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EE263 (Introduction to Linear Dynamical Systems)
Problem of the Day 9
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1 Positive-quadrant invariance

Consider a system $\dot{x} = Ax$, where $A \in \mathbb{R}^{2\times 2}$. (The results of this problem can be generalized to higher dimensions, but you only need to focus on the specific case when the state has dimension $n = 2$.) We say that the system is positive-quadrant invariant (PQI) if we have that $x_1(t) \geq 0$ and $x_2(t) \geq 0$ for all $t \geq T$ whenever we have that $x_1(T) \geq 0$ and $x_2(T) \geq 0$. In other words, if the state starts inside (or enters) the positive quadrant (also called the first quadrant), then the state remains in the positive quadrant forever.

(a) Find the precise conditions on A under which the system $\dot{x} = Ax$ is positive-quadrant invariant.

(b) Consider the following statement: if $\dot{x} = Ax$ is positive-quadrant invariant, then the eigenvalues of A are real. Either prove that this statement is true, or give a counterexample showing that it is false.