Lecture 9: Policy Gradient II [1]

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CS234 Reinforcement Learning.

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- Additional reading: Sutton and Barto 2018 Chp. 13

[1] With many slides from or derived from David Silver and John Schulman and Pieter Abbeel

Refresh Your Knowledge

- Select all that are true about policy gradients:
  1. $\nabla_\theta V(\theta) = \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a) Q^{\pi_\theta}(s,a)]$
  2. θ is always increased in the direction of $\nabla_\theta \ln(\pi(S_t, A_t, \theta)$.
  3. State-action pairs with higher estimated Q values will increase in probability on average
  4. Are guaranteed to converge to the global optima of the policy class
  5. Not sure

Refresh Your Knowledge Solutions

- Select all that are true about policy gradients:
  1. $\nabla_\theta V(\theta) = \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a) Q^{\pi_\theta}(s,a)]$
  2. θ is always increased in the direction of $\nabla_\theta \ln(\pi(S_t, A_t, \theta)$.
  3. State-action pairs with higher estimated Q values will increase in probability on average
  4. Are guaranteed to converge to the global optima of the policy class
  5. Not sure

1 and 3 are true. The direction of θ also depends on the Q-values /returns. We are only guaranteed to reach a local optima

Class Structure

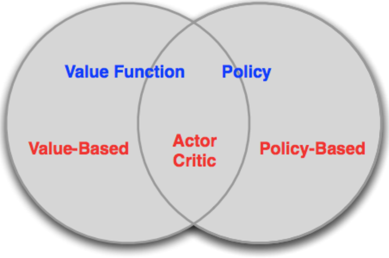
- Last time: Policy Search
- **This time: Policy Search**
- Next time: Exam
- Next next time: Exploration

Recall: Policy-Based RL

- Policy search: directly parametrize the policy

$$\pi_\theta(s, a) = \mathbb{P}[a|s; \theta]$$

- Goal is to find a policy π with the highest value function V^π
- (Pure) Policy based methods
  - No Value Function
  - Learned Policy
- Actor-Critic methods
  - Learned Value Function
  - Learned Policy



Recall: Advantages of Policy-Based RL

Advantages:

- Better convergence properties
- Effective in high-dimensional or continuous action spaces
- Can learn stochastic policies

Disadvantages:

- Typically converge to a local rather than global optimum
- Evaluating a policy is typically inefficient and high variance

Recall: Policy Gradient

- Defined $V(\theta) = V^{\pi_\theta}(s_0) = V(s_0, \theta)$ to make explicit the dependence of the value on the policy parameters
- Assumed episodic MDPs
- Policy gradient algorithms search for a *local* maximum of $V(\theta)$ by ascending the gradient of the policy, w.r.t parameters θ

$$\Delta\theta = \alpha \nabla_\theta V(\theta)$$

- Where $\nabla_\theta V(\theta)$ is the policy gradient

$$\nabla_\theta V(\theta) = \begin{pmatrix} \frac{\partial V(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial V(\theta)}{\partial \theta_n} \end{pmatrix}$$

- and α is a step-size hyperparameter

Desired Properties of a Policy Gradient RL Algorithm

- Goal: Converge as quickly as possible to a local optima
  - Incurring reward / cost as execute policy, so want to minimize number of iterations / time steps until reach a good policy

Desired Properties of a Policy Gradient RL Algorithm

- Goal: Converge as quickly as possible to a local optima
  - Incurring reward / cost as execute policy, so want to minimize number of iterations / time steps until reach a good policy
- During policy search alternating between evaluating policy and changing (improving) policy (just like in policy iteration)
- Would like each policy update to be a monotonic improvement
  - Only guaranteed to reach a local optima with gradient descent
  - Monotonic improvement will achieve this
  - And in the real world, monotonic improvement is often beneficial

Desired Properties of a Policy Gradient RL Algorithm

- Goal: Obtain large monotonic improvements to policy at each update
- Techniques to try to achieve this:
  - Last time and today: Get a better estimate of the gradient (intuition: should improve updating policy parameters)
  - Today: Change, how to update the policy parameters given the gradient

