Fading Distributions. Average Fade Duration. Wideband Fading Models.

Lecture Outline

- **Signal Envelope Distributions: Rayleigh, Rician, Nakagami**
- **Level Crossing Rate and Average Fade Duration**
- **Wideband Channel Models**
- **Scattering Function**

1. **Signal Envelope Distributions**

   - CLT approximation leads to Rayleigh distribution (in-phase and quadrature zero mean and jointly Gaussian): $p_Z(z) = \frac{2z}{\overline{P}_r}\exp[-z^2/\overline{P}_r] = \frac{z}{\sigma^2}\exp[-z^2/(2\sigma^2)], \;\; z \geq 0$.
   - A LOS component leads to a received signal with non-zero mean. The Rician distribution models signal envelope in this case, with K factor dictating the relative power of the LOS component: $p_Z(z) = \frac{z}{\sigma^2}\exp\left[\frac{-(z^2+s^2)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right), \quad z \geq 0$.
   - Experimental results support a Nakagami distribution for some environments. Similar to Rician, but can model "worse than Rayleigh." Model generally leads to closed-form expressions in BER and diversity analysis: $p_Z(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)P_r^m}\exp\left[\frac{-mz^2}{P_r}\right], \quad m \geq .5$.

2. **Level Crossing Rate and Average Fade Duration**

   - Level crossing rate L_R is the rate at which a signal envelope crosses the threshold R.
   - In Rayleigh fading the level crossing rate is $L_R = \sqrt{2\pi} f_D \rho e^{-\rho^2}$, where $\rho = R/\sqrt{\Omega_p}$.
   - Average Fade Duration (AFD) measures how long a signal's envelope or power stays below a given target threshold: derived from the level crossing rate.
   - For Rayleigh fading $\bar{t}_r = e^{\rho^2-1}/(\rho f_D\sqrt{2\pi}$, where ρ is the ratio of target envelope level to average envelope level (or square root of ratio of target power level to average power level).

3. **Wideband Channel Models**

   - In wideband multipath channels the individual multipath components can be resolved by the receiver. True if $T_m > 1/B$.
   - If the components can be resolved then they can be combined for diversity gain (e.g. using an equalizer).

4. **Channel Scattering Function:**

   - For deterministic channels, the scattering function is defined as the Fourier transform of $c(\tau, t)$ with respect to t.
   - Typically $c(\tau, t)$ is unknown, so it must be characterized statistically.
   - Since the underlying process $c(\tau, t)$ is Gaussian, we only need to characterize its mean and correlation. We assume $c(\tau, t)$ has mean zero.

- Autocorrelation of $c(\tau, t)$ is $A_c(\tau_1, \tau_2; \Delta t) = A_c(\tau_1, \tau_2; \Delta t)\delta(\tau_1 - \tau_2) = A_c(\tau; \Delta t)$ since we assume channel response associated with different scatterers is uncorrelated.
- Statistical scattering function defined as $S(\tau, \rho) = \mathcal{F}_{\Delta t}[A_c(\tau, \Delta t)]$.
- This function measures the average channel gain as a function of both delay τ and Doppler ρ.
- $S(\tau, \rho)$ easy to measure empirically and is used to get average delay spread T_M, rms delay spread σ_τ, and Doppler spread B_d for empirical channel measurements.

Main Points

- The signal envelope under narrowband fading with uniform AOA is Rayleigh. Other common distribution are Ricean (when a LOS component exists) and Nakagami.
- Average Fade Duration used to determine how long a user is in continuous outage (e.g. for coding design).
- Wideband models characterized by scattering function, which measures average channel gain relative to delay and Doppler.
- Scattering function used to obtain key channel characteristics of rms delay spread and Doppler spread, which are important for system design.