Alex Lemon
EE263 (Introduction to Linear Dynamical Systems)
Problem of the Day 7
July 15, 2014

1 Estimation with known input norm

Consider a measurement model

$$y = Ax + v,$$

where the known matrix $A \in \mathbb{R}^{m \times n}$ is skinny, and full rank, $x \in \mathbb{R}^n$ is an unknown vector that we want to estimate, $v \in \mathbb{R}^m$ is an unknown noise vector, and $y \in \mathbb{R}^m$ is a known vector of measurements. Additionally, we assume that the magnitude of v is small. Suppose we also know that $\|x\| = 1$. For example, we can think of the measurement model as a communications system, where the input signal x is normalized such that $\|x\| = 1$.

(a) Explain how to estimate x given the measurement y, and the fact that the $\|x\| = 1$. State any assumptions that are needed for your method to work. Is your estimate $\hat{x}$ a linear function of y?

(b) Apply your method to the data given in `known_input_norm_data.m`. Report your estimate $\hat{x}$, and the residual norm $\|y - A\hat{x}\|$; verify that your estimate satisfies $\|\hat{x}\| = 1$.