Course information

Overview

Modeling

Least-squares problems

The singular-value decomposition

Course components

- lecture: Tuesday/Thursday, 3.15pm – 5.05pm (Thornton 102)
- problem sessions: Thursday, 9.00am – 9.50am (Gates B03)
- office hours: TBA

Prerequisites

- necessary:
  - linear algebra (as in MATH104)
    - speaking vocabulary versus reading vocabulary
    - *The Karate Kid* analogy
  - differential equations and Laplace transforms (as in EE102A)
- *not* necessary (but may increase appreciation):
  - control systems
  - circuits and systems
  - dynamics

Course materials

- everything you need is on the course website
- some additional references (*not* necessary)
  - linear algebra: Strang, Meyer, Axler
  - dynamical systems and applied math: Luenberger, Strang
- living document on the Piazza forum
- grades (and only grades) on CourseWork

Grading

- weekly problem sets: 20 % (*usually* due on Fridays at 5pm)
- midterm exam: 30 % (24-hour take-home)
- final exam: 50 % (24-hour take-home)

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Overview

- Modeling
- Least-squares problems
- The singular-value decomposition

Modeling

- convert a practical problem into a mathematical model
- most important and most difficult part of the course
- “All models are wrong, but some are useful.” – George Box
- “Nothing at all takes place in the universe in which some rule of maximum or minimum does not appear.” – Leonhard Euler

Linear dynamical systems

- discrete-time linear dynamical system:

$$\begin{aligned} x(t+1) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{aligned}$$

  - $x(t) \in \mathbb{R}^n$ is the state
  - $u(t) \in \mathbb{R}^p$ is the input
  - $y(t) \in \mathbb{R}^m$ is the output
  - $A(t) \in \mathbb{R}^{n\times n}$ is the dynamics matrix
  - $B(t) \in \mathbb{R}^{n\times p}$ is the input matrix
  - $C(t) \in \mathbb{R}^{m\times n}$ is the measurement matrix
  - $D(t) \in \mathbb{R}^{m\times p}$ is the feedthrough matrix

Least-squares problems

$$\begin{aligned} \underset{x \in \mathbb{R}^n}{\text{minimize}} &: \|Ax - b\| \\ \text{subject to} &: Cx = d \end{aligned}$$

- system identification
- minimum-energy control
- linear-filter design

The singular-value decomposition: extremal-gain problems

$$\begin{aligned} \underset{x \in \mathbb{R}^n}{\text{minimize}} &: \|Ax\| \\ \text{subject to} &: \|x\| = 1 \end{aligned}$$

- minimum-residual subspace
- maximum-variance subspace
- analysis of robustness

The singular-value decomposition: low-rank approximation

$$\begin{aligned} \underset{X \in \mathbb{R}^{m\times n}}{\text{minimize}} &: \|A - X\| \\ \text{subject to} &: \operatorname{rank}(X) \leq r \end{aligned}$$

- latent-semantic indexing
- recommendation systems
- factor analysis