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EE263 (Introduction to Linear Dynamical Systems)
Problem of the Day 11
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1 System identification with state and input selection

Suppose we measure a time series

$$z(1), \ldots, z(N) \in \mathbb{R}^p,$$

and we want to model the evolution of the time series as a linear dynamical system:

$$x(t+1) = Ax(t) + Bu(t),$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input, and $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are parameters. We assume that each component of $z(t)$ is either a component of the state $x(t)$, or the input $u(t)$, but not both. One measure of the power of our model is the number of components assigned to the input: we can think of the input as the vector of exogenous effects, so a model with fewer inputs captures more of the structure in the data.

(a) Explain how to assign the components of $z(t)$ to either $x(t)$ or $u(t)$ in order to minimize the number of inputs m. Additionally, once you have chosen $x(t)$ and $u(t)$, explain how to estimate A and B.

(b) Carry out your method on the data given in `lds_sysid_state_selection_data.m`. Report your choices of $x(t)$ and $u(t)$, and your estimates of A and B. The root mean squared state residual is

$$R = \left(\frac{1}{N-1} \sum_{t=1}^{N-1} \| x(t+1) - (Ax(t) + Bu(t)) \|^2 \right)^{\frac{1}{2}},$$

and the RMS value of x over the same period is

$$S = \left(\frac{1}{N-1} \sum_{t=1}^{N-1} \| x(t+1) \|^2 \right)^{\frac{1}{2}}.$$

Report the relative RMS residual: $\rho = R/S$.