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Likelihood Ratio / Score Function Policy Gradient

- Recall last time (m is a set of trajectories):

$$\nabla_\theta V(s_0, \theta) \approx (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$

- Unbiased estimate of gradient but very noisy
- Fixes that can make it practical
  - Temporal structure (discussed last time)
  - Baseline
  - Alternatives to using Monte Carlo returns $R(\tau^{(i)})$ as targets

Policy Gradient: Introduce Baseline

- Reduce variance by introducing a *baseline* $b(s)$

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t;\theta) \left(\sum_{t'=t}^{T-1} r_{t'} - b(s_t) \right) \right]$$

- For any choice of b, gradient estimator is unbiased.
- Near optimal choice is the expected return,

$$b(s_t) \approx \mathbb{E}[r_t + r_{t+1} + \cdots + r_{T-1}]$$

- Interpretation: increase logprob of action a_t proportionally to how much returns $\sum_{t'=t}^{T-1} r_{t'}$ are better than expected

Baseline $b(s)$ Does Not Introduce Bias–Derivation

$$\mathbb{E}_\tau[\nabla_\theta \log \pi(a_t|s_t;\theta)b(s_t)]$$
$$= \mathbb{E}_{s_{0:t},a_{0:(t-1)}}\left[\mathbb{E}_{s_{(t+1):T},a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t;\theta)b(s_t)]\right]$$

Baseline $b(s)$ Does Not Introduce Bias–Derivation

$$
\begin{aligned}
&\mathbb{E}_\tau[\nabla_\theta \log \pi(a_t|s_t;\theta) b(s_t)] \\
&= \mathbb{E}_{s_{0:t}, a_{0:(t-1)}}\left[\mathbb{E}_{s_{(t+1):T}, a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t;\theta) b(s_t)]\right] \text{ (break up expectation)} \\
&= \mathbb{E}_{s_{0:t}, a_{0:(t-1)}}\left[b(s_t)\mathbb{E}_{s_{(t+1):T}, a_{t:(T-1)}}[\nabla_\theta \log \pi(a_t|s_t;\theta)]\right] \text{ (pull baseline term out)} \\
&= \mathbb{E}_{s_{0:t}, a_{0:(t-1)}}\left[b(s_t)\mathbb{E}_{a_t}[\nabla_\theta \log \pi(a_t|s_t;\theta)]\right] \text{ (remove irrelevant variables)} \\
&= \mathbb{E}_{s_{0:t}, a_{0:(t-1)}}\left[b(s_t)\sum_a \pi_\theta(a_t|s_t)\frac{\nabla_\theta \pi(a_t|s_t;\theta)}{\pi_\theta(a_t|s_t)}\right] \text{ (likelihood ratio)} \\
&= \mathbb{E}_{s_{0:t}, a_{0:(t-1)}}\left[b(s_t)\sum_a \nabla_\theta \pi(a_t|s_t;\theta)\right] \\
&= \mathbb{E}_{s_{0:t}, a_{0:(t-1)}}\left[b(s_t)\nabla_\theta \sum_a \pi(a_t|s_t;\theta)\right] \\
&= \mathbb{E}_{s_{0:t}, a_{0:(t-1)}}\left[b(s_t)\nabla_\theta 1\right] \\
&= \mathbb{E}_{s_0:t, a_0:(t-1)}\left[b(s_t)\cdot 0\right] = 0
\end{aligned}
$$

"Vanilla" Policy Gradient Algorithm

Initialize policy parameter θ, baseline b
for iteration$=1, 2, \cdots$ **do**
  Collect a set of trajectories by executing the current policy
  At each timestep t in each trajectory τ^i, compute
    Return $G_t^i = \sum_{t'=t}^{T-1} r_{t'}^i$, and
    Advantage estimate $\hat{A}_t^i = G_t^i - b(s_t)$.
  Re-fit the baseline, by minimizing $\sum_i \sum_t ||b(s_t) - G_t^i||^2$,
  Update the policy, using a policy gradient estimate $\hat{g}$,
    Which is a sum of terms $\nabla_\theta \log \pi(a_t|s_t, \theta)\hat{A}_t$.
    (Plug $\hat{g}$ into SGD or ADAM)
endfor

Other Choices for Baseline?

