EE263 homework 7 additional exercise

1. *Spectral mapping theorem.* Suppose $f : \mathbf{R} \to \mathbf{R}$ is analytic, *i.e.*, given by a power series expansion

$$f(u) = a_0 + a_1 u + a_2 u^2 + \cdots$$

(where $a_i = f^{(i)}(0)/(i!)$). (You can assume that we only consider values of u for which this series converges.) For $A \in \mathbf{R}^{n\times n}$, we define $f(A)$ as

$$f(A) = a_0 I + a_1 A + a_2 A^2 + \cdots$$

(again, we'll just assume that this converges).

Suppose that $Av = \lambda v$, where $v \neq 0$, and $\lambda \in \mathbf{C}$. Show that $f(A)v = f(\lambda)v$ (ignoring the issue of convergence of series). We conclude that if λ is an eigenvalue of A, then $f(\lambda)$ is an eigenvalue of $f(A)$. This is called the *spectral mapping theorem.*

To illustrate this with an example, generate a random 3×3 matrix, for example using `A=randn(3)`. Find the eigenvalues of $(I + A)(I - A)^{-1}$ by first computing this matrix, then finding its eigenvalues, and also by using the spectral mapping theorem. (You should get very close agreement; any difference is due to numerical round-off errors in the various compuations.)