Standard Probabilistic IR

Information need

query

\[ P(R | Q, d) \]

matching

d1

d2

\ldots

dn

document collection
One night in a hotel, I saw this late night talk show where Sergey Brin popped on suggesting the web search tip that you should think of some words that would likely appear on pages that would answer your question and use those as your search terms – let’s exploit that idea!
Formal Language (Model)

- Traditional generative model: generates strings
  - Finite state machines or regular grammars, etc.
- Example:

  I wish
  I wish I wish
  I wish I wish I wish I wish
  I wish I wish I wish I wish I wish I wish
  *
  *wish I wish
Stochastic Language Models

- Models *probability* of generating strings in the language (commonly all strings over $\Sigma$)

Model $M$

<table>
<thead>
<tr>
<th>Probability</th>
<th>Word</th>
<th>Probability</th>
<th>Word</th>
<th>Probability</th>
<th>Word</th>
<th>Probability</th>
<th>Word</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>the</td>
<td>0.2</td>
<td>man</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>likes</td>
</tr>
<tr>
<td>0.1</td>
<td>a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>0.01</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>woman</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>said</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>likes</td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

$\text{multiply}$

$$P(s \mid M) = 0.00000008$$
Stochastic Language Models

- Model probability of generating any string

<table>
<thead>
<tr>
<th>Model M1</th>
<th>Model M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
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<td>0.0001</td>
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<td>0.02</td>
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<td>0.01</td>
</tr>
<tr>
<td>0.0005 maiden</td>
<td>0.0001 maiden</td>
</tr>
<tr>
<td>0.01 woman</td>
<td>0.0001 woman</td>
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</table>

\[
P(s|M_2) > P(s|M_1)
\]
Stochastic Language Models

- A statistical model for generating text
  - Probability distribution over strings in a given language

\[ P(\bullet, \bullet, \bullet, \bullet | M) = P(\bullet | M) \]

\[ P(\bullet | M, \bullet) \]
\[ P(\bullet | M, \bullet) \]
\[ P(\bullet | M, \bullet, \bullet) \]
\[ P(\bullet | M, \bullet, \bullet, \bullet) \]
Unigram and higher-order models

\[ P ( \bullet \bullet \bullet \bullet ) = P ( \bullet ) P ( \bullet \bullet ) P ( \bullet \bullet \bullet ) P ( \bullet \bullet \bullet \bullet ) \]

- **Unigram Language Models**
  \[ P ( \bullet ) P ( \bullet ) P ( \bullet ) P ( \bullet ) \]

- **Bigram (generally, n-gram) Language Models**
  \[ P ( \bullet ) P ( \bullet \bullet ) P ( \bullet \bullet ) P ( \bullet \bullet \bullet ) \]

- **Other Language Models**
  - Grammar-based models (PCFGs), etc.
    - Probably not the first thing to try in IR

"Easy. Effective!"
Using Language Models in IR

- Treat each document as the basis for a model (e.g., unigram sufficient statistics)
- Rank document \( d \) based on \( P(d \mid q) \)
- \( P(d \mid q) = \frac{P(q \mid d) \times P(d)}{P(q)} \)
  - \( P(q) \) is the same for all documents, so ignore
  - \( P(d) \) (the prior) is often treated as the same for all \( d \)
    - But we could use criteria like authority, length, genre
  - \( P(q \mid d) \) is the probability of \( q \) given \( d \)'s model
- Very general formal approach
The fundamental problem of LMs

- Usually we don’t know the model $M$
  - But have a sample of text representative of that model

$$P\left(\ldots | M(\ldots)\right)$$

- Estimate a language model from a sample
- Then compute the observation probability
Language Models for IR

Language Modeling Approaches

- Attempt to model query generation process
- Documents are ranked by the probability that a query would be observed as a random sample from the respective document model
  - Multivariate approach
    \[
    P(Q|M_D) = \prod_{w \in Q} P(w|M_D) \prod_{w \notin Q} (1 - P(w|M_D))
    \]
  - Multinomial approach
    \[
    P(Q|M_D) = \prod_w P(w|M_D)^{q_w}
    \]
Retrieval based on probabilistic LM

- Treat the generation of queries as a random process.

**Approach**
- Infer a language model for each document.
- Estimate the probability of generating the query according to each of these models.
- Rank the documents according to these probabilities.
- Usually a unigram estimate of words is used
  - Some work on bigrams, paralleling van Rijsbergen
Retrieval based on probabilistic LM

- Intuition
  - Users ...
    - Have a reasonable idea of terms that are likely to occur in documents of interest.
    - They will choose query terms that distinguish these documents from others in the collection.