Initialize policy parameter θ, baseline b
for iteration$=1, 2, \cdots$ **do**
  Collect a set of trajectories by executing the current policy
  At each timestep t in each trajectory τ^i, compute
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endfor

Choosing the Baseline: Value Functions

- Recall Q-function / state-action-value function:

$$Q^\pi(s,a) = \mathbb{E}_\pi\left[r_0 + \gamma r_1 + \gamma^2 r_2 \cdots | s_0 = s, a_0 = a\right]$$

- State-value function can serve as a great baseline

$$\begin{aligned} V^\pi(s) &= \mathbb{E}_\pi\left[r_0 + \gamma r_1 + \gamma^2 r_2 \cdots | s_0 = s\right] \\ &= \mathbb{E}_{a\sim\pi}[Q^\pi(s,a)] \end{aligned}$$

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- Unbiased estimate of gradient but very noisy
- Fixes that can make it practical
  - Temporal structure (discussed last time)
  - Baseline
  - **Alternatives to using Monte Carlo returns G_t^i as estimate of expected discounted sum of returns for the policy parameterized by θ?**

Choosing the Target

- G_t^i is an estimation of the value function at s_t from a single roll out
- Unbiased but high variance
- Reduce variance by introducing bias using bootstrapping and function approximation
    - Just like in we saw for TD vs MC, and value function approximation

Actor-critic Methods

- Estimate of V/Q is done by a **critic**
- **Actor-critic** methods maintain an explicit representation of policy and the value function, and update both
- A3C (Mnih et al. ICML 2016) is a very popular actor-critic method

Policy Gradient Formulas with Value Functions

- Recall:

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t;\theta) \left(\sum_{t'=t}^{T-1} r_{t'} - b(s_t) \right) \right]$$

$$\nabla_\theta \mathbb{E}_\tau[R] \approx \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t;\theta) \left(Q(s_t, a_t; \boldsymbol{w}) - b(s_t) \right) \right]$$

- Letting the baseline be an estimate of the value V, we can represent the gradient in terms of the state-action advantage function

$$\nabla_\theta \mathbb{E}_\tau[R] \approx \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t;\theta) \hat{A}^\pi(s_t, a_t) \right]$$

- where the advantage function $A^\pi(s,a) = Q^\pi(s,a) - V^\pi(s)$

Choosing the Target: N-step estimators

$$\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} \sum_{t=0}^{T-1} R_t^i \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$

- Note that critic can select any blend between TD and MC estimators for the target to substitute for the true state-action value function.

Choosing the Target: N-step estimators

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- Note that critic can select any blend between TD and MC estimators for the target to substitute for the true state-action value function.

$$\hat{R}_t^{(1)} = r_t + \gamma V(s_{t+1}) \quad \text{td}$$

$$\hat{R}_t^{(2)} = r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2}) \quad \cdots$$

$$\hat{R}_t^{(\text{inf})} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots$$

- If subtract baselines from the above, get advantage estimators

$$\hat{A}_t^{(1)} = r_t + \gamma V(s_{t+1}) - V(s_t)$$

$$\hat{A}_t^{(\text{inf})} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+1} + \cdots - V(s_t)$$

Check Your Understanding: Blended Advantage Estimators

$$\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} \sum_{t=0}^{T-1} R_t^i \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$

- If subtract baselines from the above, get advantage estimators

$$\hat{A}_t^{(1)} = r_t + \gamma V(s_{t+1}) - V(s_t)$$
$$\hat{A}_t^{(\text{inf})} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+1} + \cdots - V(s_t)$$

- Select all that are true
- $\hat{A}_t^{(1)}$ has low variance & low bias.
- $\hat{A}_t^{(1)}$ has high variance & low bias.
- $\hat{A}_t^{(\infty)}$ low variance and high bias.
- $\hat{A}_t^{(\infty)}$ high variance and low bias.
- Not sure

