mid s) = P(a_t = a \mid s_t = s)$
- Trajectory: $T = (s_1, a_1, r_1, s_2, a_2, r_2, \dots, s_L, a_L, r_L)$
- Historical data: $D = \{T_1, T_2, \dots, T_n\}$
- Historical data from behavior policy, π_b
- Objective:

$$V^\pi = \mathbb{E}[\sum_{t=1}^L \gamma^t R_t \mid \pi]$$

Safe batch reinforcement learning algorithm

$$\mathcal{A}(D) \rightarrow \pi$$

- Reinforcement learning algorithm, $\mathcal{A}$
- Historical data, D , which is a random variable
- Policy produced by the algorithm, $\mathcal{A}(D)$, which is a random variable
- a safe batch reinforcement learning algorithm, $\mathcal{A}$, satisfies:

$$\Pr(V^{\mathcal{A}(D)} \geq V^{\pi_b}) \geq 1 - \delta$$

value of current policy used in default

value of policy output by $\mathcal{A}$

or, in general

$$\Pr(V^{\mathcal{A}(D)} \geq V_{min}) \geq 1 - \delta$$

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1 Notation

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Create a safe batch reinforcement learning algorithm

- Off-policy policy evaluation (OPE)
 - For any evaluation policy, π_e , Convert historical data, D , into n independent and unbiased estimates of V^{π_e}
- High-confidence off-policy policy evaluation (HCOPE)
 - Use a concentration inequality to convert the n independent and unbiased estimates of V^{π_e} into a $1 - \delta$ confidence lower bound on V^{π_e}
- Safe policy improvement (SPI)
 - Use HCOPE method to create a safe batch reinforcement learning algorithm,
- Methods today focused on work by Philip Thomas UAI and ICML 2015 papers.

Off-policy policy evaluation (OPE)



High-confidence off-policy policy evaluation (HCOPE)



Safe policy improvement (SPI)



Create a safe batch reinforcement learning algorithm

- **Off-policy policy evaluation (OPE)**
 - For any evaluation policy, π_e , Convert historical data, D , into n independent and unbiased estimates of V^{π_e}
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 - Use a concentration inequality to convert the n independent and unbiased estimates of V^{π_e} into a $1 - \delta$ confidence lower bound on V^{π_e}
- **Safe policy improvement (SPI)**
 - Use HCOPE method to create a safe batch reinforcement learning algorithm,

Monte Carlo (MC) Off Policy Evaluation

- Aim: estimate value of policy π_1 , $V^{\pi_1}(s)$, given episodes generated under behavior policy π_2
 - $s_1, a_1, r_1, s_2, a_2, r_2, \dots$ where the actions are sampled from π_2
- $G_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \gamma^3 r_{t+3} + \dots$ in MDP M under policy π
- $V^\pi(s) = \mathbb{E}_\pi[G_t | s_t = s]$ unbiased
high var
- Have data from a different policy, behavior policy π_2
- If π_2 is stochastic, can often use it to estimate the value of an alternate policy (formal conditions to follow)
- Again, no requirement that have a model nor that state is Markov

Monte Carlo (MC) Off Policy Evaluation: Distribution Mismatch

coverage

- Distribution of episodes & resulting returns differs between policies

*imitation
learning
IRL*

Importance Sampling

- Goal: estimate the expected value of a function $f(x)$ under some probability distribution $p(x)$, $\mathbb{E}_{x \sim p}[f(x)]$
- Have data $x_1, x_2, \dots, x_n$ sampled from distribution $q(s)$
- Under a few assumptions, we can use samples to obtain an unbiased estimate of $\mathbb{E}_{x \sim q}[f(x)]$

$$= \frac{1}{N} \sum_{i=1}^N f(x_i)$$

$x_i \sim q$

Importance Sampling

"Overlap"

