EE263 homework 2 additional exercises

1. *Temperatures in a multi-core processor.* We are concerned with the temperature of a processor at two critical locations. These temperatures, denoted $T = (T_1, T_2)$ (in degrees C), are affine functions of the power dissipated by three processor cores, denoted $P = (P_1, P_2, P_3)$ (in W). We make 4 measurements. In the first, all cores are idling, and dissipate 10W. In the next three measurements, one of the processors is set to full power, 100W, and the other two are idling. In each experiment we measure and note the temperatures at the two critical locations.

P_1	P_2	P_3	T_1	T_2
10W	10W	10W	27°	29°
100W	10W	10W	45°	37°
10W	100W	10W	41°	49°
10W	10W	100W	35°	55°

Suppose we operate all cores at the same power, p. How large can we make p, without T_1 or T_2 exceeding 70°?

You must fully explain your reasoning and method, in addition to providing the numerical solution.

2. *Relative deviation between vectors.* Suppose a and b are nonzero vectors of the same size. The relative deviation of b from a is defined as the distance between a and b, divided by the norm of a,

$$\eta_{ab} = \frac{\|a - b\|}{\|a\|}.$$

This is often expressed as a percentage. The relative deviation is not a symmetric function of a and b; in general, $\eta_{ab} \neq \eta_{ba}$.

Suppose $\eta_{ab} = 0.1$ (*i.e.*, 10%). How big and how small can be η_{ba} be? How big and how small can $\angle(a, b)$ be? Explain your reasoning. For bounding $\angle(a, b)$, you can just draw some pictures; you don't have to give a formal argument.

3. *Single sensor failure detection and identification.* We have $y = Ax$, where $A \in \mathbf{R}^{m \times n}$ is known, and $x \in \mathbf{R}^n$ is to be found. Unfortunately, up to one sensor may have failed (but you don't know which one has failed, or even whether any has failed). You are given $\tilde{y}$ and not y, where $\tilde{y}$ is the same as y in all entries except, possibly, one (say, the kth entry). If all sensors are operating correctly, we have $y = \tilde{y}$. If the kth sensor fails, we have $\tilde{y}_i = y_i$ for all $i \neq k$.

The file `one_bad_sensor.m`, available on the course web site, defines A and $\tilde{y}$ (as `A` and `ytilde`). Determine which sensor has failed (or if no sensors have failed). You must explain your method, and submit your code.

For this exercise, you can use the Matlab code `rank([F g])==rank(F)` to check if $g \in \mathcal{R}(F)$. (We will see later a much better way to check if $g \in \mathcal{R}(F)$.)