Voting in metric spaces

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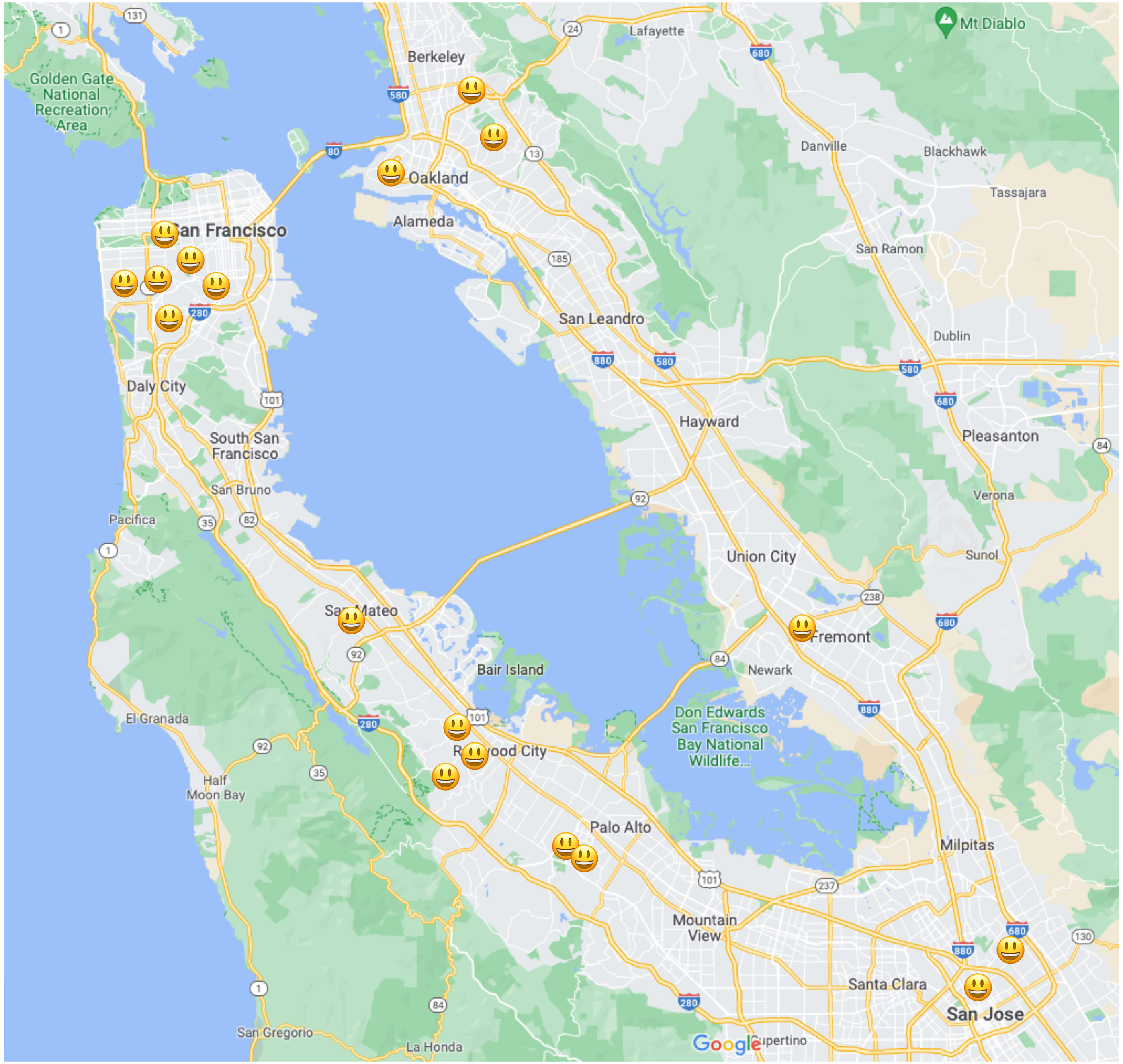
based on joint work with Moses Charikar

CS265

Problem set up

- You are a coffee company opening a new shop
- From candidate locations, choose one
- If you knew the customers' locations, pick candidate with min total distance
- Instead: ask each customer to rank candidates by distance
- How do you choose? Can randomness help?