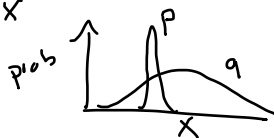
$$\cancel{E_{x \sim q}[f(x)] = \int q(x) f(x) dx}$$
$$E_{x \sim p}[f(x)] = \int x p(x) f(x) dx$$

$$= \int_x \frac{q(x)}{p(x)} p(x) f(x) dx$$

$$= \int_x q(x) \left[\frac{p(x)}{q(x)} f(x) \right] dx$$

$$= E_{x \sim q} \left[\frac{p}{q} f \right]$$

$$\approx \frac{1}{N} \sum_{\substack{i=1 \\ x_i \sim q}}^N \frac{p(x_i)}{q(x_i)} f(x_i)$$



unbiased &

$\forall x \text{ s.t. } p(x) > 0 \text{ then } q(x) > 0$

Checking Your Understanding: Importance Sampling

We can use importance sampling to do batch bandit policy evaluation. Consider we have a dataset for pulls from 3 arms. Consider that arm 1 is a Bernoulli where with probability 0.98 we get 0 and with probability 0.02 we get 100. Arm 2 is a Bernoulli where with probability 0.55 the reward is 2 else the reward is 0. Arm 3 has a probability of yielding a reward of 1 with probability 0.5 else it gets 0. Select all that are true.

- Data is sampled from π_1 where with probability 0.8 it pulls arm 3 else it pulls arm 2.
T The policy we wish to evaluate, π_2 , pulls arm 2 with probability 0.5 else it pulls arm 1. π_2 has higher true reward than π_1 .
- We cannot use π_1 to get an unbiased estimate of the average reward π_2 using importance sampling.
T π_1 never pulls arm 1
- We can use π_1 to get a lower bound on the average reward of π_2 using importance sampling.
T if all rewards are pos
- If rewards can be positive or negative, we can still get a lower bound on π_2 using data from π_1 using importance sampling
F
- Now assume π_1 selects arm1 with probability 0.2 and arm2 with probability 0.8. We can use importance sampling to get an unbiased estimate of π_2 using data from π_1 .
T
- Still with the same π_1 , it is likely with $N=20$ pulls that the estimate using IS for π_2 will be higher than the empirical value of π_1 .
F
- Not sure

Importance Sampling (IS) for RL Policy Evaluation

- Let h_j be episode j (history) of states, actions and rewards

$$h_j = (s_{j,1}, a_{j,1}, r_{j,1}, s_{j,2}, a_{j,2}, r_{j,2}, \dots, s_{j,L_j}(\text{terminal}))$$

Importance Sampling (IS) for Policy Evaluation

- Let h_j be episode j (history) of states, actions and rewards

$$h_j = (s_{j,1}, a_{j,1}, r_{j,1}, s_{j,2}, a_{j,2}, r_{j,2}, \dots, s_{j,L_j}(\text{terminal}))$$

$$\begin{aligned} p(h_j | \pi, s = s_{j,1}) &= p(a_{j,1} | s_{j,1}) p(r_{j,1} | s_{j,1}, a_{j,1}) p(s_{j,2} | s_{j,1}, a_{j,1}) \\ &\quad p(a_{j,2} | s_{j,2}) p(r_{j,2} | s_{j,2}, a_{j,2}) p(s_{j,3} | s_{j,2}, a_{j,2}) \dots \\ &= \prod_{t=1}^{L_j-1} p(a_{j,t} | s_{j,t}) p(r_{j,t} | s_{j,t}, a_{j,t}) p(s_{j,t+1} | s_{j,t}, a_{j,t}) \\ &= \prod_{t=1}^{L_j-1} \pi(a_{j,t} | s_{j,t}) p(r_{j,t} | s_{j,t}, a_{j,t}) p(s_{j,t+1} | s_{j,t}, a_{j,t}) \end{aligned}$$

Importance Sampling (IS) for Policy Evaluation

- Let h_j be episode j (history) of states, actions and rewards, where the actions are sampled from π_2

$$h_j = (s_{j,1}, a_{j,1}, r_{j,1}, s_{j,2}, a_{j,2}, r_{j,2}, \dots, s_{j,L_j}(\text{terminal}))$$

$$V^{\pi_1}(s) \approx \sum_{j=1}^n \frac{p(h_j | \pi_1, s)}{p(h_j | \pi_2, s)} G(h_j)$$

$$= \prod_{i=1}^{L-1} \frac{p(a_i | \pi_1, s_i)}{p(a_i | \pi_2, s_i)}$$

policy gradient

Importance Sampling for Policy Evaluation

- Aim: estimate $V^{\pi_1}(s)$ given episodes generated under policy π_2
 - $s_1, a_1, r_1, s_2, a_2, r_2, \dots$ where the actions are sampled from π_2
- Have access to $G_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \gamma^3 r_{t+3} + \dots$ in MDP M under policy π_2
- Want $V^{\pi_1}(s) = \mathbb{E}_{\pi_1}[G_t | s_t = s]$
- IS = Monte Carlo estimate given off policy data
- Model-free method
- Does not require Markov assumption
- Under some assumptions, unbiased & consistent estimator of V^{π_1}
- Can be used when agent is interacting with environment to estimate value of policies different than agent's control policy