Check Your Understanding: Blended Advantage Estimators Answers

$$\nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} \sum_{t=0}^{T-1} R_t^i \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)})$$

- If subtract baselines from the above, get advantage estimators

$$\hat{A}_t^{(1)} = r_t + \gamma V(s_{t+1}) - V(s_t)$$
$$\hat{A}_t^{(\text{inf})} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+1} + \cdots - V(s_t)$$

result

- ~~Solution~~: $\hat{A}_t^{(1)}$ has low variance & high bias. $\hat{A}_t^{(\infty)}$ high variance but low bias.

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    (**Plug $\hat{g}$ into SGD or ADAM**)
endfor

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Policy Gradient and Step Sizes

- Goal: Each step of policy gradient yields an updated policy π' whose value is greater than or equal to the prior policy π: $V^{\pi'} \geq V^{\pi}$
- Gradient descent approaches update the weights a small step in direction of gradient
- **First order** / linear approximation of the value function's dependence on the policy parameterization
- Locally a good approximation, further away less good

Why are step sizes a big deal in RL?

- Step size is important in any problem involving finding the optima of a function
- Supervised learning: Step too far $\rightarrow$ next updates will fix it
- Reinforcement learning
  - Step too far $\rightarrow$ bad policy
  - Next batch: collected under bad policy
  - **Policy is determining data collection!** Essentially controlling exploration and exploitation trade off due to particular policy parameters and the stochasticity of the policy
  - May not be able to recover from a bad choice, collapse in performance!

Simple Step Size

- Simple step-sizing: Line search in direction of gradient
  - Simple but expensive (perform evaluations along the line)
  - Naive: ignores where the first order approximation is good or bad

Policy Gradient Methods with Auto-Step-Size Selection

- Can we automatically ensure the updated policy π' has value greater than or equal to the prior policy π: $V^{\pi'} \geq V^{\pi}$?
- Consider this for the policy gradient setting, and hope to address this by modifying step size

Objective Function

- Goal: find policy parameters that maximize value function[1]

$$V(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t); \pi_\theta\right]$$

- where $s_0 \sim P(s_0)$, $a_t \sim \pi(a_t|s_t), s_{t+1} \sim P(s_{t+1}|s_t, a_t)$
- Have access to samples from the current policy π_θ (param. by θ)
- Want to predict the value of a different policy (off policy learning!)

[1] For today we will primarily consider discounted value functions

Objective Function

- Goal: find policy parameters that maximize value function[1]

$$V(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t); \pi_\theta\right]$$

- where $s_0 \sim P(s_0)$, $a_t \sim \pi(a_t|s_t), s_{t+1} \sim P(s_{t+1}|s_t, a_t)$
- Express value of $\tilde{\pi}$ in terms of advantage over π

$$\begin{aligned} V(\tilde{\theta}) &= V(\theta) + \mathbb{E}_{\pi_{\tilde{\theta}}}\left[\sum_{t=0}^{\infty} \gamma^t A_\pi(s_t, a_t)\right] && (1) \\ &= V(\theta) + \sum_s \mu_{\tilde{\pi}}(s) \sum_a \tilde{\pi}(a|s) A_\pi(s, a) && (2) \\ \mu_{\tilde{\pi}}(s) &= E_{\tilde{\pi}} \sum_{t=0}^{\infty} \gamma^t I(s_t = s) && (3) \end{aligned}$$

- $\mu_{\tilde{\pi}}(s)$ is the discounted weighted frequency of state s under policy $\tilde{\pi}$