Problem set up

- Suppose we have an election with n voters and m candidates
- Voters and candidates lie in a metric space
- Ideal: find candidate whose total distance to the voters is minimal

Social cost of candidate ***i***: $SC(i) = \sum_{v \in V} d(i, v)$

- Smaller cost => better candidate
- Catch: don’t see metric space; only voters’ ranking of candidates by distance
- Can no longer guarantee we pick the best candidate

Problem set up

- Modified goal: guarantee that for any metric, our candidate is only a small factor worse. i.e., minimize the approximation ratio (*distortion*)

$$\frac{SC(\hat{i})}{\min_{i \in C} SC(i)}$$

- Or for a randomized mechanism…

$$\frac{\mathbb{E}[SC(\hat{i})]}{\min_{i \in C} SC(i)}$$

- ***Question***: what is the smallest distortion we can guarantee?

Problem summary

- We have an election with n voters and m candidates, in a metric space
- Social Cost of a candidate: total distance to voters

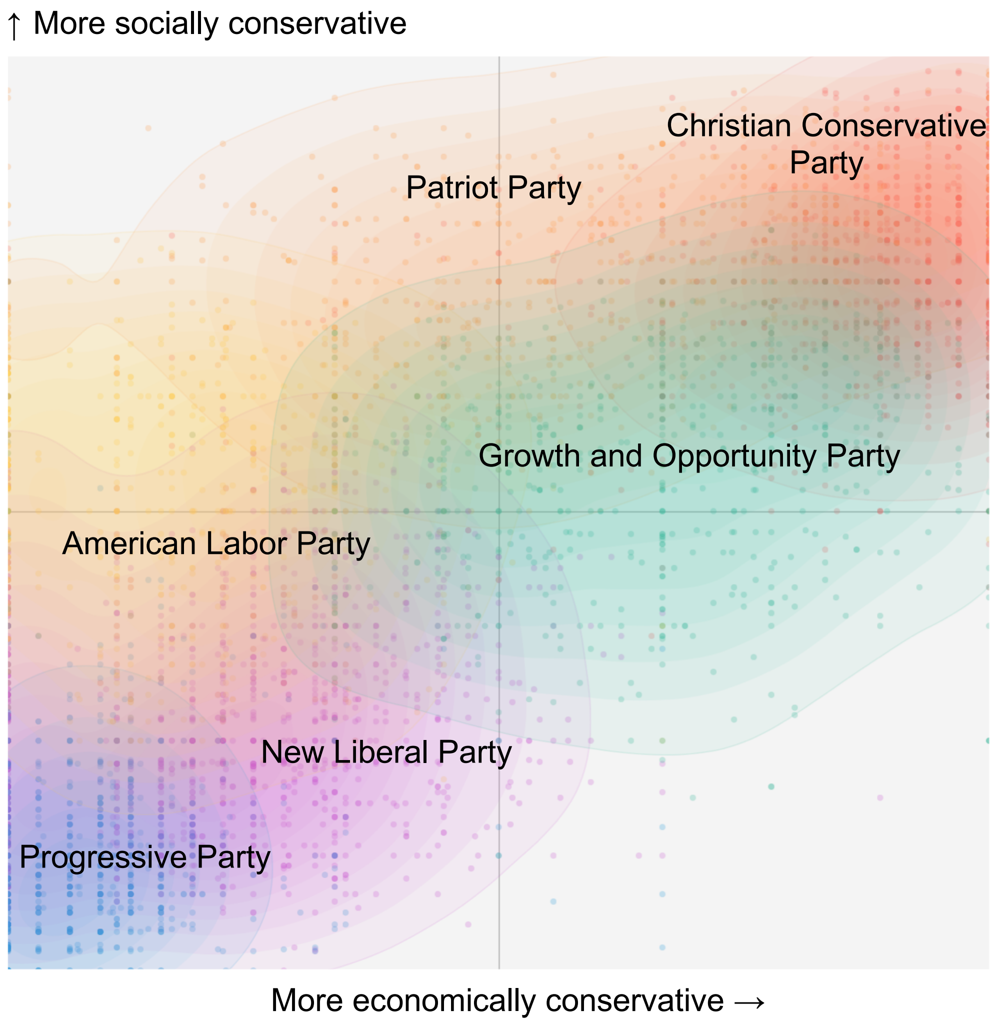
$$SC(i) = \sum_{v \in V} d(i, v)$$

- **Input:** each voter's ranking of candidates by distance
- **Goal:** mechanism that chooses a candidate with low distortion

