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EE263 (Introduction to Linear Dynamical Systems)
Problem of the Day 12
July 31, 2014

1 Minimum-residual affine subspace

You are given data points $x_1, \ldots, x_N \in \mathbb{R}^n$, and want to find an affine subspace $S \subseteq \mathbb{R}^n$ of dimension k that minimizes the total residual

$$\sum_{i=1}^{N} \|x_i - \hat{x}_i\|^2,$$

where $\hat{x}_i$ denotes the projection of x_i onto S.

(a) We can parameterize S by a vector $a \in \mathbb{R}^n$, and a matrix $Q \in \mathbb{R}^{n \times k}$ with orthonormal columns:

$$S = a + \text{range}(Q) = \{a + Qz : z \in \mathbb{R}^k\}.$$

Give an expression for the projection $\hat{x}$ of a vector $x \in \mathbb{R}^n$ onto the affine subspace S. Your expression for $\hat{x}$ should be in terms of x, a and Q.

(b) For a fixed value of the matrix Q, explain how to choose the vector a in order to minimize the total residual.

(c) Explain how to choose the vector a, and the matrix Q in order to minimize the total residual.