[1] For today we will primarily consider discounted value functions

Objective Function

- Goal: find policy parameters that maximize value function[1]

$$V(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t); \pi_\theta\right]$$

- where $s_0 \sim \mu(s_0)$, $a_t \sim \pi(a_t|s_t), s_{t+1} \sim P(s_{t+1}|s_t, a_t)$
- Express expected return of another policy in terms of the advantage over the original policy

$$V(\tilde{\theta}) = V(\theta) + \mathbb{E}_{\pi_{\tilde{\theta}}}\left[\sum_{t=0}^{\infty} \gamma^t A_\pi(s_t, a_t)\right] = V(\theta) + \sum_s \mu_{\tilde{\pi}}(s) \sum_a \tilde{\pi}(a|s) A_\pi(s, a)$$

- where $\mu_{\tilde{\pi}}(s)$ is defined as the discounted weighted frequency of state s under policy $\tilde{\pi}$
- We know the advantage A_π and $\tilde{\pi}$
- But we can't compute the above because we don't know $\mu_{\tilde{\pi}}$, the state distribution under the new proposed policy

[1] For today we will primarily consider discounted value functions

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Local approximation

- Can we remove the dependency on the discounted visitation frequencies under the new policy?
- Substitute in the discounted visitation frequencies under the current policy to define a new objective function:

$$L_\pi(\tilde{\pi}) = V(\theta) + \sum_s \mu_\pi(s) \sum_a \tilde{\pi}(a|s) A_\pi(s,a)$$

- Note that $L_{\pi_{\theta_0}}(\pi_{\theta_0}) = V(\theta_0)$
- Gradient of L is identical to gradient of value function at policy parameterized evaluated at θ_0: $\nabla_\theta L_{\pi_{\theta_0}}(\pi_\theta)|_{\theta=\theta_0} = \nabla_\theta V(\theta)|_{\theta=\theta_0}$

Conservative Policy Iteration

- Is there a bound on the performance of a new policy obtained by optimizing the surrogate objective?
- Consider mixture policies that blend between an old policy and a different policy

$$\pi_{new}(a|s) = (1-\beta)\pi_{old}(a|s) + \beta\pi'(a|s)$$

- In this case can guarantee a lower bound on value of the new π_{new}:

$$V^{\pi_{new}} \geq L_{\pi_{old}}(\pi_{new}) - \frac{2\epsilon\gamma}{(1-\gamma)^2}\beta^2$$

- where $\epsilon = \max_s \left|\mathbb{E}_{a\sim\pi'(a|s)}\left[A_\pi(s,a)\right]\right|$

Check Your Understanding: Conservative Policy Iteration

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What can we say about this lower bound? (Select all)

1. It is tight if $\pi_{new} = \pi_{old}$
2. It is most loose if $\beta = 1$
3. It is most tight if $\beta = 1$
4. It is most tight if $\beta = 0$
5. Not sure

Check Your Understanding: Conservative Policy Iteration

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It is tight when $\pi_{new} = \pi_{old}$ and most loose if $\beta = 1$ and most tight if

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- where $\epsilon = \max_s \left|\mathbb{E}_{a\sim\pi'(a|s)}\left[A_\pi(s,a)\right]\right|$

Find the Lower-Bound in General Stochastic Policies

- Would like to similarly obtain a lower bound on the potential performance for general stochastic policies (not just mixture policies)
- Recall $L_\pi(\tilde{\pi}) = V(\theta) + \sum_s \mu_\pi(s) \sum_a \tilde{\pi}(a|s) A_\pi(s, a)$

Theorem

Let $D_{TV}^{\max}(\pi_1, \pi_2) = \max_s D_{TV}(\pi_1(\cdot|s), \pi_2(\cdot|s))$. *Then*

$$V^{\pi_{new}} \geq L_{\pi_{old}}(\pi_{new}) - \frac{4\epsilon\gamma}{(1-\gamma)^2}(D_{TV}^{\max}(\pi_{old}, \pi_{new}))^2$$

where $\epsilon = \max_{s,a} |A_\pi(s, a)|$.

Find the Lower-Bound in General Stochastic Policies

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- Recall $L_\pi(\tilde{\pi}) = V(\theta) + \sum_s \mu_\pi(s) \sum_a \tilde{\pi}(a|s) A_\pi(s,a)$

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$$V^{\pi_{new}} \geq L_{\pi_{old}}(\pi_{new}) - \frac{4\epsilon\gamma}{(1-\gamma)^2}(D_{TV}^{\max}(\pi_{old}, \pi_{new}))^2$$

where $\epsilon = \max_{s,a} |A_\pi(s,a)|$.