$$\frac{\mathbb{E}[SC(\hat{i})]}{\min_{i \in C} SC(i)}$$

Another motivation: Democracy

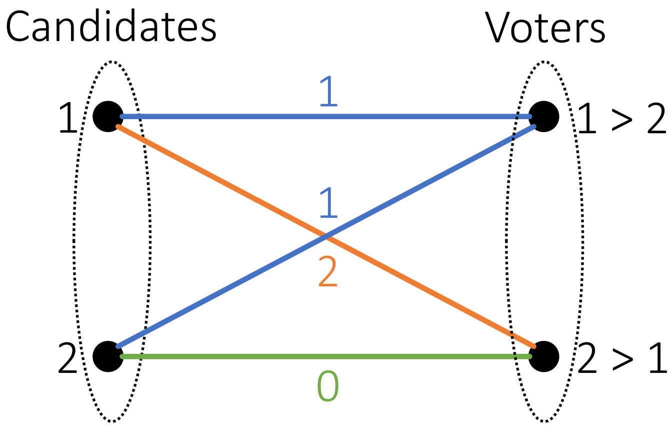
- Metric space corresponds to the *political spectrum*
- Hidden for conceptual, practical reasons



The New York Times

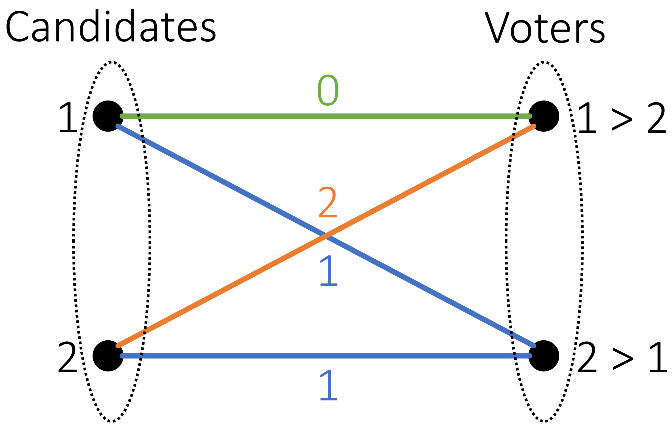
Easy lower bound

- Two candidates: 1, 2. Half the voters rank 1 > 2, half rank 2 > 1.