Leveraging Future Can't Influence Past Rewards

- Importance sampling (IS):

$$IS(D) = \frac{1}{n} \sum_{i=1}^n \left(\prod_{t=1}^L \frac{\pi_e(a_t | s_t)}{\pi_b(a_t | s_t)} \right) \left(\sum_{t=1}^L \gamma^t R_t^i \right)$$

*policy
grad*

*don't
need
dynamics
model*

- Per-decision importance sampling (PDIS)

$$PSID(D) = \sum_{t=1}^L \gamma^t \frac{1}{n} \sum_{i=1}^n \left(\prod_{\tau=1}^t \frac{\pi_e(a_\tau | s_\tau)}{\pi_b(a_\tau | s_\tau)} \right) R_t^i$$

Off-policy policy evaluation

*Note: we only went carefully through slides before this point. The remaining slides are kept for those interested but will not be material required for the quiz. See the last slide for a summary of what you should know

- Importance sampling (IS):

$$IS(D) = \frac{1}{n} \sum_{i=1}^n w_i \left(\sum_{t=1}^L \gamma^t R_t^i \right)$$

- Weighted importance sampling (WIS)

$$WIS(D) = \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i \left(\sum_{t=1}^L \gamma^t R_t^i \right)$$

Off-policy policy evaluation

- Weighted importance sampling (WIS)

$$WIS(D) = \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i \left(\sum_{t=1}^L \gamma^t R_t^i \right)$$

- Biased or unbiased?

Off-policy policy evaluation

- Weighted importance sampling (WIS)

$$WIS(D) = \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i \left(\sum_{t=1}^L \gamma^t R_t^i \right)$$

- Biased. When $n = 1$, $\mathbb{E}[WIS] = V(\pi_b)$
- Strongly consistent estimator of V^{π_e}
 - i.e. $\Pr(\lim_{n \rightarrow \infty} WIS(D) = V^{\pi_e}) = 1$
 - If
 - Finite horizon
 - One behavior policy, or bounded rewards

Control variates

- Given: X
- Estimate: $\mu = \mathbb{E}[X]$
- $\hat{\mu} = X$
- Unbiased: $\mathbb{E}[\hat{\mu}] = \mathbb{E}[X] = \mu$
- Variance: $\text{Var}(\hat{\mu}) = \text{Var}(X)$

Control variates

- Given: $X, Y, \mathbb{E}[Y]$
- Estimate: $\mu = \mathbb{E}[X]$
- $\hat{\mu} = X - Y + \mathbb{E}[Y]$
- Unbiased:
 $\mathbb{E}[\hat{\mu}] =$
- Variance:

$$\text{Var}(\hat{\mu}) = \text{Var}(X - Y + \mathbb{E}[Y]) = \text{Var}(X - Y)$$

Control variates

- Given: $X, Y, \mathbb{E}[Y]$
- Estimate: $\mu = \mathbb{E}[X]$
- $\hat{\mu} = X - Y + \mathbb{E}[Y]$
- Unbiased:
$$\mathbb{E}[\hat{\mu}] = \mathbb{E}[X - Y + \mathbb{E}[Y]] = \mathbb{E}[X] - \mathbb{E}[Y] + \mathbb{E}[Y] = \mathbb{E}[X] = \mu$$
- Variance:

$$\begin{aligned}\text{Var}(\hat{\mu}) &= \text{Var}(X - Y + \mathbb{E}[Y]) = \text{Var}(X - Y) \\ &= \text{Var}(X) + \text{Var}(Y) - 2\text{Cov}(X, Y)\end{aligned}$$

- Lower variance if $2\text{Cov}(X, Y) > \text{Var}(Y)$
- We call Y a control variate
- We saw this idea before: baseline term in policy gradient estimation

