How to use a Practice Exam

CS265/CME309, Winter 2025

We have posted a practice exam, which is pretty close to an exam that was given for this class a few years ago. There is a reference sheet at the back of that exam; we will include the same reference sheet in our final exam. This practice exam is drawn from approximately the same distribution as our exam will be. Inspired by this, here is a warm-up question about how best to use this practice exam.

(Note: this question is *not* drawn from that same distribution :))

0. **(0 pt.)** Suppose that a class covers n topics. Each student i in the class has studied up on a set $S(i) \subseteq \{1, \ldots, n\}$ of topics. To create the exam, the instructors choose k topics to test, independently and uniformly at random (with replacement) among the n topics, and give $100/k$ points to each. Suppose that n and k are sufficiently large[1], and that $k = o(n)$.

   (a) What is the expected score of student i, in terms of $|S(i)|$? (Assuming that student i aces any problem on a topic they have studied).

   (b) Suppose that a student wants to use a practice exam to assess if they have studied enough. That is, the student studies first, and then scores s on the practice exam taken under the same conditions as the actual final. Bound the probability that they score worse than $s - 1$ (out of 100) on the actual final, asymptotically in terms of k and/or n.

   (c) Suppose on the other hand that a student wants to use a practice exam to help guide their study. That is, they student studies the topics that appear on the practice exam as they take it. Say the student scores s on the practice exam taken in this open-book way. What can you say about how well the student will score on the actual final?

   (d) How many topics should a student study up on to maximize their expected score on the final exam?

Solutions on next page!

[1] It seems reasonable to us that there would be upwards of a million topics on the exam...

SOLUTION:

(a) $100 \cdot |S(i)|/n$.

(b) Let $\mu = 100 \cdot |S(i)|/n$. Let s_p be the score on the practice exam; let s_r be the score on the real exam. Because the two exams are independent, s_p and s_r are identically distributed. For each of them, we can apply a Chernoff bound to conclude that

$$\Pr[s_p \geq \mu + .5] \leq \exp(-\Omega(k))$$

and

$$\Pr[s_r \leq \mu - .5] \leq \exp(-\Omega(k)).$$

If neither of these bad events happen (which by a union bound is with probability at least $1 - 2e^{-\Omega(k)}$, which is very close to 1), then

$$s_r > \mu - 0.5 > s_p - 1.$$

So the probability that $s_r \leq s_p - 1$ is at most $\exp(-\Omega(k))$. So if the student uses the exam like this, they have a very good probability of decently assessing whether or not they are prepared for the real final exam. (Assuming that k is sufficiently large...)

(c) You can't say much, since the topics on the practice exam and the final are independent.

(d) All of them!