- Note that $D_{TV}(p,q)^2 \leq D_{KL}(p,q)$ for prob. distrib p and q.
- Then the above theorem immediately implies that

$$V^{\pi_{new}} \geq L_{\pi_{old}}(\pi_{new}) - \frac{4\epsilon\gamma}{(1-\gamma)^2} D_{KL}^{\max}(\pi_{old}, \pi_{new})$$

- where $D_{KL}^{\max}(\pi_1, \pi_2) = \max_s D_{KL}(\pi_1(\cdot|s), \pi_2(\cdot|s))$

Guaranteed Improvement[1]

- Goal is to compute a policy that maximizes the objective function defining the lower bound:

[1] $L_\pi(\tilde{\pi}) = V(\theta) + \sum_s \mu_\pi(s) \sum_a \tilde{\pi}(a|s) A_\pi(s,a)$

Guaranteed Improvement[1]

- Goal is to compute a policy that maximizes the objective function defining the lower bound:

$$
\begin{aligned}
M_i(\pi) &= L_{\pi_i}(\pi) - \frac{4\epsilon\gamma}{(1-\gamma)^2} D_{KL}^{\max}(\pi_i, \pi) \\
V^{\pi_{i+1}} &\geq L_{\pi_i}(\pi_{i+1}) - \frac{4\epsilon\gamma}{(1-\gamma)^2} D_{KL}^{\max}(\pi_i, \pi_{i+1}) = M_i(\pi_{i+1}) \\
V^{\pi_i} &= M_i(\pi_i) = L_{\pi_i}(\pi_i) \\
V^{\pi_{i+1}} - V^{\pi_i} &\geq M_i(\pi_{i+1}) - M_i(\pi_i)
\end{aligned}
$$

- So as long as the new policy π_{i+1} is equal or an improvement compared to the old policy π_i with respect to the lower bound, we are guaranteed to to monotonically improve!
- The above is a type of Minorization-Maximization (MM) algorithm

[1] $L_\pi(\tilde{\pi}) = V(\theta) + \sum_s \mu_\pi(s) \sum_a \tilde{\pi}(a|s) A_\pi(s,a)$

Guaranteed Improvement[1]

$$V^{\pi_{new}} \geq L_{\pi_{old}}(\pi_{new}) - \frac{4\epsilon\gamma}{(1-\gamma)^2} D_{KL}^{\max}(\pi_{old}, \pi_{new})$$

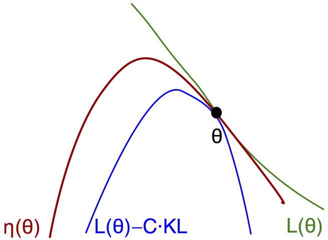

Figure: Source: John Schulman, Deep Reinforcement Learning, 2014

[1] $L_{\pi}(\tilde{\pi}) = V(\theta) + \sum_s \mu_{\pi}(s) \sum_a \tilde{\pi}(a|s) A_{\pi}(s,a)$

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Optimization of Parameterized Policies[1]

- Goal is to optimize

$$\max_{\theta_{new}} L_{\theta_{old}}(\theta_{new}) - \frac{4\epsilon\gamma}{(1-\gamma)^2} D_{KL}^{\max}(\theta_{old}, \theta_{new}) = L_{\theta_{old}}(\theta_{new}) - C D_{KL}^{\max}(\theta_{old}, \theta_{new})$$

- where C is the penalty coefficient
- In practice, if we used the penalty coefficient recommended by the theory above $C = \frac{4\epsilon\gamma}{(1-\gamma)^2}$, the step sizes would be very small
- New idea: Use a trust region constraint on step sizes. Do this by imposing a constraint on the KL divergence between the new and old policy.