- Collection statistics ...
  - Are integral parts of the language model.
  - Are not used heuristically as in many other approaches.
    - In theory. In practice, there’s usually some wiggle room for empirically set parameters.
Query generation probability (1)

- Ranking formula
  \[ p(Q, d) = p(d) p(Q | d) \]
  \[ \approx p(d) p(Q | M_d) \]
  The probability of producing the query given the language model of document \( d \) using MLE is:
  \[ \hat{p}(Q | M_d) = \prod_{t \in Q} \hat{p}_{ml}(t | M_d) \]

  \[ = \prod_{t \in Q} \frac{tf_{(t,d)}}{dl_d} \]

  Unigram assumption:
  Given a particular language model, the query terms occur independently

\( M_d \): language model of document \( d \)
\( tf_{(t,d)} \): raw tf of term \( t \) in document \( d \)
\( dl_d \): total number of tokens in document \( d \)
Insufficient data

- **Zero probability**  
  \( p(t \mid M_d) = 0 \)
  
  May not wish to assign a probability of zero to a document that is missing one or more of the query terms (gives conjunction semantics)

- **General approach**
  
  A non-occurring term is possible, but no more likely than would be expected by chance in the collection.
  
  If \( tf_{(t,d)} = 0 \)  
  \( p(t \mid M_d) = \frac{cf_t}{cs} \)

  \( cf_t \) : raw count of term \( t \) in the collection

  \( cs \) : raw collection size (total number of tokens in the collection)
Insufficient data

- There’s a wide space of approaches to smoothing probability distributions to deal with this problem, such as adding 1, \( \frac{1}{2} \) or \( \varepsilon \) to counts, Dirichlet priors, discounting, and interpolation
  - (See FSNLP ch. 6 or CS224N if you want more)
- A simple idea that works well in practice is to use a mixture between the document multinomial and the collection multinomial distribution
Mixture model

- \( P(w) = \lambda P_{\text{mle}}(w|M_d) + (1 - \lambda) P_{\text{mle}}(w|M_c) \)
- Mixes the probability from the document with the general collection frequency of the word.
- Correctly setting \( \lambda \) is very important
- A high value of lambda makes the search “conjunctive-like” – suitable for short queries
- A low value is more suitable for long queries
- Can tune \( \lambda \) to optimize performance
  - Perhaps make it dependent on document size (cf. Dirichlet prior or Witten-Bell smoothing)
Basic mixture model summary

- General formulation of the LM for IR
  \[
p(Q, d) = p(d) \prod_{t \in Q} ((1 - \lambda) p(t) + \lambda p(t \mid M_d))
  \]

- The user has a document in mind, and generates the query from this document.
- The equation represents the probability that the document that the user had in mind was in fact this one.
Example

- Document collection (2 documents)
  - \(d_1\): Xerox reports a profit but revenue is down
  - \(d_2\): Lucent narrows quarter loss but revenue decreases further

- Model: MLE unigram from documents; \(\lambda = \frac{1}{2}\)

- Query: revenue down
  - \(P(Q|d_1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \times \frac{3}{32} = \frac{3}{256}\)
  - \(P(Q|d_2) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \times \frac{1}{32} = \frac{1}{256}\)

- Ranking: \(d_1 > d_2\)
Ponte and Croft Experiments

- **Data**
  - TREC topics 202-250 on TREC disks 2 and 3
    - Natural language queries consisting of one sentence each
  - TREC topics 51-100 on TREC disk 3 using the concept fields
  - Lists of good terms

```
<num>Number: 054
<dom>Domain: International Economics
<title>Topic: Satellite Launch Contracts
<desc>Description:
... </desc>

<con>Concept(s):
1. Contract, agreement
2. Launch vehicle, rocket, payload, satellite
3. Launch services, ... </con>
```
Precision/recall results 202-250

<table>
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<tr>
<th></th>
<th>tf.idf</th>
<th>LM</th>
<th>%chg</th>
<th>I/D</th>
<th>Sign</th>
<th>Wilc.</th>
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<td>6501</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rret:</td>
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<td>3364</td>
<td>5.09</td>
<td>36/43</td>
<td>0.0000*</td>
<td>0.0002*</td>
</tr>
<tr>
<td>Prec.</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>0.7590</td>
<td>+2.0</td>
<td>10/22</td>
<td>0.7383</td>
<td>0.5709</td>
</tr>
<tr>
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<td>0.4521</td>
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<td>24/42</td>
<td>0.2204</td>
<td>0.0761</td>
</tr>
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<td>27/44</td>
<td>0.0871</td>
<td>0.0051*</td>
</tr>
<tr>
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<td>28/43</td>
<td>0.0330*</td>
<td>0.0054*</td>
</tr>
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<td>0.0541</td>
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<td>24/35</td>
<td>0.0205*</td>
<td>0.0018*</td>
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<td>0.0008*</td>
<td>0.0027*</td>
</tr>
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<td>0.0222*</td>
<td>0.0003*</td>
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<table>
<thead>
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<td>0.0001*</td>
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<td>+16.32</td>
<td>34/43</td>
<td>0.0001*</td>
<td>0.0000*</td>
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</table>


Precision/recall results 51-100

<table>
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<tr>
<th></th>
<th>tf.idf</th>
<th>LM</th>
<th>%chg</th>
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<table>
<thead>
<tr>
<th>Prec.</th>
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<td>0.0382*</td>
<td>0.0076*</td>
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<td>0.0011*</td>
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<td>0.0005*</td>
<td>0.0003*</td>
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<td>0.0052*</td>
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</table>
LM vs. Prob. Model for IR

- The main difference is whether “Relevance” figures explicitly in the model or not.