$SC(1) = 3$

$SC(2) = 1$

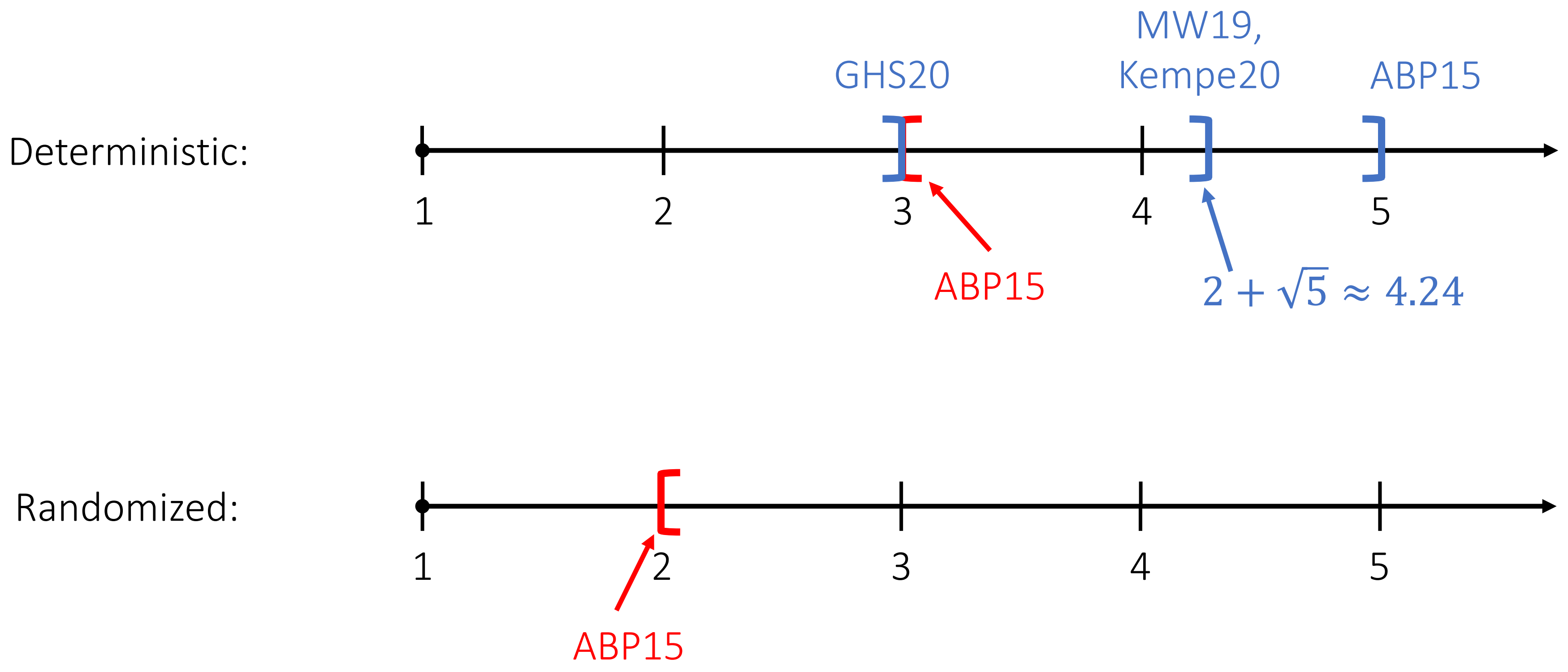


$SC(1) = 1$

$SC(2) = 3$

- Deterministic lower bound: 3
- Randomized lower bound: 2

Upper Bounds

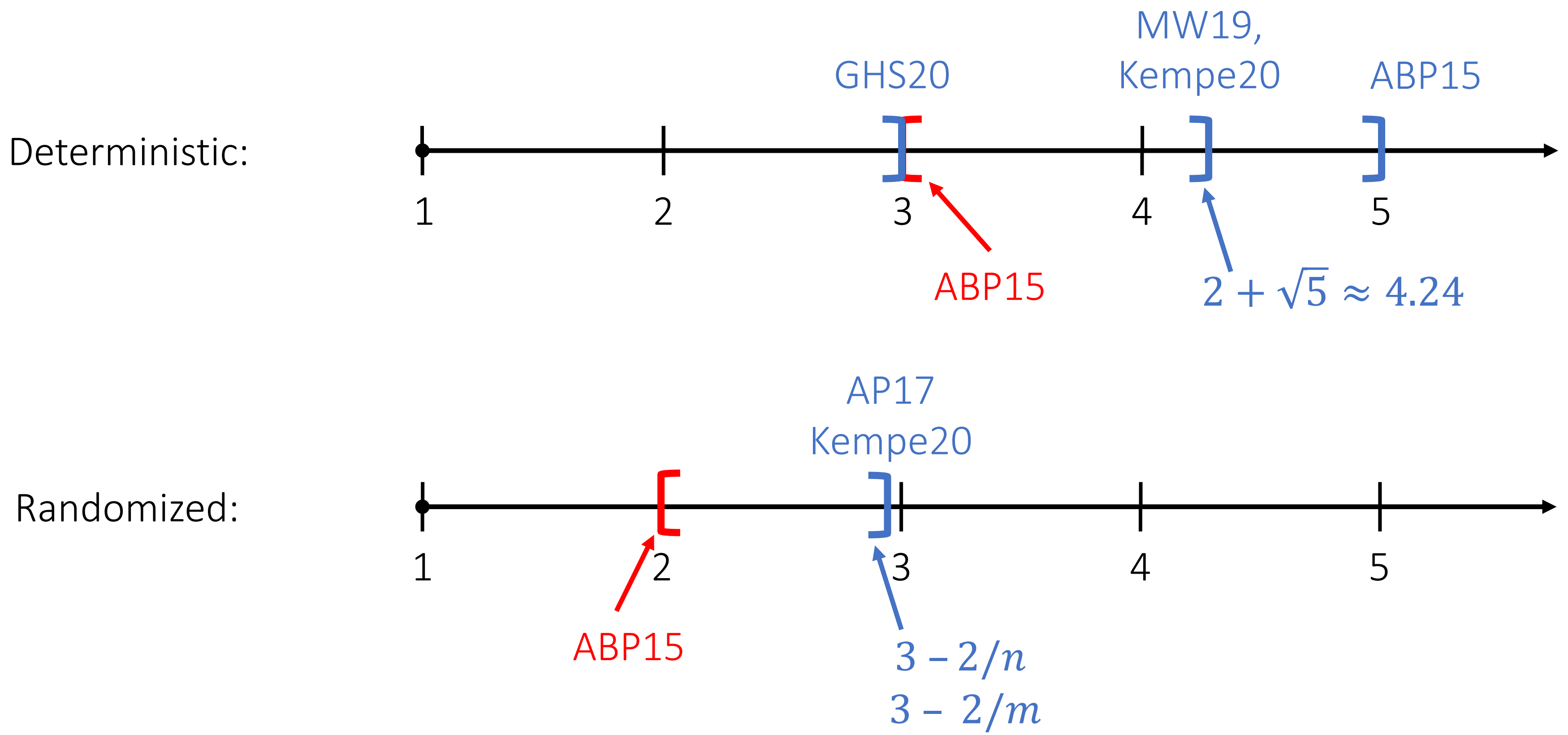


Aside: PluralityVeto (Kizilkaya-Kempe 22)