$$\max_{\theta} L_{\theta_{old}}(\theta)$$
$$\text{subject to } D_{KL}^{s \sim \mu_{\theta_{old}}}(\theta_{old}, \theta) \leq \delta$$

- This uses the average KL instead of the max (the max requires the KL is bounded at all states and yields an impractical number of constraints)

[1] $L_{\pi}(\tilde{\pi}) = V(\theta) + \sum_s \mu_{\pi}(s) \sum_a \tilde{\pi}(a|s) A_{\pi}(s,a)$

TRPO Overview

- Policy gradient approach
- Uses surrogate optimization function
- Automatically constrains the weight update to a trusted region, to approximate where the first order approximation is valid
- Empirically consistently does well
- Very influential: $+4500$ citations since introduced in 2015

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Practical Algorithm: TRPO

Applied to

- Locomotion controllers in 2D

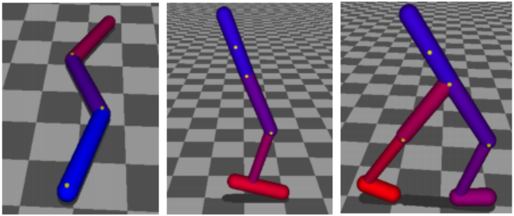
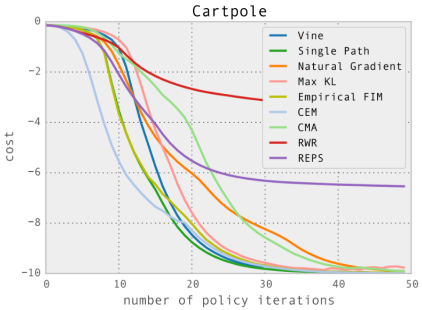


Figure: Trust Region Policy Optimization, Schulman et al, 2015

- Atari games with pixel input

TRPO Results



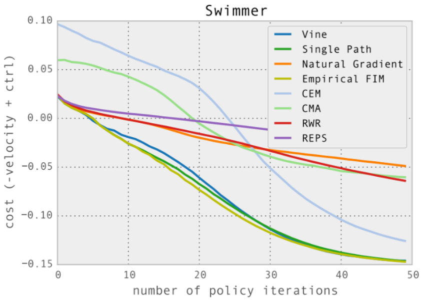


Figure: Trust Region Policy Optimization, Schulman et al, 2015

From Theory to Practice

- Prior objective:

$$\max_{\theta} L_{\theta_{old}}(\theta)$$

$$\text{subject to } D_{KL}^{s \sim \mu_{\theta_{old}}}(\theta_{old}, \theta) \leq \delta$$

where $L_{\pi}(\tilde{\pi}) = V(\theta) + \sum_s \mu_{\pi}(s) \sum_a \tilde{\pi}(a|s) A_{\pi}(s, a)$

- Don't know the visitation weights nor true advantage function
- In TRPO implementation do several substitutions

From Theory to Practice

- Prior objective:

$$\max_\theta L_{\theta_{old}}(\theta)$$
$$\text{subject to } D_{KL}^{s\sim\mu_{\theta_{old}}}(\theta_{old},\theta) \leq \delta$$

  where $L_\pi(\tilde{\pi}) = V(\theta) + \sum_s \mu_\pi(s) \sum_a \tilde{\pi}(a|s) A_\pi(s,a)$

- Don't know the visitation weights nor true advantage function
- Instead do the following substitutions:

$$\sum_s \mu_\pi(s) \rightarrow \frac{1}{1-\gamma}\mathbb{E}_{s\sim\mu_{\theta_{old}}}[\ldots],$$

From Theory to Practice

- Next substitution:

$$\sum_a \pi_\theta(a|s_n) A_{\theta_{old}}(s_n, a) \to \mathbb{E}_{a\sim q}\left[\frac{\pi_\theta(a|s_n)}{q(a|s_n)} A_{\theta_{old}}(s_n, a)\right]$$

