STANFORD UNIVERSITY
Department of Electrical Engineering

FINAL EXAMINATION
Fall Quarter, 2001

EE214

10 December 2001

CLOSED BOOK; Two std. 8.5” x 11” sheets of notes permitted

CAUTION: Useful information may be included at the end of a problem statement, so be sure to read each problem COMPLETELY before beginning to work on it. **We will be incredibly unforgiving about failures to read the problems thoroughly.**

Count the number of pages you have, and make sure it equals the page count shown at the bottom, just in case the copy machine decided to omit a page here or there to trip you up.

All problem questions carry roughly equal weight.

NOTES: Be sure to summarize your answers in the spaces provided to help us determine correct answers quickly during grading. However, turn in and show all work, as partial credit will be given. **You may make reasonable approximations, but be sure to state all assumptions. If you are unsure about “reasonableness,” ask.**

Put your name on each booklet and loose page you intend to turn in (now might be a good time).

You may read the following after the exam to save time:

**Regrade policy:** We photocopy a percentage of final exams on a random sampling basis to give the teaching staff a chance to spot systematic grading errors even after we’ve handed back exam booklets. Despite our best efforts, though, some mistakes in grading occasionally evade detection by our extensive quality control procedures. If you feel that you have been the victim of a (hopefully rare) grading error, submit a written regrade request, along with your entire exam, by January 5, 2002. The entire exam will be studied for regrading.

We **hope** to have graded final exams and final course grades available by noon on Wednesday, December 12, at the office of EE214 administrative associate Ann Guerra (CIS-207). **PLEASE DO NOT CALL.** We will not release exam and final course grades over the phone. To get your exam, you must show up in person and present your student ID. SITN students will receive their exams and final grades through regular SITN channels. Final exam solutions will not be distributed, but may be viewed when picking up your exam.

Happy holidays!
PROBLEM 1: Output impedance, current matching, and compliance are among the more important characteristics of current mirrors. Compliance here refers to the range of output voltages over which a current source maintains a high output resistance (e.g., is in saturation).

a) Consider first a simple textbook mirror:

FIGURE 1. Simple current mirror

Provide expressions for the current ratio, small-signal output resistance and output voltage compliance. Neglect back-gate bias, but not channel-length modulation. Assume for this and all subsequent parts of the problem that we characterize the latter with $\lambda$ for large signals, and with $r_0$ for small signals. The transistors have a threshold voltage of $V_T$.

Ans: The current ratio $I_{out}/I_{in}$ in saturation is ____________________________.

Ans: The small signal output resistance is $r_0 = __________$ (give the simplest answer, involving only the output current and $\lambda$).

Ans: The current source drops out of saturation when $V_{OUT} = __________$.

b) The inability of any one mirror circuit to satisfy all requirements in all cases has led to a proliferation of mirror topologies. We’ve seen, for example, ordinary cascodes, super cascodes, and low headroom cascodes. We consider here an alternative mirror, originally developed by Wilson in bipolar form. For this circuit (see the figure on the next page), provide expressions for the nominal current ratio, small-signal output resistance, and identify the lower compliance voltage. Again, consider only channel-length modulation. In particular, neglect body effect (back-gate effects). (And be sure to note that the input and output ports have reversed position relative to the previous figure.)
Ans: The current ratio $I_{out}/I_{in}$ in saturation is ________________________________.

Ans: The small-signal output resistance is ________________________________.

(You may assume that the input current is supplied by a perfect current source. You may also simplify your derivation, without incurring much error, by neglecting the $r_0$ of the diode-connected MOSFET only.)

Ans: The output voltage compliance has a lower limit given by ________________.

c) You should have found in part b) that the Wilson mirror has a current ratio that is not quite unity. Modify the circuit with the addition of at most one transistor to eliminate this systematic mismatch.

Ans: My circuit is:
**PROBLEM 2**: The method of open-circuit time constants is extremely powerful, but it is an approximate method, and therefore limited. This problem explores one of these limitations.

Consider the following cascade of $n$ identical amplifiers (biasing details omitted for simplicity; also assume that the $n$th stage is loaded in a capacitance equal to that of each of the other stages):

**FIGURE 3. Cascaded MOS amplifiers**

In all that follows, neglect all resistive parasitics, and neglect all capacitances except $c_{gs}$.

a) Derive an expression for the overall small-signal transfer function:

**Ans**: $H(s) =$ __________________________________________________________________________.

b) Using open-circuit time constants, provide an estimate of the –3dB bandwidth:

**Ans**: $BW_{oct} =$ ___________________________________________________________________.

c) Your expression for a) is simple enough to permit a formal derivation of a formula for the actual bandwidth. Please derive it.

**Ans**: $BW_{real} =$ ___________________________________________________________________.
d) To facilitate a comparison with open-circuit time constants, simplify your expression from part c), assuming large (but not infinite) $n$. In your simplification, you may (or may not) find useful some the following mathematical facts:

\[
\frac{b}{a} = e^{\ln(a^k)}
\]  

(EQ 1)

and

\[
e^x = 1 + x + \frac{x^2}{2!} + \ldots
\]  

(EQ 2)

Ans: $BW_{\text{large}_n} =$

\[
\text{-----------------------------------------------}
\]

e) Comment as quantitatively as possible on the error of the open-circuit time constant estimate, relative to the estimate based on your answer to d), in the limit as $n$ grows very large.
PROBLEM 3: Ok, you’ve just gotten your Stanford engineering degree, so what’s next? This being the Valley, you decide to join a startup, of course! Sadly, though, the dot-com implosion has temporarily limited your options. On your first day at SubOptimal™ Products, you are given ownership of the following circuit, intended to be an oscillator:

FIGURE 4. SubOptimal™ oscillator/amplifier

a) As with all ideal textbook op-amps, assume that the op-amps here have infinite bandwidth, infinite impedance, and zero output impedance. What is the overall loop transmission of this circuit if the “magic network” is initially a wire? Be sure to keep track of signs!

\[ L(s) = \frac{Vin}{Vout} = \frac{1}{s^2RC} \]

b) Can this circuit oscillate with a wire as the magic network? If so, provide an expression for the oscillation frequency. If not, provide an argument for why not, in terms of gain or phase margin concepts.

\textbf{Ans:}
c) Suppose now that you are given two choices for the “magic network.” One of these is a simple RC high-pass filter, and the other is a simple RC low-pass filter:

**FIGURE 5. Possible magic networks**

Provide expressions for the overall loop transmissions when each of these two choices is used in the oscillator circuit:

**Ans:** $L_{\text{high}}(s) =$

**Ans:** $L_{\text{low}}(s) =$

**d) Assume that you choose to use the low-pass filter as the magic network. Does the circuit oscillate? If so, provide an expression for the oscillation frequency. If not, provide an argument for why not, again in terms of gain or phase margin concepts.**

**Ans:**

Reminder: Make sure that you have put your name on all work to be turned in, and that you have all the pages of this exam!