AKA: my preferred way to pick a board game ☺

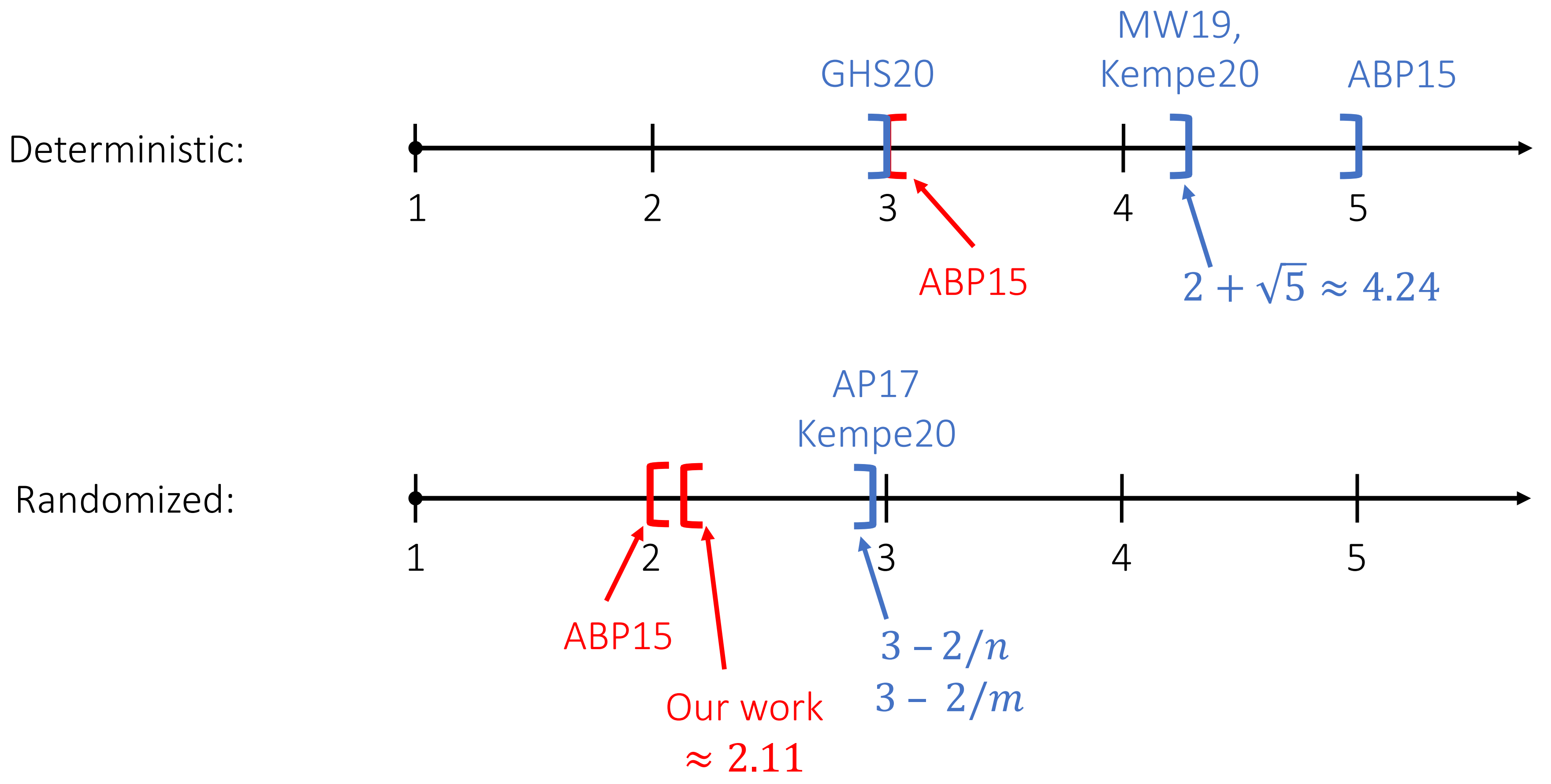
- Simple deterministic distortion 3 voting rule!
- Each candidate has a *score*, initially # of voters that rank them first
- In (any) order, each voter decrements the score (*veto*) of their least favorite candidate with positive score
- Once only one candidate has positive score, they win!
- (!!) Only requires $2 \log m$ bits of communication per voter (full ranking requires $\Omega(m \log m)$ bits)