- where q is some sampling distribution over the actions and s_n is a particular sampled state.
- This second substitution is to use importance sampling to estimate the desired sum, enabling the use of an alternate sampling distribution q (other than the new policy π_θ).
- Third substitution:

$$A_{\theta_{old}} \to Q_{\theta_{old}}$$

- Note that the above substitutions do not change solution to the above optimization problem

Selecting the Sampling Policy

- Optimize

$$\max_{\theta} \mathbb{E}_{s \sim \mu_{\theta_{old}}, a \sim q} \left[\frac{\pi_\theta(a|s)}{q(a|s)} Q_{\theta_{old}}(s,a) \right]$$

$$\text{subject to } \mathbb{E}_{s \sim \mu_{\theta_{old}}} D_{KL(\pi_{\theta_{old}}(\cdot|s), \pi_\theta(\cdot|s))} \leq \delta$$

- Standard approach: sampling distribution is $q(a|s)$ is simply $\pi_{old}(a|s)$
- For the vine procedure see the paper

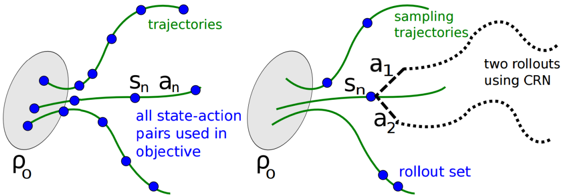


Figure: Trust Region Policy Optimization, Schulman et al, 2015

Searching for the Next Parameter

- Use a linear approximation to the objective function and a quadratic approximation to the constraint
- Constrained optimization problem
- Use conjugate gradient descent

Practical Algorithm: TRPO

1: **for** iteration$=1, 2, \ldots$ **do**
2: Run policy for T timesteps or N trajectories
3: Estimate advantage function at all timesteps
4: Compute policy gradient g
5: Use CG (with Hessian-vector products) to compute $F^{-1}g$ where F is the Fisher information matrix
6: Do line search on surrogate loss and KL constraint
7: **end for**

Common Template of Policy Gradient Algorithms

for iteration$=1, 2, \ldots$ **do**
    Run policy for T timesteps or N trajectories
    At each timestep in each trajectory, compute target $Q^{\pi}(s_t, a_t)$, and baseline $b(s_t)$
    Compute estimated policy gradient $\hat{g}$
    Update the policy using $\hat{g}$, potentially constrained to a local region
end for

Policy Gradient Summary

- Extremely popular and useful set of approaches
- Can incorporate prior knowledge by choosing the policy parameterization
- You should be very familiar with REINFORCE and the policy gradient template on the prior slide
- Understand where different estimators can be slotted in (and implications for bias/variance)
- Don't have to be able to derive or remember the specific formulas in TRPO for approximating the objectives and constraints
- Will have the opportunity to practice with these ideas in homework 3

Class Structure

- Last time: Policy Search
- This time: Policy Search
- **Next time: Exam**
- Next next time: Exploration

Practical Implementation with Auto differentiation

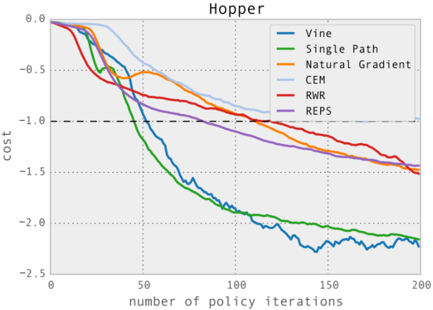
- Usual formula $\sum_t \nabla_\theta \log \pi(a_t|s_t;\theta)\hat{A}_t$ is inefficient–want to batch data
- Define "surrogate" function using data from current batch

$$L(\theta) = \sum_t \log \pi(a_t|s_t;\theta)\hat{A}_t$$

- Then policy gradient estimator $\hat{g} = \nabla_\theta L(\theta)$
- Can also include value function fit error

$$L(\theta) = \sum_t \left(\log \pi(a_t|s_t;\theta)\hat{A}_t - ||V(s_t) - \hat{G}_t||^2\right)$$

TRPO Results



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