Off-policy policy evaluation

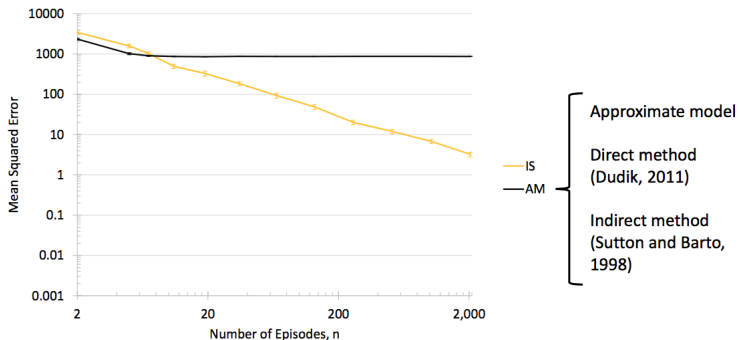
- Idea: add a control variate to importance sampling estimators
 - X is the importance sampling estimator
 - Y is a control variate build from an approximate model of the MDP
- Called the doubly robust estimator (Jiang and Li, 2015)
 - Robust to (1) poor approximate model, and (2) error in estimates of π_b
 - If the model is poor, the estimates are still unbiased
 - If the sampling policy is unknown, but the model is good, MSE will still be low
- Non-recursive and weighted forms, as well as control variate view provided by Thomas and Brunskill (ICML 2016)

Off-policy policy evaluation

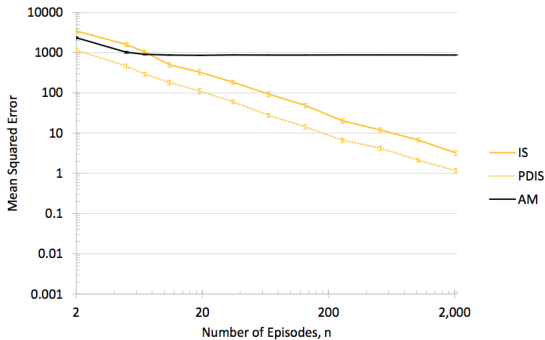
$$DR(\pi_e \mid D) = \frac{1}{n} \sum_{i=1}^n \sum_{t=0}^{\infty} \gamma^t w_t^i (R_t^i - \hat{q}^{\pi_e}(S_t^i, A_t^i)) + \gamma^t \rho_{t-1}^i \hat{v}^{\pi_e}(S_t^i),$$

where $w_t^i = \prod_{\tau=1}^t \frac{\pi_e(a_\tau \mid s_\tau)}{\pi_b(a_\tau \mid s_\tau)}$

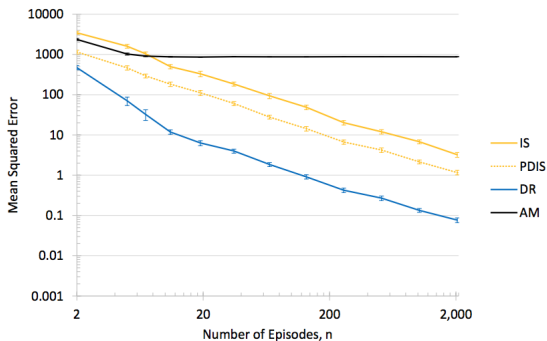
Empirical Results (Gridworld)



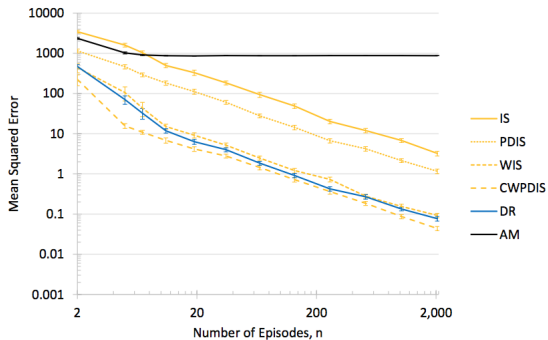
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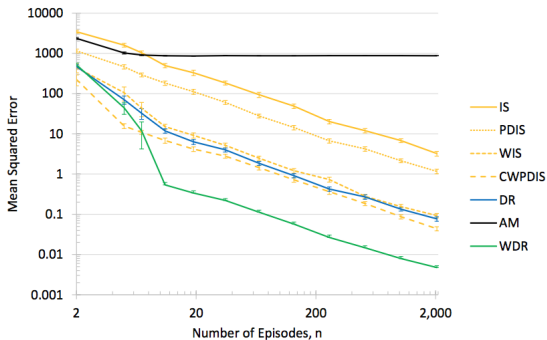
Empirical Results (Gridworld)



Empirical Results (Gridworld)