- Problems in LM approach
  - Relevance feedback is difficult to integrate into the LM approach.
  - As are user preferences, and other general issues of relevance.
  - Can’t easily accommodate phrases, passages, Boolean operators.

- Current extensions focus on putting relevance back into the model, etc.
Extensions: 3-level model

- 3-level model
  1. Whole collection model (MC)
  2. Specific-topic model; relevant-documents model (MT)
  3. Individual-document model (M_d)

- Relevance hypothesis
  - A request(query; topic) is generated from a specific-topic model {MC, MT}
  - Iff a document is relevant to the topic, the same model will apply to the document.
    - It will replace part of the individual-document model in explaining the document.
  - The probability of relevance of a document
    - The probability that this model explains part of the document
    - The probability that the {MC, MT, MD} combination is better than the {MC, M_d} combination

\[
\text{CM}_{CM} \quad \text{TM}_{CM} \quad \text{dM}_{CM}
\]
3-level model

\[
P(Q | M_C, M_T)
\]

\[
P(Q | M_C, M_T, M_d)
\]
Alternative Models of Text Generation

Searcher \[ P(M \mid Searcher) \]

Query Model \[ P(Query \mid M) \]

Is this the same model?

Writer \[ P(M \mid Writer) \]

Doc Model \[ P(Doc \mid M) \]
Retrieval Using Language Models

![Diagram](attachment:image.png)

Retrieval: Query likelihood (1), Document likelihood (2), Model comparison (3)
Query Likelihood

- $P(Q | D_m)$
- Major issue is estimating document model
  - i.e. smoothing techniques instead of tf.idf weights
- Good retrieval results
  - e.g. UMass, BBN, Twente, CMU
- Problems dealing with relevance feedback, query expansion, structured queries
Document Likelihood

- Rank by likelihood ratio $P(D|R)/P(D|NR)$
  - treat as a generation problem
  - $P(w|R)$ is estimated by $P(w|Q_m)$
  - $Q_m$ is the query or relevance model
  - $P(w|NR)$ is estimated by collection probabilities $P(w)$

- Issue is estimation of query model
  - Treat query as generated by mixture of topic and background
  - Estimate relevance model from related documents (query expansion)
  - Relevance feedback is easily incorporated

- Good retrieval results
  - e.g. UMass at SIGIR 01
  - inconsistent with heterogeneous document collections
Model Comparison

- Estimate query and document models and compare
- Obvious measure is KL divergence $D(Q_m \| D_m)$
  - equivalent to query-likelihood approach if simple empirical distribution used for query model
- More general risk minimization framework has been proposed
  - Zhai and Lafferty 2001
- Better results than query-likelihood or document-likelihood approaches
Other Approaches

- HMMs (BBN) – really just a renaming of the mixture model present earlier
- Probabilistic Latent Semantic Indexing (Hofmann)
  - assume documents are generated by a mixture of “aspect” models
  - estimation more difficult
  - probabilistic foundations questionable
- Translation model (Berger and Lafferty)
  - Lets you generate query words not in document via “translation” to synonyms etc.
Language models: pro & con

- Novel way of looking at the problem of text retrieval based on probabilistic language modeling
  - Conceptually simple and explanatory
  - Formal mathematical model
  - Natural use of collection statistics, not heuristics (almost...)
- LMs provide effective retrieval and can be improved to the extent that the following conditions can be met
  - Our language models are accurate representations of the data.
  - Users have some sense of term distribution.*
    - *Or we get more sophisticated with translation model
Comparison With Vector Space

- There's some relation to traditional tf.idf models:
  - (unscaled) term frequency is directly in model
  - the probabilities do length normalization of term frequencies
  - the effect of doing a mixture with overall collection frequencies is a little like idf: terms rare in the general collection but common in some documents will have a greater influence on the ranking
Comparison With Vector Space

- Similar in some ways
  - Term weights based on frequency
  - Terms often used as if they were independent
  - Inverse document/collection frequency used
  - Some form of length normalization useful

- Different in others
  - Based on probability rather than similarity
    - Intuitions are probabilistic rather than geometric
  - Details of use of document length and term, document, and collection frequency differ
Resources


(Several relevant newer papers at *SIGIR 23–25*, 2000–2002.)
