EE 278 Lecture 1

- Logistics
- What this course is about
- Law of large numbers

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Control

Communication

*Estimation

Continuous

Clean signal

Noisy signal

Discrete

Clean signal

* Detection / Hypothesis testing / Classification

Communication

Statistics

Machine learning
Law of large numbers.

\[ X_1, \ldots, X_n, \ldots, \text{ i.i.d.} \rightarrow \text{ independent and identically distributed} \]

E.g., \( X_i = \) the result of the \( i \)-th coin flip (0 for H, 1 for T)

\[ S_n = \frac{1}{n} \sum_{i=1}^{n} X_i \quad \text{running sum} \]

Law of large numbers says:

\[ \frac{1}{n} S_n \xrightarrow{\text{in probability}} \mathbb{E}[X_1] = \mu \]

running average.

Weak law of large number. In other words,

\[ \Pr \left\{ \left| \frac{1}{n} S_n - \mu \right| > \varepsilon \right\} \xrightarrow{n \to \infty} 0 \]

\[ \mathbb{E}\left( \frac{1}{n} S_n \right) = \mu \rightarrow \text{by linearity of expectation} \]

the probability that \( \frac{1}{n} S_n \) goes out of the range \( \rightarrow 0 \)

If \( \text{var} [X_i] = \sigma^2 < \infty \), \( \mathbb{E}[X_i] = \mu < \infty \), the weak LLN holds.

E.g. \( X_i \) is the \( i \)-th coin flip.

model: \( p = \Pr [X_i = \text{Tail}] \) - parameter

flipping coins - generating data

\[ \hat{p} = \frac{\# \text{ of Tails}}{\# \text{ of flips}} \]
The estimator can be stated as:

\[ \hat{p}_n = \frac{X_1 + X_2 + \ldots + X_n}{n} \]

\[ X_i = 1 \text{ if the outcome is } T \]

\[ = \frac{1}{n} S_n \]

\[ \xrightarrow{P} \mu = \mu \]

Sample average

\[ \xrightarrow{P} \] convergence in probability. L.L.N.: law of large numbers

\[ \text{Data} \rightarrow \text{model parameters} \]

\[ \text{Data} \rightarrow \overset{\text{model parameters}}{\text{L.L.N.}} \rightarrow \text{predict performance} \]

\[ \frac{X_1 + \ldots + X_n}{n} = \text{long term performance}. \]

\[ E \left[ \frac{1}{n} \sum_{i=1}^{n} X_i \right] = \mu \]

\[ \text{var} \left[ \frac{1}{n} \sum_{i=1}^{n} X_i \right] = \frac{1}{n^2} \text{ var} \left( \sum_{i=1}^{n} X_i \right) = \frac{1}{n} \text{ var} (X_1) = \frac{\sigma^2}{n} \]