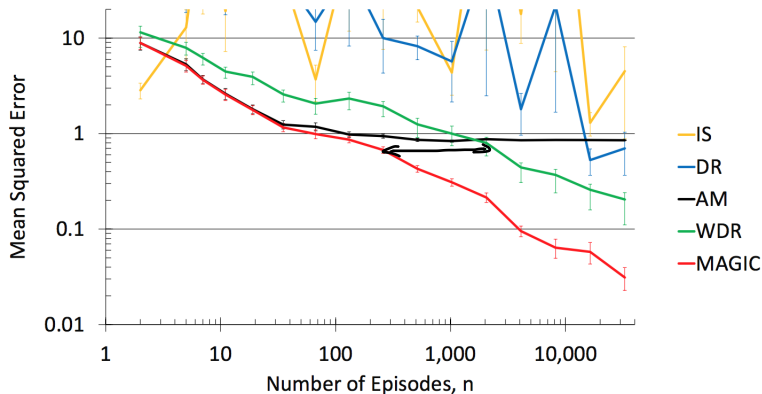
Empirical Results (Gridworld)



Off-policy policy evaluation : Blending

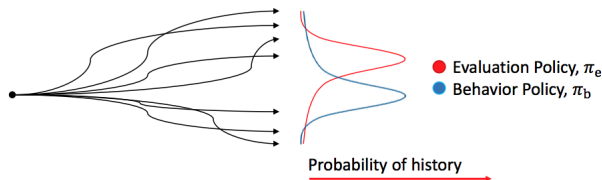
- Importance sampling is unbiased but high variance
- Model based estimate is biased but low variance
- Doubly robust is one way to combine the two
- Can also trade between importance sampling and model based estimate within a trajectory
- MAGIC estimator (Thomas and Brunskill ICML 2016)
- Can be particularly useful when part of the world is non-Markovian in the given model, and other parts of the world are Markov

Can Need an Order of Magnitude Less Data To Get Good Estimates



Off-policy policy evaluation

- What if $\text{supp}(\pi_e \subset \text{supp}(\pi_b))$
- There is a state-action pair, (s, a) , such that $\pi_e(a | s) = 0$, but $\pi_b(a | s) \neq 0$.
- If we see a history where (s, a) occurs, what weight should we give it?
- $IS(D) = \frac{1}{n} \sum_{i=1}^n \left(\prod_{t=1}^L \frac{\pi_e(a_t | s_t)}{\pi_b(a_t | s_t)} \right) \left(\sum_{t=1}^L \gamma^t R_t^i \right)$

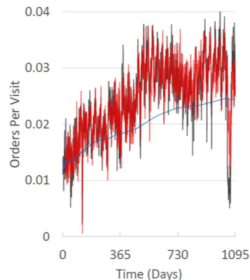
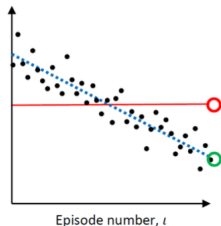


Off-policy policy evaluation

- What if there are zero samples ($n = 0$)?
 - The importance sampling estimate is undefined
- What if no samples are in $\text{supp}(\pi_e)$ (or $\text{supp}(p)$ in general)?
 - Importance sampling says: the estimate is zero
 - Alternate approach: undefined
- Importance sampling estimator is unbiased if $n > 0$
- Alternate approach will be unbiased given that at least one sample is in the support of p
- Alternate approach detailed in Importance Sampling with Unequal Support (Thomas and Brunskill, AAAI 2017)

Off-policy policy evaluation

- Thomas et. al. Predictive Off-Policy Policy Evaluation for Nonstationary Decision Problems, with Applications to Digital Marketing (AAAI 2017)



Create a safe batch reinforcement learning algorithm

- Off-policy policy evaluation (OPE)
 - For any evaluation policy, π_e , Convert historical data, D , into n independent and unbiased estimates of V^{π_e}
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 - Use HCOPE method to create a safe batch reinforcement learning algorithm,

*Heffding for
UCB*

High-confidence off-policy policy evaluation

- Consider using IS + Hoeffding's inequality for HCOPE on mountain car

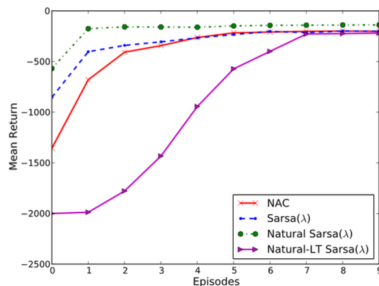
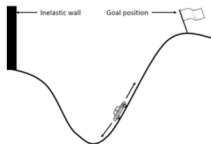


