Course information

Course components

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- lecture: Tuesday/Thursday, 3.15pm – 5.05pm (Thornton 102)

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- office hours: TBA

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  - dynamics

Course materials

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- final exam: 50 % (24-hour take-home)

Course information

Overview

- Modeling
- Least-squares problems
- The singular-value decomposition

Modeling

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- convert a practical problem into a mathematical model

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Modeling

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- "All models are wrong, but some are useful." – George Box
- "Nothing at all takes place in the universe in which some rule of maximum or minimum does not appear." – Leonhard Euler

Linear dynamical systems

Linear dynamical systems

- discrete-time linear dynamical system:

Linear dynamical systems

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$$x(t+1) = A(t)x(t) + B(t)u(t)$$

Linear dynamical systems

- discrete-time linear dynamical system:

$$\begin{aligned} x(t+1) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{aligned}$$

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Least-squares problems

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- system identification

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The singular-value decomposition: extremal-gain problems

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- minimum-residual subspace

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The singular-value decomposition: low-rank approximation

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- latent-semantic indexing

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- recommendation systems

The singular-value decomposition: low-rank approximation

- latent-semantic indexing
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The singular-value decomposition: low-rank approximation

$$\begin{aligned} \underset{X \in \mathbb{R}^{m\times n}}{\text{minimize}} &: \|A - X\| \\ \text{subject to} &: \operatorname{rank}(X) \leq r \end{aligned}$$

- latent-semantic indexing
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