Upper Bounds



n: # of voters
m: # of candidates

New lower bound



n: # of voters
m: # of candidates

Our results

- Stronger lower bounds for all $m \geq 3$

m	Lower Bound
3	2.026
4	2.049
5	2.063
∞	2.112

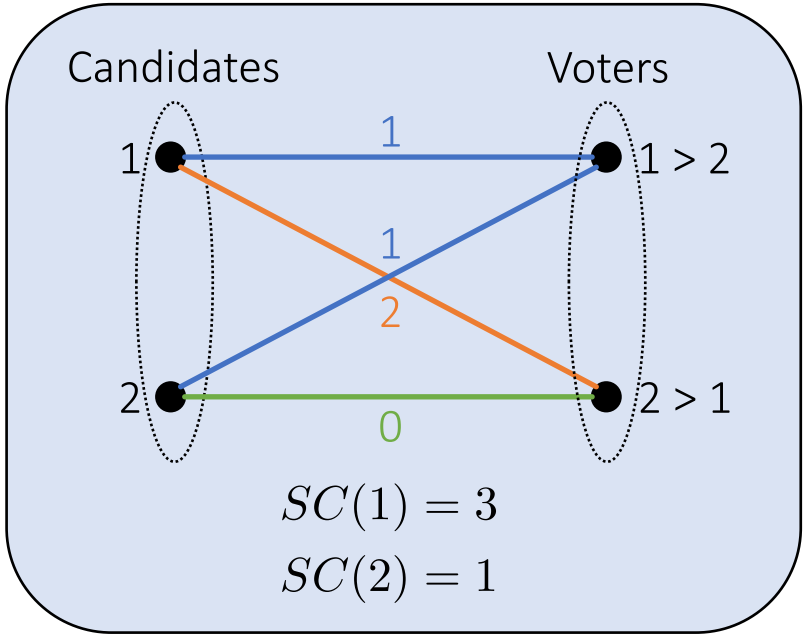
local max of

$$1 + \frac{8x(x^2 - 1)}{x^4 - 6x^3 - x^2 + 2x - 3}$$

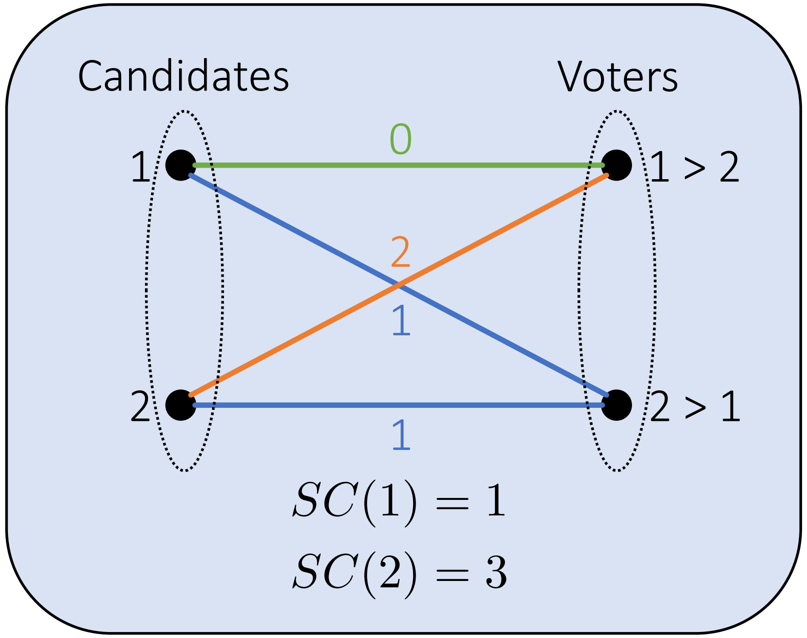
- ***Conjecture***: these bounds are optimal!
- One indication: matching upper bound when $m = 3$
- ***Pressing open problem***: get distortion $3 - \varepsilon$

Intuition for lower bound

- Metrics in simple lower bound:



If too little weight on 2, mechanism has high distortion on this metric

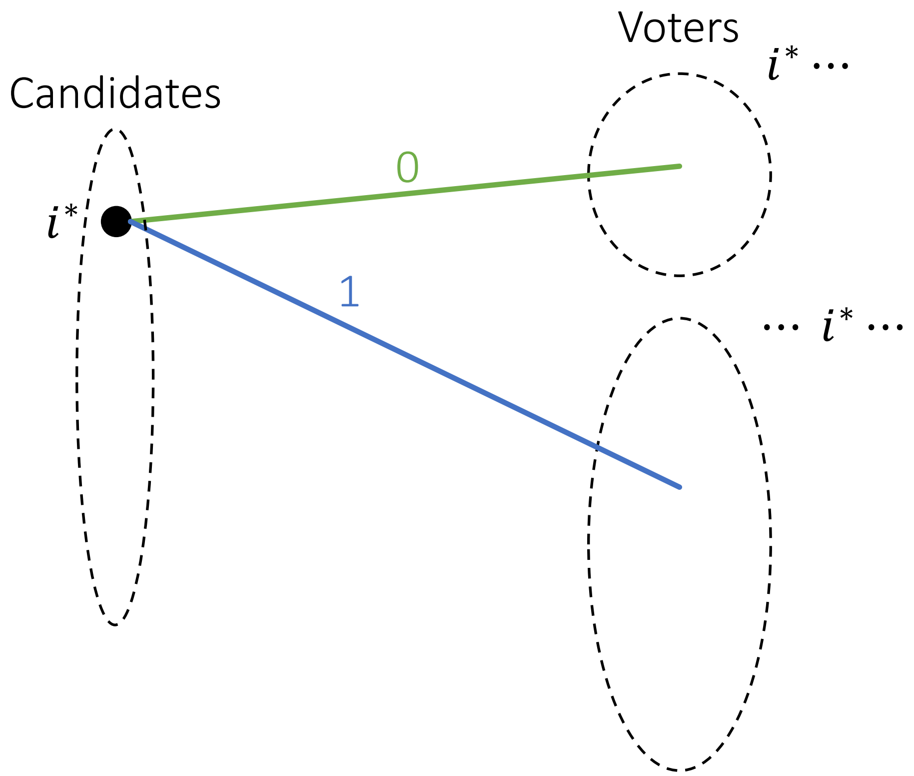


If too little weight on 1, mechanism has high distortion on this metric

- Idea: design metrics that force high weight for one candidate at a time

The (0,1,2,3)-metrics

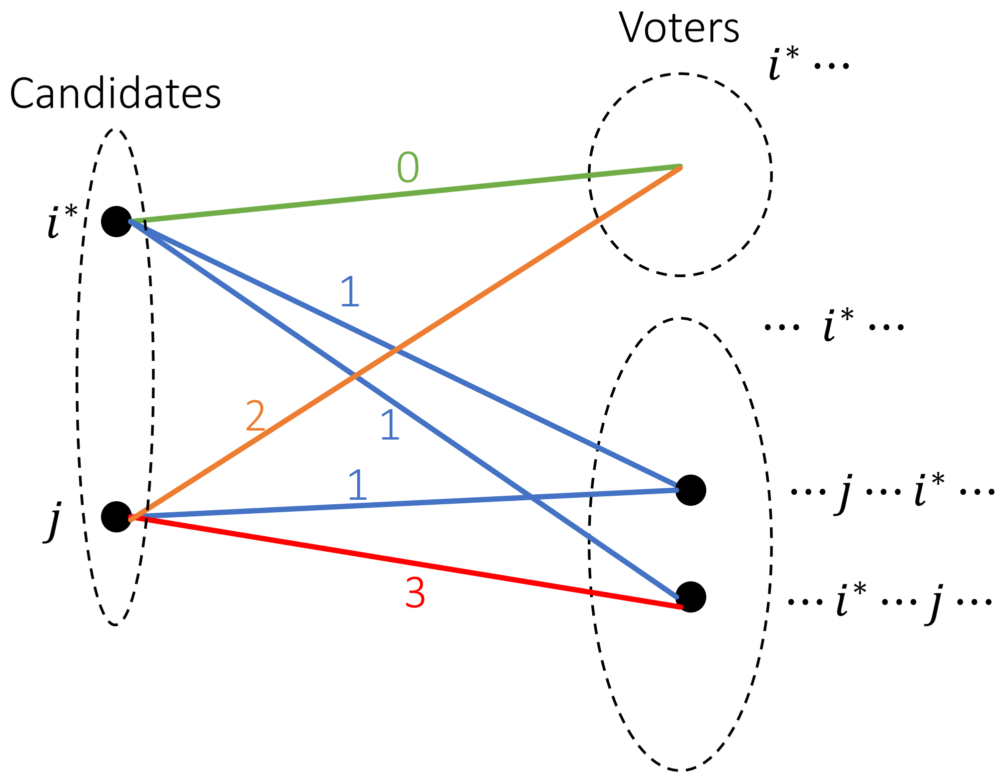
- One metric for each candidate i^*



Distance 0 to voters that rank i^* first, and 1 to other voters

The (0,1,2,3)-metrics

- One metric for each candidate i^*

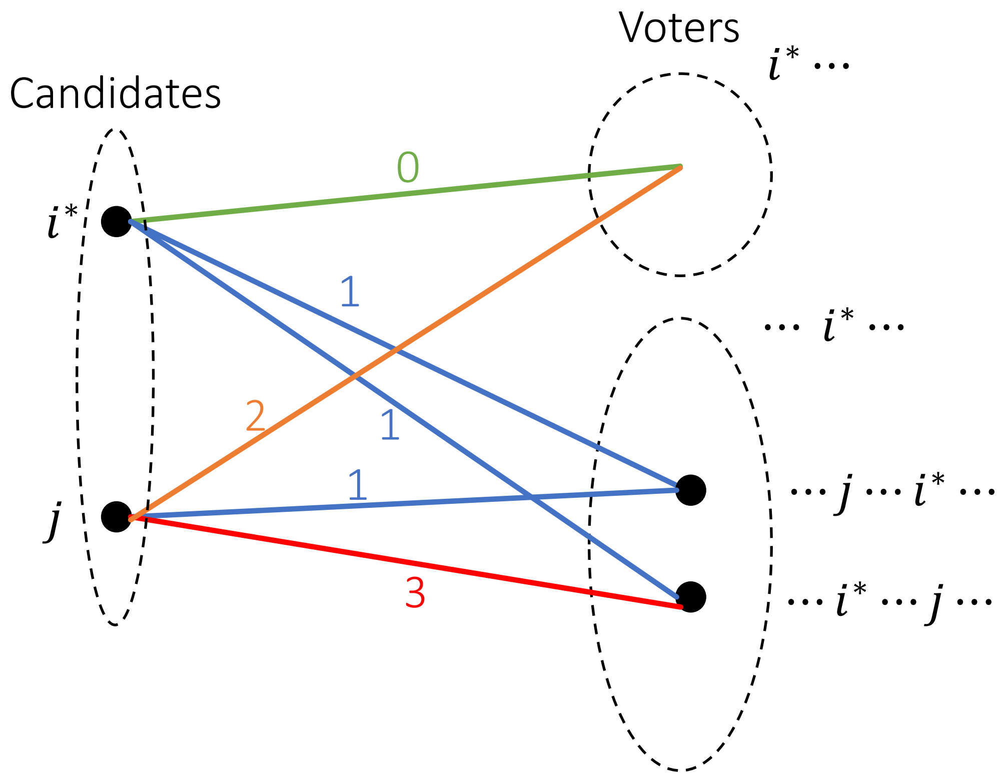


Make all other distances as large as possible, subject to

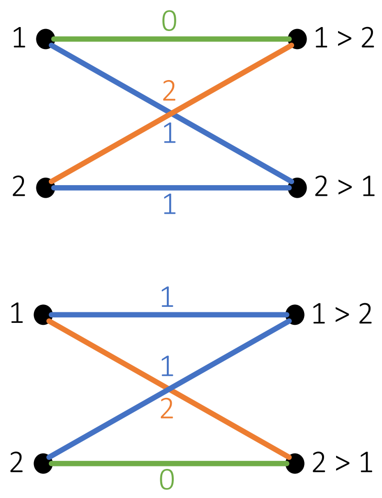
- Ranking constraints
- Triangle inequality

The (0,1,2,3)-metrics

- One metric for each candidate i^*



Note: these generalize previous lower bound



Discussion – one possible mechanism

- Proposed mechanism: imagine you only have to worry about (0,1,2,3)-metrics, and minimize distortion over those
- Mechanism only needs comparisons matrix and plurality vector
  - Comparisons matrix – results of pairwise elections
  - Plurality vector – proportion of first choices
- n^2 parameters instead of $n!$ – easier to sample!
- If optimal, surprising given some lower bounds:
  - GKM17 -- with only comparisons matrix, can't do better than distortion 3
  - GHS20 – with only plurality vector, can't do better than distortion $3 - 2/m$
- ***Conjecture***: with both, you can get optimal distortion

Thank you!