Figure 3: Mountain Car (Sarsa(λ))
Natural Temporal Difference Learning, Dabney and Thomas, 2014

Hoeffding's inequality

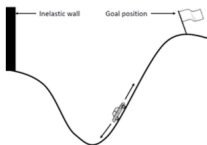
- Let $X_1, \dots, X_n$ be n independent identically distributed random variables such that $X_i \in [0, b]$
- Then with probability at least $1 - \delta$:

$$\mathbb{E}[X_i] \geq \frac{1}{n} \sum_{i=1}^n X_i - b \sqrt{\frac{\ln(1/\delta)}{2n}},$$

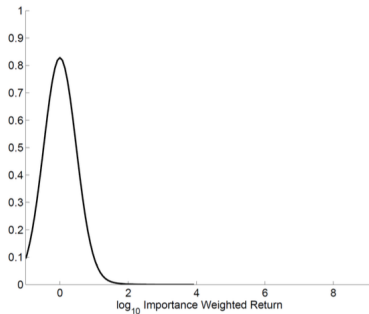
where $X_i = \frac{1}{n} \sum_{i=1}^n (w_i \sum_{t=1}^L \gamma^t R_t^i)$ in our case.

High-confidence off-policy policy evaluation

- Using 100,000 trajectories
- Evaluation policy's true performance is $0.19 \in [0, 1]$
- We get a 95% confidence lower bound of: -5,8310,000



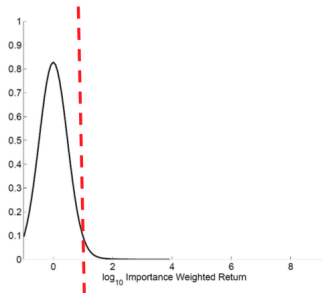
What went wrong



$$w_i = \prod_{t=1}^L \frac{\pi_e(a_t | s_t)}{\pi_b(a_t | s_t)}$$

High-confidence off-policy policy evaluation

- Removing the upper tail only decreases the expected value.



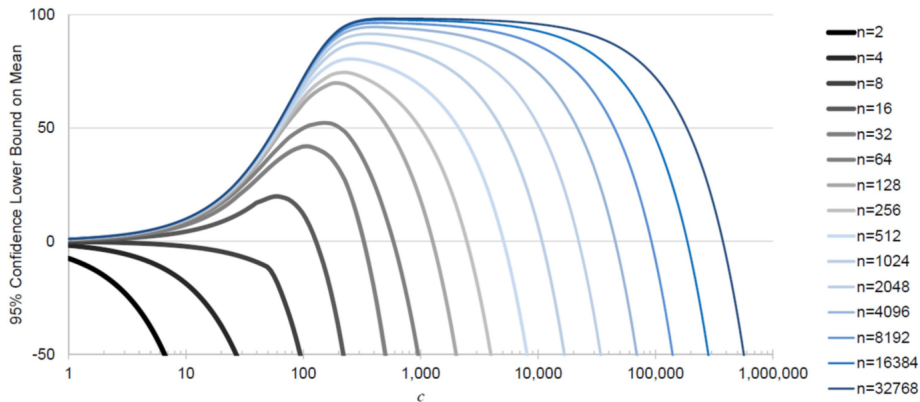
High-confidence off-policy policy evaluation

- Thomas et. al, High confidence off-policy evaluation, AAAI 2015

Theorem 1. Let $X_1, \dots, X_n$ be n independent real-valued random variables such that for each $i \in \{1, \dots, n\}$, we have $\mathbb{P}[0 \leq X_i] = 1$, $\mathbb{E}[X_i] \leq \mu$, and some threshold value $c_i > 0$. Let $\delta > 0$ and $Y_i := \min\{X_i, c_i\}$. Then with probability at least $1 - \delta$, we have

$$\mu \geq \underbrace{\left(\sum_{i=1}^n \frac{1}{c_i} \right)^{-1} \sum_{i=1}^n \frac{Y_i}{c_i}}_{\text{empirical mean}} - \underbrace{\left(\sum_{i=1}^n \frac{1}{c_i} \right)^{-1} \frac{7n \ln(2/\delta)}{3(n-1)}}_{\text{term that goes to zero as } 1/n \text{ as } n \rightarrow \infty} - \underbrace{\left(\sum_{i=1}^n \frac{1}{c_i} \right)^{-1} \sqrt{\frac{\ln(2/\delta)}{n-1} \sum_{i,j=1}^n \left(\frac{Y_i}{c_i} - \frac{Y_j}{c_j} \right)^2}}_{\text{term that goes to zero as } 1/\sqrt{n} \text{ as } n \rightarrow \infty}. \quad (3)$$

