EE 278: Statistical Signal Processing
Homework 7
Due: Saturday, May 27, 2017 at 5pm

1. Exercise 10.5 in Gallager.

2. (a) In Q. 1 (10.5 in Gallager), fix $\bar{X}_1 = 0$, $\sigma^2_Z = 1$, $h = 1$, $\sigma^2_{X_1} = 1$ and $\beta = 1$. Consider 4 possible values for $\alpha$: $\alpha = 0.1, 0.5, 0.9, 0.99$. For each of these values, simulate the system and plot a realization of $\{X_n\}$ and a realization of $\{\hat{X}_n\}$ on the same plot. Plot them for long enough time that the system reaches steady state. Explain how the plot qualitatively changes as $\alpha$ varies.

(b) In Q. 1, compute for general parameter values the impulse response of the LTI system from $\{W_n\}$ to $\{X_n\}$ and the impulse response of the estimator from $\{Y_n\}$ to $\{\hat{X}_n\}$. For the parameter values in part (a), plot the two impulse responses. How do the impulse responses qualitatively change with $\alpha$? Is this consistent with your answer to part (a)?

3. Consider the vector dynamical system:

$$
\begin{align*}
X_1 &\sim \mathcal{N}(0, K_1) \\
X_{n+1} &= AX_n + BW_{n+1} \\
Y_n &= CX_n + DZ_n
\end{align*}
$$

where $W_1, \ldots, Z_1, \ldots$, are independent and $W_n \sim \mathcal{N}(0, K_w)$ and $Z_n \sim \mathcal{N}(0, K_z)$.

(a) Reformulate the scalar system:

$$
\begin{align*}
X &\sim \mathcal{N}(0, 1) \\
X_{n+1} &= X_n + 0.5 X_{n-1} + 0.2 X_{n-2} + W_{n+1} \\
Y_n &= X_n + 0.4 X_{n-1} + Z_n
\end{align*}
$$

as a vector dynamical system. Here, $W_n$’s and $Z_n$’s are independent and $\mathcal{N}(0, 1)$ distributed.

(b) For the general vector dynamical system, derive the recursion for the Kalman filter estimates and for the covariance matrices of the errors. Assuming that the error covariance matrices approach a limit as $n \to \infty$, characterize the limit in terms of the solution of an equation.