1. (15 points) **Q. 4 of midterm continued**
   Compute the variance of the total time James Bond takes before escaping.

2. (10 points) **Leftovers from homework 6**
   (a) Homework 6, Q. 3 (b).
   (b) Homework 6, Q. 5(c).

3. (25 points) **Probabilistically Buying Probability Books**
   Chuck will go shopping for probability books for $K$ hours. Here, $K$ is a random variable and is equally likely to be 1, 2, or 3. The number of books $N$ that he buys is random and depends on how long he shops.
   We are told that
   $$P(N = n | K = k) = \frac{c}{k}, \quad \text{for } n = 1, \ldots, k$$
   for some constant $c$.
   (a) Compute $c$.
   (b) Find the joint pmf of $K$ and $N$.
   (c) Find the marginal pmf of $N$.
   (d) Find the conditional pmf of $K$ given that $N = 1$.
   (e) We are now told that he bought at least 1 but no more than 2 books. Find the conditional mean and variance of $K$, given this piece of information.
   (f) The cost of each book is a random variable with mean 3 and is independent of the number of books he buys. What is the expectation of his total expenditure? **Hint:** Condition on events $N = 1, \ldots, N = 3$ and use the total expectation theorem you proved in Q. 4 of the midterm.

4. (25 points) **Bus waiting times**
   You show up at a Marguerite shuttle stop at a random time. There are three shuttles running on a periodic schedule and they all go to where you want to go. Each of the three shuttles arrives at your stop once every 10 minutes, but at different offsets from each other. The offsets of the three buses are uniformly random and independent, and independent of your arrival time. You will get on the first shuttle to arrive at the stop. Let the random variable $T$ denote the number of minutes you have to wait until the first shuttle arrives.
   (a) Compute an expression for the probability density function (pdf) and the cumulative distribution function (cdf) for $T$.
   (b) Plot the pdf and the cdf.
   (c) Compute $E[T]$ and $\text{Var}(T)$.

5. (25 points) **Exponential Distribution**
   We begin by proving two very useful properties of the exponential distribution. We then use them to solve a problem in photography.
(a) Let r.v. $X$ have geometric distribution with parameter $p$. Show that, for any positive $m, n$, we have

$$P(X > m + n \mid X > m) = P(X > n).$$

This is the “memoryless” property of the geometric distribution. Why do you think this property is called memoryless?

(b) Let r.v. $X$ have exponential distribution with parameter $\lambda$. Show that, for any positive $s, t$, we have

$$P(X > s + t \mid X > t) = P(X > s).$$

[This is the “memoryless” property of the exponential distribution.]

(c) Let r.v.'s $X_1, X_2$ be independent and exponentially distributed with parameters $\lambda_1, \lambda_2$. Show that the r.v. $Y = \min\{X_1, X_2\}$ is exponentially distributed with parameter $\lambda_1 + \lambda_2$. [Hint: work with cdf’s.]

(d) You have a digital camera that requires two batteries to operate. You purchase $n$ batteries, labeled $1, 2, \ldots, n$, each of which has a lifetime that is exponentially distributed with parameter $\lambda$ and is independent of all the other batteries. Initially you install batteries 1 and 2. Each time a battery fails, you replace it with the lowest-numbered unused battery. At the end of this process you will be left with just one working battery. What is the expected total time until the end of the process? Justify your answer.

(e) In the scenario of part (c), what is the probability that battery $i$ is the last remaining working battery, as a function of $i$?