High-confidence off-policy policy evaluation



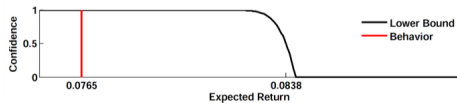
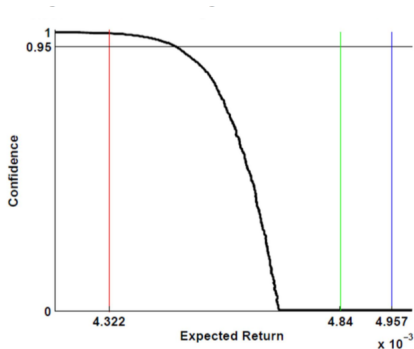
High-confidence off-policy policy evaluation

- Use 20% of the data to optimize c (cutoff)
- Use 80% to compute lower bound with optimized c
- Mountain car results:

	CUT	Chernoff-Hoeffding	Maurer	Anderson	Bubeck et al.
95% Confidence lower bound on the mean	0.145	-5,831,000	-129,703	0.055	-.046

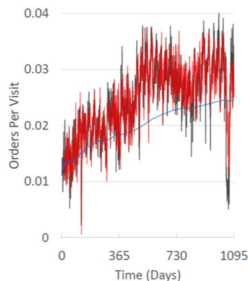
High-confidence off-policy policy evaluation

Digital marketing:



High-confidence off-policy policy evaluation

Cognitive dissonance:



$$\mathbb{E}[X_i] \geq \frac{1}{n} \sum_{i=1}^n X_i - b \sqrt{\frac{\ln(1/\delta)}{2n}}$$

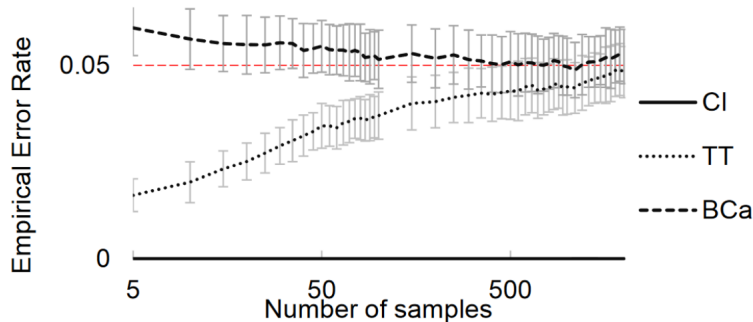
High-confidence off-policy policy evaluation

- Student's t-test
 - Assumes that $IS(D)$ is normally distributed
 - By the central limit theorem, it (is as $n \rightarrow \infty$)

$$\Pr \left(\mathbb{E} \left[\frac{1}{n} \sum_{i=1}^n X_i \right] \geq \frac{1}{n} \sum_{i=1}^n X_i \right) = \frac{\sqrt{\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2}}{\sqrt{n}} t_{1-\delta, n-1} \\ \geq 1 - \delta$$

- Efron's Bootstrap methods (e.g., BCa)
 - Also, without importance sampling: Hanna, Stone, and Niekum, AAMAS 2017

High-confidence off-policy policy evaluation



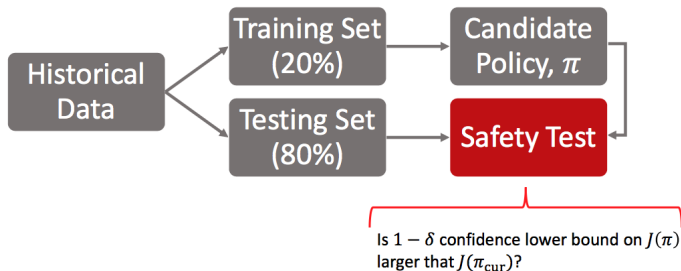
P. S. Thomas. Safe reinforcement learning (PhD Thesis, 2015)

Create a safe batch reinforcement learning algorithm

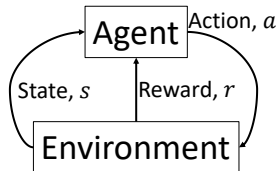
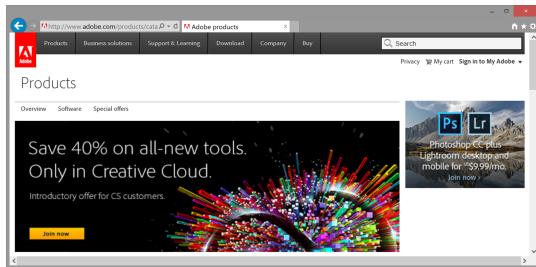
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Safe policy improvement

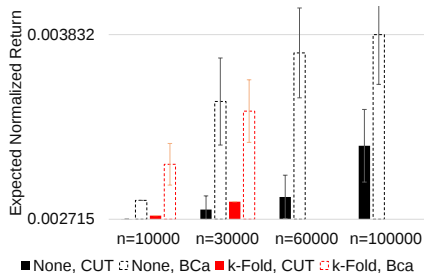
Thomas et. al, ICML 2015



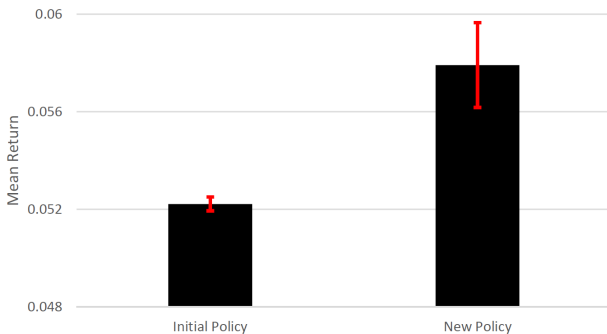
Empirical Results: Digital Marketing



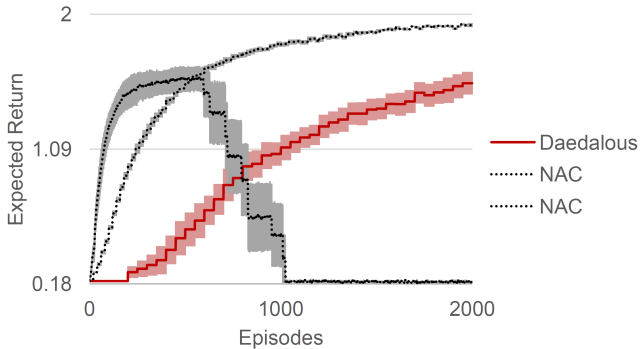
Empirical Results: Digital Marketing



Empirical Results: Digital Marketing



Empirical Results: Digital Marketing



Other Relevant Work

- How to deal with long horizons? (Guo, Thomas, Brunskill NIPS 2017)
- How to deal with importance sampling being “unfair”? (Doroudi, Thomas and Brunskill, best paper UAI 2017)
- What to do when the behavior policy is not known? (Liu, Gottesman, Raghu, Komorowski, Faisal, Doshi-Velez, Brunskill NeurIPS 2018)
- What to do when the behavior policy is deterministic?
- What to do when care about safe exploration?
- What to do when care about performance on a single trajectory
- Many others also doing great work in this space, including the groups of Yisong Yue, Susan Murphy, Finale Doshi-Velez, Marco Pavone, Pieter Abbeel, Shie Mannor, Sergey Levine and Claire Tomlin, amongst others

Off Policy Policy Evaluation and Selection

- Very important topic: healthcare, education, marketing, ...
- Insights are relevant to on policy learning
- Big focus of my lab
- A number of others on campus also working in this area (e.g. Stefan Wager, Susan Athey...)
- Very interesting area at the intersection of causality and control
- Our Science 2019 paper show how to do safe policy improvement for a high fidelity diabetes simulator and discusses the need for ensuring good behavior

What You Should Know: Off Policy Policy Evaluation and Selection

- Be able to define and apply importance sampling for off policy policy evaluation]
- Define some limitations of IS (variance) coverage / overlap / finite data
- Define why we might want safe reinforcement learning batch

Class Structure

- Last time: Fast Reinforcement Learning
- **This time: Batch RL**
- Next time: Guest Lecture